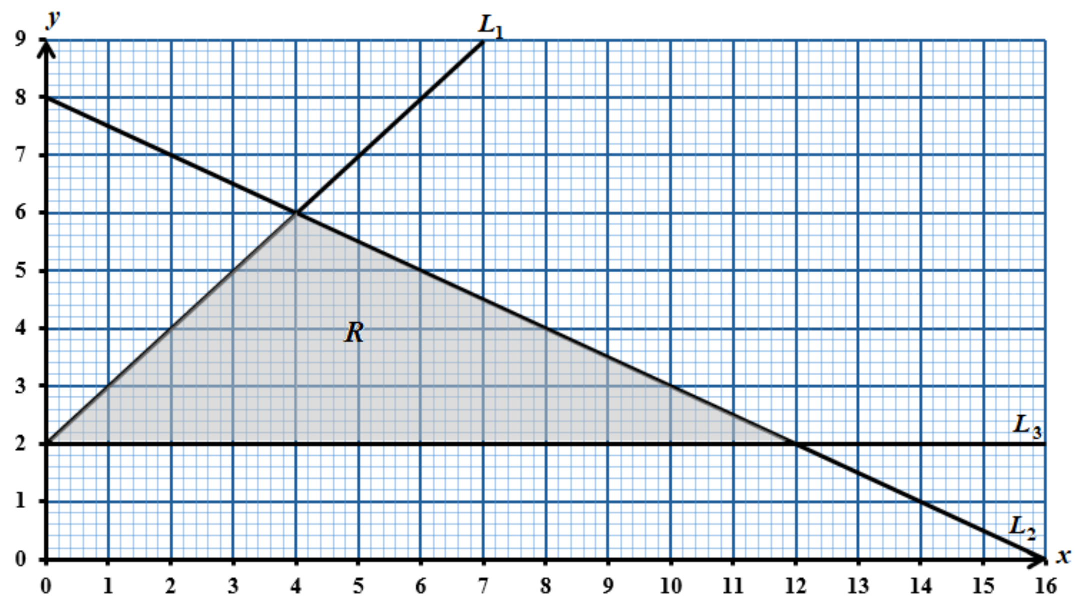
CSEC MATHEMATICS JANUARY 2024 PAPER 3

1. (a) The diagram below shows the graphs of 3 lines, L_1, L_2 and L_3, and a shaded region, *R*, which represents the common region for the 3 inequalities associated with the lines L_1, L_2 and L_3, that define *R*.



(i) The table below shows the equation of L_3, and the respective inequality associated with the shaded region, *R*.

Line	Equation of Line	Inequality Associated with Line
L_1		
L_2		
L_3	$y = 2$	$y \geq 2$

Complete the table above by filling in the missing information for L_1 and L_2. Write the equations in the form $y = mx + c$.

SOLUTION:
Data: Diagram showing the graphs of three lines L_1, L_2 and L_3, and a shaded region, *R*. An incomplete table showing the equation and the associated inequality of L_3.
Required to complete: The table given for L_1 and L_2.

