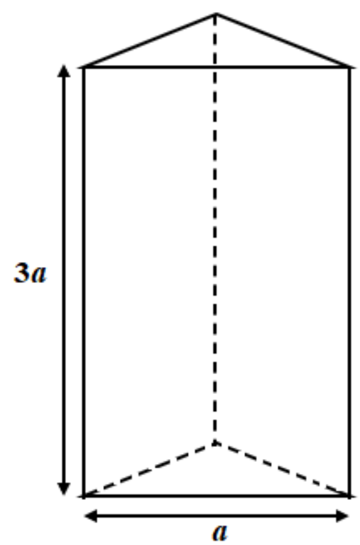
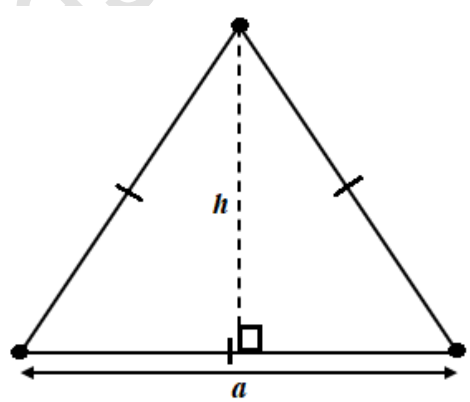
CSEC MATHEMATICS JANUARY 2021 PAPER 3

1. The diagram below shows a container of length $3a$ that John made to store cooking oil. The cross-section of the container is an equilateral triangle of side a and perpendicular to height h.



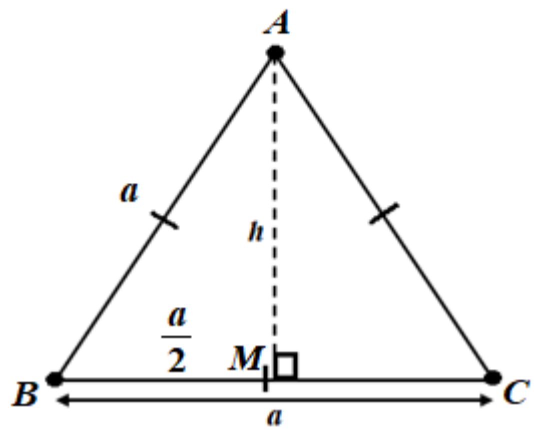
(a) An enlarged diagram of the triangular cross-section of the container is shown below.



Find the perpendicular height , h, in terms of a.

SOLUTION:
Data: Diagram showing a container with a cross-section in the shape of a equilateral triangle of length $3a$. The equilateral triangle has a side of length a and perpendicular height h.
Required to find: The perpendicular height of the equilateral triangle, h, in terms of a.
Solution:

The cross-sectional area is named *ABC* for convenience.
M is the midpoint of *BC*.



$AM^2 + BM^2 = AB^2$ (Pythagoras' Theorem)

$$h^2 + \left(\frac{a}{2}\right)^2 = a^2$$

$$h^2 = a^2 - \frac{a^2}{4}$$

$$= \frac{3a^2}{4}$$

$$\therefore h = \sqrt{\frac{3a^2}{4}}$$

$$= \frac{\sqrt{3}a}{2} \text{ units (in exact form)}$$

(b) Show that the area of the triangular cross-section, in terms of a, is $\frac{\sqrt{3}}{4}a^2$.

[**Note:** $\frac{\sqrt{3}}{4} \approx 0.433$]

SOLUTION:

Required to show: Area of the triangular cross-section, in terms of a, is $\frac{\sqrt{3}}{4}a^2$.

Proof:

Area of the triangular cross-section $= \dfrac{a \times \frac{\sqrt{3}}{2} a}{2}$

$$= \frac{\frac{\sqrt{3}}{2}a^2}{2}$$

$$= \frac{\sqrt{3}}{4}a^2 \text{ square units (in exact form)}$$

(c) Determine, in terms of a, the volume of the container. **Simplify your answer**.

