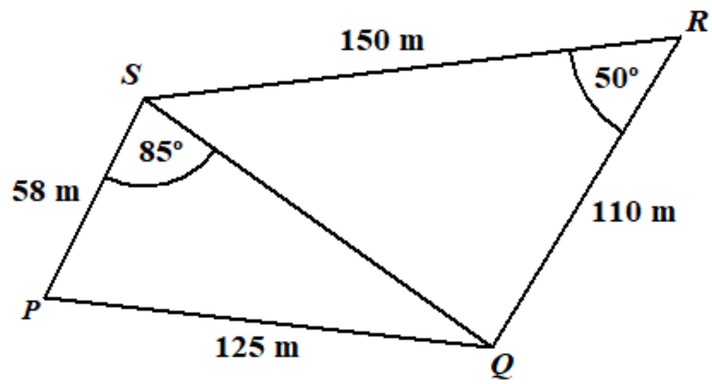
CSEC MATHEMATICS MAY/JUNE 2021 PAPER 3

1. The quadrilateral *PQRS* shown below represents a piece of land. A straight plywood wall is erected from *Q* to *S*, which divides the land into 2 triangular portions. The portion *QRS* is a flea market. *PQS* is a storage facility and a parking lot to serve the flea market.

The side $PS = 58$ m, $PQ = 125$ m, $SR = 150$ m and $QR = 110$ m.
Angle $QRS = 50°$ and angle $PSQ = 85°$.

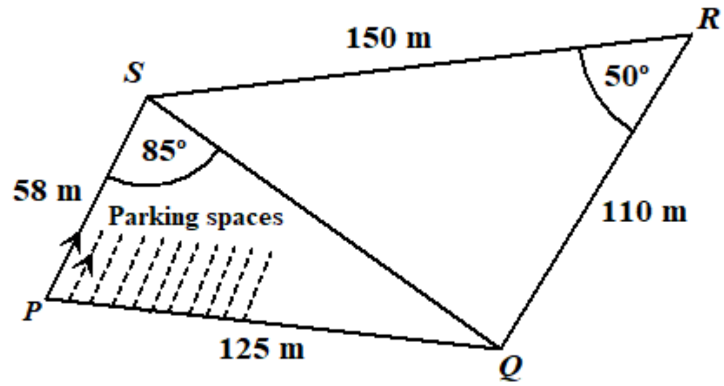


(a) Show that the length of the wall, *QS*, is 116 m, correct to the nearest metre.

SOLUTION:
Data: Diagram showing a piece of land in the shape of a quadrilateral *PQRS*, which is divided into two triangular sections by a ply wall erected from *Q* to *S*.
The side $PS = 58$ m, $PQ = 125$ m, $SR = 150$ m and $QR = 110$ m.
Angle $QRS = 50°$ and angle $PSQ = 85°$.
Required to show: The length of *QS* is 116 m, correct to the nearest metre.
Proof:
Consider triangle *QRS*:

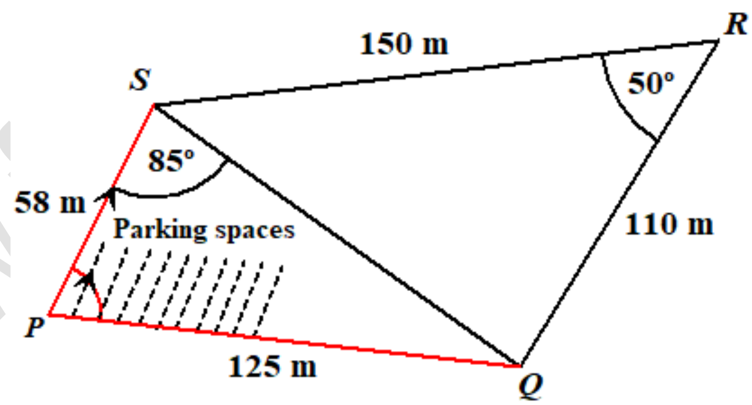
$$QS^2 = (110)^2 + (150)^2 - 2(110)(150)\cos 50° \qquad (\text{Cosine law})$$
$$= 12100 + 22500 - 21211.99$$
$$= 13388.01$$
$$QS = \sqrt{13388.01}$$
$$= 115.\underline{\underline{7}} \text{ m}$$
$$\approx 116 \text{ m (correct to the nearest metre)}$$

(b) A parking lot with a number of parking spaces is to be installed along the side *PQ*. All the parking spaces run EXACTLY parallel to *PS*.