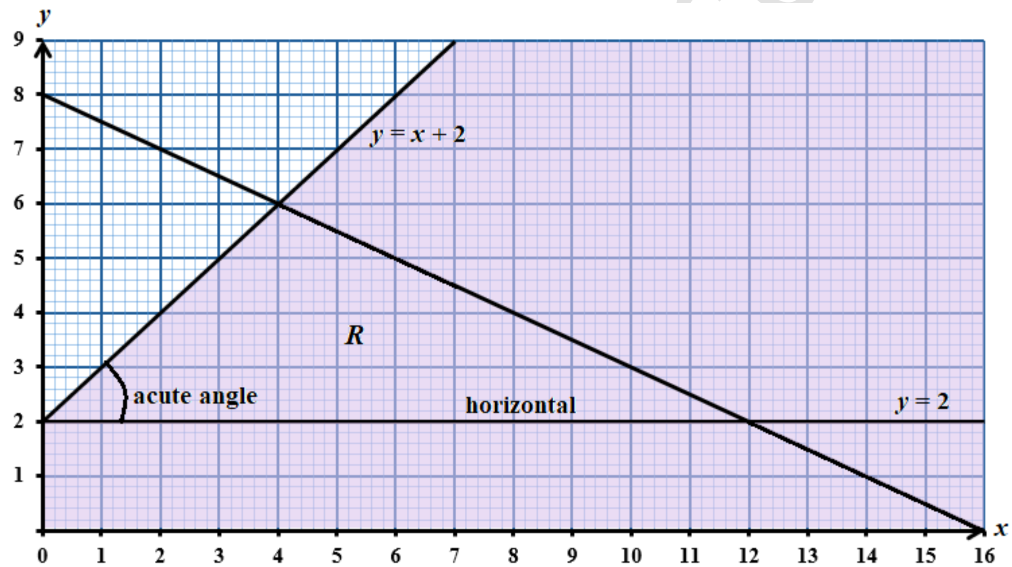
Solution:

Consider the line L_1:

Two points on L_1 are $(0, 2)$ and $(7, 9)$.

Hence, the gradient of $L_1 = \frac{9-2}{7-0}$
$= 1$

The equation of L_1 is $y = mx + c$ where $m = 1$ is the gradient and $c = 2$ is the intercept on the vertical axis.
The equation of L_1 is $y = 1x + 2$, that is, $y = x + 2$.



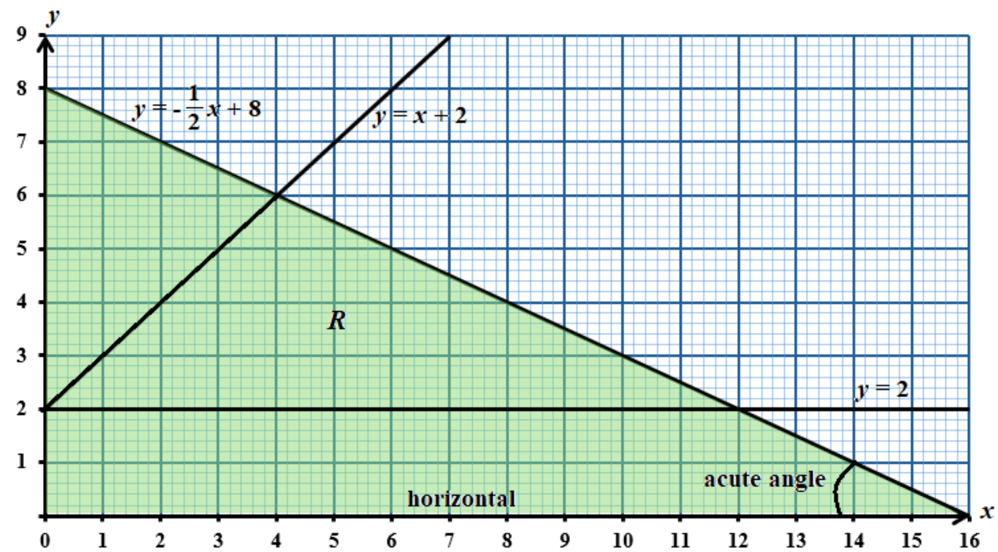
R is part of the shaded region which makes an acute angle with the horizontal and L_2. Hence, the inequality associated with L_1 is $y \le x + 2$.
The sign $\le$ implies that the line is included in the region described as *R*.

Consider the line L_2:

Two points on L_2 are $(16, 0)$ and $(0, 8)$.

Hence, the gradient of $L_2 = \frac{8-0}{0-16}$
$= -\frac{1}{2}$

The equation of L_2 is $y = mx + c$, where m is the gradient and c is the intercept on the vertical axis. Hence, $m = -\frac{1}{2}$ and $c = 8$. So, the equation of L_2 is $y = -\frac{1}{2}x + 8$.