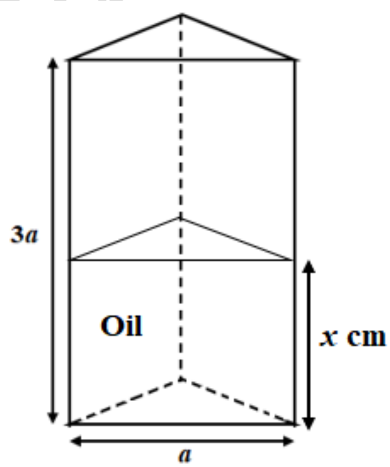
SOLUTION:
Required to determine: The volume of the container, in terms of a.
Solution:
Volume of the container $= \text{Area of cross} - \text{section} \times \text{Height}$

$$= \frac{\sqrt{3}}{4}a^2 \times 3a$$

$$= \frac{3\sqrt{3}a^3}{4} \text{ cubic units (in exact form)}$$

(d) Given that $a = 10$cm, calculate the depth of the oil in the container when John pours 800 cm^3 of oil into it.

SOLUTION:
Data: $a = 10$cm and John pours 800 cm^3 of oil into the container
Required to calculate: The depth of oil in the container
Calculation:



$a = 10$cm

$$\therefore \text{Cross-sectional area } = \frac{\sqrt{3}}{4}(10)^2 \text{ cm}^2$$

$$= 25\sqrt{3} \text{ cm}^2$$

Let the height of the oil be x cm.

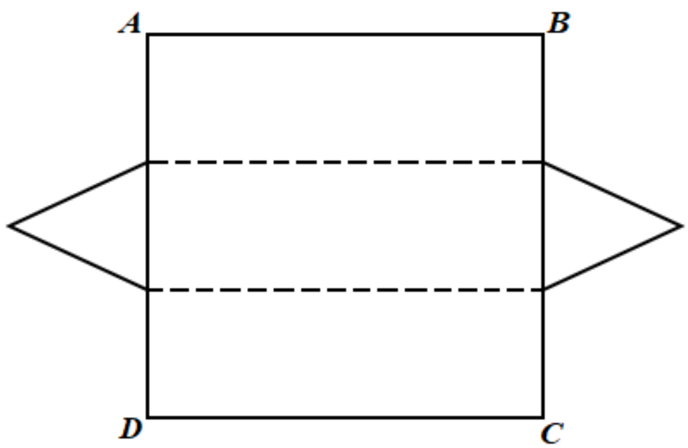
$$\therefore 25\sqrt{3} \times x = 800$$

$$x = \frac{800}{25\sqrt{3}}$$

$$= \frac{32}{\sqrt{3}} \text{ cm (in exact form)}$$

$$= 18.5\ cm$$

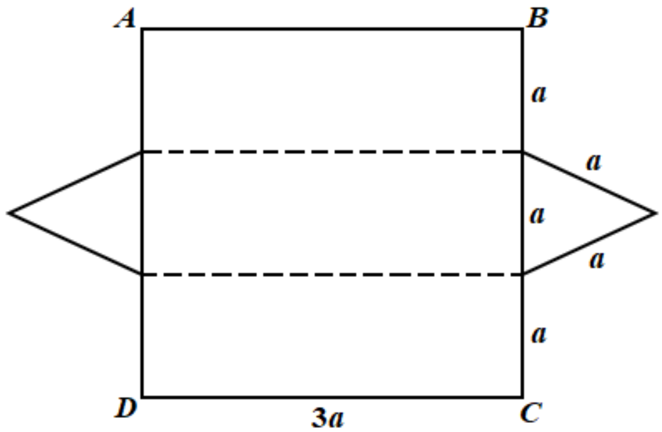
(e) The diagram below shows the **net** of the container when it is opened.



Show that the area of the net, in terms of a, is $\frac{1}{2}a^2\left(18+\sqrt{3}\right) \approx 9.866a^2$.

