CSEC MATHEMATICS JUNE 2023 PAPER 3

1. (a) (i) Write down, in ASCENDING order, the two missing factors of 16.

1, ……………, 4, ……………., 16

SOLUTION:
Data: An incomplete set of the factors of 16, in ascending order
Required To Write: The missing factors of 16 from the set
Solution:
1, 2, 4, 8, 16

(ii) Write down the missing factors of 16, in ASCENDING order, as powers of 2.

2^0,……………, 2^2, ……………, 2^4

SOLUTION:
Required To Write: The missing factors of 16, in ascending order, as powers of 2.
Solution:
$2^0,\ 2=2^1, 2^2,\ 8=2^3,\ 2^4$

(b) Given that r is a prime number.

(i) state the four factors of r^3 as powers of r (One has been written for you.)

r^0, ……………, ……………, ……………

SOLUTION:
Data: r is a prime number
Required To State: The four factors of r^3 as powers of r
Solution:
$r^3 = r \times r \times r$
So the factors of r^3 are $1, r, r^2$ and r^3.

The four factors of r^3 are:
$r^0,\ r^1 = r,\ r^2 = r \times r,\ r^3 = r \times r \times r$

(ii) state in terms of n, the number of factors of r^n.

SOLUTION:
Required To State: The number of factors of r^n, in terms of n
Solution:
r has 2 factors.

r^2 has 3 factors.
r^3 has 4 factors.
r^n has $(n+1)$ factors.

(iii) **a)** Express 2 187 in the form 3^p.

SOLUTON:
Required To Express: 2 187 in the form 3^p
Solution:
$2187 = 3\times3\times3\times3\times3\times3\times3$
$= 3^7$, where $p = 7$

b) Hence, determine the number of factors of 2 187. **Do NOT write them out.**

SOLUTION:
Required To Determine: The number of factors of 2 187
Solution:
$2187 = 3^7$ will have $7+1=8$ factors.

(c) 40 is not a prime number.
$40 = 2^3 \times 5^1$ where 2 and 5 are prime numbers.

(i) Complete the table below by finding the factors of 40 that are missing.

Power of 5

Power of 2	5^0	5^1
2^0	$2^0 \times 5^0 =$	$2^0 \times 5^1 = 5$
2^1	$2^1 \times 5^0 = 2$	$2^1 \times 5^1 = 10$
2^2	$2^2 \times 5^0 = 4$	$2^2 \times 5^1 = 20$
2^3	$2^3 \times 5^0 = 8$	$2^3 \times 5^1 =$ ....

SOLUTION:
Data: 40 is not a prime number and $40 = 2^3 \times 5^1$ where 2 and 5 are prime numbers. An incomplete table showing the factors of 40.
Required To Complete: The table given
Solution:

		Power of 5: 5^0	5^1
Power of 2	2^0	$2^0 \times 5^0 = 1 \times 1 = 1$	$2^0 \times 5^1 = 5$
	2^1	$2^1 \times 5^0 = 2$	$2^1 \times 5^1 = 10$
	2^2	$2^2 \times 5^0 = 4$	$2^2 \times 5^1 = 20$
	2^3	$2^3 \times 5^0 = 8$	$2^3 \times 5^1 = 8 \times 5 = 40$

(ii) The table above has 4 rows and 2 columns.

Describe how to find the number of factors of 40 using the number of rows and the number of columns.

SOLUTION:
Data: The table given in part (i) above has 4 rows and 2 columns.
Required To Describe: The method to find the number of factors of 40 using the number of rows and the number of columns in the table.
Solution:
The factors of 40 are represented as a 4×2 matrix.

$$\begin{pmatrix} 1 & 5 \\ 2 & 10 \\ 4 & 20 \\ 8 & 40 \end{pmatrix}$$

$$\text{Number of elements } = 4 \times 2$$
$$= 8$$

(iii) a) Given that $5\,000 = 2^3 \times 5^4$, determine the number of factors of 5 000.

