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CSEC MATHS P2

2025 JANUARY

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CSEC MATHEMATICS JANUARY 2025 PAPER 2

SECTION I

Answer ALL questions.

All working MUST be clearly shown.

1. (a) Using a calculator or otherwise, determine the value of

(i) $\frac{2}{3}$ of $\left(\frac{1}{8}+\frac{5}{12}\div\frac{1}{9}\right)$, giving your answer in EXACT form

SOLUTION:

Required to calculate: $\frac{2}{3}$ of $\left(\frac{1}{8}+\frac{5}{12}\div\frac{1}{9}\right)$

Calculation:

The question is best written as $\frac{2}{3}\left\{\frac{1}{8}+\left(\frac{5}{12}\div\frac{1}{9}\right)\right\}$.

First, we calculate $\frac{5}{12}\div\frac{1}{9}=\frac{5}{12}\times\frac{9}{1}$

$$=\frac{15}{4}$$

$$\left\{\frac{1}{8}+\left(\frac{5}{12}\div\frac{1}{9}\right)\right\}=\frac{1}{8}+\frac{15}{4}$$

$$=\frac{1+30}{8}$$

$$=\frac{31}{8}$$

Now, we find $\frac{2}{3}$ of $\frac{31}{8}=\frac{2}{3}\times\frac{31}{8}$

$$=\frac{2}{3}\times\frac{31}{8}$$

$$=\frac{31}{12}$$

$$=2\frac{7}{12}\text{ (in exact form)}$$

(ii) $314.2-\frac{26082}{52164}$, giving your answer in standard form.

SOLUTION:

Required to calculate: $314.2-\frac{26082}{52164}$, giving the answer in standard form

Calculation:

$$314.2-\frac{26082}{52164}=314.2-0.5$$
$$=313.7$$

We shift the decimal point 2 places to the left to get

$=3.137\times10^{2}$ in standard form

(b) Jim packed several cases of fruit juice for sale. Each case contained 24 boxes of juice in 3 different varieties, apple, orange and pineapple, in the ratio $2:5:1$ respectively.

(i) How many boxes of **pineapple** juice were packed in each case?

SOLUTION:

Data: A case of juice contains 24 boxes of juice in 3 different varieties, apple, orange and pineapple, in the ratio $2:5:1$ respectively.

Required to find: The number of boxes of pineapple juice packed in each case

Solution:

Ratio of boxes of juices according to variety is

Apple : Orange : Pineapple

2 : 5 : 1

Hence, the actual numbers of boxes of juice in each case is $2x$ of apple, $5x$ of orange and x of pineapple, where x is a positive integer

$\therefore 2x+5x+x=24$, (since there are 24 boxes in each of the cases)

$$8x=24$$
$$x=\frac{24}{8}$$
$$x=3$$

So, the number of boxes of pineapple juice in each case is 3.

(ii) The profit gained from selling ALL of the boxes of pineapple juice is \$35.64. Each box of pineapple juice was sold at \$3.34.

a) Show that the cost price of a box of pineapple juice is \$2.35.

SOLUTION:
Data: A profit of \$35.64 is made from selling all the boxes of pineapple juice. Each box of pineapple juice was sold at \$3.34.
Required to show: The cost price of a box of pineapple juice is \$2.35
Proof:
The question never gave the number of cases of juice sold, which $\times 3 =$ Number of boxes of pineapple juice sold

$$\text{Total profit} \div \text{Number of boxes sold} = \text{Profit on one box}$$
$$\text{Selling price of 1 box} - \text{Profit on 1box} = \text{Cost price of 1 box}$$

For the question to be solved it should be given that 12 cases were sold.
Profit on one box = \$35.64 ÷ (12×3) = \$0.99
Hence, cost price = \$3.4 - \$0.99 = \$2.35
The question is unsolvable.

b) Calculate the percentage profit made on the sale of the boxes of pineapple juice.

SOLUTION:
Required to calculate: The percentage profit made on the sale of boxes of pineapple juice
Calculation:

$$\begin{aligned}\text{Profit on each box of pineapple juice} &= \text{Selling price} - \text{Cost price}\\ &= \$3.34 - \$2.35\\ &= \$0.99\end{aligned}$$

$$\begin{aligned}\therefore \text{Profit} &= \frac{0.99}{2.35} \times 100\\ &= 42.12\underline{\underline{7}}\%\\ &\approx 42.13\% \text{ (correct to 2 decimal places)}\end{aligned}$$

2. (a) (i) Factorise EACH of the following algebraic expressions

a) $x^2 - 49$

SOLUTION:
Required to factorise: $x^2 - 49$
Solution:
$x^2 - 49 = (x)^2 - (7)^2$ which is a difference of two squares
So, $x^2 - 49 = (x-7)(x+7)$

b) $x^2 + 2x - 35$

SOLUTION:
Required to factorise: $x^2 + 2x - 35$
Solution:
7 + (– 5) = +2 and 7 × -5 = -35
So, $x^2 + 2x - 35 = (x+7)(x-5)$

(ii) Hence, simplify the expression $\dfrac{x^2 - 49}{x^2 + 2x - 35}$

SOLUTION:

Required to simplify: $\dfrac{x^2 - 49}{x^2 + 2x - 35}$

Solution:

Using what we obtained from (i)

$$\frac{x^2 - 49}{x^2 + 2x - 35} = \frac{(x-7)\cancel{(x+7)}}{\cancel{(x+7)}(x-5)}$$
$$= \frac{x-7}{x-5}$$

(b) Rearrange the formula below to make *m* the subject.

$$s = k - m^2$$

SOLUTION:
Data: $s = k - m^2$
Required to rearrange: The given formula to make *m* the subject
Solution:
$$s = k - m^2$$
$$s - k = -m^2$$
$(\times -1)$ and switching sides to get
$$m^2 = k - s$$
$$\therefore m = \sqrt{k - s}$$

(c) Lisa has \$56 to buy a total of no more than 70 red balloons and green balloons for her party.

She buys more green balloons than red balloons but must buy at least 15 red balloons. Each red balloon costs \$0.75 and each green balloon costs \$0.50.

Let x and y represent the number of red balloons and the number of green balloons respectively. Write TWO inequalities in x and y, other than $x \geq 0$ and $y \geq 0$, to represent the information above.

SOLUTION:
Data: Lisa has \$56 to buy no more than 70 red balloons and green balloons for her party. She buys more green balloons than red balloons but must buy at least 15 red balloons. Each red balloon costs \$0.75 and each green balloon costs \$0.50. x represents the number of red balloons bought and y represents the number of green balloons bought.

Required to write: TWO inequalities to represent the information given.
Solution:
The total number of red balloons and green balloons is no more than 70, that is ≤ 70

Hence $x + y \leq 70$

The number of green balloons must be more than the number of red balloons
So, $y > x$

There must be at least 15 red balloons.
Therefore, $x \geq 15$

The cost of x red balloons at \$0.75 and y green balloons at \$0.50 each must be at most \$56.00
Using cents we get

$75x + 50y \leq 5600$

$(\div 25)$

$2y + 3x \leq 224$

Hence, four possible inequalities are $x + y \leq 70, y > x, x \geq 15$ and $3x + 2y \leq 224$.

Note: If Lisa buys red balloons and green balloons, then she buys at least 1 of each colour. Therefore, it should be $x > 0,\ y > 0$ and NOT $x \geq 0,\ y \geq 0$.

(d) Given that y is inversely proportional to $(x-2)$ and $x = 11$ when $y = 9$, find the value of y when $x = 29$.

SOLUTION:
Data: y is inversely proportional to $(x-2)$ and $x=11$ when $y=9$
Required to find: y when $x=29$
Solution:
y is inversely proportional to $(x-2)$

$$\therefore y \propto \frac{1}{(x-2)}$$

$\therefore y = k \times \frac{1}{(x-2)}$, where k is the constant of proportionality

When $x=11$, $y=9$:

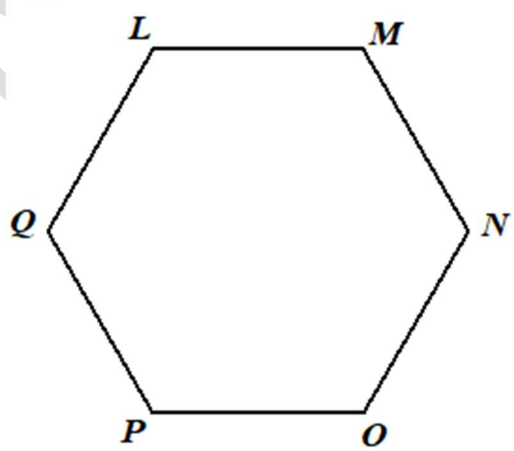
$$\therefore 9 = k \times \frac{1}{(11-2)}$$

$$\therefore k = 9 \times 9$$
$$= 81$$

When $x = 29$:

$$y = 81 \times \frac{1}{(29-2)}$$
$$= \frac{81}{27}$$
$$= 3$$

3. (a) The diagram below shows a **regular** hexagon, *LMNOPQ*, whose side is 8 cm.



(i) Show that the value of angle *PQL* is 120º.

SOLUTION:
Data: Diagram showing a regular hexagon *LMNOPQ*.

Required to show: The size or measure of Angle PQL is 120º.
Solution:
The sum of the interior angles of a polygon of n sides
$= (2n - 4) \times 90^0$ OR
$= (n-2) \times 180°$
In a hexagon, $n = 6$ so the sum of the 6 interior angles
$= (6-2) \times 180°$
$= 720°$

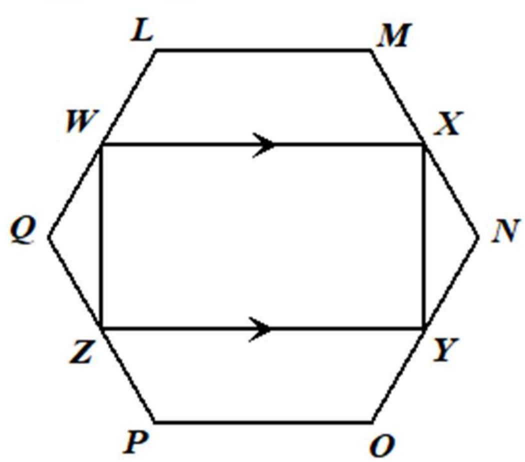
In a regular polygon, the sizes of all the interior angles are equal.

$$\text{Angle } PQL = \frac{720°}{6}$$
$$= 120°$$

Q.E.D.