(**Hint:** *ABCD* is a square)

SOLUTION:
Data: Diagram showing the net of the container.
Required To Show: The area of the net, in terms of a, is $\frac{1}{2}a^2\left(18+\sqrt{3}\right) \approx 9.866a^2$
Proof:



Area of $ABCD = 3a \times 3a$
$= 9a^2$

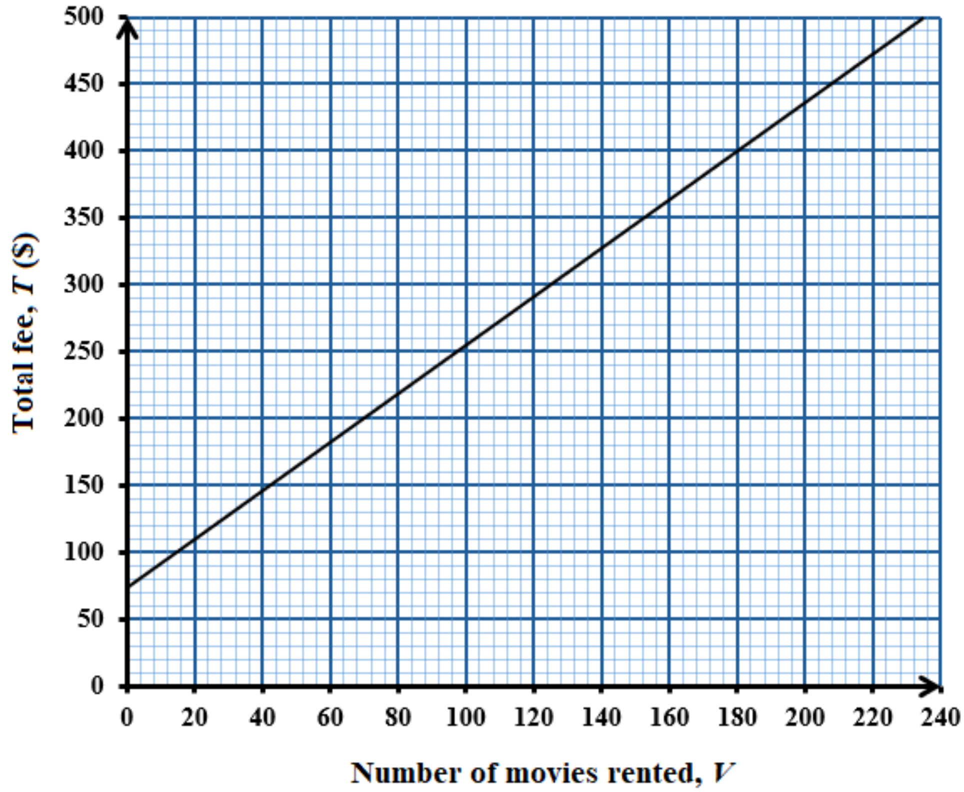
Area of the triangular sides of the net $= 2\left(\dfrac{a \times \frac{\sqrt{3}}{2}a}{2}\right)$
$= \dfrac{\sqrt{3}}{2}a^2$

The total area of the net $= 9a^2 + \dfrac{\sqrt{3}}{2}a^2$
$= \dfrac{1}{2}a^2\left(18 + \sqrt{3}\right)$

Q.E.D.

