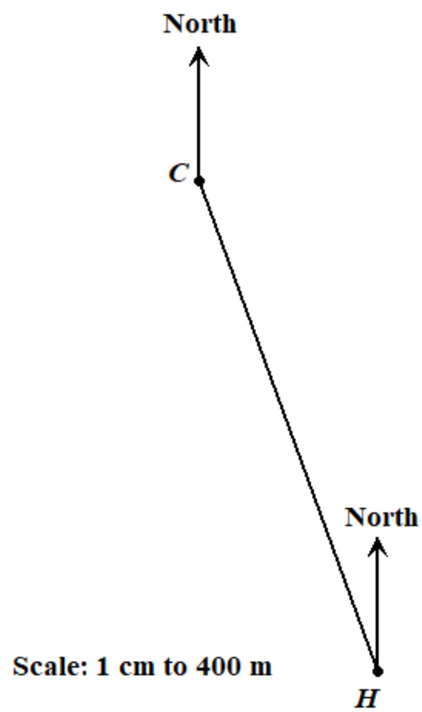
CSEC MATHEMATICS JANUARY 2022 PAPER 3

1. The Crusaders and the Sea Hawks are two teams in a basketball tournament.

(a) The scale drawing below shows the relative position of the Crusaders Club, C, and the Sea Hawks Club, H, on a map. The scale on the drawing is 1 centimetre to 400 metres.



(i) Determine, CH, the **actual distance, in kilometres**, between the two clubs.

SOLUTION:

Data: Scale drawing showing the relative position of the Crusaders Club, C, and the Sea Hawks Club, H, on a map. The scale is 1 centimetre to 400 metres.

Required to determine: The actual distance, in kilometres between the two clubs.

Solution:
Measuring from the outer ends of the 'dots':
$CH = 8$ cm

$$1\text{ cm} \equiv 400\text{ m}$$
$$\therefore 8\text{ cm} = 8 \times 400\text{ m}$$
$$= 3200\text{ m}$$
$$1000\text{ m} = 1\text{ km}$$
$$1\text{ m} = \frac{1}{1000}\text{ km}$$
$$\therefore 3200\text{ m} = \frac{1}{1000} \times 3200\text{ km}$$
$$= 3.2\text{ km}$$

So, the actual distance of *CH* is 3.2 km

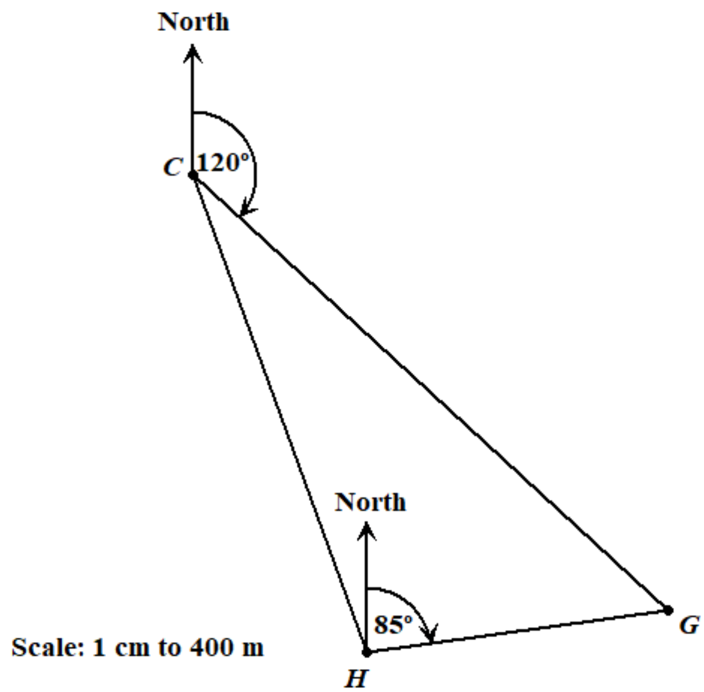
(ii) The Greenwich Park Sports Arena, *G*, the venue for the tournament, is on a bearing of 120° from *C* and 085° from *H*.

