

CSEC MATHEMATICS JUNE 2019 PAPER 3

1. A company which employs 10 unskilled and 8 skilled workers produces two models of a toy, Model A and Model B.
- Model A requires 20 hours of unskilled labour while Model B requires 15 hours of unskilled labour
 - Model A requires 10 hours of skilled labour while Model B requires 25 hours of skilled labour
- Each worker provides 40 hours of labour per week.

- (a) Calculate the maximum number of hours available per week for (i) unskilled labour (ii) skilled labour

SOLUTION:

Data: The company employs 10 unskilled workers and each worker provides 40 hours of labour per week. The company also employs 8 skilled workers and each worker provides 40 hours of labour per week.

Required to calculate: the maximum number of hours available for (i) unskilled labour and (ii) skilled labour.

Calculation:

$$\begin{aligned} \text{Maximum number of hours available for unskilled labour} &= 40 \text{ hours} \times 10 \\ &= 400 \text{ hours} \end{aligned}$$

$$\begin{aligned} \text{Maximum number of hours available for skilled labour} &= 40 \text{ hours} \times 8 \\ &= 320 \text{ hours} \end{aligned}$$

- (b) Assume that the factory makes x Model A and y Model B toys per week. Show that The inequalities for the hours of labour are:

(i) unskilled labour, $4x + 3y \leq 80$ (ii) skilled labour, $2x + 5y \leq 64$.

SOLUTION:

Data:

The factory makes x Model A and y Model B toys per week

Model A requires 20 hours of unskilled labour while Model B requires 15 hours of unskilled labour.

Model A requires 10 hours of skilled labour while Model B requires 25 hours of skilled labour.

Required to show: the inequalities for the hours of labour are:

(i) unskilled labour: $4x + 3y \leq 80$ and (ii) skilled labour: $2x + 5y \leq 64$.

Solution:

The information can be represented in the following table:

	No. of hours		Maximum number of hours
	Model A (x)	Model B (y)	
Unskilled labour	20	15	400
Skilled labour	10	25	320

(i) Unskilled labour:

Each Model A toy require 20 hours of unskilled labour
 x Model A toys will require $20x$ hours of unskilled labour

Each Model B toy require 15 hours of unskilled labour
 y Model A toys will require $15y$ hours of unskilled labour

Total number of hours required for unskilled labour is: $20x + 15y$

The maximum number of hours available is 400.

Therefore, the inequality that represents this condition is: $20x + 15y \leq 400$

Dividing by 5, this simplifies to: $4x + 3y \leq 80$

(ii) Skilled labour:

Each Model A toy require 10 hours of skilled labour
 x Model A toys will require $10x$ hours of skilled labour

Each Model B toy require 25 hours of skilled labour
 y Model A toys will require $25y$ hours of skilled labour

Total number of hours required for skilled labour is: $10x + 25y$

The maximum number of hours available is 320.

Therefore, the inequality that represents this condition is: $10x + 25y \leq 320$

Dividing by 5, this simplifies to: $2x + 5y \leq 64$

(c) **Show, by calculation that it is possible to make 15 Model A toys and 5 Model B toys.**

SOLUTION:

Data: For x Model A toys and y Model B toys, the following requirements for hours of labour must be satisfied:

Unskilled labour: $4x + 3y \leq 80$

Skilled labour: $2x + 5y \leq 64$

Required: To show by calculation that it is possible to make 15 Model A toys and 5 Model B toys.

Calculation:

The number of Model A toys = $x = 15$ and the number of Model B toys = $y = 5$

<p>For unskilled labour, $4x + 3y \leq 80$ So, $4(15) + 3(5) = 60 + 15 = 75$ Since $75 \leq 80$, we conclude that it is possible to make 15 Model A toys and 5 model B toys.</p>	<p>For skilled labour, $2x + 5y \leq 64$ So, $2(15) + 5(5) = 30 + 25 = 55$ Since $55 \leq 64$, we conclude that it is possible to make 15 Model A toys and 5 model B toys.</p>
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Note also, we could have substituted these values in the original inequalities to obtain the same conclusion. The calculations are shown below:

<p>For unskilled labour, $20x + 15y \leq 400$ So, $20(15) + 15(5) = 300 + 75 = 375$ Since $375 \leq 400$, we conclude that it is possible to make 15 Model A toys and 5 model B toys.</p>	<p>For skilled labour, $10x + 25y \leq 320$ So, $10(15) + 25(5) = 150 + 125 = 275$ Since $275 \leq 320$, we conclude that it is possible to make 15 Model A toys and 5 model B toys.</p>
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- (d) **The factory makes a profit of \$40 on a Model A toy and \$60 on a Model B toy each week.**

(i) Write an equation for the TOTAL profit, P , per week, for making x Model A and y Model B toys.