2. Flagship Movie Rentals charges an annual membership fee plus an additional fee for the weekly rental of each movie.

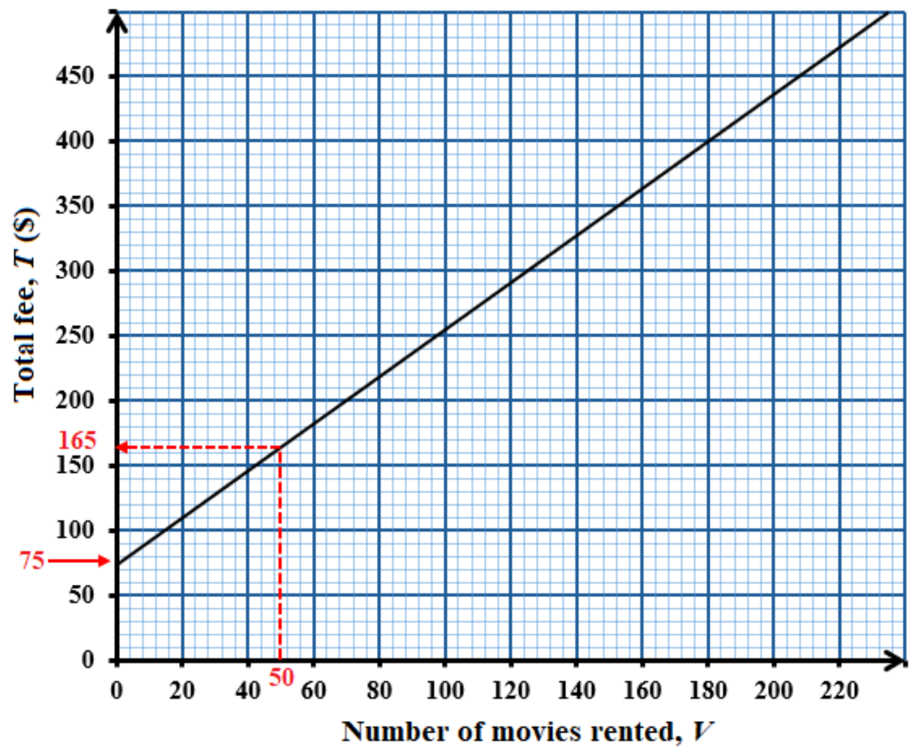
The graph below shows the total fee, (T), as it varies with the number of movies rented, (V).



(a) Using the graph, fill in the missing values in the table below.

Number of Moves Rented (*V*)	**Total Fee, *T* ($)**
________	75
30	130
50	________
124	300

SOLUTION:
Data: Graph showing the total fee, (*T*), as it varies with the number of movies rented, (*V*) by Flagship Movie Rentals. An annual membership fee is also charged.
Required to complete: The table given showing the number of movies rented (*V*) with the corresponding total fee (*T*), in dollars
Solution:



The completed table looks like:

Number of Moves Rented (V)	**Total Fee, T ($)**
0	75
30	130
50	**165**
124	300

(b) The total fee, T, is related to the number of movies rented, V, by the equation $T = mV + c$. Determine the value of:

(i) c

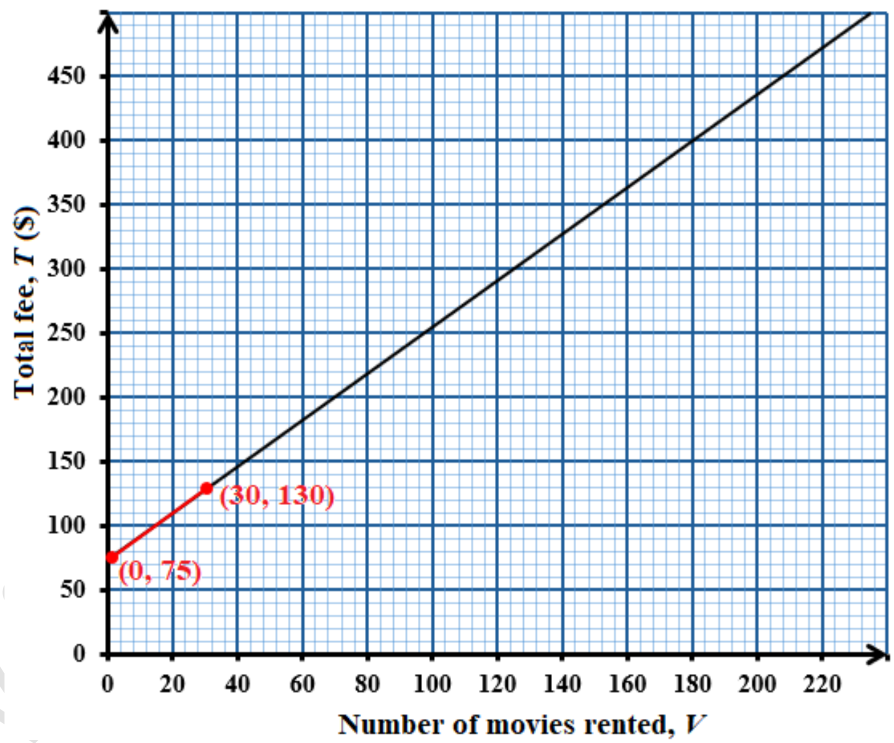
SOLUTION:
Data: The total fee, T, is related to the number of movies rented, V, by the equation $T = mV + c$.
Required to determine: The value of c