On the scale drawing above, indicate the position of *G*, the venue for the basketball tournament. Show ALL reference lines and angles.

SOLUTION:
Data: The Greenwich Park Sports Arena, *G*, the venue for the tournament, is on a bearing of 120° from *C* and 085° from *H*.

Required to indicate: The position of G on the scale drawing, showing all reference lines and angles.

Solution:
G is on a bearing of 120° from *C* and 085° from *H*. The line drawn from *C* at an angle of 120^0 with the North line and the line drawn from *H* at an angle of 85^0 with the North line will meet at *G*. This is shown in the diagram.



(b) Teams are given points for winning or drawing games. A win is given x points and a draw is given y points. No points are given when a team loses.

(i) The Crusaders have 24 points after winning 2 games and drawing 5 games. The Sea Hawks have 29 points after winning 3 games and drawing 4 games.

Write TWO equations, in x and y, to represent the information given above.

SOLUTION:
Data: A win is given x points and a draw is given y points. No points are given when a team loses. The Crusaders have 24 points after winning 2 games and drawing 5 games. The Sea Hawks have 29 points after winning 3 games and drawing 4 games.

Required to write: Two equations, in x and y, to represent the information given.

Solution:
For the Crusaders:

$$2(x)+5(y)=24$$

$$2x+5y=24 \quad \ldots❶$$

For the Sea Hawks:

$$3(x)+4(y)=29$$

$$3x+4y=29 \quad \ldots❷$$

(ii) Using a matrix method, solve your two equations to find the number of points for a win and the number of points for a draw. Show ALL working.

SOLUTION:

Required to solve: The two equations for *x* and *y* using a matrix method

Solution:

$$2x+5y=24$$
$$3x+4y=29$$

$$\begin{pmatrix}2 & 5\\ 3 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}24\\ 29\end{pmatrix} \quad \text{…matrix equation}$$

Let $A=\begin{pmatrix}2 & 5\\ 3 & 4\end{pmatrix}$

$$\begin{aligned}|A| &= (2\times 4)-(5\times 3)\\ &= 8-15\\ &= -7\end{aligned}$$

$$\begin{aligned}A^{-1} &= \frac{1}{-7}\begin{pmatrix}4 & -(5)\\ -(3) & 2\end{pmatrix}\\ &= \begin{pmatrix}-\frac{4}{7} & \frac{5}{7}\\ \frac{3}{7} & -\frac{2}{7}\end{pmatrix}\end{aligned}$$

Pre-multiply the matrix equation by A^{-1}:

$$\underbrace{\underbrace{\begin{pmatrix}-\frac{4}{7} & \frac{5}{7}\\ \frac{3}{7} & -\frac{2}{7}\end{pmatrix}\begin{pmatrix}2 & 5\\ 3 & 4\end{pmatrix}}_{I}\begin{pmatrix}x\\ y\end{pmatrix}}_{\begin{pmatrix}x\\ y\end{pmatrix}}=\underbrace{\begin{pmatrix}-\frac{4}{7} & \frac{5}{7}\\ \frac{3}{7} & -\frac{2}{7}\end{pmatrix}\begin{pmatrix}24\\ 29\end{pmatrix}}_{\substack{2\times 2\,\times\, 2\times 1=2\times 1\\ \begin{pmatrix}e_{11}\\ e_{21}\end{pmatrix}}}$$

$$e_{11} = \left(-\frac{4}{7}\times 24\right) + \left(\frac{5}{7}\times 29\right)$$
$$= -\frac{96}{7} + \frac{145}{7}$$
$$= \frac{49}{7}$$
$$= 7$$

$$e_{21} = \left(\frac{3}{7}\times 24\right) + \left(-\frac{2}{7}\times 29\right)$$
$$= \frac{72-58}{7}$$
$$= \frac{14}{7}$$
$$= 2$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$$

Equating corresponding entries we obtain
$x = 7$ and $y = 2$

$\therefore$ A win scores 7 points and a draw scores 2 points.

2. Karla invests in a poultry farm business. She buys x number of chicks and y number of ducklings to start up her business. She must consider the following information in determining the number of each of these birds that she must buy to ensure she gains maximum profit.