SOLUTION:

Data: The profit per week on one Model A toy is \$40 and the profit on one Model B toy is \$60.

Required: To write an equation for the Total profit made per week on x model A toys and y Model B toys.

Profit Equation:

Profit per week on one Model A toy is \$40. Profit per week on x Model A toys is $40x$

Profit per week on one Model B toy is \$60. Profit per week on y Model B toys is $60y$

Total profit per week on both type of toys in dollars is : $P = 40x + 60y$

(ii) What is the profit to the company when it makes 15 Model A toys and 5 Model B toys?

SOLUTION:

Data: The total profit, P , per week on x Model A toys and y Model B toys is $40x + 60y$.

Required: To calculate the profit per week made on 15 model A toys and 5 Model B toys.

Calculation:

$$P = 40x + 60y$$

Substitute, $x = 15$, and $y = 5$

$$P = \$40(15) + \$60(5) = \$600 + \$300 = \$900$$

- (e) Show that the profit calculated in (d) (ii) is NOT the maximum profit.

SOLUTION:

Data: Total profit per week on both type of toys in dollars is : $P = 40x + 60y$

Required: To show that the profit of \$900 made on 15 model A toys and 5 Model B toys is not the maximum profit.

Solution:

In order to show that the profit calculated when $x = 15$ and $y = 5$ is not the maximum, we must find a pair of values for x and y that would result in a profit that is more than \$900.

Consider the point, $x = 14$ and $y = 6$. We need to calculate the profit on making 14 Model A toys and 6 Model B toys.

Before doing so, we must first test to see that this point satisfies the given conditions:

$$x \geq 0, y \geq 0, 4x + 3y \leq 80 \text{ and } 2x + 5y \leq 62$$

Letting $x = 14$ and $y = 6$, we note that, $14 \geq 0$ and $6 \geq 0$

$$\text{Also, } 4(14) + 3(6) = 56 + 18 = 74 \leq 80$$

$$\text{And, } 2(14) + 5(6) = 28 + 30 = 58 \leq 62$$

Since (14, 6) satisfies all conditions and lies within the feasible region, it is possible to make 14 Model A toys and 6 Model B toys.

The profit on 14 Model A toys and 6 Model B toys is:

$$P = 40x + 60y$$

Substitute, $x = 14$, and $y = 6$

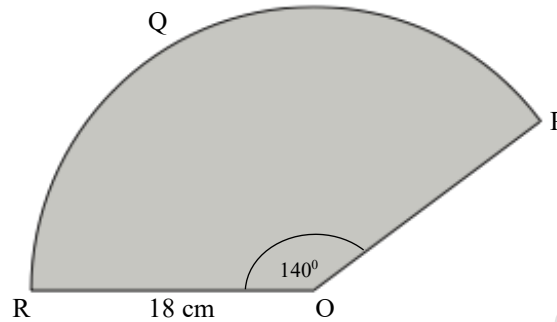
$$P = \$40(14) + \$60(6) = \$560 + \$360 = \$920$$

Since $\$920 > \900 , the maximum profit cannot be \$900.

Q.E.D.

2. The diagram below, not drawn to scale, shows a flexible piece of card in the shape of a sector of a circle with centre O and radius 18 cm.

Use $\pi = \frac{22}{7}$



- (a) Show that the perimeter of the card is 80 cm.

SOLUTION:

Data: Diagram showing a card in the shape of a sector of a circle of radius 18 cm and angle at the centre equal to 140° .

Required to show: The perimeter of the sector is 80 cm.