Solution:
When $V = 0,\ T = 75$

$$\therefore 75 = m(0) + c$$
$$c = 75$$

(ii) m

SOLUTION:
Required to determine: The value of m
Solution:
Consider the portion of the graph from $(0,\ 75)$ to $(30,\ 130)$:



$$\text{Gradient, } m = \frac{130 - 75}{30 - 0}$$
$$= \frac{55}{30}$$
$$= \frac{11}{6}$$

(c) Complete the following statement.

According to the graph, the annual membership fee is \$ ………………. and the fee to rent one movie for a week is \$ …………………………..

SOLUTION:
Required To Complete: The statement given
Solution:
The figures on the table show a discrepancy.
30 movies cost a total rental fee of \$130 (inclusive of monthly rental of \$75)
To calculate the rental fee for 1 movie we must subtract the rental fee from \$130
30 movies cost $= \$130 - \$75 = \$55$
1 movie will cost $= \frac{\$55}{30} = \$1.83$

Note that the gradient of the graph is the rate for renting one movie for the week.

Using another pair of values on the given table, we found the rate to be \$1.81

$$\begin{aligned}\text{124 movies cost a total of \$300 } &= \frac{300-75}{124}\\ &= \frac{225}{124}\\ &= \$1.81 \text{ each}\end{aligned}$$

The different rental rates indicate that there is a discrepancy in the table values because the gradient of a line is constant.

The completed statement is
According to the graph, the annual membership fee is **\$75** and the fee to rent one move for a week is **\$1.81 to \$1.83**.

(d) Brooke and her family budgeted \$700 for renting movies for the year.

(i) Using an equation, or otherwise, determine the MAXIMUM number of movies they can rent.

SOLUTION:
Data: Brooke and her family budgeted \$700 for the rental of movies for the year
Required to determine: The maximum number of movies they can rent.
Solution:
Using the equation:

$$T = \frac{11}{6}V + 75$$

We have:

$$700 = \frac{11}{6}V + 75$$

$$\frac{11}{6}V = 625$$

$$V = \frac{625 \times 6}{11}$$
$$= 340.9$$

But, $n \in Z^{+}$, so the maximum number of movies that can be rented is the integer just before 340.9, which is 340.

(ii) At Rightstar Movie Rentals, there is no membership fee but Brooke will pay $1.95 to rent a movie for one week. From which of the two rental clubs would Brooke be able to rent more movies on the same budget? **Show calculations to justify your answer**.

SOLUTION:
Data: At Rightstar Movie Rentals, there is no membership fee but Brooke will pay $1.95 to rent a movie for one week.
Required to determine: The rental club that Brooke will be able to rent more movies from on the same budget.
Solution:
On $700, Brooke can rent $\frac{700}{1.95} = 358.97$ movies at Rightstar Movie Rental, that is, a maximum of 358 movies.

So, Brooke would be able to rent more movies at Rightstar Movie Rentals than at Flagship Movie Rentals on the same budget.