R is part of the shaded region which makes an acute angle with the horizontal and L_2. Hence, the inequality associated with L_2 is $y \leq -\frac{1}{2}x + 8$. The sign $\leq$ implies that the line is included in R.

The completed table looks like:

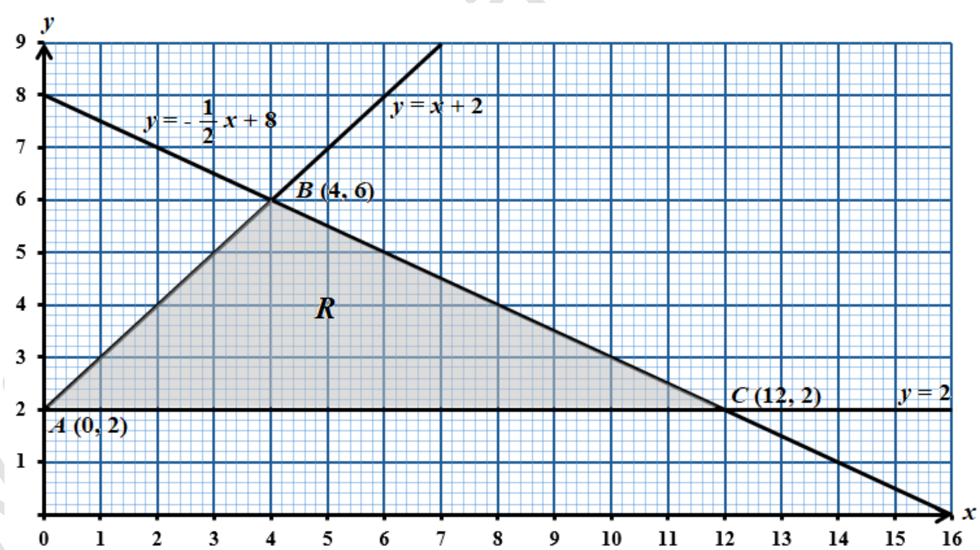
Line	Equation of Line	Inequality Associated with Line
L_1	$y = x + 2$	$y \leq x + 2$
L_2	$y = -\frac{1}{2}x + 8$	$y \leq -\frac{1}{2}x + 8$
L_3	$y = 2$	$y \geq 2$

(ii) A toy manufacturer makes x phones and y trucks. The cost to make a phone is \$75 and the cost to make a truck is \$200. The shaded region, R, in the diagram above, represents the only possible number of toy phones and toy trucks the manufacturer can make.

Using the graph, determine the number of phones and the number of trucks which give the GREATEST total cost of productivity. Write down the greatest possible total cost of production.

SOLUTION:
Data: A toy manufacturer makes x phones costing \$75 each and y trucks costing \$200 each. The region, R, in the given diagram, represents the only number of phones and trucks that can be made by the manufacturer.
Required to determine: The number of phones and trucks that would give the greatest possible cost of production and the greatest possible cost of production.
Solution:
Number of phones $= x$
Number of trucks $= y$

Cost to make a phone $= \$75$
Cost to make a truck $= \$200$



We consider the vertices $A(0, 2)$, $B(4, 6)$ and $C(12, 2)$ of the triangle that comprises region R.

Since the manufacturer makes both phones and trucks, we ignore $A(0, 2)$ since $x = 0$.

At B, $x = 4$, $y = 6$:
Cost $= (\$75 \times 4) + (\$200 \times 6)$
$= \$1500$

At C, $x=12$, $y=2$:
Cost $=(\$75\times12)+(\$200\times2)$
$=\$1\ 300$

Hence, the greatest cost of production is $1500 when the production consists of 4 phones $(x=4)$ and 6 trucks $(y=6)$.

(b) Over time, the company makes a total of 120 toys. Each toy, whether a phone or a truck, is either blue, red or green in colour. Other information regarding the toys is as follows.

