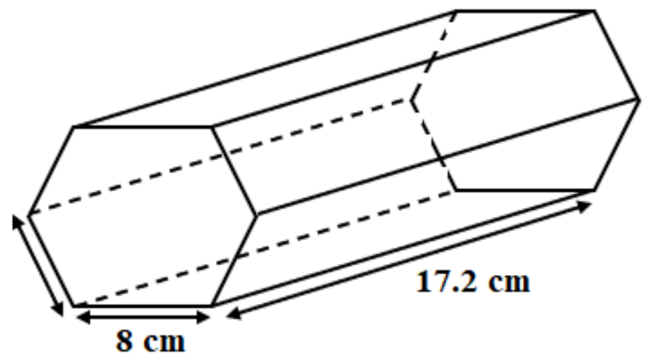
FAS-PASS Maths

CSEC MATHEMATICS PAPER 3 MAY/JUNE 2022

1. (Use $\pi = \frac{22}{7}$ where required.)

The diagram below shows a solid bar of gold in the shape of a hexagonal prism of length 17.2 cm. The vertical cross-section of this prism is a **regular** hexagon with side of length 8 cm.



(a) **(i)** The formula for the area of any regular polygon is given by

$$\text{Area} = \frac{s^2 n}{4\tan\left(\frac{180}{n}\right)}$$

where s is the length of any side
n is the number of sides of the regular polygon, and
tan is the tangent function calculated in **degrees**.

Using the formula above, calculate the area of the vertical cross-section of the hexagonal bar of gold.

SOLUTION:
Data: Diagram showing a solid bar of gold in the shape of a hexagonal prism of length 17.2 cm with a regular hexagonal cross-section with each side of length 8 cm. The area of any regular polygon is given by

$$\text{Area} = \frac{s^2 n}{4\tan\left(\frac{180}{n}\right)}$$

Required To Calculate: The area of the vertical cross-section of the hexagonal bar of gold
Calculation:

$$\text{Area } = \frac{s^2 n}{4\tan\left(\frac{180}{n}\right)}$$

In the hexagon, $s = 8,\ n = 6$

$\therefore$ Area of the vertical hexagonal cross-section

$$= \frac{(8)^2(6)}{4\tan\left(\frac{180^\circ}{6}\right)}$$

$$= \frac{64\times 6}{4\tan 30^\circ}$$

$$= \frac{96}{\frac{1}{\sqrt{3}}}$$

$$= 96\sqrt{3}\ \text{cm}^2\ \text{(in exact form)}$$

$$\approx 166.28\ \text{cm}^2\ \text{(correct to 2 decimal places)}$$

(ii) Determine the volume of the bar of gold.

SOLUTION:
Required To Determine: The volume of the bar of gold
Solution:
Volume of the bar of gold $= \text{Cross-sectional area} \times \text{length}$

$$= 166.28 \times 17.2\ \text{cm}^3$$

$$= 2860\ \text{cm}^3$$

(iii) Given that the density of gold is 19.3 g/cm^3, calculate the mass of the bar of gold, **to the nearest kilogram**.

(Density $= \frac{\text{mass}}{\text{volume}}$)

SOLUTION:

Data: The density of gold is 19.3 g/cm^3 and density $= \frac{\text{mass}}{\text{volume}}$

Required To Calculate: The mass of the bar of gold, correct to the nearest kilogram

Calculation:

$$\text{Mass} = \text{Density} \times \text{Volume}$$

$$= 2860 \times 19.3\ \text{g}$$

$$= \frac{2860 \times 19.3}{1000} \text{ kg} \qquad (\text{Re}: 1000 \text{ g} = 1 \text{ kg})$$
$$= 55.198 \text{ kg}$$
$$\approx 55 \text{ kg (correct to nearest kilogram)}$$

(iv) Calculate the TOTAL surface area of the bar of gold.

SOLUTION:
Required To Calculate: The total surface area of the bar of gold
Calculation:
Area of the bar
$$= \text{Area of hexagonal face} \times 2 + \text{Area of one rectangular face} \times 6$$
$$= 2(166.28) + 6(17.2 \times 8)$$
$$= 332.56 + 825.6$$
$$= 1158.16 \text{ cm}^2$$

(b) A jeweller melted 393 cm^3 of gold to make 6 identical spheres. Calculate the radius of EACH sphere.

