CSEC MATHEMATICS JANUARY 2020 PAPER 3

1. (a) A hardware store has a sale on hammers, drills and spanners. A hammer is sold for \$15, a drill for \$75 and a spanner for \$25. **Customer A** bought 9 hammers, 14 drills and 2 spanners, and **Customer B** bought 8 hammers, 6 drills and 7 spanners.

(i) Construct the following matrices.

a) Matrix *P*, of order 3×1, to show the prices of the items on sale

SOLUTION:
Data: Customer A bought 9 hammers, 14 drills and 2 spanners. Customer B bought 8 hammers, 6 drills and 7 spanners. The cost of a hammer is \$15, the cost of a drill is \$75 and the cost of a spanner is \$25.
Required to construct: The 3×1 matrix *P* to show the prices of the items on sale.
Solution:

$$P=\underset{3\times1}{\begin{pmatrix}15\\75\\25\end{pmatrix}}$$

b) Matrix *N*, of order 2×3, to show the number of items bought by the two customers.

SOLUTION:
Required to construct: The 2×3 matrix N, to show the number of items bought by the two customers.
Solution:

$$N=\underset{2\times3}{\begin{pmatrix}9&14&2\\8&6&7\end{pmatrix}}$$

(ii) Calculate, using a matrix method, the TOTAL amount spent by EACH of the customers.

SOLUTION:
Required to calculate: The total amount spent by each of the customers.
Calculation:

$$N \times P = \underset{2\times 3}{\begin{pmatrix} 9 & 14 & 2 \\ 8 & 6 & 7 \end{pmatrix}} \underset{3\times 1}{\begin{pmatrix} 15 \\ 75 \\ 25 \end{pmatrix}}$$

$$= \underset{2\times 1}{\begin{pmatrix} e_{11} \\ e_{21} \end{pmatrix}}$$

$$e_{11} = (9\times 15) + (14\times 75) + (2\times 25)$$
$$= \$1235$$
$$e_{21} = (8\times 15) + (6\times 75) + (7\times 25)$$
$$= \$745$$

$$N \times P = \begin{pmatrix} 1235 \\ 745 \end{pmatrix}$$

Hence, Customer **A** spent \$1235 and Customer **B** spent \$745.

(b) Teacher Mabel is providing a meal for all the junior students in her school. Each student will have either a hamburger or a pizza. The cost of a hamburger is \$4 while the cost of a pizza is \$5. Let x represent the number of hamburgers and y the number of pizzas that she buys.

(i) Fill in the missing equations/inequalities in the table below to represent the condition given in Column 2.

	Condition	Equation/Inequality
a)	Each of 220 students must get one meal.	
b)	Teacher Mabel has no more than \$900 to cater for meals for all of the students.	
c)	She must buy more hamburgers than pizzas.	

SOLUTION:
Data: Teacher Mabel is providing a meal of either hamburger at \$4 each or pizza at \$5 each for all the junior students at her school. The number of hamburgers that she buys is denoted by x and the number of pizzas that she buys is denoted by y.
Required to complete: The table given, showing some conditions and the equation or inequality associated with each condition.

Solution:

a) Number of hamburgers $= x$
Number of pizzas $= y$

Each of the 220 students gets one meal.
Hence, $x + y = 220$

b) Cost of x hamburgers at \$4 each and y pizzas at \$5 each
$= \$(4 \times x) + \$(5 \times y)$
$= \$(4x + 5y)$