- The number of chicks must be **at least twice** the number of ducklings.
- She must buy **at least** 80 ducklings.
- **Altogether**, Karla must buy **more than** 500 birds.

(a) Using the information above, complete the table below by inserting the inequality that represents each of the restrictions.

Restriction/Consideration	**Inequality**
The number of chicks must be **at least twice** the number of ducklings.	
She must buy **at least** 80 ducklings.	$y \geq 80$
Altogether, Karla must buy **more than** 500 birds.	

SOLUTION:
Data: Karla invests in a poultry farm by buying x chicks and y ducklings, such that, the number of chicks must be at least twice the number of ducklings, she buys at least 80 ducklings and she must buy more than 500 birds altogether.

Required to complete: The table given with the inequalities that represent the restrictions.

Solution:
The number of chicks must be **at least twice** the number of ducklings.

$$x \qquad \geq \qquad 2 \times y$$

$$x \geq 2y$$
$$2y \leq x$$
$$y \leq \frac{1}{2}x$$

Altogether, Karla must buy **more than** 500 birds.
Number of chicks and number of ducklings > 500
$x + y > 500$
Note the equality is NOT included since it is 'more than'

The completed table looks like:

Restriction/Consideration	Inequality
The number of chicks must be **at least twice** the number of ducklings.	$y \leq \frac{1}{2}x$
She must buy **at least** 80 ducklings.	$y \geq 80$
Altogether, Karla must buy **more than** 500 birds.	$x + y > 500$

(b) Each chick costs \$2 and each duckling costs \$4. The maximum amount of money Karla can spend is \$1400.

Show that $x + 2y \leq 700$.

SOLUTION:
Data: Each chick costs \$2 and each duckling costs \$4. The maximum amount of money Karla can spend is \$1400.

Required to show: $x + 2y \leq 700$

Solution:
Cost of x chicks at \$2 each and cost of y ducklings at \$4 costs not more than \$1400.