- 48 toys are phones
- 77 toys are blue
- 5 toy trucks are red
- 36 toys are green
- $\frac{4}{9}$ of the green toys are phones

(i) Complete the table below to show this information.

	Blue	Red	Green	Total
Phone				
Truck				
Total				120

SOLUTION:
Data: The company makes a total of 120 toys such that 48 toys are phones, 77 toys are blue, 5 toy trucks are red, 36 toys are green and $\frac{4}{9}$ of the green toys are phones.
Required to complete: The table given, showing the numbers of different types of toys.

Solution:

	Blue	Red	Green	Total
Phone	$77-47=30$	$7-5=2$	$\frac{4}{9}(36)=16$	48
Truck	$72-(5+20)$ $=47$	5	$36-16=20$	$120-48=72$
Total	77	$120-(77+36)$ $=7$	36	120

The completed table looks like:

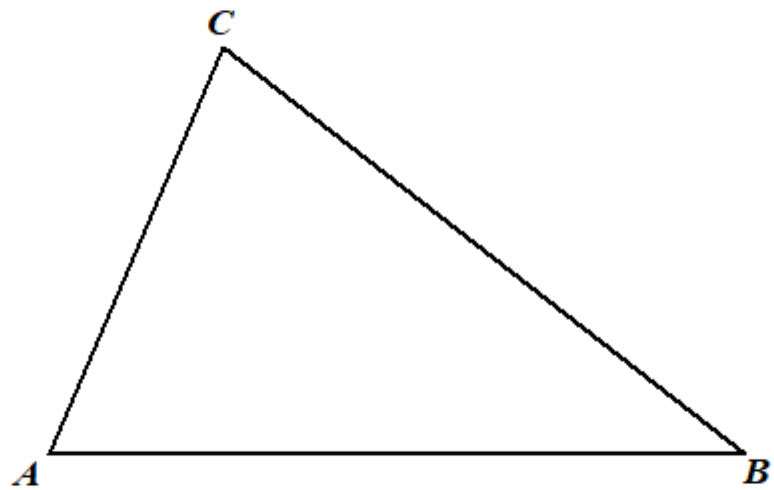
	Blue	Red	Green	Total
Phone	30	2	16	48
Truck	47	5	20	72
Total	77	7	36	120

(ii) One of the 120 toys is chosen at random. Determine the probability that this toy is a red truck.

SOLUTION:
Required to determine: The probability that a toy chosen at random is a red truck.
Solution:

$$\text{P(toy chosen at random is a red truck)} = \frac{\text{Number of red trucks}}{\text{Total number of toys}}$$
$$= \frac{5}{120}$$
$$= \frac{1}{24}$$

2. Farmer Jay owns a triangular field, *ABC*. A scale diagram of this field is shown below (1 centimetre represents 10 metres).



(a) **(i)** **a)** Complete the following statement.

The side of the field, *AC*, is ………………. metres long.

SOLUTION:
Data: Scale of diagram showing a triangular field *ABC*, where 1 centimetre represents 10 metres.
Required to complete: The statement given, by stating the actual length of the side *AC*.
Solution:
By measurement, $AC = 4.8$ cm.

$1 \text{ cm} \equiv 10 \text{ m}$

$\therefore AC = 4.8 \times 10 \text{ m}$

$= 48 \text{ m}$

The side of the field, *AC*, is 48 metres.

b) Measure, in degrees, the angle *ACB*.