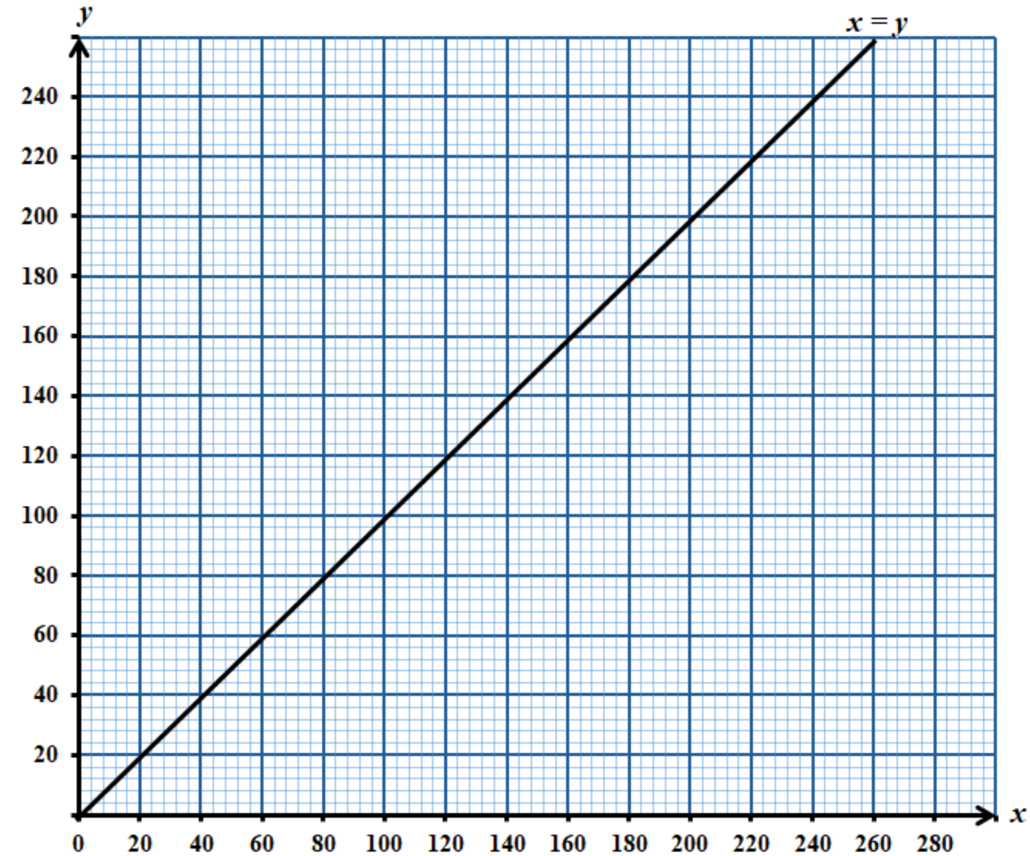
Hence, $4x + 5y \leq 900$

c) More burgers than pizzas.
Hence, $x > y$.

The completed table looks like:

	Condition	Equation/Inequality
a)	Each of 220 students must get one meal.	$x + y = 220$
b)	Teacher Mabel has no more than \$900 to cater for meals for all of the students.	$4x + 5y \leq 900$
c)	She must buy more hamburgers than pizzas.	$x > y$

(ii) The line corresponding to the inequality $x > y$ is shown on the graph below. On the graph, draw the lines corresponding to the other two equations/inequalities in the table in part (i) above.