$\therefore (2 \times x) + (4 \times y) \not> 1400$

$2x + 4y \not> 1400$

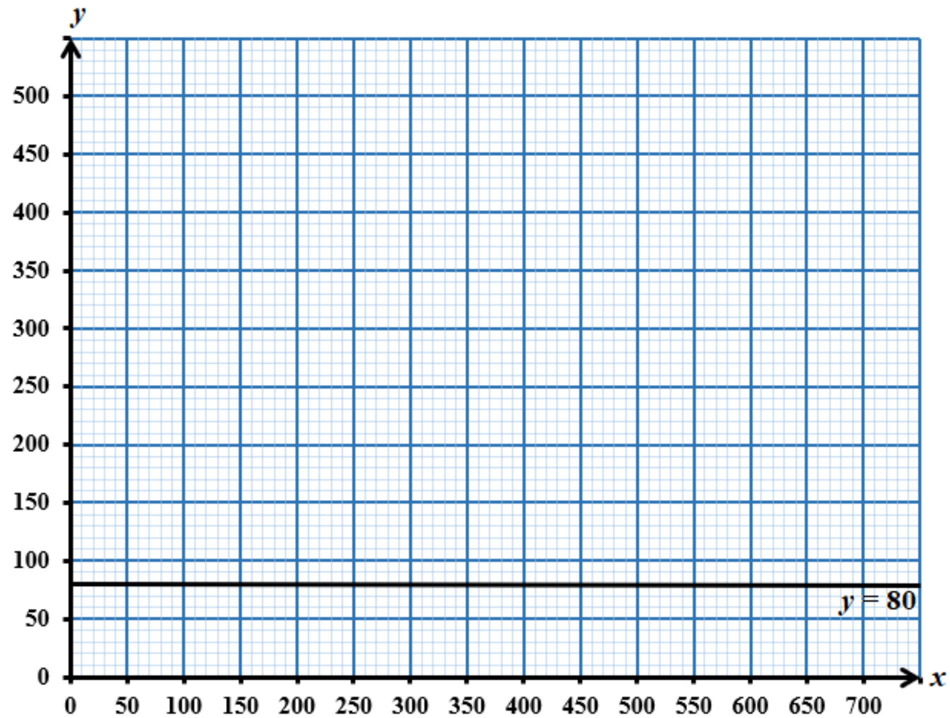
$2x + 4y \leq 1400$

$(\div 2)$

Hence, $x + 2y \leq 700$

Q.E.D

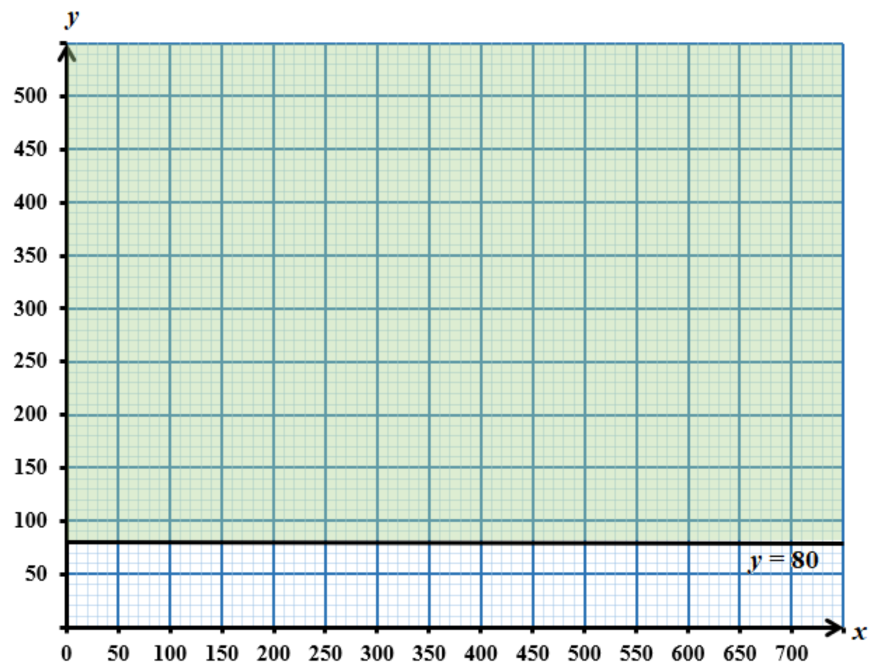
(c) The line $y = 80$ is drawn on the grid below. Draw the THREE lines associated with the other inequalities given in (a) and (b). Label, as R, the REQUIRED region which satisfies ALL four inequalities.



SOLUTION:
Data: Grid with the line $y = 80$ drawn on it.

Required to draw: The three lines associated with the other inequalities and label the critical region of all four inequalities *R*.

Solution:
The region $y \geq 80$ includes the line and is shown coloured in green below.