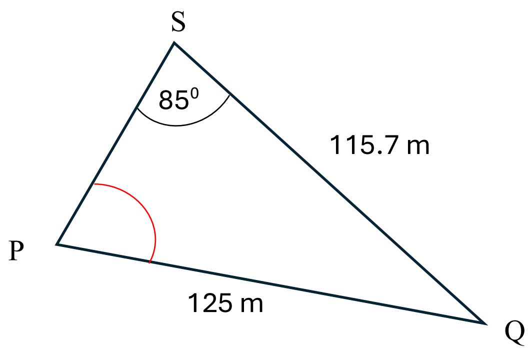


At what angle would the side of each parking space be inclined to *PQ*?

SOLUTION:
Data: A parking lot with a number of parking spaces is to be installed along the side *PQ*. All the parking spaces run EXACTLY parallel to *PS*.
Required to find: The angle at which each parking space would be inclined to *PQ*
Solution:
The angle required is $S\hat{P}Q$.



Consider ΔPQS:



$$\frac{125}{\sin 85^\circ} = \frac{115.7}{\sin \hat{P}} \quad (\text{Sin}\,e\ rule)$$

$$\sin \hat{P} = \frac{115.7 \times \sin 85^\circ}{125}$$

$$= 0.9221$$

$$\hat{P} = \sin^{-1}(0.9221)$$

$$\approx 67.2^\circ \text{ (correct to the nearest } 0.1^\circ\text{)}$$

(b) Stalls are to be built on the land for the flea market shown in triangle *QRS*. Each stall is to be built on at least 20.4 m^2 of land and 2000 m^2 of land MUST be used to build the aisles.

Calculate the MAXIMUM number of stalls which can be built. Show ALL working.

SOLUTION:
Data: Stalls are to be built on the land for the flea market in triangle *QRS*. Each stall is to be built on at least 20.4 m^2 of land and 2000 m^2 of land MUST be used to build the aisles.
Required to calculate: The maximum number of stalls which can be built.
Calculation:
Consider ΔQRS:

$$\text{Area } = \frac{1}{2}(150)(110)\sin 50^\circ$$

$$= 6319.87 \text{ m}^2$$

Area to be used for the aisles $= 2000 \text{ m}^2$

$$\therefore \text{Area used for stalls } = 6319.87 - 2000 \text{ m}^2$$

$$= 4319.87 \text{ m}^2$$

$$\therefore \text{Number of stalls} = \frac{\text{Area available for stalls}}{\text{Area of 1 stall}}$$
$$= \frac{4319.87}{20.4}$$
$$= 211.75$$

However, the number of stalls $\in Z^+$ and we cannot have a fraction of a stall. Hence, the maximum number of stalls is 211.

(d) The company wishes to enclose the land, *PQRS*, with chain-link fencing. The table below shows the advertised cost price for chain-link fencing.

Quantity (m)	**Price per Metre of Fencing ($)**
200 or less	25
Between 200 and 500	21.50
500 or more	18.50

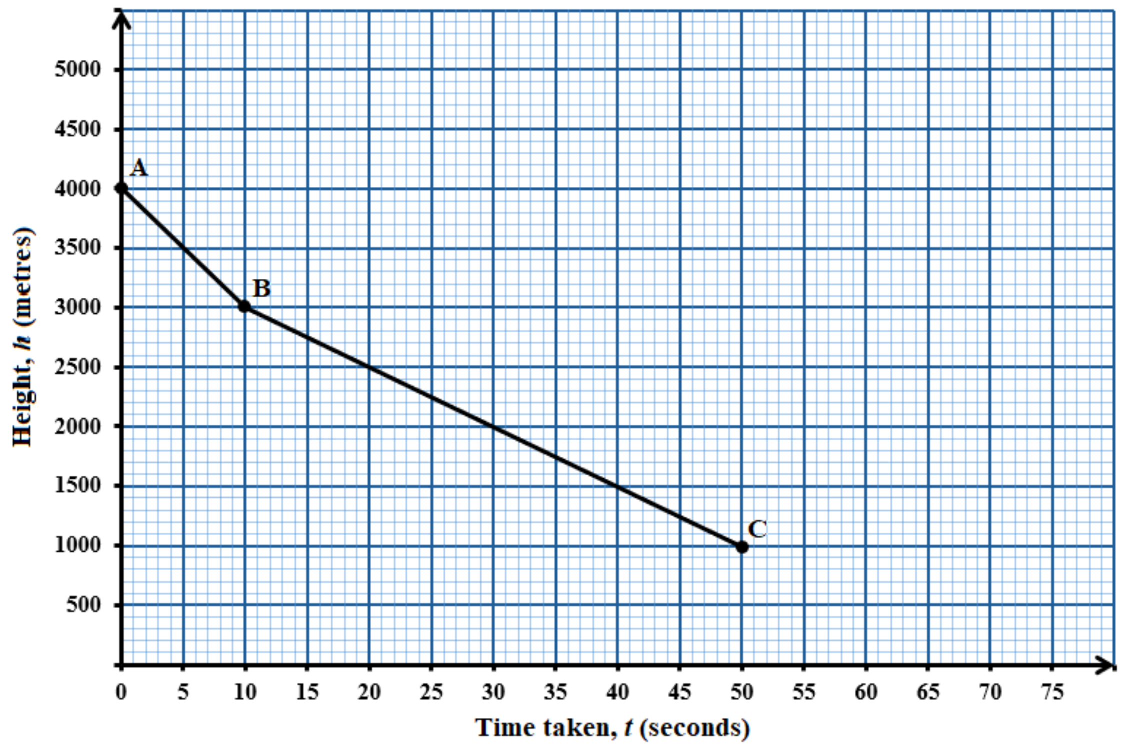
Calculate the COST for the chain-link fencing required to enclose the land, *PQRS*. Show ALL working.