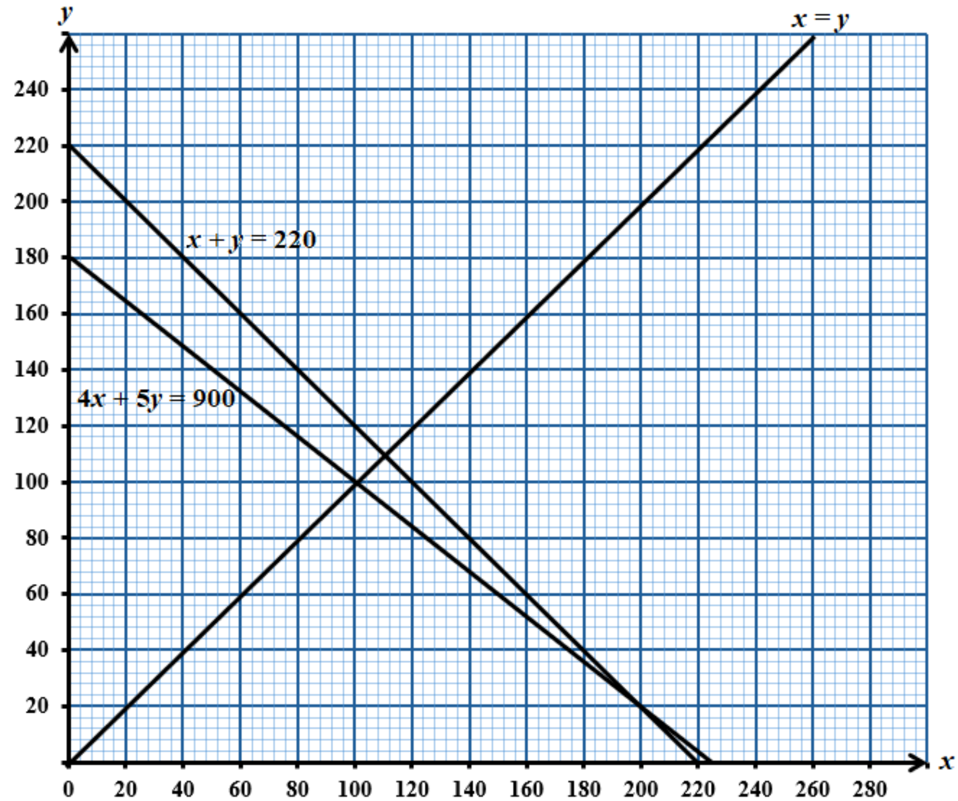


SOLUTION:
Data: Graph showing the line corresponding to the inequality $x > y$.
Required to draw: The lines corresponding to the other two equations/inequalities.
Solution:
Consider the line $x + y = 220$