SOLUTION:
Data: $5\,000 = 2^3 \times 5^4$
Required To Determine: The number of factors of 5 000
Solution:

$$5\,000 = 2^3 \times 5^4 \text{ will have } (3+1)\times(4+1) = 4 \times 5$$
$$= 20 \text{ factors}$$

b) Write 1 944 in the form $2^p \times 3^q$, where p and q are integers, given that 1 944 has 24 factors.

SOLUTION:
Data: 1 944 has 24 factors.
Required To Write: 1 944 in the form $2^p \times 3^q$, where p and q are integers.
Solution:

$$\begin{array}{r|r}
2 & 1944 \\ \hline
2 & 972 \\ \hline
2 & 486 \\ \hline
3 & 243 \\ \hline
3 & 81 \\ \hline
3 & 27 \\ \hline
3 & 9 \\ \hline
3 & 3 \\ \hline
 & 1
\end{array}$$

$1944 = 2^3 \times 3^5$ is of the form $2^p \times 3^q$ where $p = 3 \in \mathbb{Z}$ and $q = 5 \in \mathbb{Z}$

The number of factors $(p+1)\times(q+1) = (3+1)\times(5+1)$
$= 4 \times 6$
$= 24$ factors

Alternative Method

$$1944 = 2^p \times 3^q$$

From (iii)(a), we know that:

$$(p+1)(q+1) = 24$$

This implies that solutions for p and q can be:

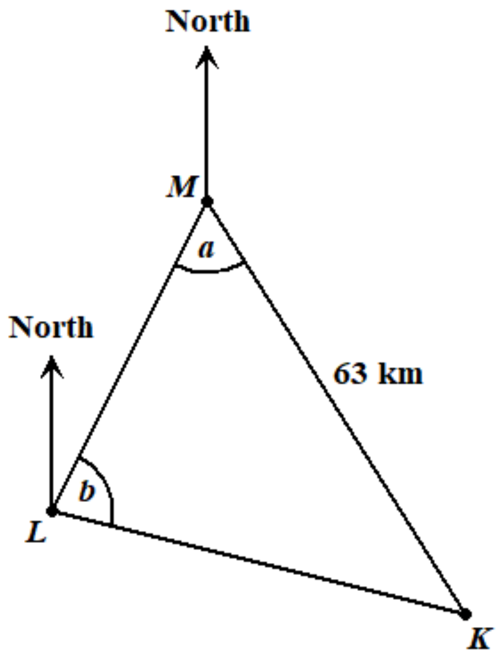
p	q	$p+1$	$q+1$
5	3	6	4
3	5	4	6
7	2	8	3
2	7	3	8
11	1	12	2
1	11	2	12

Try $p = 5$ and $q = 3$, $2^5 \times 3^3 = 864 \neq 1944$

Try $p = 3$ and $q = 5$, $2^3 \times 3^5 = 1944$

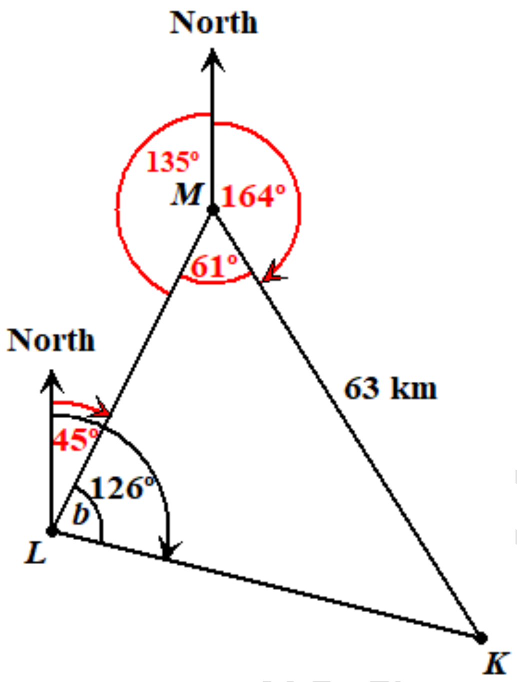
Hence, $1944 = 2^3 \times 3^5$

2. (a) The diagram below shows the positions of 3 small islands , *L*, *M* and *K*, located in a river. The bearing of *M* from *L* is 045°. The bearing of *K* from *L* is 126°. The bearing of *K* from *M* is 164°. The distance *MK* is 63 km.