NOTE: The length of the side 8 cm was not used anywhere in the question. Unused numerical data in a question is misleading.

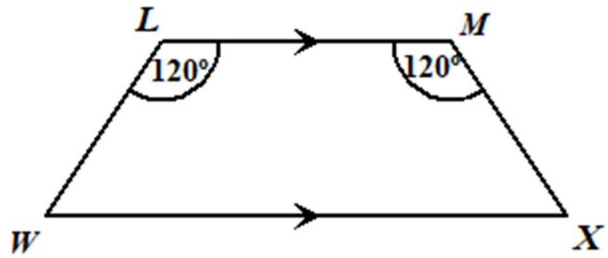
(ii) The vertices of a rectangle, $WXYZ$, touch the sides PQ, QL, MN and NO of the hexagon in Part (a). ZY and WX are parallel to LM and PO.

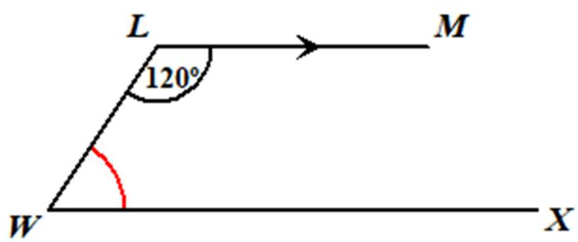


Calculate the value of Angle LWX.

SOLUTION:
Data: Diagram showing a rectangle $WXYZ$ touching the sides PQ, QL, MN and NO of the hexagon $LMNOPQ$. ZY and WX are parallel to LM and PO.
Required to calculate: The value of angle LWX.
Calculation:

Consider the trapezium ***LMXW***





The transversal LW meets the parallel lines LM and WX

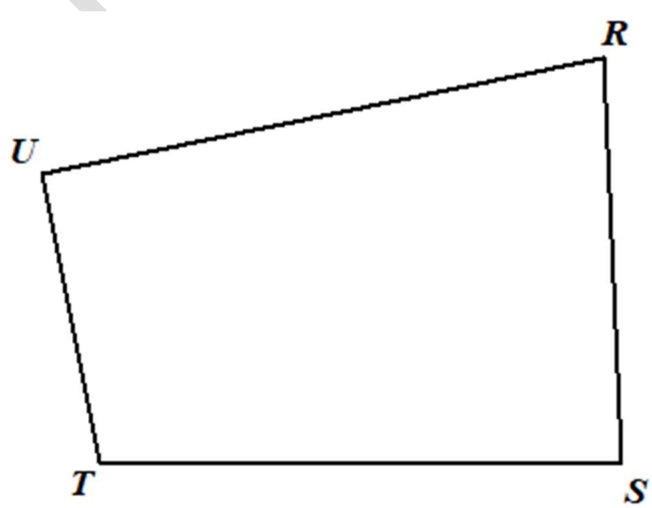
$M\hat{L}W$ and $L\hat{W}X$ are co-interior angles which are supplementary. Hence,

$$\begin{aligned} L\hat{W}X &= 180° - 120° \\ &= 60° \end{aligned}$$

(What is the purpose of drawing the rectangle *WXYZ* when only the line *WX* is useful in the question)

(b) In the following parts, show all your construction lines where required.

The field of sports club is in the shape of a quadrilateral, *RSTU*. A scale diagram of this field is shown below.



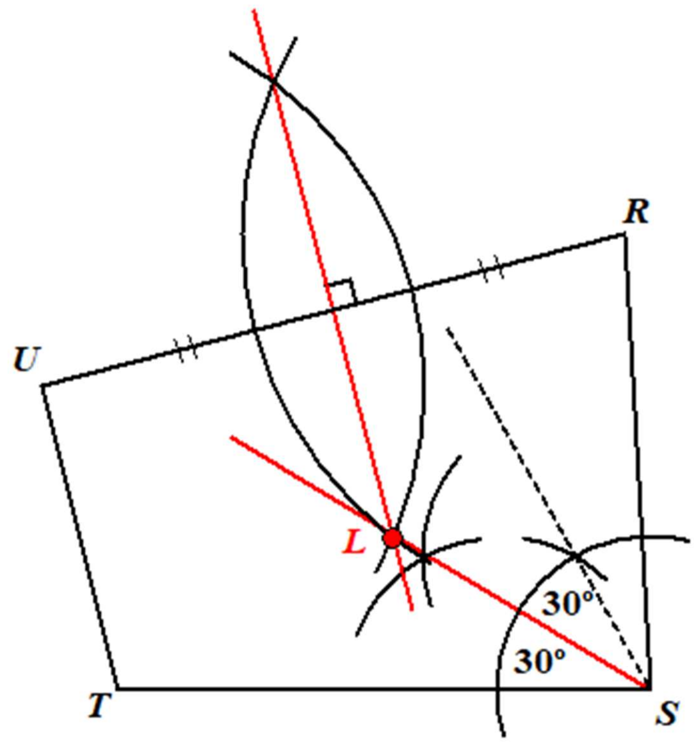
A lamppost is to be erected on the field at a point marked L so that floodlights can be installed. The point L should be located in such a way that L lies on the perpendicular bisector of the line UR and angle LST equals 30º.

Using a ruler and compasses only, locate the point L on the field.

SOLUTION:
Data: Scale diagram showing a field in the shape of a quadrilateral $RSTU$. A lamppost, L, is to be erected on the field such that L lies on the perpendicular bisector of the line UR and Angle LST equals 30º.

Required to locate: Point L on the field, using ruler and compasses only.
Solution:

We construct the perpendicular bisector of $\boldsymbol{UR}$.
At $\boldsymbol{S}$ we construct a 60^0 angle using TS as one side.
We bisect the 60^0 angle to create a 30^0 with $\boldsymbol{TS}$ as one side.
The perpendicular bisector of $\boldsymbol{UR}$ and the bisector of the 60^0 angle will meet at $\boldsymbol{L}$
These lines bisecting at $\boldsymbol{L}$ are shown in red on the diagram below.



4. A line segment joins the points $C\,(-5,\,6)$ and $D\,(7,\,2)$.

(a) Calculate the midpoint of the line segment CD.

SOLUTION:

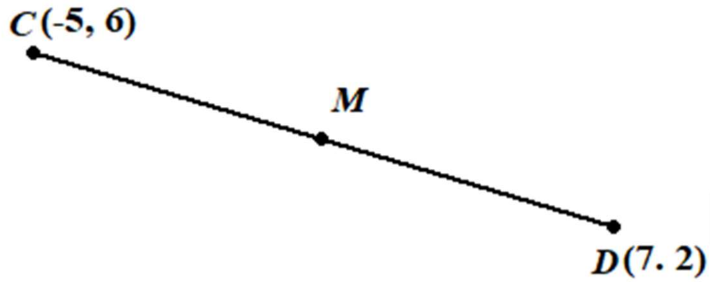
Data: $C\left(-5,\ 6\right)$ and $D\left(7,\ 2\right)$ are two points on a line segment.

Required to calculate: The midpoint of the line segment *CD*

Calculation:

Let the midpoint of *CD* be *M*.

We create a sketch which may be helpful.



Using the mid-point formula, the coordinates of $M = \left(\frac{-5+7}{2},\ \frac{6+2}{2}\right)$

$$= \left(\frac{2}{2},\ \frac{8}{2}\right)$$

$$= \left(1,\ 4\right)$$

(b) Find the gradient of the line segment *CD*.

SOLUTION:

Required to find:

The gradient of the line segment *CD*.

Solution:

Using the formula when given the coordinates of two points on the line.

$$\text{Gradient of } CD = -\frac{6-2}{-5-7}$$

$$= \frac{4}{-12}$$

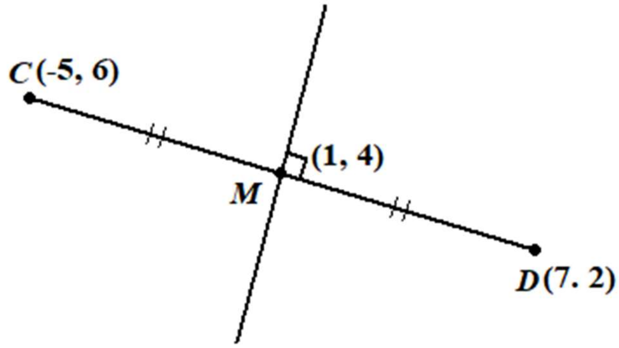
$$= -\frac{1}{3}$$

(c) Determine the equation of the perpendicular bisector of *CD*.

SOLUTION:

Required to determine: The equation of the perpendicular bisector of *CD*.

Solution:



The gradient of any perpendicular line to CD is $\frac{-1}{-\frac{1}{3}} = 3$

(Product of the gradients of perpendicular lines $= -1$.)