x	*y*
0	220
220	0

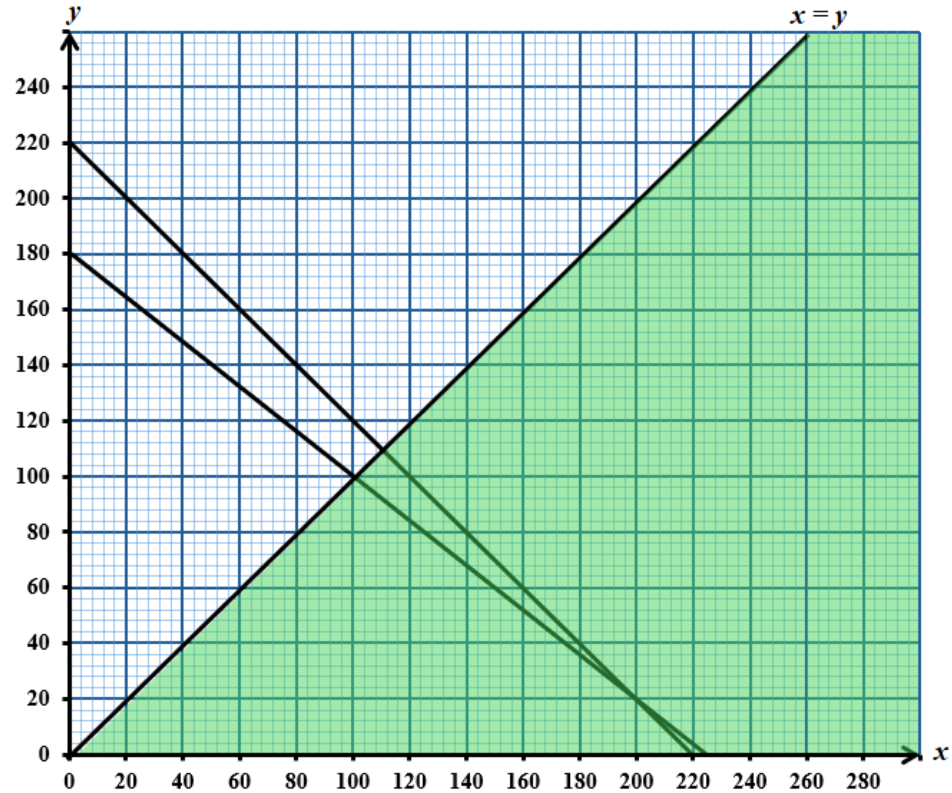
Consider the line $4x + 5y = 900$

x	*y*
0	180
225	0

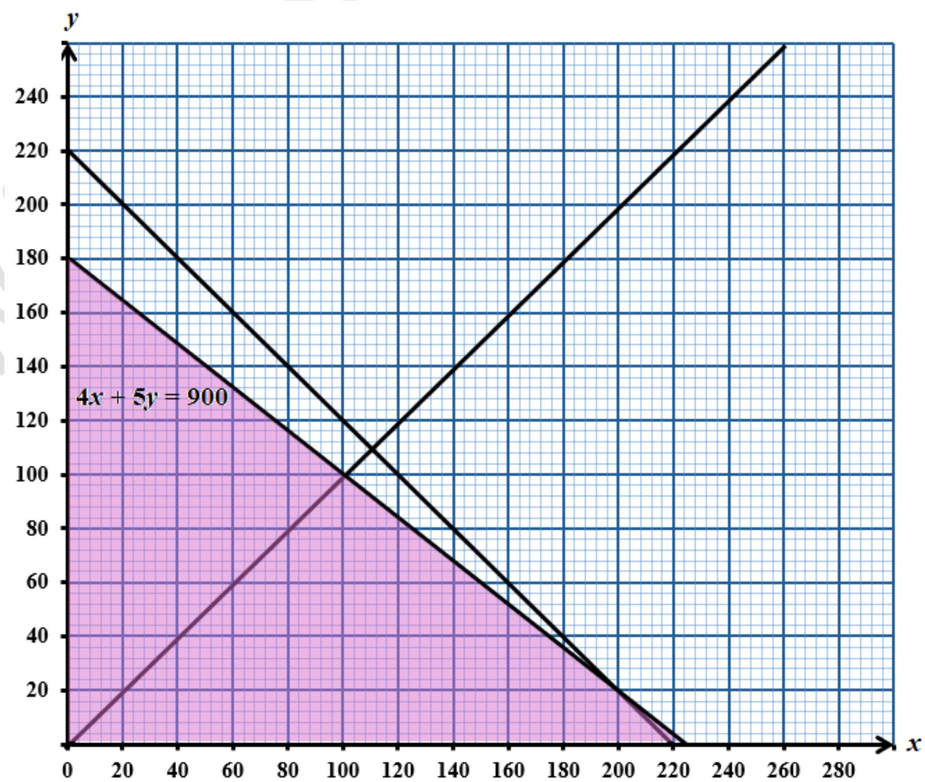


(iii) Using your graph, or otherwise, determine the MAXIMUM number of hamburgers and pizzas that Teacher Mabel can buy.

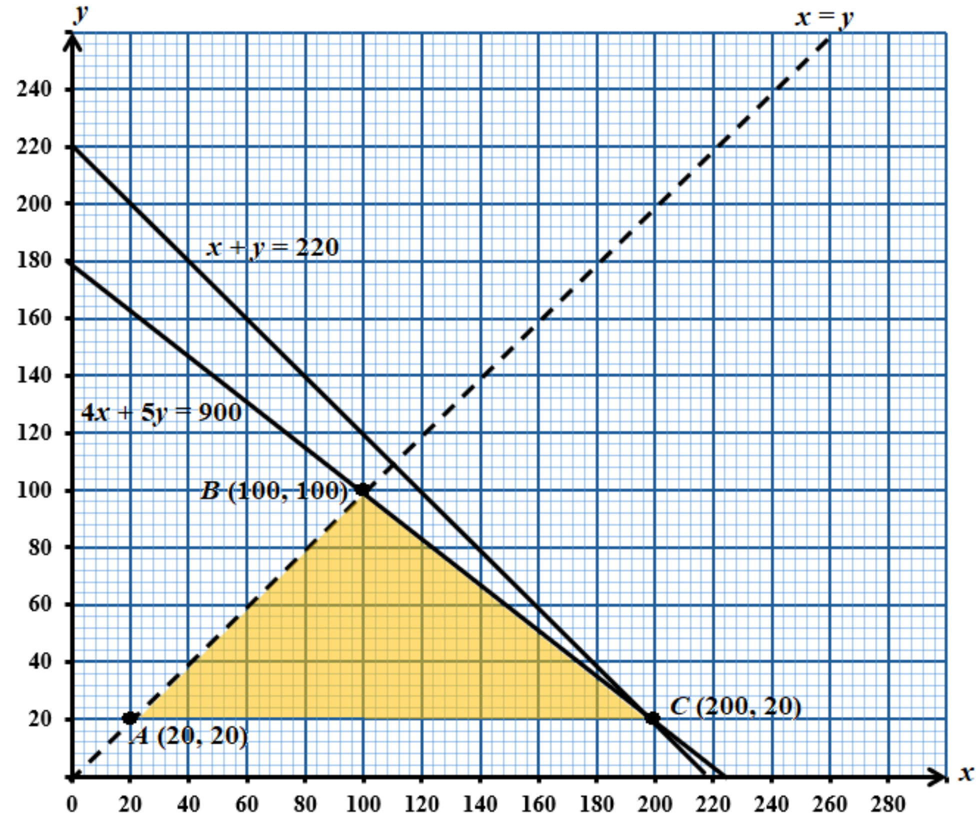
SOLUTION:
Required to determine: The maximum number of hamburgers and pizzas that Teacher Mabel can buy.
Solution:
Finding the feasible region
For $x > y$:



For $4x+5y\leq 900$:



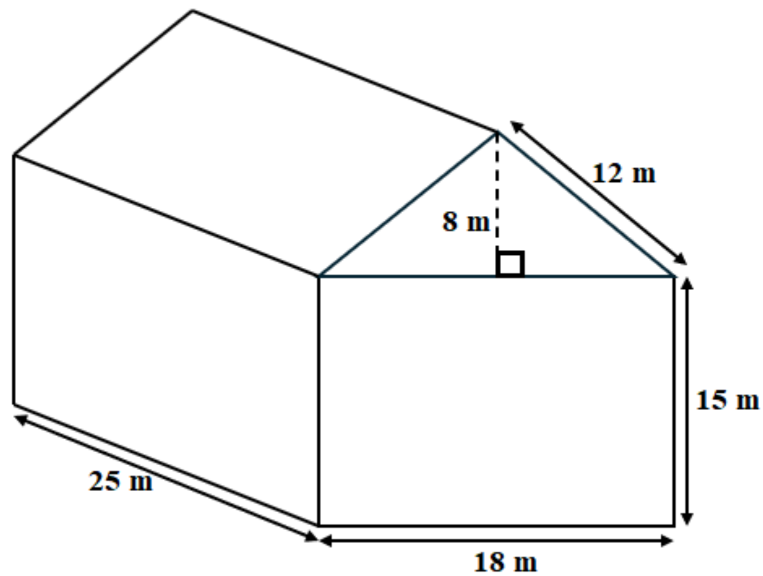
The feasible region is



The vertices of the feasible region (the region accepted by all the inequalities) are $A\,(20,\,20)$, $B\,(100,\,100)$ and $C\,(200,\,20)$.

Clearly, the maximum number of hamburgers and pizzas will be 100 of each.

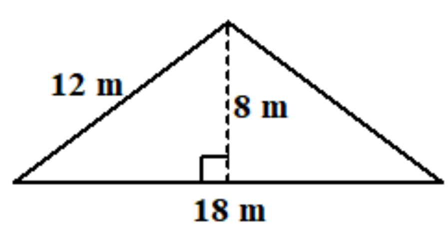
2. John makes a wooden barn to store grains for his farm animals. The barn is in the shape of a prism with a pentagonal cross-section and has dimensions as shown in the diagram below.



(a) Show that the TOTAL outer surface area of the barn (excluding the floor) is 2034 m^2.

SOLUTION:
Data: Diagram showing the dimensions of a wooden barn in the shape of a prism with a pentagonal cross-section.
Required to show: The total outer surface area of the barn, excluding the floor, is 2034 m^2.
Proof:
Area of the both sides of the rectangular roof $= 2(25 \times 12)$
$= 600 \text{ m}^2$

Area of left and right rectangular walls $= 2(25 \times 15)$
$= 750 \text{ m}^2$



Area of front and back walls $= 2(18 \times 15) + 2\left(\frac{18 \times 8}{2}\right)$

$= 540 + 144 \text{ m}^2$

$= 684 \text{ m}^2$

Total outer surface area $= 600 + 750 + 684$

$= 2034 \text{ m}^2$

Q.E.D.

(b) **(i)** Given that 1 gallon of paint covers approximately 28 square metres of surface, determine the TOTAL amount of paint, **in litres**, that is needed to paint the outer surface area of the barn. (1 gallon ≈ 3.79 litres)

SOLUTION:
Data: 1 gallon of paint covers approximately 28 square metres of surface
Required to determine: The total amount of paint, in litres, is needed to paint the outer surface area of the barn.
Solution:
Area to be painted (assuming there are no windows)