(The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

SOLUTION:
Data: 6 identical metal spheres are made with 393 cm^3 of melted gold. The volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere.
Required To Calculate: The radius of each sphere.
Calculation:
Volume of each sphere $= \frac{393}{6} \text{ cm}^3$
$$= 65.5 \text{ cm}^3$$

$$\therefore \frac{4}{3}\pi r^3 = 65.5$$
$$r^3 = \frac{65.5 \times 3}{4\pi}$$
$$= \frac{196.5}{12.568}$$
$$= 15.635$$
$$r = \sqrt[3]{15.635}$$
$$= 2.501$$
Hence, the radius of each sphere ≈ 2.50 cm (correct to 2 decimal places)

2. (a) The distance a bus travels on a journey from City A to City B is 800 km.

(i) Write an expression, in terms of x, for the average speed of the bus, in km/h, when the journey takes

a) x hours

SOLUTION:
Data: A bus travels 800 km from City A to City B
Required To Write: An expression, in terms of x, for the average speed of the bus when the journey takes x hours.
Solution:
Distance $= 800$ km
Time $= x$ h

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$
$$= \frac{800}{x} \text{ km/h}$$

b) $(x+2)$ hours

SOLUTION:
Required To Write: An expression, in terms of x, for the average speed of the bus when the journey takes $(x+2)$ hours.
Solution:

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$
$$= \frac{800}{x+2} \text{ km/h}$$

(ii) The difference between the average speeds in (a) (i) is 20 km/h.

Show that $x^2 + 2x - 80 = 0$.

SOLUTION:
Data: The difference between the average speeds in (a) (i) is 20 km/h.
Required To Show: $x^2 + 2x - 80 = 0$
Proof:
Difference between the speeds $= 20$ km/h

$$\frac{800}{x} - \frac{800}{x+2} = 20$$

$$\frac{800(x+2) - 800(x)}{x(x+2)} = 20$$

$$\frac{800x + 1600 - 800x}{x(x+2)} = 20$$

$$\frac{1600}{x(x+2)} = 20$$

$$1600 = 20x(x+2)$$

$$20x^2 + 40x - 1600 = 0$$

$(\div 20)$

$$x^2 + 2x - 80 = 0$$

Q.E.D.

(iii) Solve the quadratic equation $x^2 + 2x - 80 = 0$ and hence, determine the average speed of the bus if the journey takes $(x+12)$ hours.

SOLUTION:
Data: The journey takes $(x+12)$ hours.
Required To Determine: The average speed of the bus by solving $x^2 + 2x - 80 = 0$
Solution:

$$x^2 + 2x - 80 = 0$$

$$(x+10)(x-8) = 0$$

$$x = -10 \text{ or } 8$$

$$x \neq -ve$$

$$\therefore x = 8 \text{ only}$$

When $x = 8$:

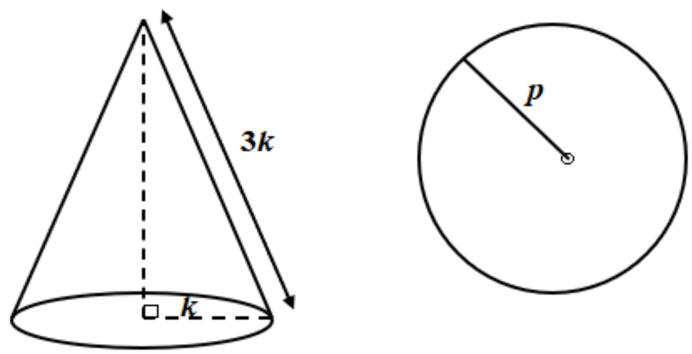
Average speed of the bus $= \dfrac{800 \text{ km}}{(8+12) \text{ h}}$

$= 40$ km/h

(b) The total surface area of a cone with radius k and slant height $3k$ is equal to the area of a circle with radius p.

Show that $p = 2k$.

[The total surface area, A, of a cone with radius r and slant height l is $A = \pi r^2 + \pi rl$.]



SOLUTION:
Data: The total surface area of a cone with radius k and slant height $3k$ is equal to the area of a circle with radius p. The total surface area, A, of a cone with radius r and slant height l is $A = \pi r^2 + \pi rl$.
Required To Show: $p = 2k$
Proof:

$$\begin{aligned}\text{Area of cone} &= \text{Area of curved surface} + \text{Area of base}\\ &= \pi(k)^2 + \pi(k)(3k)\\ &= \pi k^2 + 3\pi k^2\\ &= 4\pi k^2\end{aligned}$$

Area of the circle $= \pi p^2$

Hence, $4\pi k^2 = \pi p^2$

$(\div\pi)$ $4k^2 = p^2$

$$4k^2 = p^2$$

$$p = \sqrt{4k^2}$$

$$= \pm 2k$$

Taking the positive value since p is the radius of the circle.
$p = 2k$

Q.E.D.