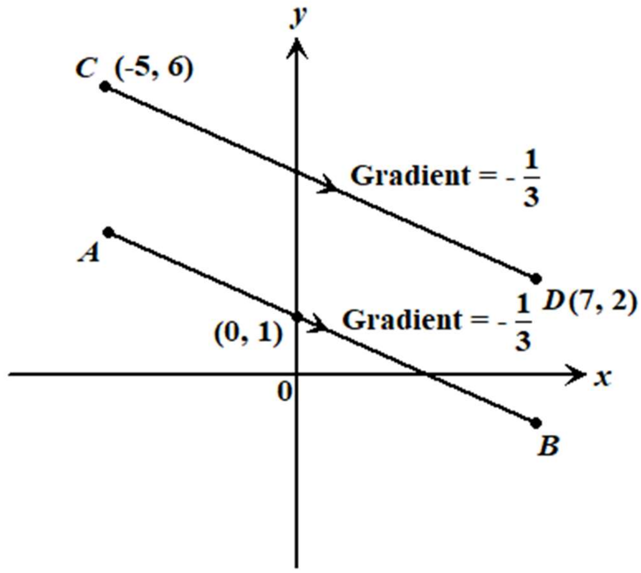
M lies on the perpendicular bisector.

Hence, the equation of the perpendicular bisector of CD is $\frac{y-4}{x-1} = 3$

$$y - 4 = 3(x-1)$$
$$y - 4 = 3x - 3$$
$$y = 3x + 1$$

(d) Another line, AB, is parallel to CD and passes through the point $(0, 1)$. Write down the equation of the line AB.

SOLUTION:
Data: Another line, AB, is parallel to CD and passes through the point $(0, 1)$.
Required to write: The equation of the line AB
Solution:



Gradient of $AB = -\frac{1}{3}$
(Parallel lines have the same gradient.)

The equation of a line is $y = mx + c$, where m is the gradient of the line and c is the intercept on the $y-$axis.
In this case, $m = -\frac{1}{3}$ and $c = 1$

Hence, the equation of AB is $y = -\frac{1}{3}x + 1$.

5. (a) The table below shows the marks, out of 10, that 40 students in a class gained on an essay writing test.

Marks (x)	**Number of Students (f)**
4	3
6	9
7	8
8	7
9	8
10	5

(i) Calculate the students' mean score on the test.

SOLUTION:
Data: Ungrouped frequency table showing the marks, out of 10, that 40 students in a class gained on an essay writing test.
Required to calculate: The students' mean score on the test

Calculation:
The mean $\bar{x}$ can be obtained from the formula

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$= \frac{(3\times 4)+(9\times 6)+(8\times 7)+(7\times 8)+(8\times 9)+(5\times 10)}{40}$$

$$= \frac{12+54+56+56+72+50}{40}$$

$$= \frac{300}{40}$$

$$= 7\frac{1}{2} \text{ or } 7.5$$

(ii) Determine the

a) modal mark

SOLUTION:
Required to determine: The modal mark scored by students
Solution:
The modal mark is the mark that occurs most often. The mark of 6 occurs more often than any other mark and so the modal mark is 6.

b) the median mark.

SOLUTION:
Required to determine: The median mark scored by students.

Solution:

Marks (x)	Number of Students (f)	Cumulative Frequency
4	3	3
6	9	12
7	8	20
8	7	27
9	8	35
10	5	40

The middle of 40 is 20 and 21.
From the revised table we observe that the 20th mark is 7 and the 21st mark is 8.

$$\therefore \text{Median mark} = \frac{7+8}{2}$$
$$= 7.5$$

(iii) Using the information in the table below, a pie chart is constructed to represent the marks students gained.

Marks (x)	Number of Students (f)
$3 \le x \le 4$	3
$5 \le x \le 6$	9
$7 \le x \le 8$	15
$9 \le x \le 10$	13

Calculate the angle for the sector representing the interval marks, $5 \le x \le 6$, in the pie chart.

SOLUTION:
Data: Grouped frequency table showing the number of students scoring marks in various intervals.
Required to calculate: The angle of the sector representing the interval $5 \le x \le 6$, if a pie chart is to be drawn.

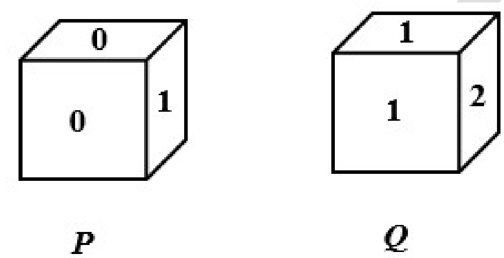
Calculation:

$$\begin{aligned}\text{The number of students } &= 3+9+15+19\\ &= 40\end{aligned}$$

Number of students who acquired $5 \le x \le 6$ marks $= 9$

Hence, the angle of the sector on a pie-chart representing $5 \le x \le 6$ marks would be $\frac{9}{40} \times 360° = 81°$

(b) The diagram below shows two fair six-sided dice, *P* and *Q*.



The six numbers on Die *P* are 0, 0, 1, 1, 2, 3.
The six numbers on Die *Q* are 1, 1, 1, 2, 2, 3.
When a die is rolled, the score is the number on the top face.

(i) Die *P* is rolled once. What is the probability that the score is NOT 2?

SOLUTION:
Data: Diagrams showing two fair six-sided dice, *P* and *Q*, such that the six numbers on Die *P* are 0, 0, 1, 1, 2, 3 and the six numbers on Die *Q* are 1, 1, 1, 2, 2, 3. When a die is rolled, the score is the number on the top face.

Required to find: The probability that the score is not 2 when Die P is rolled once.

Solution:

$$\begin{aligned}P(\text{score is not 2}) &= \frac{\text{No. of scores that are not 2}}{\text{No. of possible scores}}\\ &= \frac{5}{6}\end{aligned}$$

Alternatively:

$$P(\text{score is not } 2) = 1 - P(\text{score is } 2)$$
$$= 1 - \frac{1}{6}$$
$$= \frac{5}{6}$$

(ii) Die Q is rolled **twice**. What is the probability that the score is 1 both times?

SOLUTION:
Data: Die Q is rolled twice.
Required to find: The probability that the score is 1 both times.

Solution:

$$P(\text{score is 1 when } Q \text{ is rolled}) = \frac{\text{No. of scores of 1}}{\text{No. of possible scores}}$$
$$= \frac{3}{6}$$
$$= \frac{1}{2}$$

$\therefore$ On the two rolls, $P(\text{both scores are } 1) = P(1) \times P(1)$
$$= \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{4}$$

(iii) Die Q is rolled 72 times. Calculate an estimate of the number of times the score is 3.

SOLUTION:
Data: Die Q is rolled 72 times.
Required to calculate: An estimate of the number of times the score is 3.

Calculation:

$$P(3 \text{ on Die } Q) = \frac{\text{No. of 3's}}{\text{No. of possible scores}}$$
$$= \frac{1}{6}$$

$\therefore$ In 72 rolls the number of 3's expected $= \frac{1}{6} \times 72$
$$= 12$$

(iv) Each die is rolled once. The product of the scores is recorded. The sample space diagram is shown below.

Die *Q*	**3**	0	0	3	3	6	9
	2	0	0	2	2	4	6
	2	0	0	2	2	4	6
	1	0	0	1	1	2	3
	1	0	0	1	1	2	3
	1	0	0	1	1	2	3
		0	**0**	**1**	**1**	**2**	**3**
$\times$				**Die *P***			

Find the probability that the product of the scores is 2 or 3.

SOLUTION:
Data: Sample space diagram showing scores obtained when each die is rolled and their outcomes are multiplied.
Required to find: The probability that the product of the scores is 2 or 3.

Solution:
The number of scores possible $=36$, though they are not all different.

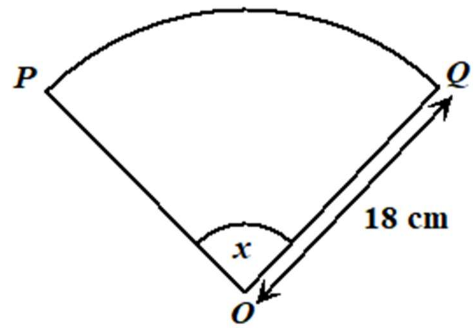
$$P(\text{Obtaining 2 or 3}) = \frac{\text{No. of scores that are 2 or 3}}{36}$$

$$= \frac{7+5}{36}$$

$$= \frac{1}{3}$$

6. [In this question, use $\pi = \dfrac{22}{7}$.]

(a) A piece of wire, 61 cm long, is bent to form a sector, as shown in the diagram below. The sector of the circle, *OPQ*, has centre *O* and a radius of 18 cm.



(i) Show that the value of x is approximately 80º.

SOLUTION:
Data: Diagram showing a sector of a circle, with centre O, sector angle xº and radius 18 cm, formed from a bent piece of wire of length 61 cm.
Required to show: x is approximately 80º
Proof:
The perimeter of $OPQ = 61\,\text{cm}$

$\therefore$ Length of arc, $PQ = 61 - (18 + 18)$ cm
$= 25$ cm

$$\therefore \frac{x^\circ}{360^\circ} \times 2\pi \times 18 = 25$$

$$x = \frac{25 \times 360^\circ}{18 \times 2\pi}$$
$$= \frac{25 \times 360^\circ}{18 \times 2 \times \frac{22}{7}}$$
$$= \frac{25 \times 360^\circ \times 7}{18 \times 2 \times 22}$$
$$= 79.5\underline{\underline{4}}^\circ$$
$$\approx 80^\circ \text{ to the nearest degree}$$

Q.E.D.

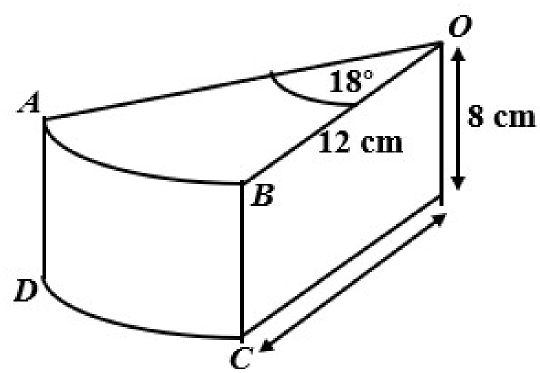
(ii) Calculate the area enclosed by the wire.