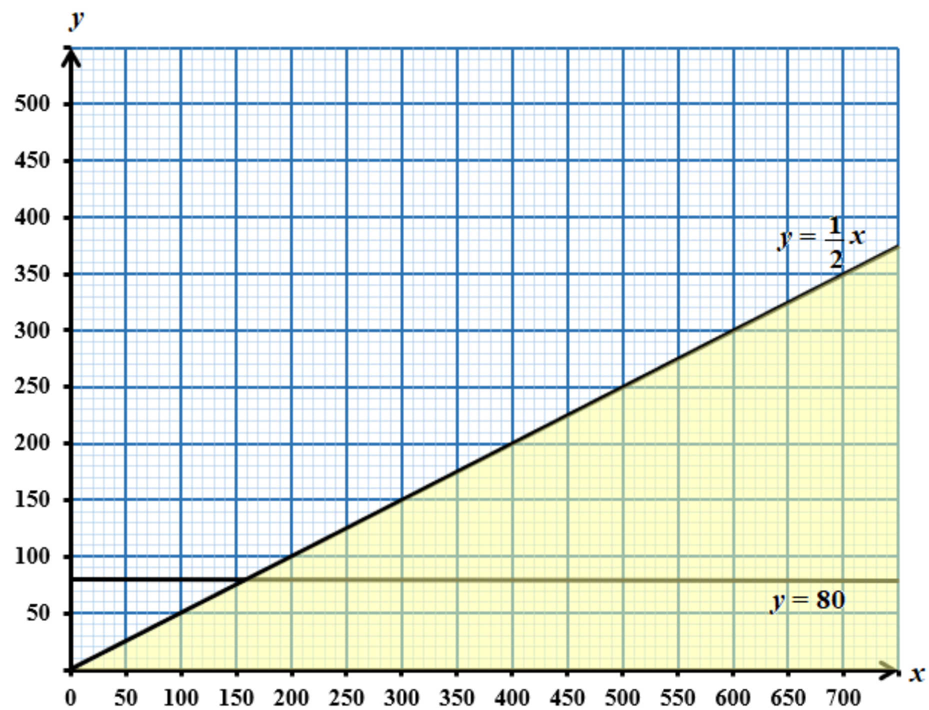
$y \leq \frac{1}{2}x$

To draw the line $y = \frac{1}{2}x$, use the following points

x	y
0	0
700	350

Plot the points $(0,\ 0)$ and $(700,\ 350)$.

The region $x \geq 2y$ or $y \leq \frac{1}{2}x$ includes the line and is shown coloured in green.



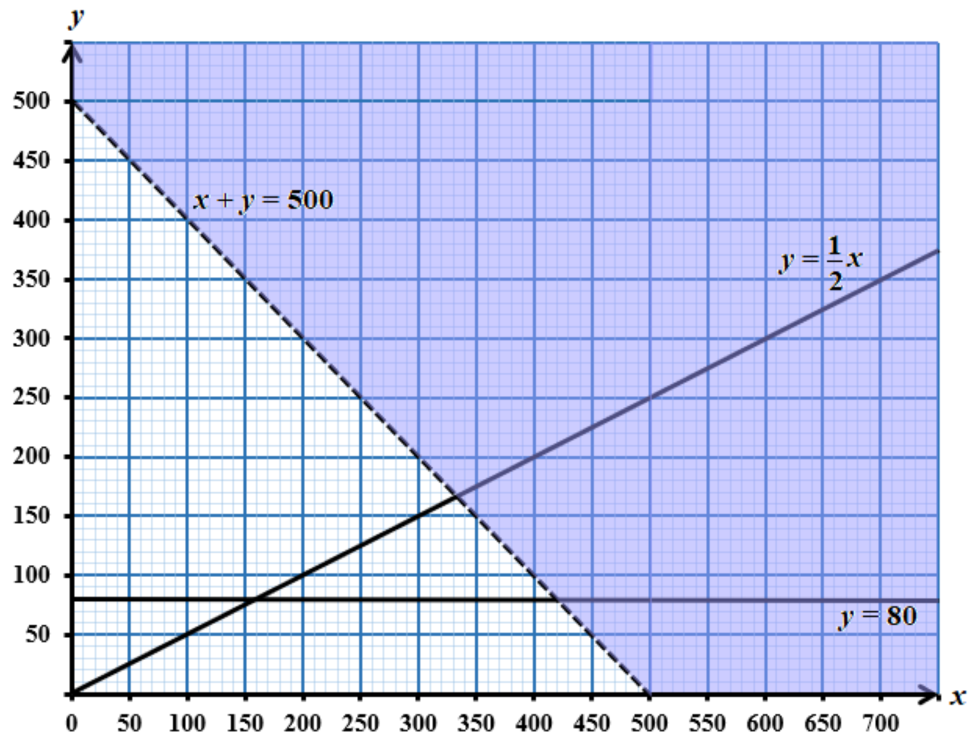
$x + y \geq 500$

To draw the line $x + y = 500$ use the following points

x	y
0	500
500	350

Plot the points $(0, 500)$ and $(500, 0)$.

The region $x + y > 500$ does not include the line and is shown coloured purple. The line is shown dotted indicating it is not to be included in the inequality.