Amount of paint required

$= \frac{\text{Surface area of the barn}}{28}$ gallons

$= \frac{2038}{28}$

$= 72.643$ gallons

$= 72.643 \times 3.79$ L

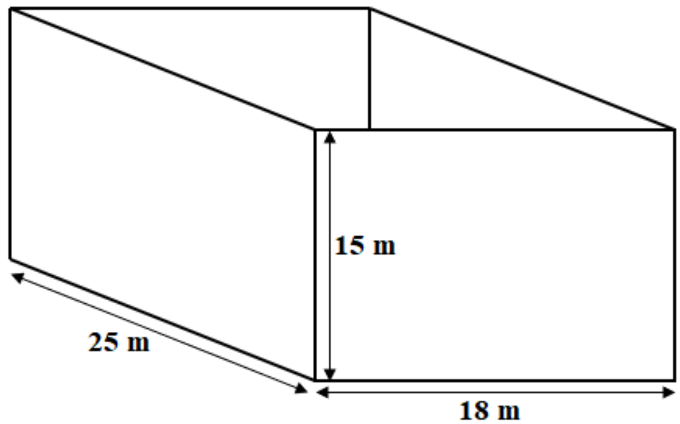
$= 275.31\ L$

(ii) If the paint is sold in one-gallon containers only, how many containers of paint are needed to complete the job?

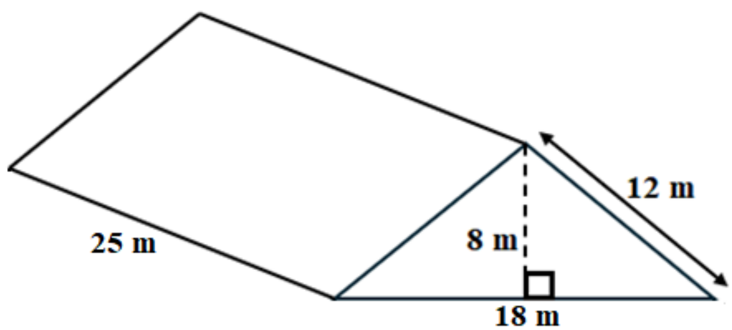
SOLUTION:
Data: Paint is sold in one-gallon containers
Required To Find: The number of containers of paint needed to complete the job
Solution:
The number of gallons required is 72.643.
Since the paint is sold in one-gallon containers, 72 gallons will be insufficient one would have to buy 73 gallons to complete the job.

(c) Determine the capacity (volume) of the barn.

SOLUTION:
Required To Determine: The capacity (volume) of the barn
Solution:



Volume of the rectangular base of the barn $= 25 \times 18 \times 15 \text{ m}^3$
$= 6750 \text{ m}^3$



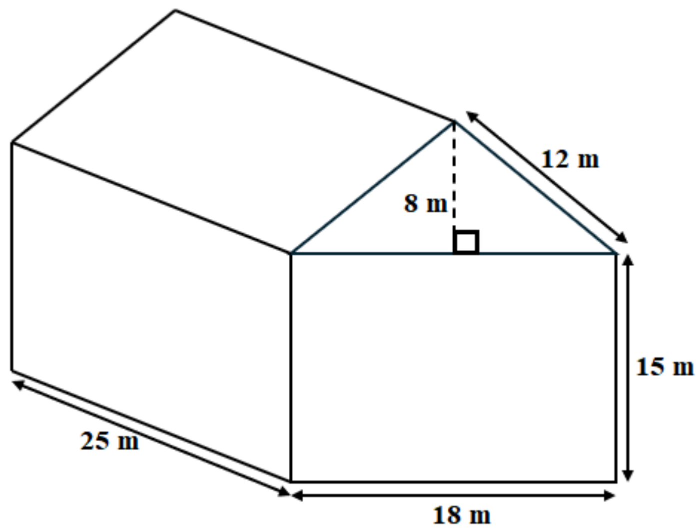
Volume of the roof region $= \text{Cross} - \text{sectional area} \times \text{Length}$

$$= \left(\frac{18 \times 8}{2}\right) \times 25$$

$$= 1800 \text{ m}^3$$

$\therefore$ Total volume of the barn $= 6750 + 1800$
$= 8550 \text{ m}^3$

Alternatively, we could have found the area of cross section of the barn and multiply this area by 25 m (length)



Area of cross-section

$$= (18 \times 15) + (\frac{1}{2} \times 18 \times 8)$$
$$= 270 + 72$$
$$= 342\ m^2$$

Volume of the barn

= Area of cross-section × length
$= 342 \times 25\ m^3$
$= 8550\ m^3$