SOLUTION:
Required to calculate: The area enclosed by the wire
Calculation:

The area enclosed by the wire $= \frac{79.54^\circ}{360^\circ} \times \pi (18)^2$

$$= \frac{79.54 \times 22 \times 18 \times 18}{360 \times 7} \text{ cm}^2$$
$$= 224.\underline{\underline{9}}8 \text{ cm}^2$$
$$\approx 225 \text{ cm}^2 \text{ to the nearest whole number}$$

(b) A cylindrical block of cheese has a radius of 12 cm and a height of 8 cm. The cheese is divided into equal slices. The uniform cross-section of a slice of cheese is a sector whose angle is 18º, as shown in the diagram below.



(i) Calculate the length of the arc *AB*.

SOLUTION:
Data: Diagram showing a slice of a cylindrical block of cheese of radius 12 cm and height 8 cm. The cross-section of the slice of cheese is a sector with an angle of 18º.
Required to calculate: The length of arc *AB*
Calculation:

$$\text{Length of arc } AB = \frac{18}{360} \times 2 \times \frac{22}{7} \times 12$$
$$= 3.77\underline{\underline{1}} \text{ cm}$$
$$\approx 3.77 \text{ cm (correct to 2 decimal places)}$$

(ii) Determine the area of the curved face, *ABCD*.

SOLUTION:
Required to determine: The area of the curved face *ABCD*
Solution:

$$\text{Area of } ABCD = \text{Length of } AB \times 8 \text{cm}^2$$
$$= 3.771 \times 8 \text{ cm}^2$$
$$= 30.16\underline{\underline{8}} \text{ cm}^2$$
$$\approx 30.17 \text{ cm}^2 \text{ (correct to 2 decimal places)}$$

(iii) Given that the area of OAB is 22.6 cm^2, calculate the volume of the ENTIRE block of cheese.

SOLUTION:
Data: Area of OAB is 22.6 cm^2
Required to calculate: The volume of the entire block of cheese
Calculation:
The volume of the given slice $= \text{Area of } OAB \times 8$
$= 22.6 \times 8 \text{ cm}^3$

The number of such slices that were cut from the cylinder $= \dfrac{360°}{18°}$
$= 20$

Hence, the volume of the entire block of cheese $= 22.6 \times 8 \times 20 \text{ cm}^3$
$= 3\,616 \text{cm}^3$

7. A sequence of patterns is made of dots and lines of unit length. Some of these lines form squares. The first three diagrams in the sequence are shown below.

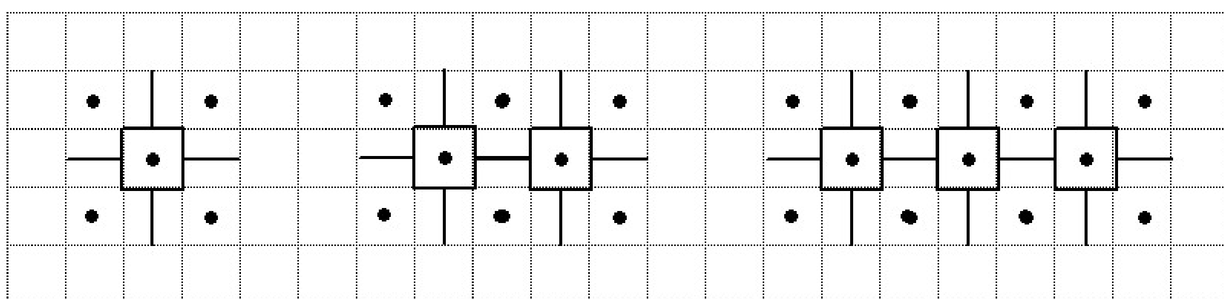


Diagram 1 **Diagram 2** **Diagram 3**

(a) Add more lines and dots to the diagram below to show Diagram 4 of the sequence.

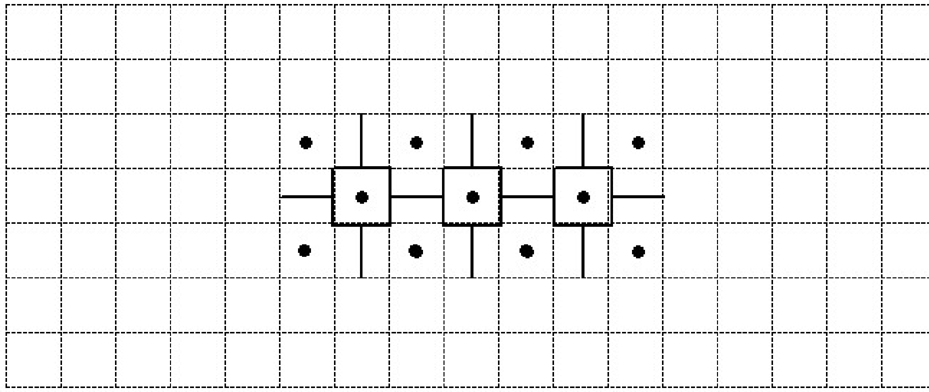


Diagram 4

SOLUTION:
Data: Diagram showing the sequence of patterns of dots and lines of unit length.
Required to complete: The diagram given by adding more lines and dots to form Diagram 4 in the sequence.
Solution:

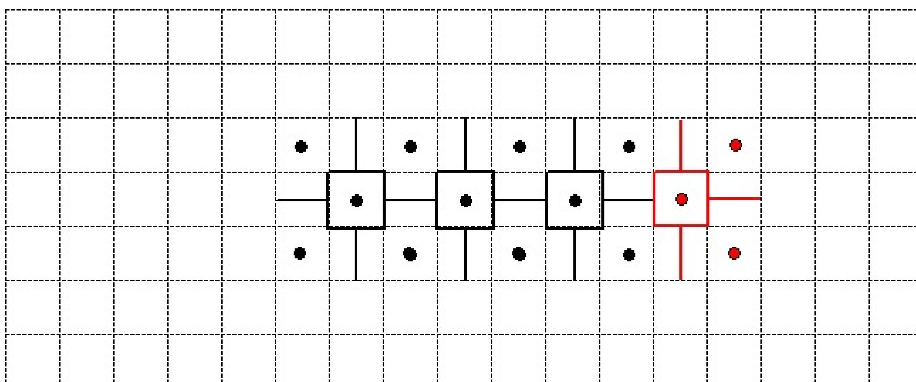


Diagram 4

(b) The number of dots, D, and the number of unit lines that form each diagram, L, form a pattern. The values for D and L for the first 3 diagrams are written in the table below. Study the pattern of numbers in each row of the table.

Complete the rows numbered (i), (ii) and (iii).

	Diagram	Number of Dots (D)	Number of Lines (L)
	1	5	8
	2	8	15
	3	11	22
(i)	4	__________	__________
	⋮	⋮	⋮
(ii)	__________	59	__________
	⋮	⋮	⋮
(iii)	n	__________	__________

The number of dots, D, for Diagrams 1, 2 and 3 are 5, 8, 11, …
These numbers increase by 3.
Hence, $D = 3 \times n + k$ (where $k =$ a constant and $n =$ diagram number

$D = 3n + k$
When $n = 1$ $\quad D = 5 = 3(1) + 2$ $\quad$ Hence $k = 2$
And $D = 3n + 2$

Testing:
When $n = 2$ $\quad D = 3(2) + 2$
$= 8$ (correct)

When $n = 3$ $\quad D = 3(3) + 2$
$= 11$ (correct)

(i) $n = 4$ $\quad D = 3(4) + 2$
$= 14$

(ii) When $D = 59$ $\quad 3n + 2 = 59$
$3n = 57$
Divide by 3 to get $n = 19$

(iii) Column 2: $D = 3n + 2$

For diagrams 1, 2 and 3, the number of lines, L, are 8, 15, 22, …

These numbers differ by 7.
So, $L = 7 \times n + c$ (where c is a constant)
$L = 7n + c$

When $n = 1$ $\quad L = 7(1) + 1$ $\quad$ Hence $c = 1$
So, $L = 7n + 1$

Testing:
When $n = 2$ $\quad L = 7(2) + 1$
$= 15$ (correct)

When $n = 3$ $\quad L = 7(3) + 1$
$= 22$ (correct)

(i) When $n = 4$ $\quad L = 7(4) + 1$
$= 29$

(ii) When $n = 19$ $\quad L = 7(19) + 1$
$= 134$

(iii) $L = 7n + 1$

The completed table looks like this:

	Diagram	**Number of Dots (D)**	**Number of Lines (L)**
	1	5	8
	2	8	15
	3	11	22
(i)	4	**14**	**29**
	⋮	⋮	⋮
(ii)	**19**	59	**134**
	⋮	⋮	⋮
(iii)	n	$3n + 2$	$7n + 1$

(b) One of the diagrams in the sequence has 148 **lines**. Calculate the number of **dots** in this diagram.

SOLUTION:
Data: The number of lines in one of the diagrams is 148.
Required to calculate: The number of dots in this diagram.
Calculation:
When $L = 148$

$$7n + 1 = 148$$
$$7n = 147$$
$$n = 21$$

$$D = 3n + 2$$
$$= 3(21) + 2$$
$$= 65$$

$\therefore$ This diagram has 65 dots.

SECTION II

Answer ALL questions.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) The functions *f* and *g* are defined as follows

$$f : x \to \frac{1+3x}{x-1},\ x \neq 1$$
$$g : x \to 5 - x$$

(i) Calculate the value of $f(-2)$.

SOLUTION:
Data: $f : x \to \frac{1+3x}{x-1},\ x \neq 1$ and $g : x \to 5 - x$
Required to calculate: $f(-2)$
Calculation:

$$f(x) = \frac{1+3x}{x-1}$$

$$f(-2) = \frac{1+3(-2)}{-2-1}$$

$$= \frac{-5}{-3}$$

$$= \frac{5}{3}$$

(ii) Determine a simplified expression for $fg(x)$.