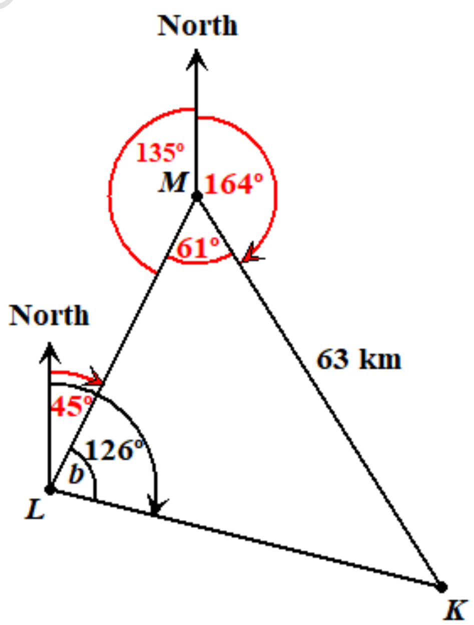
(i) Determine the values of the angles *a* and *b*.

SOLUTION:
Data: Diagram showing the positions of 3 islands, *K*, *L* and *M*, on a river. The bearing of *M* from *L* is 045°. The bearing of *K* from *L* is 126°. The bearing of *K* from *M* is 164°. The distance *MK* is 63 km.
Required To Determine: The values of angles *a* and *b*
Solution:



$N\hat{M}L = 180° - 45°$
$= 135°$
(Co-interior angles are supplementary.)

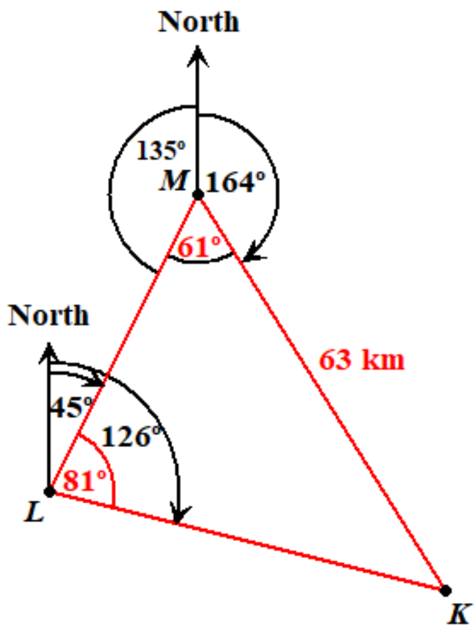
$135° + a + 164° = 360°$
$a = 61°$
(Angles in a complete turn $= 360°$)



$$N\hat{L}K = 126°$$
$$N\hat{L}M = 45°$$
$$\therefore M\hat{L}K = 126° - 45°$$
$$= 81°$$
$$\therefore b = 81°$$

(ii) Calculate the distance LK.

SOLUTION:
Required To Calculate: The distance LK
Calculation:

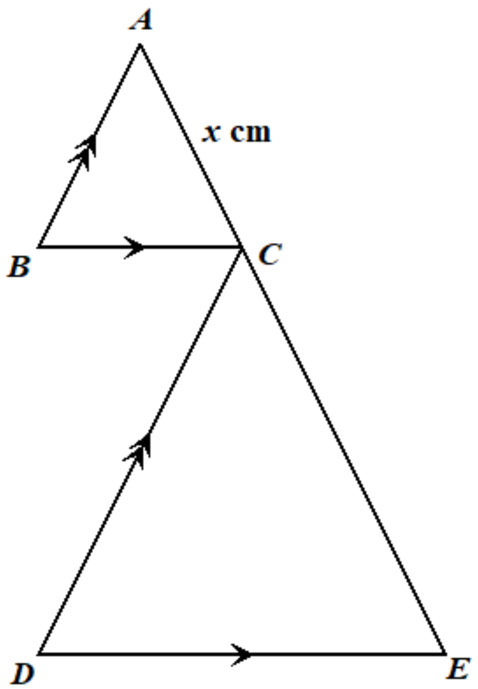


Using the sine rule:

$$\frac{LK}{\sin 61°} = \frac{63}{\sin 81°}$$
$$LK = \frac{63 \times \sin 61°}{\sin 81°}$$
$$= 55.78\underline{\underline{7}}$$
$$\approx 55.79 \text{ km (correct to 2 decimal places)}$$

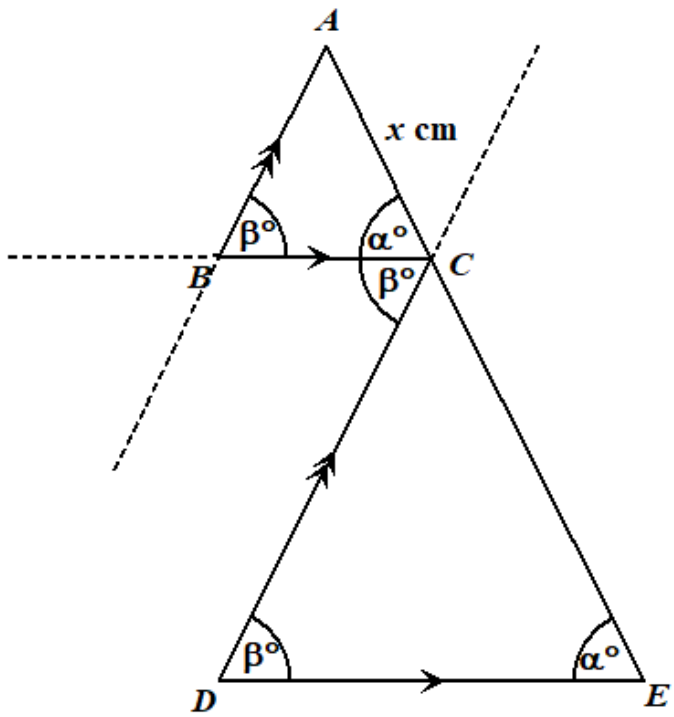
(b) The diagram below shows a scaled drawing for part of a building plan. In the diagram, BC is parallel to DE and BA is parallel to DC. ACE is a straight line.

$AC = x$ cm, $BC = 3.5$ cm, $DE = 6.5$ cm and $AE = 12$ cm.



(i) Calculate the length AC.

SOLUTION:
Data: Diagram showing a scaled drawing for part of a building plan. In the diagram, BC is parallel to DE and BA is parallel to DC. ACE is a straight line. $AC = x$ cm, $BC = 3.5$ cm, $DE = 6.5$ cm and $AE = 12$ cm
Required To Calculate: The length AC
Calculation:

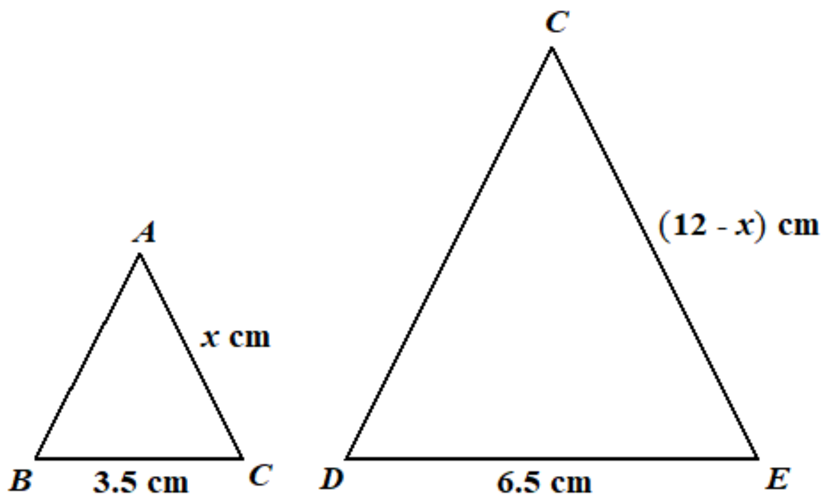


$$A\hat{C}B = C\hat{E}D \quad \text{(Corresponding angles)}$$
$$= \alpha°$$

$$A\hat{B}C = B\hat{C}D \quad \text{(Alternate angles)}$$

$$B\hat{C}D = C\hat{D}E \quad \text{(Alternate angles)}$$

$$B\hat{A}C = D\hat{C}E$$
$$= 180° - (\alpha + \beta)°$$



ΔABC is similar to ΔCDE.

$$AE = 12 \text{ cm}$$
$$\therefore CE = (12 - x) \text{ cm}$$

When figures are similar, the ratio of their corresponding sides are equal.

$$\therefore \frac{BC}{DE} = \frac{AC}{CE} = \frac{AB}{CD}$$

$$\frac{3.5}{6.5} = \frac{x}{12-x}$$

$$3.5(12-x) = 6.5 \times x$$

$$42.0 - 3.5x = 6.5x$$

$$42 = 6.5x + 3.5x$$

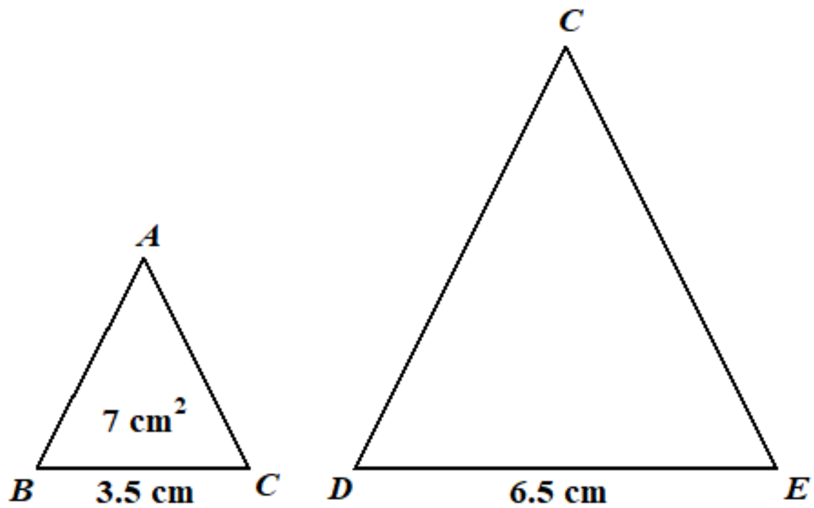
$$10x = 42$$

$$x = 4.2$$

$$\therefore AC = 4.2 \text{ cm}$$

(ii) If the area of triangle *ABC* is 7 cm^2, determine the TOTAL area of the portion of the building, *ABCDE*, shown above.

SOLUTION:
Data: Area of triangle *ABC* is 7 cm^2
Required To Determine: The total area of the *ABCDE*
Solution:



$$\frac{DE}{BC} = \frac{6.5}{3.5}$$

$$\therefore \text{Area of } \Delta CDE = \left(\frac{6.5}{3.5}\right)^2 \times 7 \text{ cm}^2$$
$$= 24.143 \text{ cm}^2$$

$$\therefore \text{Area of } ABCDE = (7 + 24.143) \text{ cm}^2$$
$$= 31.14 \text{ cm}^2 \text{ (correct to 2 decimal places)}$$