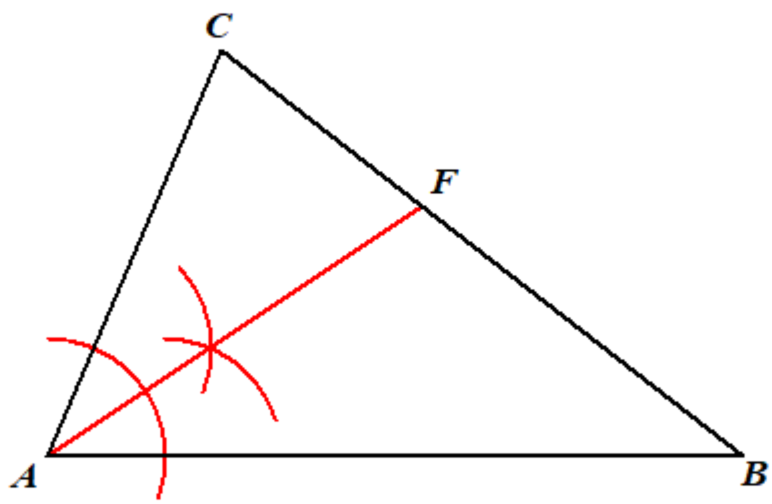
SOLUTION:
Required to measure: the size of angle *ACB*
Solution:
Angle $ACB = 73°$ (Measurement taken by a protractor)

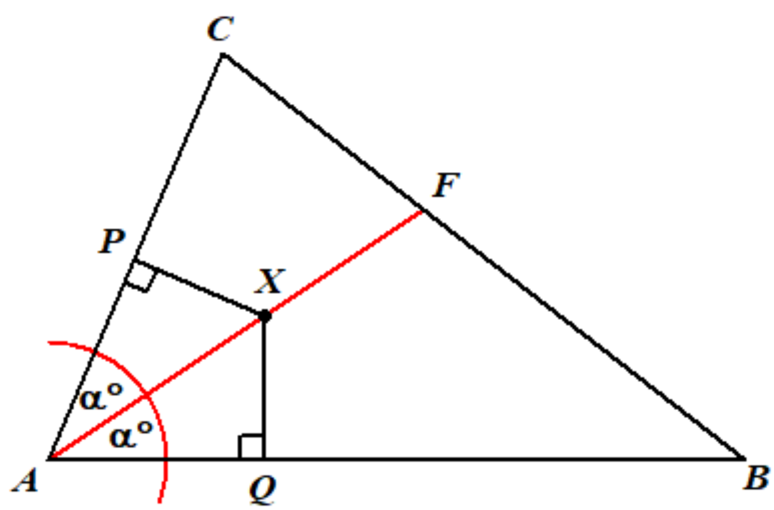
Construct the responses to parts (ii) and (iii) on the scaled drawing shown above. Show ALL construction lines.

(ii) Farmer Jay divides the field with a fence from *A* to the side *BC*. Each point on the fence is the same distance from *AB* as from *AC*.

Using a straight edge and compasses only, construct the line representing the fence. Label the fence *AF*.

SOLUTION:
Data: A fence from *A* to *BC* is to divide the field such that the distance from any point on the fence to *AB* and *AC* is the same.
Required to construct: The line representing the fence and label the fence *AF*
Solution:
We bisect the angle at *A* to meet *AB* at *F*.





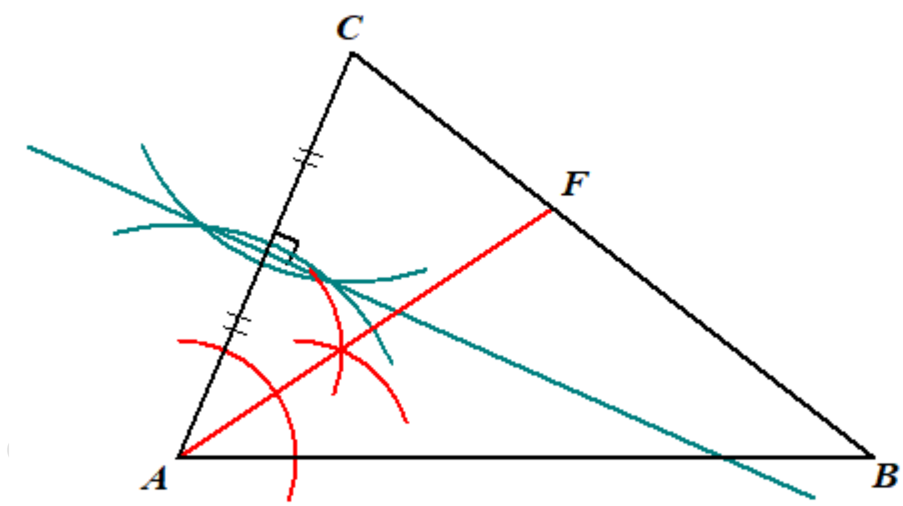
Any point on *AF* will be the same (perpendicular) distance to *AC* and to *AB*.