SOLUTION:
Required to determine: An expression for $fg(x)$
Solution:

$$f(x) = \frac{1+3x}{x-1}$$

$$fg(x) = \frac{1+3(g(x))}{g(x)-1}$$

$$= \frac{1+3(5-x)}{(5-x)-1}$$

$$= \frac{1+15-3x}{4-x}$$

$$= \frac{16-3x}{4-x},\ x \neq 4$$

(iii) Derive an expression in terms of x for the inverse function, $f^{-1}(x)$.

SOLUTION:
Required to derive: An expression for $f^{-1}(x)$
Solution:

$$f(x) = \frac{1+3x}{x-1}$$

Let $y = \dfrac{1+3x}{x-1}$

$$\therefore y(x-1) = 1+3x$$

$$xy - y = 1 + 3x$$

$$xy - 3x = 1 + y$$

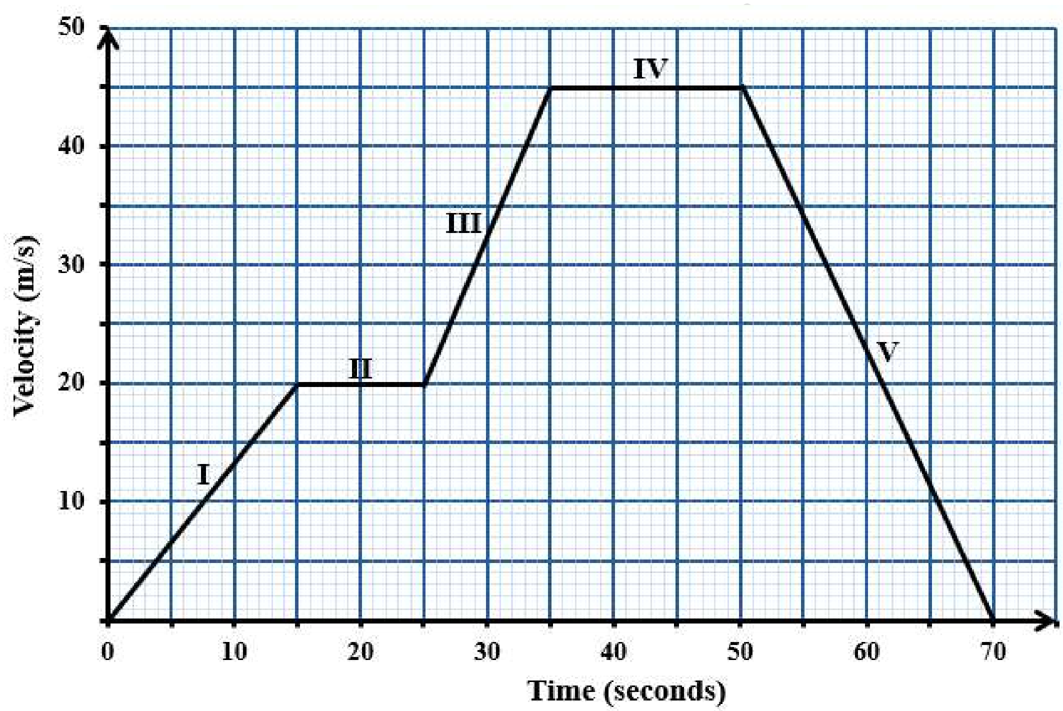
$$x(y-3) = 1 + y$$

$$x = \frac{1+y}{y-3}$$

Replace y by x to get

$$f^{-1}(x) = \frac{1+x}{x-3}, x \neq 3$$

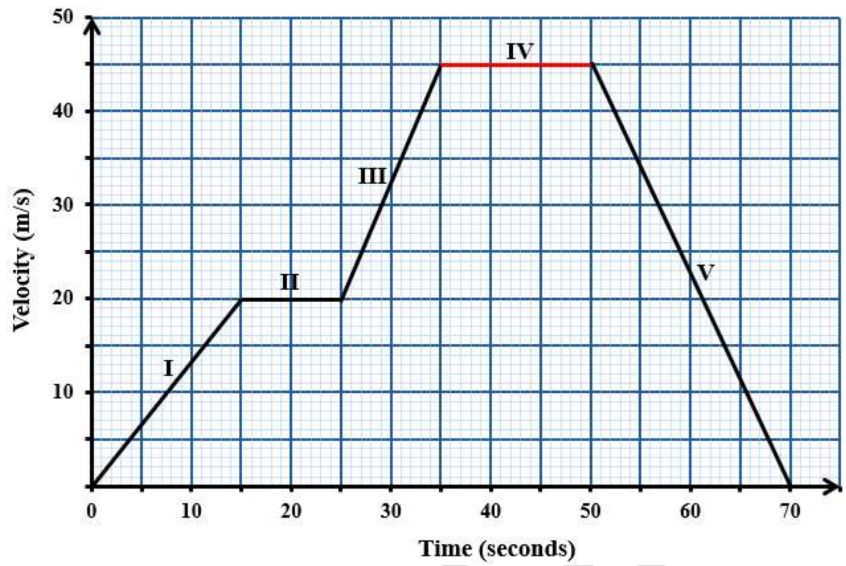
(b) The velocity – time graph below describes the journey of a car over a period of 70 seconds. The journey is represented by 5 stages labelled I to V.



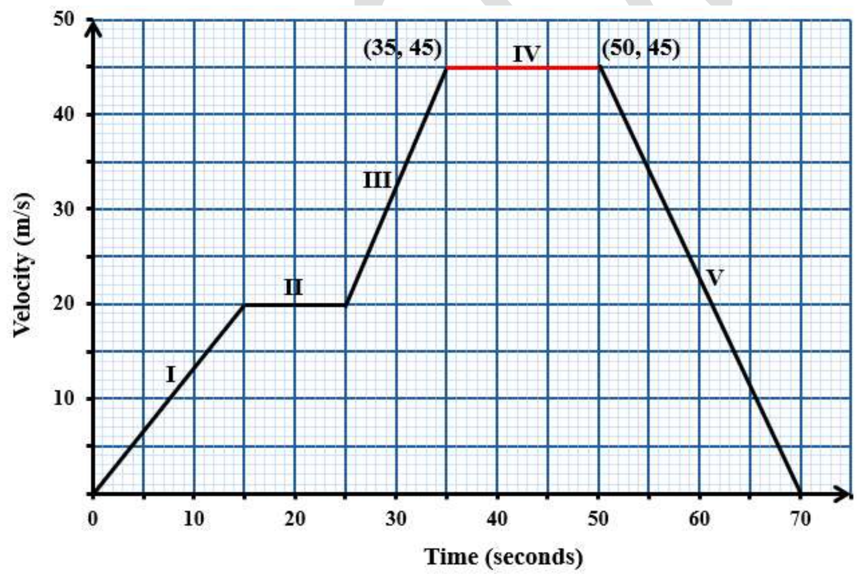
(i) Complete the following statement.

During Stage IV, the car is travelling at …………………… m/s with an acceleration of ………………….. m/s^2.

SOLUTION:
Data: A velocity – time graph showing the journey of a car over a period of 70 seconds. The various stages of the journey are labelled I to V.
Required to complete: The statement given
Solution:



During Stage IV, the car is travelling at 45 ms^{-1} obtained by a read off.



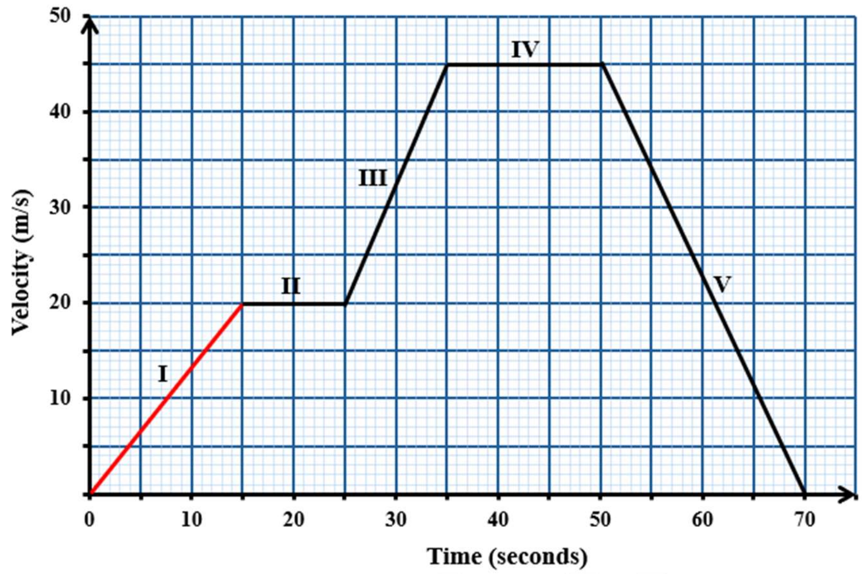
Acceleration $=$ Gradient of the branch that illustrates Stage IV

$= \frac{45-45}{50-35} = 0$(as expected for a horizontal line)

Acceleration $= 0$ ms^{-2}

(ii) Determine the MAXIMUM acceleration of the car during the 70 seconds.