SOLUTION:
Data: Table showing the advertised cost of chain-link fencing
Required to calculate: The cost of chain-link fencing required to enclose *PQRS*
Calculation:
Perimeter of *PQRS* $= (150 + 110 + 125 + 58)$ m
$= 443$ m

According to the table, the cost should be \$21.50 per metre.

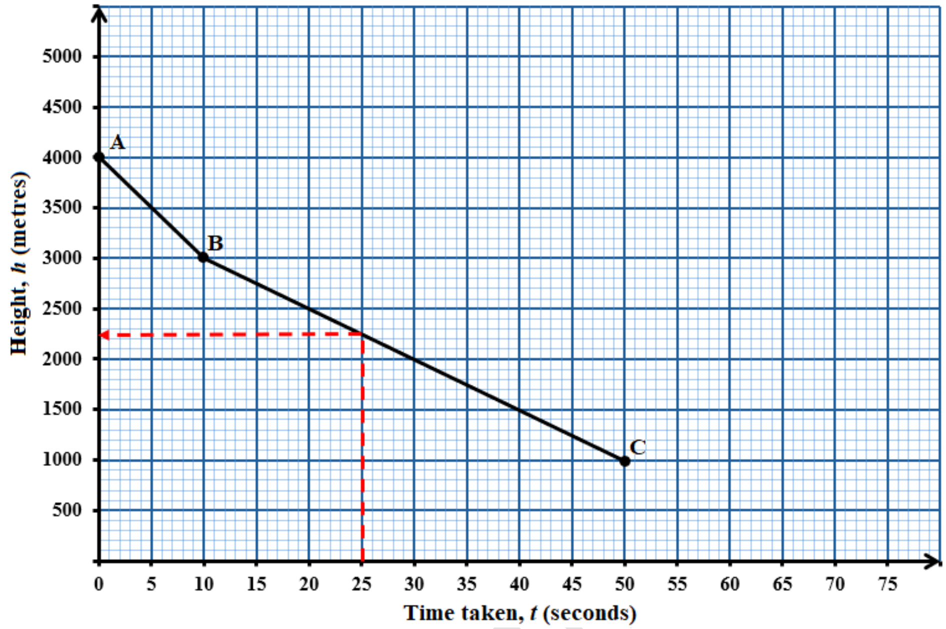
$$\therefore \text{Cost of chain-link fencing} = \$21.50 \times 443$$
$$= \$9524.50$$

2. (a) A skydiver jumps from an aeroplane at Point *A* and after 10 seconds opens his parachute and falls at a constant rate. The graph below shows the height, *h*, that the skydiver is above the ground after *t* seconds.



(i) How far above the ground is the skydiver after 25 seconds?