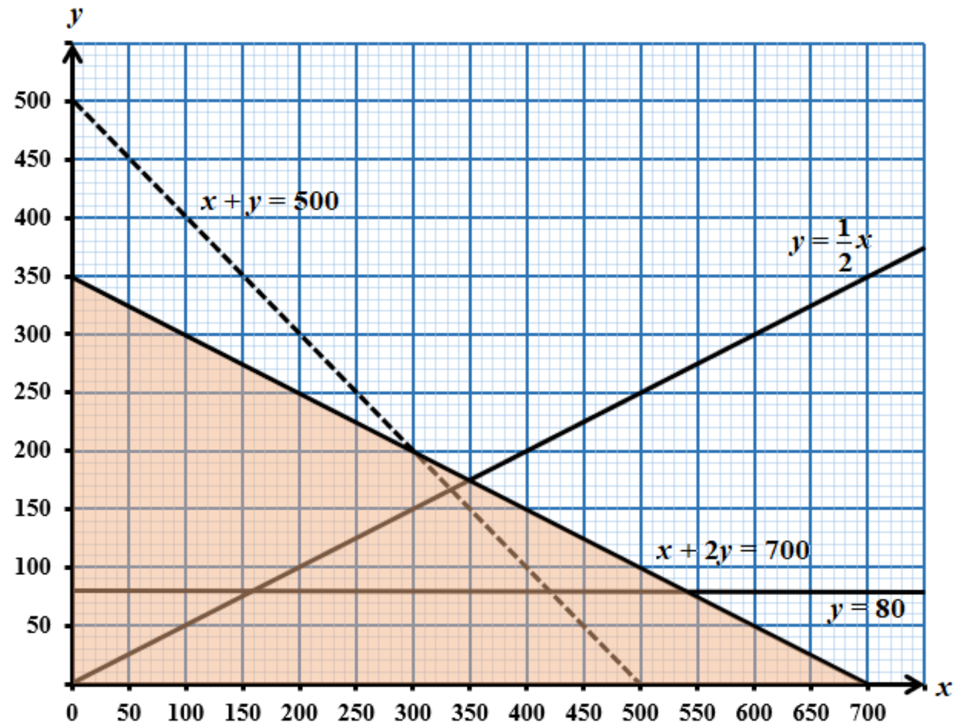
$x + 2y \le 700$

To draw the line, $x + 2y = 700$, use the following points

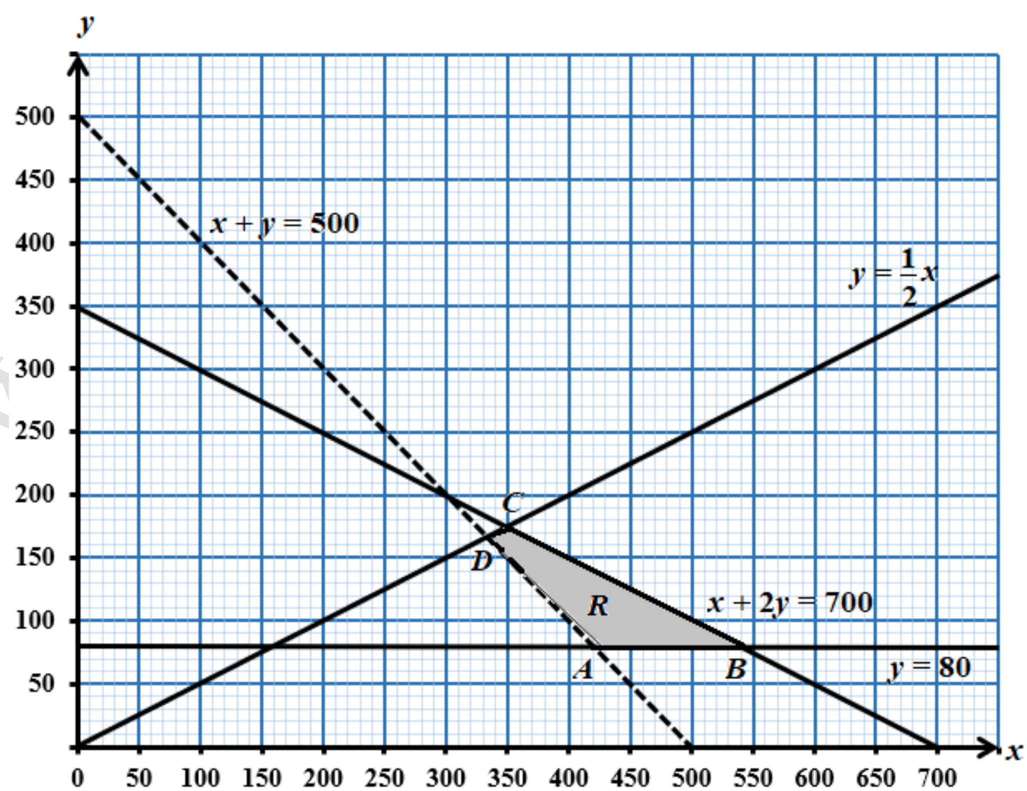
x	y
700	0
0	350

Plot the points $(700,\ 0)$ and $(0,\ 350)$.

The region $x + 2y \le 700$ includes the line and is shown coloured in orange.



ABCD is the feasible region, *R*, that satisfies $2y \le x$, $x + y \ge 500$, $x + 2y \le 700$ and $y \ge 80$.



(d) The average weight of a chicken at the time of harvest is 2.75 kg and for a duck 2.28 kg. The current market trend indicates that the selling price per kilogram of chicken is \$11.00 and for a kilogram of duck \$12.50.

(i) Assuming that the information given above holds true, write an equation in x and y that represents Karla's TOTAL sales when she sells x chickens and y ducks.

SOLUTION:

Data: The average weight of a chicken is 2.75 kg and the average weight of a duck is 2.28 kg. Chicken is sold at \$11.00 per kilogram and duck is sold at \$12.50 per kilogram.

Required to write: An equation, in terms of x and y, that represents Karla's TOTAL sales.

Solution:

Cost of a 2.75 kg chicken at \$11.00 per kg $= 2.75 \times \$11.00$

Cost of x such chickens $= \$(2.75 \times 11.00 \times x)$

$= \$30.25\ x$

Cost of a 2.28 kg duck at \$12.50 per kg $= 2.28 \times \$12.50$

Cost of y such ducks $= \$(2.28 \times 12.50 \times y)$

$= \$28.50\ y$

Total sales of chickens and ducks $= \$(30.25x + 28.50y)$

(ii) Given that none of the birds will die prior to harvesting, advise Karla on the number of chicks and ducklings that she must buy to yield MAXIMUM sales.