SOLUTION:
Required to determine: The maximum acceleration of the car during the 70 seconds.
Solution:
Stage I:

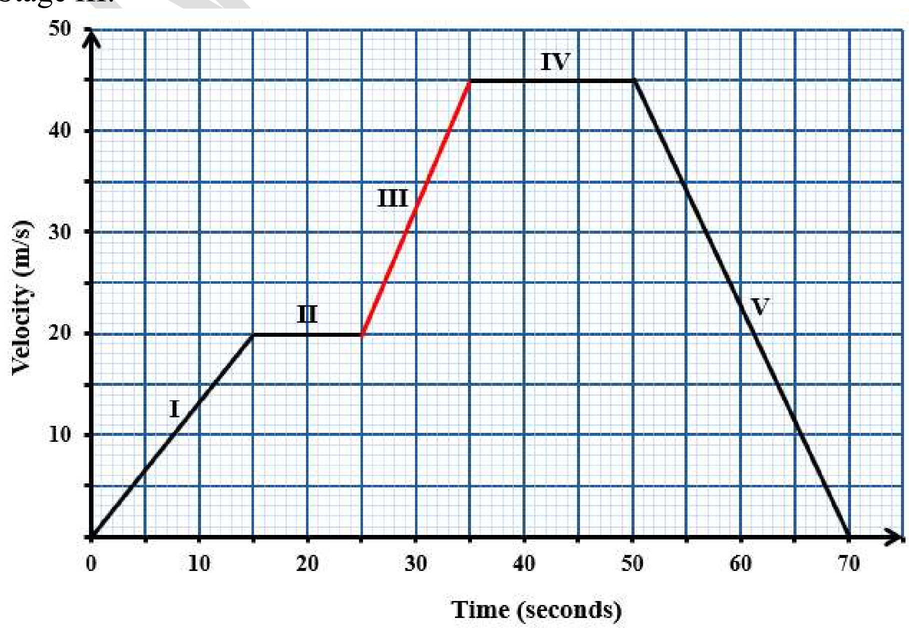


$$\text{Gradient } = \frac{20-0}{15-0}$$
$$= \frac{4}{3} \text{ ms}^{-2}$$
$$= 1\frac{1}{3} \text{ ms}^{-2}$$

Stage II:
This stage is represented by a horizontal line.
$\Rightarrow$ Gradient $= 0$

Acceleration $= 0$ ms^{-2}

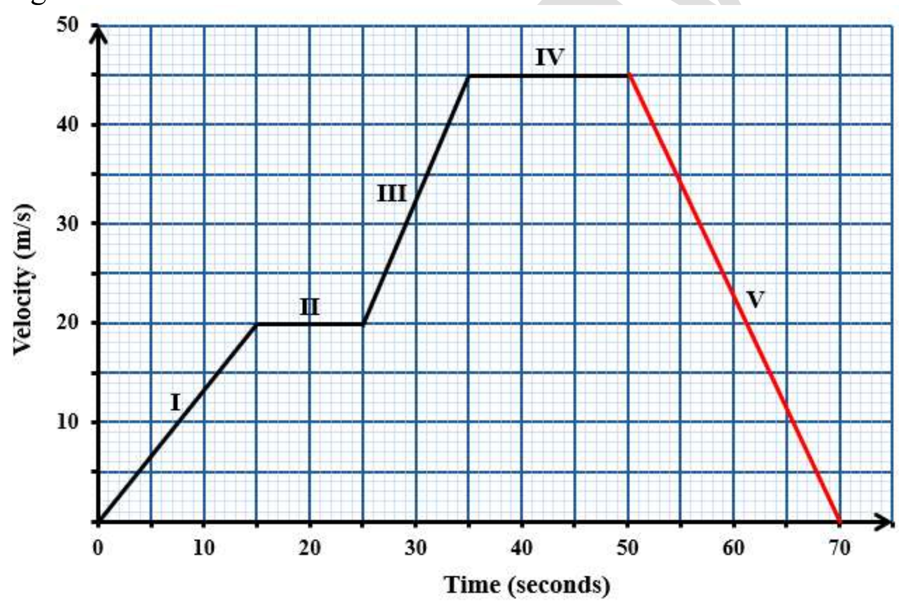
Stage III:



$$\text{Gradient } = \frac{45-20}{35-25}$$
$$= \frac{25}{10}$$
$$= 2\frac{1}{2} \text{ ms}^{-2}$$

Stage IV:
Stage IV has an acceleration of 0 ms^{-2}.

Stage V:



$$\text{Gradient } = \frac{45-0}{50-70}$$
$$= \frac{45}{-20}$$
$$= -2\frac{1}{4} \text{ ms}^{-2}$$

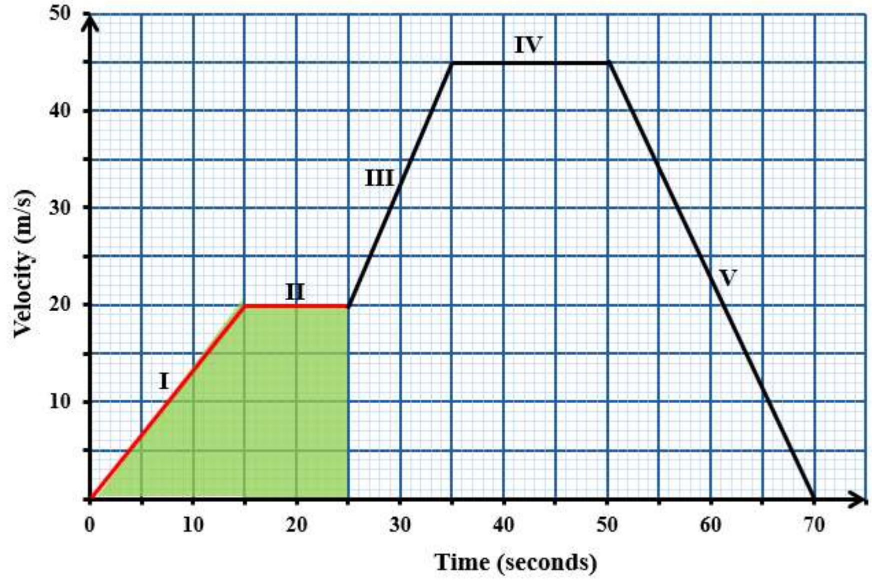
A negative acceleration indicates a deceleration.

$$2\frac{1}{2} > 1\frac{1}{3} > 0 > -2\frac{1}{4}$$

$\therefore$ The maximum acceleration is $2\frac{1}{2}$ ms^{-2}.

(iii) Calculate the distance travelled by the car during the **first 25 seconds** of its journey.

SOLUTION:
Required to calculate: The distance travelled by the car during the first 25 seconds of the journey
Calculation:

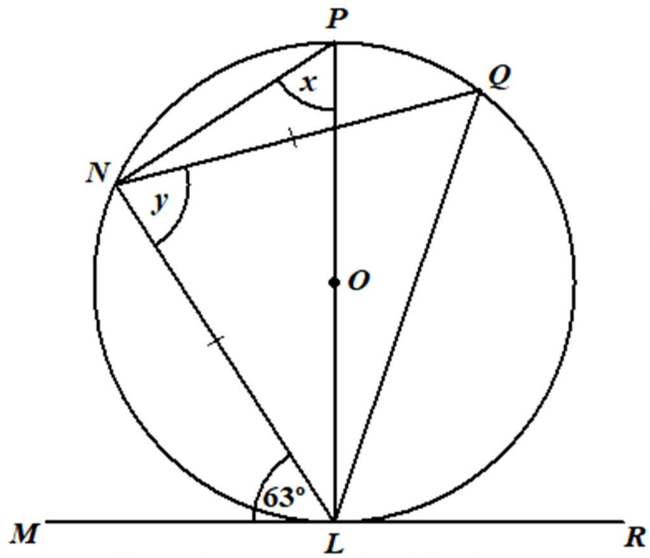


The area under the graph $=$ Distance covered
The distance covered in the first 25 s is indicated by the shaded region, which is defined by a trapezium.

Hence, the distance covered in the first 25 seconds $= \frac{1}{2}(10+25)\times 20$ m
$= 350$ m

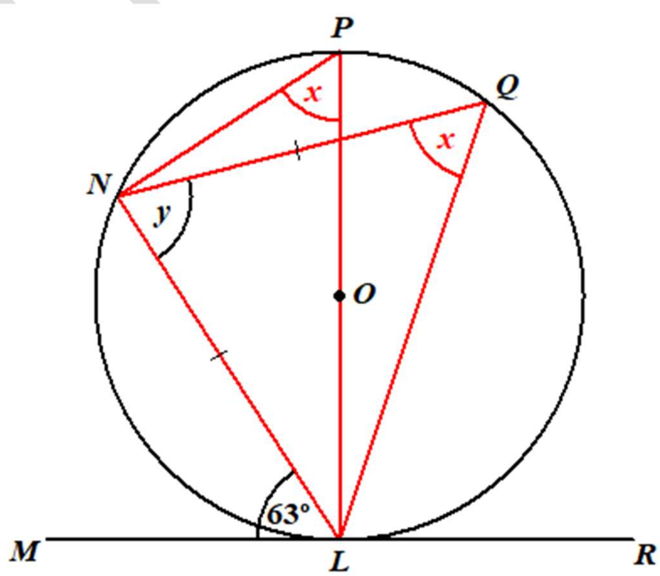
GEOMETRY AND TRIGONOMETRY

9. (a) The diagram below shows a circle with its centre O and the points P, Q, L and N lying on its circumference. $LN = NQ$ and RM is a tangent to the circle at L. Angle $MLN = 63°$.



(i) Explain why Angle x and Angle NQL are equal.