When X is any point on AF. So, $\Delta APX \equiv \Delta AQX$. Therefore, $PX = QX$.

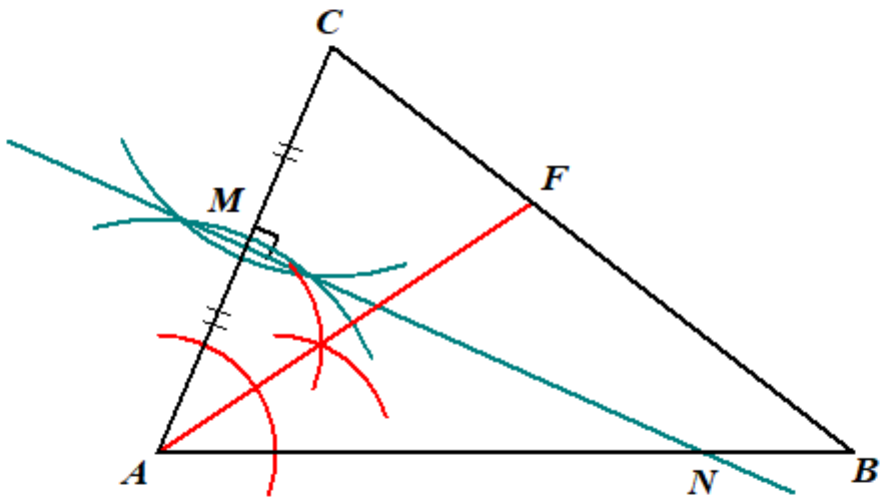
(iii) Farmer Jay puts another fence along the perpendicular bisector of the side AC. He decides to keep cows in the region of the field which is closer to AC than to AB and closer to A than to C.

Using a ruler and compasses only, construct the line representing this fence and shade and label as W the region in the field where cows are kept.

SOLUTION:
Data: Farmer is erecting another fence along the perpendicular bisector of the side AC and intends to keep cows in the region that is closer to AC than AB and closer to A than to C.
Required to construct: The line representing this second fence and shade and label the region as W where the cows are to be kept.
Solution:
We construct the perpendicular bisector of AC.

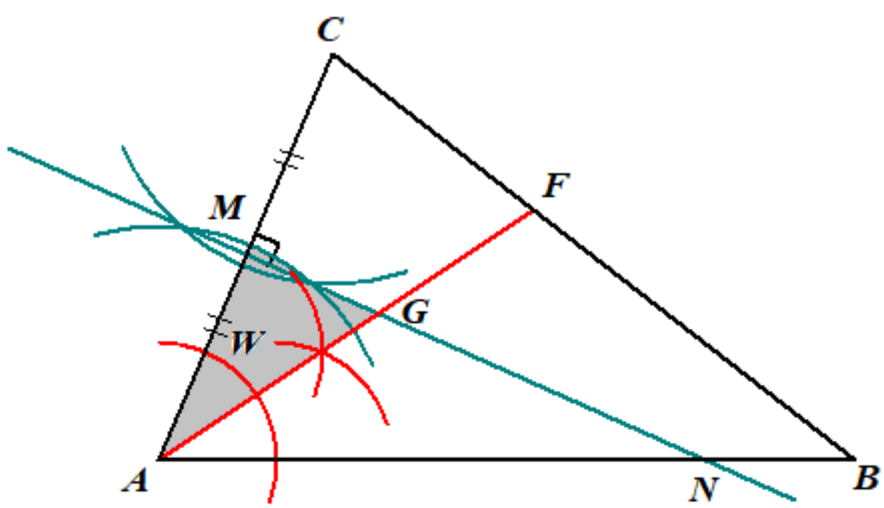


The two fences are AF and MN.