SOLUTION:

Required to find: The number of chicks and ducklings that Karla must buy in order to yield maximum sales, provided that no chickens or ducks die before harvesting.

Solution:

The maximum will always occur at one of the vertices of the common region (by the theorem of Linear Programming)

Consider the vertices of R, that is, A, B, C and D.

A (420, 80) B (540,80) C (350, 175) D (333, 167)

We substitute each pair of values in the expression $30.25x + 28.50y$ to determine which pair yields the maximum.

Total Sales: $= \$(30.25x + 28.50y)$

A = (420, 80)

When $x = 420$ and $y = 80$

Total Sales $= \$(30.25 \times 420) + \$(28.5 \times 80)$

$= \$14\,985$

$B = (540, 80)$

When $x = 540$ and $y = 80$:

Sales $= \$(30.25 \times 540) + \$(28.50 \times 80)$

$= \$18615$

NOTE: The point *B* has the same *y*-coordinate as *A* but a higher *x*-coordinate. Hence, the point *A* need not have been tested.

$C = (350, 175)$

When $x = 350$ and $y = 175$:

Sales $= \$(30.25 \times 350) + \$(28.50 \times 175)$

$= \$15575$

D = (333, 167)

When $x = 333$ and $y = 167$

Total Sales $= \$(30.25 \times 333) + \$(28.5 \times 167)$

$= \$14\,833$

NOTE: The point *C* has both a higher *x*-coordinate and a higher *y*-coordinate than the point *D*. Hence, the point *D* need not have been tested.

By observation, we need only test the points *B* and *D*.

The maximum sales will be \$18 615 when $x = 540$ and $y = 80$

Hence, Karla would be advised to buy 540 chicks and 80 ducklings.

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.