SOLUTION:
Data: Diagram showing a circle with its centre O and the points P, Q, L and N lying on its circumference. $LN = NQ$ and RM is a tangent to the circle at L. Angle $MLN = 63°$.
Required to explain: Why Angle x and Angle NQL are equal.
Solution:



$N\hat{P}L = x$

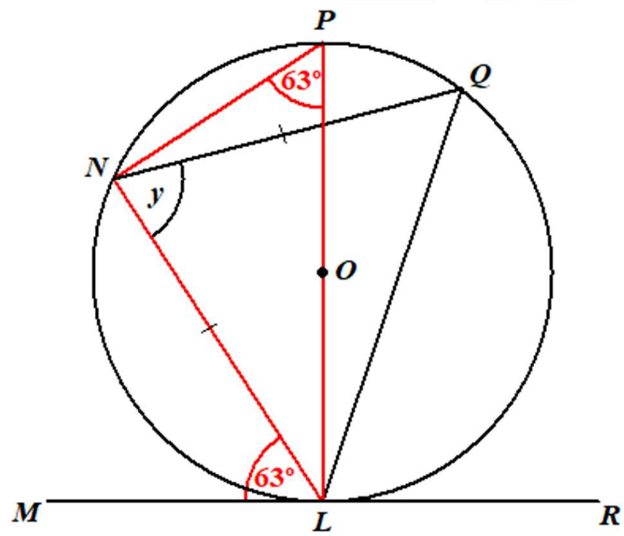
$N\hat{Q}L = x$

(The angles subtended by a chord (NL) at the circumference of a circle (angles $N\hat{P}L$ and $N\hat{Q}L$) and standing on the same arc are equal.

(ii) Determine the value of EACH of the following angles. Show detailed working where possible and give a **reason** for your answer.

a) Angle x

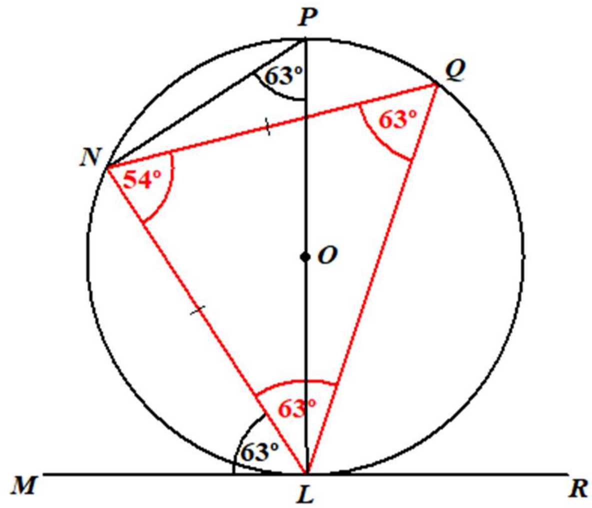
SOLUTION:
Required to determine: The value of angle x
Solution:



$x = 63°$
(The angle made by a tangent to a circle (ML) and a chord (NL) at the point of contact (L) is equal to the angle in the alternate segment ($N\hat{P}L = x$).

b) Angle y

SOLUTION:
Required to determine: The value of angle y
Solution:



Consider ΔNQL :

$N\hat{Q}L = 63°$ (See (i) above)

$\therefore N\hat{L}Q = 63°$

(Base angles in an isosceles triangle are equal.)

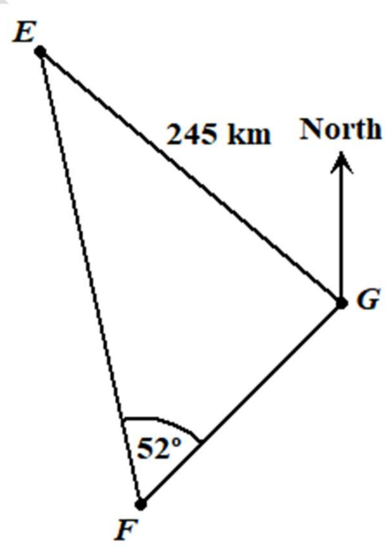
$L\hat{N}Q = 180° - (63° + 63°)$

$= 54°$

$y = 54$

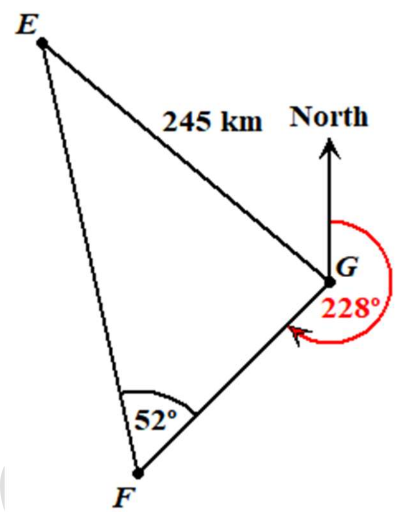
(Sum of the interior angles in a triangle $= 180°$).

(b) Two ports, E and G, are on level ground, 245 km apart. The bearing of E from G is 302º. A ship is anchored at F, some distance away from G, on a bearing of 228º. Angle $EFG = 52°$. This information is shown in the diagram below.



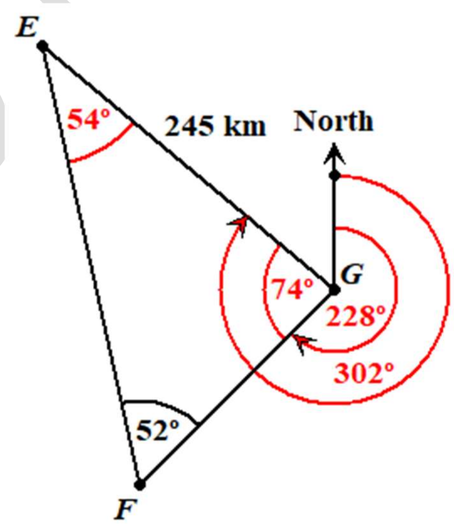
(i) **a)** On the diagram above, insert the angle 228º, the bearing of *F* from *G*.

SOLUTION:
Data: Incomplete diagram showing two ports, *E* and *G*, on level ground, 245 km apart. The bearing of *E* from *G* is 302º. A ship is anchored at *F*, some distance away from *G*, on a bearing of 228º. Angle $EFG = 52°$.
Required to insert: The angle 228º, the bearing of F from G
Solution:



b) Determine the value of Angle *FEG*.

SOLUTION:
Required to determine: The value of Angle *FEG*
Solution:

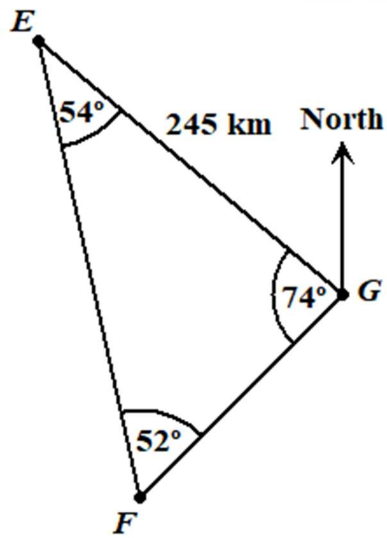


$E\hat{G}F = 302° - 228°$
$= 74°$

$F\hat{E}G = 180° - (52° + 74°)$
$= 54°$
(Sum of the interior angles in a triangle $= 180°$).

(ii) Calculate *GF*, the distance the ship is from Port *G*.

SOLUTION:
Required to calculate: *GF*, the distance the shape is from Port *G*
Calculation:
Consider ΔEGF :



$$\frac{245}{\sin 52°} = \frac{GF}{\sin 54°} \quad \text{(Sine Rule)}$$

$$\therefore GF = \frac{245 \times \sin 54°}{\sin 52°}$$

$$= \frac{198.209}{0.788}$$

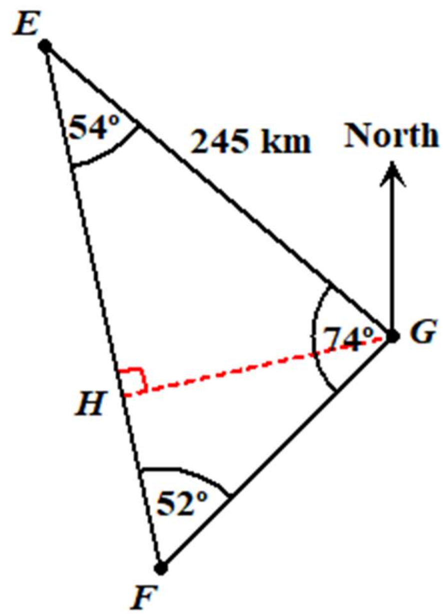
$$= 251.53\underline{\underline{4}}$$

$$\approx 251.53 \text{ km (correct to 2 decimal places)}$$

(iii) **a)** Indicate the point *H* on the line *EF*, such that *GH* is the SHORTEST distance from *G* to *EF*.

SOLUTION:

Required to indicate: The point *H* on the line *EF*, such that *GH* is the SHORTEST distance from *G* to *EF*.
Solution:



The perpendicular from *G* to *EF* meets *EF* at *H*.

b) Determine the distance *GH*.

SOLUTION:
Required To Determine: The distance *GH*
Solution:
Consider ΔFGH :

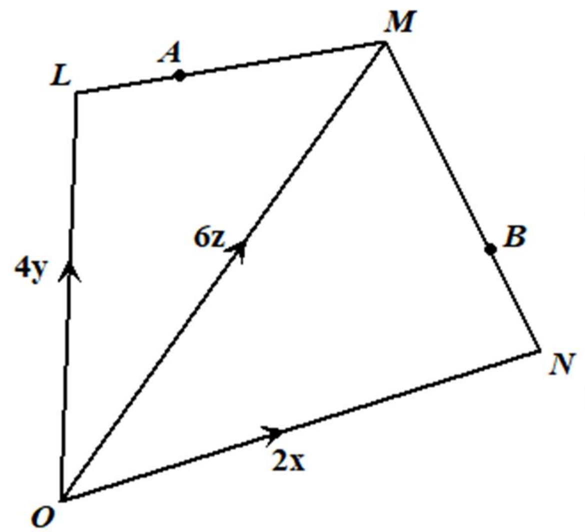
$$\sin 52° = \frac{GH}{251.53}$$

$$GH = 251.53 \times \sin 52°$$

$$= 198.20\underline{\underline{8}}$$

$$\approx 198.21 \text{ (correct to 2 decimal places)}$$

10. (a) The diagram below shows quadrilateral *OLMN*, in which *O* is the origin $\overrightarrow{OL} = \mathbf{4y}$, $\overrightarrow{OM} = \mathbf{6z}$ and $\overrightarrow{ON} = \mathbf{2x}$. The point *A* lies on *LM* such that $LA : AM = 1 : 2$ and the point *B* on *MN* such that $MB : BN = 2 : 1$.