Proof:

Perimeter of the sector = length of arc RQP + length of radius PO + length of radius OR

Arc length RPQ = $\frac{\theta}{360} \times 2\pi r$, where θ is the angle in the sector and r is the radius of the circle.

$$\text{Arc length RPQ} = \frac{140}{360} \times 2 \times \frac{22}{7} \times 18 = 44 \text{ cm}$$

Since PO and OR are radii of the circle, PO=OR= 18 cm

Perimeter of the sector = Arc length RQP + PO + OR

$$= 44 + 18 + 18 \text{ cm}$$

$$= 80 \text{ cm}$$

Q.E.D.

- (b) Calculate the area of the card $OPQR$.

Required to calculate: The area of the card $OPQR$

Calculation:

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$

$$\text{Area of card} = \frac{140}{360} \times \frac{22}{7} \times 18 \times 18 = 396 \text{ cm}^2$$

(c) The card is bent and the edges OP and OR are taped together so that the card forms the curved surface of a cone with a circular base, PQR.

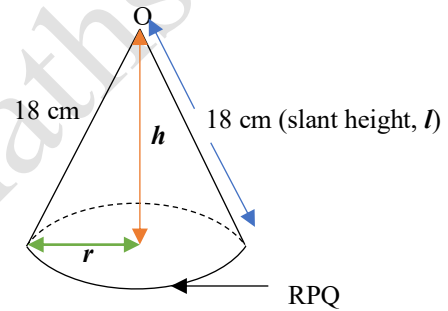
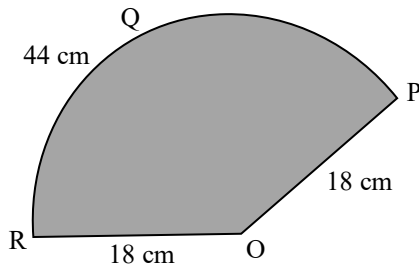
(i) Draw a diagram of the cone formed, showing clearly the measurement 18 cm, the perpendicular height, h , and the radius, r , of the base of the cone.

SOLUTION:

Data: A diagram of a sector of a circle with radius 18 cm.

Required: To draw a diagram of the cone formed when the card is taped.

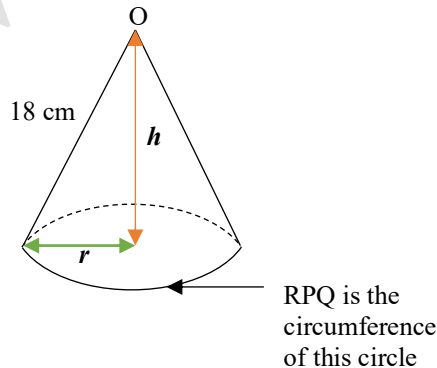
Drawing:



Sector or net of the cone.
When folded OP and OQ will coincide.

Cone – the arc of the sector, RPQ becomes its circular base. The radius OP or OR becomes the slant height, l . The distance from the apex of the cone to the center of its circular base, h , is the perpendicular height and r is the radius of the cone.

(ii) Calculate the radius of the circular base of the cone.



Circumference of the circular base of the cone, RPQ = 44 cm

$$\begin{aligned} 2\pi r &= 44 \\ 2 \times \frac{22}{7} \times r &= 44 \\ r &= \frac{44 \times 7}{2 \times 22} = 7 \text{ cm} \end{aligned}$$

Radius of base of cone = 7 cm

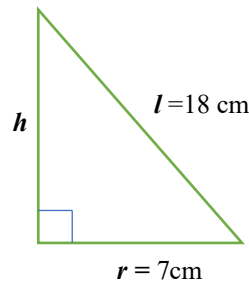
- (iii) Using Pythagoras' theorem, or otherwise, determine the perpendicular height of the resulting cone.

SOLUTION:

Data: The radius, $r = 7$ cm and slant height, $l = 18$ cm of a cone.

Required: To calculate the perpendicular height of the cone.

Calculation:



By Pythagoras' Theorem

$$h^2 + r^2 = l^2$$

$$h^2 = l^2 - r^2$$

$$h^2 = l^2 - r^2$$

$$h^2 = 18^2 - 7^2 = 324 - 49$$

$$h^2 = 275$$

$$h = \sqrt{275} = \sqrt{25 \times 11} = 5\sqrt{11} \quad (\text{in exact form})$$

$$h = 16.58 \text{ cm} \quad (\text{correct to 2 decimal places})$$