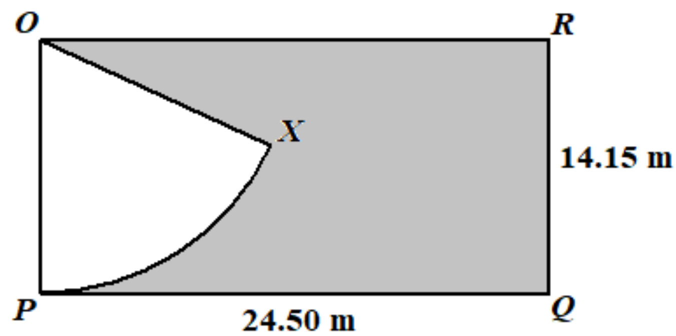
We shade W, such that any point within W is closer to A than to C, that is, $WA < WC$ for all points within W.

The final diagram looks like:



Let AF and MN meet at G.
Any point within ΔCAF will be closer to AC than to AB. The line AF is not included in the region W since any point on AF is equidistant from AB and AC.

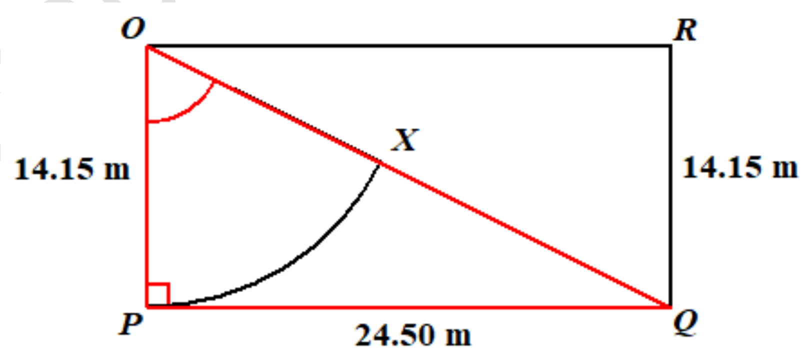
(b) Farmer Macy has a rectangular garden, *OPQR*, which is shown below.



The length of the garden is 24.50 m and the width is 14.15 m. *OQ* is a diagonal and *OPX* is a sector of a circle, with centre *O*. The unshaded sector represents a pond where Farmer Macy stores water to irrigate her farm. The shaded area is planted with crops.

(i) Determine the value of Angle *POQ*, to the nearest degree.

SOLUTION:
Data: Diagram showing a rectangular garden *OPQR* with dimensions 24.50 m by 14.15 m. The garden is such that *OQ* is a diagonal of the rectangle and *OPX* is a sector of a circle.
Required to determine: The value of angle POQ, correct to the nearest degree.
Solution:
Consider triangle *OPQ*:



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan P\hat{O}Q = \frac{24.50}{14.15}$$

$$P\hat{O}Q = \tan^{-1}\left(\frac{24.50}{14.15}\right)$$

$$= 59.99°$$

$$\approx 60° \text{ (correct to the nearest degree)}$$

(ii) Calculate the area of the farmland that is planted with crops.

SOLUTION:
Required to calculate: The area of the farmland that is planted in crops.
Calculation:
Area of entire field, $OPQR = (24.50 \times 14.15)$

$$= 346.675 \text{ m}^2$$

Area of the pond, $OPX = \frac{60°}{360°} \times \pi(14.15)^2$

$$=104.878$$

The area that is planted in crops
= Area of rectangle *OPQR* – Area of pond *OPX*
= 346.675 – 104.878
= 241.797 m^2