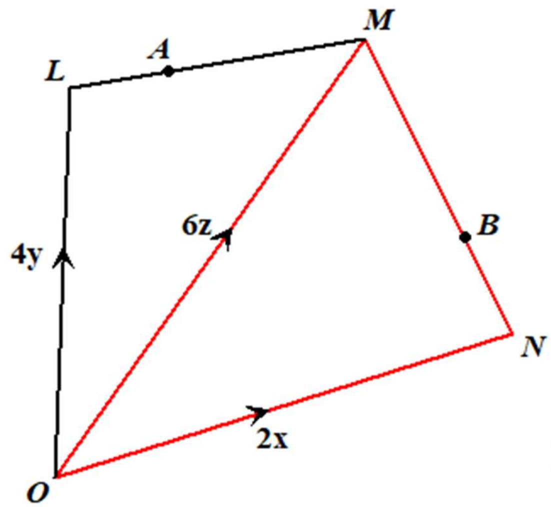
(i) Express, in its simplest form, $\overrightarrow{MN}$ in terms of **x** and **z**.

SOLUTION:
Data: Diagram showing a quadrilateral *OLMN*, in which *O* is the origin $\overrightarrow{OL} = \mathbf{4y}$, $\overrightarrow{OM} = \mathbf{6z}$ and $\overrightarrow{ON} = \mathbf{2x}$. The point *A* lies on *LM* such that $LA : AM = 1 : 2$ and the point *B* on *MN* such that $MB : BN = 2 : 1$.
Required to express: $\overrightarrow{MN}$ in terms of **x** and **z**.
Solution:
$LA : AM = 1 : 2$

$$\therefore LA = \frac{1}{3}LM \text{ and } AM = \frac{2}{3}LM$$

$MB : BN = 2 : 1$

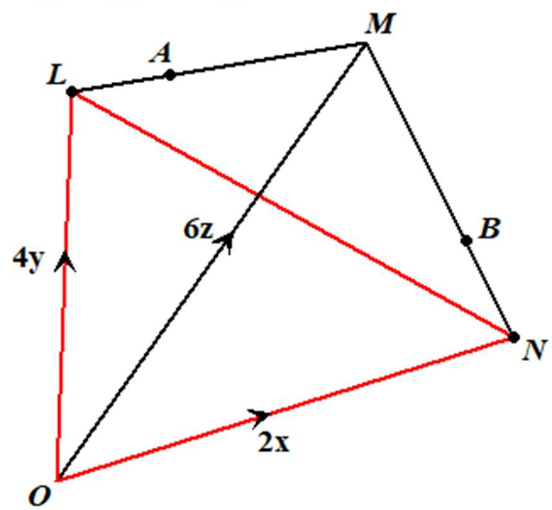
$$\therefore MB = \frac{2}{3}MN \text{ and } BN = \frac{1}{3}MN$$



$\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON}$
$= -(6\mathbf{z}) + 2\mathbf{x}$
$= \mathbf{-6}z + \mathbf{2x}$ or $\mathbf{2x} - \mathbf{6z}$

(ii) **a)** Find, in terms of **x** and **y**, in its simplest form, an expression for $\overrightarrow{LN}$.

SOLUTION:
Required to find: $\overrightarrow{LN}$, in terms of **x** and **y**
Solution:



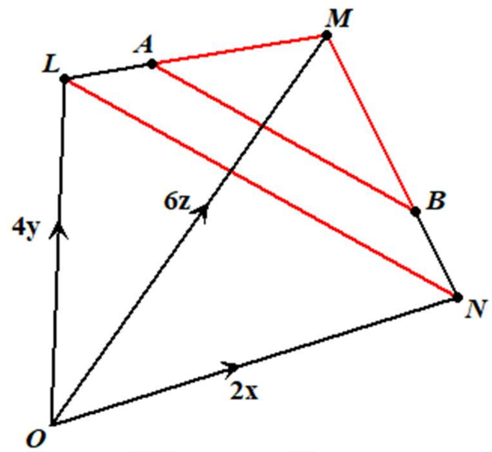
$\overrightarrow{LN} = \overrightarrow{LO} + \overrightarrow{ON}$
$= -(4\mathbf{y}) + 2\mathbf{x}$
$= \mathbf{2x} - \mathbf{4y}$

b) Show that $\overrightarrow{AB}$ equals $\frac{2}{3}(\mathbf{2x}-\mathbf{4y})$.

SOLUTION:

Required to show: $\overrightarrow{AB}$ equals $\frac{2}{3}(\mathbf{2x}-\mathbf{4y})$

Proof:



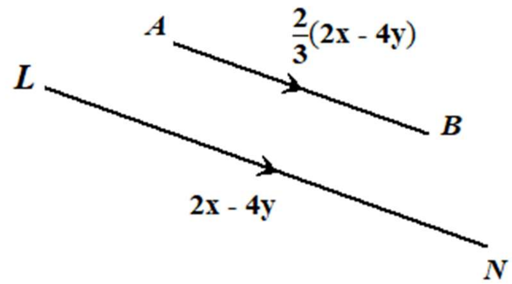
$$\overrightarrow{LM} = \overrightarrow{LO} + \overrightarrow{OM}$$
$$= -\mathbf{4y} + \mathbf{6z}$$

$$\overrightarrow{AB} = \overrightarrow{AM} + \overrightarrow{MB}$$
$$= \frac{2}{3}\overrightarrow{LM} + \frac{2}{3}\overrightarrow{MN}$$
$$= \frac{2}{3}(-\mathbf{4y} + \mathbf{6z}) + \frac{2}{3}(\mathbf{2x} - \mathbf{6z})$$
$$= \frac{2}{3}(-\mathbf{4y} + \mathbf{6z} + \mathbf{2x} - \mathbf{6z})$$
$$= \frac{2}{3}(\mathbf{2x} - \mathbf{4y})$$

Q.E.D.

(iii) Based on your results in Part (ii), state TWO geometric properties relating *LN* to *AB*.

SOLUTION:
Required to state: Two geometrical properties relating *LN* to *AB*.
Solution:



$\overrightarrow{AB} = \frac{2}{3}\left(\overrightarrow{LN}\right)$, that is, a scalar multiple of $\overrightarrow{LN}$.

$\therefore \overrightarrow{AB}$ is parallel to $\overrightarrow{LN}$ and $\left|\overrightarrow{AB}\right| = \frac{2}{3}\left|\overrightarrow{LN}\right|$.

(b) Determine the values of the unknowns in EACH of the matrix equations below:

(i) $\begin{pmatrix} 4 & 0 \\ -2 & 5 \end{pmatrix} + \begin{pmatrix} x & 2 \\ 8 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ y & 4 \end{pmatrix}$

SOLUTION:

Data: $\begin{pmatrix} 4 & 0 \\ -2 & 5 \end{pmatrix} + \begin{pmatrix} x & 2 \\ 8 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ y & 4 \end{pmatrix}$

Required to determine: The value of x and the value of y

Solution:

$$\begin{pmatrix} 4 & 0 \\ -2 & 5 \end{pmatrix} + \begin{pmatrix} x & 2 \\ 8 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ y & 4 \end{pmatrix}$$

$$\begin{pmatrix} 4+x & 0+2 \\ -2+8 & 5+(-1) \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ y & 4 \end{pmatrix}$$

$$\begin{pmatrix} 4+x & 2 \\ 6 & 4 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ y & 4 \end{pmatrix}$$

Since the matrices are equal we can equate corresponding entries.

$4 + x = -3$

$$\therefore x = -7$$

And, $y = 6$

$\therefore x = -7, y = 6$

(ii) $\begin{pmatrix} 5 & -3 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} a & 2 \\ c & -1 \end{pmatrix} = \begin{pmatrix} -10 & 13 \\ 17 & 1 \end{pmatrix}$

SOLUTION:

Data: $\begin{pmatrix} 5 & -3 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} a & 2 \\ c & -1 \end{pmatrix} = \begin{pmatrix} -10 & 13 \\ 17 & 1 \end{pmatrix}$

Required to determine: The value of a and the value of c

Solution:

$$\underset{2\times 2}{\begin{pmatrix} 5 & -3 \\ 2 & 3 \end{pmatrix}}\underset{2\times 2}{\begin{pmatrix} a & 2 \\ c & -1 \end{pmatrix}} = \begin{pmatrix} -10 & 13 \\ 17 & 1 \end{pmatrix}$$

$$=2\times 2$$

$$\begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$$

$$\begin{aligned} e_{11} &= (5\times a)+(-3\times c) \\ &= 5a-3c \\ e_{12} &= (5\times 2)+(-3\times -1) \\ &= 13 \\ e_{21} &= (2\times a)+(3\times c) \\ &= 2a+3c \\ e_{22} &= (2\times 2)+(3\times -1) \\ &= 1 \end{aligned}$$

$$\therefore \begin{pmatrix} 5a-3c & 13 \\ 2a+3c & 1 \end{pmatrix} = \begin{pmatrix} -10 & 13 \\ 17 & 1 \end{pmatrix}$$

Equating corresponding entries:

$5a-3c=-10 \quad \ldots$ ❶

$2a+3c=17 \quad \ldots$ ❷

Equation ❶ + Equation ❷:

$$7a=7$$
$$a=1$$

When $a=1$ into equation ❶:

$$\begin{aligned} 5(1)-3c &= -10 \\ -3c &= -15 \\ c &= 5 \end{aligned}$$

$$\therefore a=1,\ c=5$$