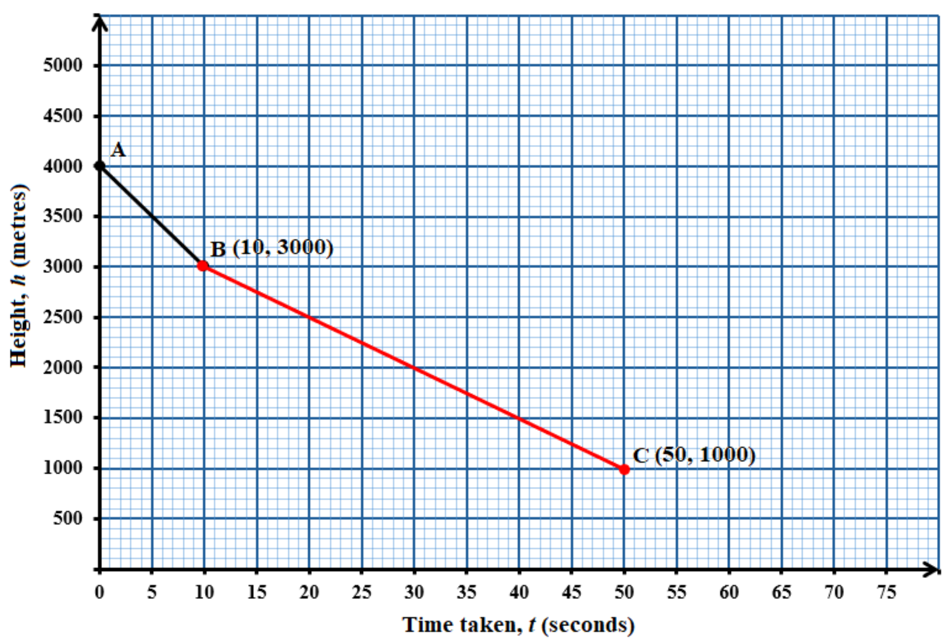
SOLUTION:
Data: Graph showing the height, *h*, that a skydiver is above ground after *t* seconds. The skydiver jumps from an aeroplane at Point *A* and opens his parachute after 10 seconds.
Required to find: The height above the ground that the skydiver is after 25 seconds.
Solution:



After 25 seconds, the skydiver is 2250 m above the ground (obtained by a read-off shown in red).

(ii) Calculate the slope of the line *BC* and explain what this value means in terms of *h* and *t*.

SOLUTION:
Required to calculate: The slope of *BC* and explain its meaning in terms of *h* and *t*
Calculation:



Gradient or slope of BC $= \text{velocity}$

$$= \frac{3000 - 1000}{10 - 50}$$

$$= \frac{2000}{-40}$$

$$= -50 \text{ ms}^{-1}$$

Explanation:
A gradient of $BC = -50 \text{ ms}^{-1}$ indicates that the rate of descent is 50 ms^{-1}. The negative sign indicates the direction of motion. Upwards would have been positive and so, downwards is negative. This is because his height from the ground decreases as time increases.

(iii) How long (in seconds) will the skydiver take to reach to the ground, falling at the same rate as from B to C?

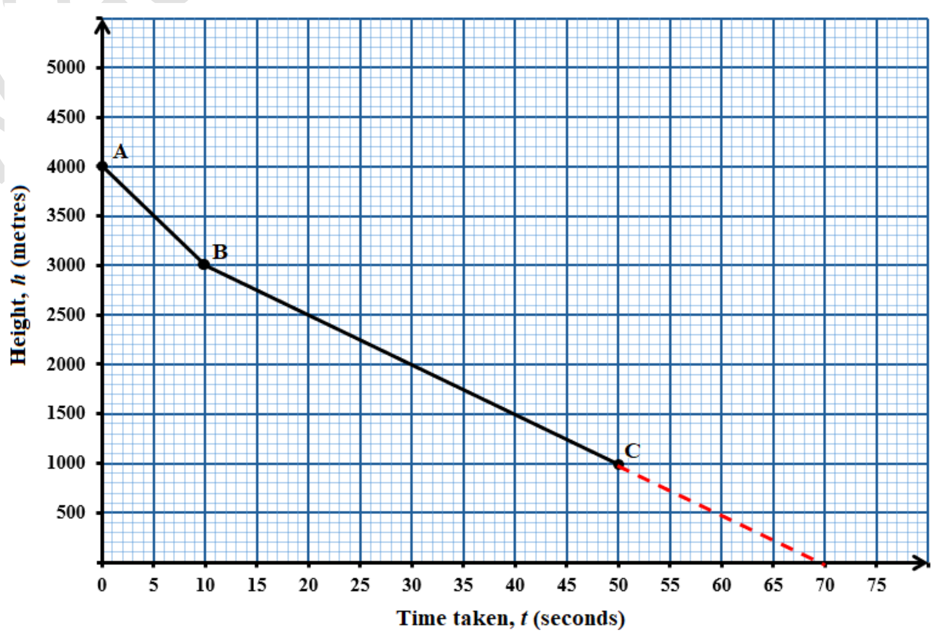
SOLUTION:
Required to find: The length of time it would take the skydiver to reach to the ground, falling as the same rate as from B to C
Solution:
From C to the ground, the skydiver will take

$$= \frac{\text{Distance above the ground}}{\text{Speed of descent}}$$

$$= \frac{1000}{50}$$

$$= 20 \text{ s}$$

Total time taken to reach the ground is $(50 + 20) = 70$ seconds



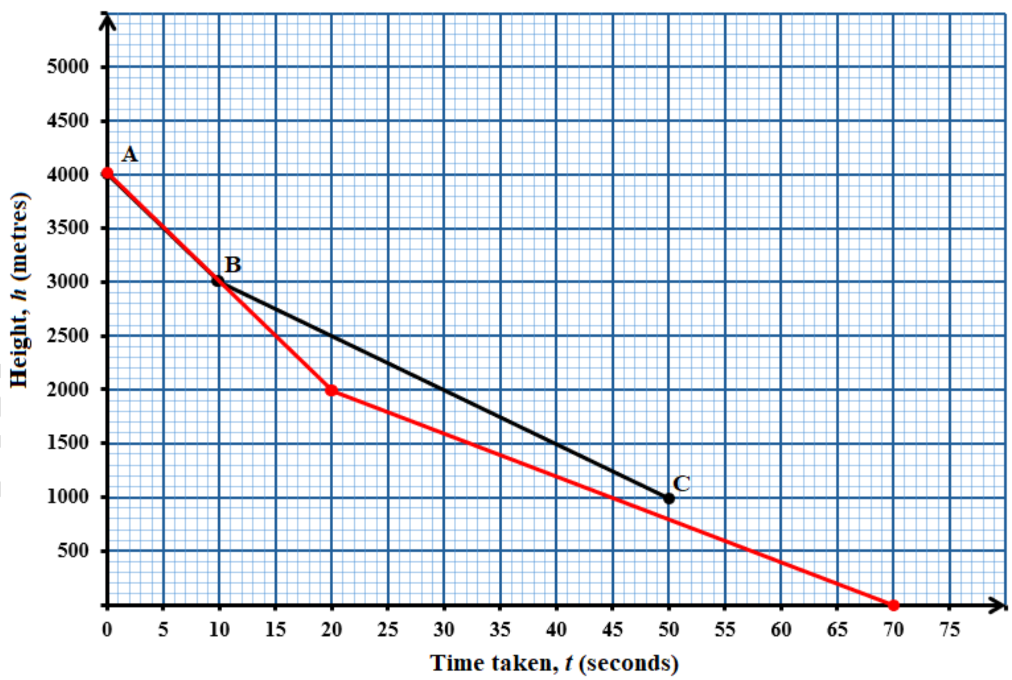
This could also have been found by extending the line BC to meet the horizontal axis at 70 s.
Time taken from B to C = $(70 - 50) = 20\ seconds$

Total time taken to reach the ground is 70 seconds

(iv) Another skydiver jumps from the aeroplane at A, falls at the same rate from A as the first skydiver, but opens his parachute after 20 seconds. He reached the ground at the same time found in (a) (iii), falling at a new constant rate after opening his parachute.

On the same pair of axes, draw the graph to represent this information.

SOLUTION:
Data: Another skydiver jumps from the aeroplane at A, falling at the same rate from A as the first skydiver and opens his parachute after 20 seconds. He reached the ground at the same time found in (a) (iii), falling at a new constant rate after opening his parachute.
Required to draw: A graph to represent this information using the same pair of axes
Solution:
The descent of the second skydive is shown in red on the same axes.



(b) Given that $\begin{pmatrix} x+y & 5 \\ 0 & x-y \end{pmatrix} = \begin{pmatrix} 9 & 5 \\ 0 & 7 \end{pmatrix}$, determine the values of x and y.

SOLUTION:

Data: $\begin{pmatrix} x+y & 5 \\ 0 & x-y \end{pmatrix} = \begin{pmatrix} 9 & 5 \\ 0 & 7 \end{pmatrix}$

Required to determine: x and y

Solution:

$$\underset{2\times 2}{\begin{pmatrix} x+y_{11} & 5_{12} \\ 0_{21} & x-y_{22} \end{pmatrix}} = \underset{2\times 2}{\begin{pmatrix} 9_{11} & 5_{12} \\ 0_{21} & 7_{22} \end{pmatrix}}$$

Since the matrices are equal, we can equate corresponding entries:

$x+y=9 \quad \ldots❶$

$x-y=7 \quad \ldots❷$

Equation ❶ + Equation ❷:

$2x=16$

$x=\frac{16}{2}$

$x=8$

Substitute $x=8$ in equation ❶:

$x+y=9$

$8+y=9$

$y=9-8$

$y=1$

Hence, $x=8$ and $y=1$.