FAS-PASS
Maths

CSEC MATHEMATICS JANUARY 2019 PAPER 3

1. (a) The information below represents the minimum temperatures, in °C, recorded in Country A for the first 20 days in a particular month.

21	23	25	22	24	25	23	26	23	24
25	26	23	25	23	25	24	25	25	25

(i) Complete the frequency table below, using the information above.

Temperature (°C)	Tally	Frequency
21	|	
22	|	
23		
24	|||	
25		
26		

SOLUTION:
Data: Set of raw data showing the minimum temperatures, in °C, recorded in Country A for the first 20 days in a month.
Required to complete: The frequency table given.

Solution:

Temperature (°C)	Tally	Frequency
21	|	1
22	|	1
23	$\not{\|\|\|}$	5
24	|||	3
25	$\not{\|\|\|}$ |||	8
26	||	2
		$\sum f = 20$

(ii) Determine the median temperature.

SOLUTION:
Required to determine: The median temperature
Solution:

The middle values of the set of 20 data values are the 10^{th} and the 11^{th} values when arranged in ascending or descending order of magnitude.
The 10^{th} value is 24.
The 11^{th} value is 25.

The median value will be $\frac{24+25}{2} = 24.5$.

(iii) Calculate the mean temperature for the twenty-day period.

SOLUTION:
Required to calculate: The mean temperature
Calculation:

Mean, $\bar{x} = \frac{\sum fx}{\sum f}$, where x =score, f =frequency and $\sum$ = the sum of

$$= \frac{(1\times 21)+(1\times 22)+(5\times 23)+(3\times 24)+(8\times 25)+(2\times 26)}{20}$$

$$= \frac{21+22+115+72+200+52}{20}$$

$$= \frac{482}{20}$$

$$= 24.1\ ^\circ\text{C}$$

(b) The diagrams below, represent the cross-sections of two circular pizzas, *A* and *B*. The pizzas are similar but vary in size. Pizza *A* has a diameter of 15 cm and Pizza *B* has a diameter of 30 cm.

Pizza *A*

Pizza *B*

(i) Determine, by calculation, if Pizza *B* is twice the size of Pizza *A*.

SOLUTION:
Data: Diagram showing the cross-sections of Pizza *A* with diameter 15 and Pizza *B* with diameter 30 cm.
Required to determine: If Pizza *B* is twice the size of Pizza *A*.
Solution:

When comparing the size of linear figures we compare lengths; in two-dimensional figures we compare their areas and in three-dimensional figures we compare their volumes.

To determine if Pizza *B* is twice the size of Pizza *A*, we have to compare the volumes since the slices are three dimensional in nature. However, in this case, the thickness was not given, **so we are going to assume that the thickness is the same in both**.

Note: This would not be true if the pizzas are 'similar' since the thickness of Pizza *B* and the thickness of Pizza *A* will be in the ratio of their diameters, that is, 30:15 and which reduces to 2:1.

Assuming that both pizzas have the same thickness

If h =thickness of both pizzas
Volume of Pizza $B = \pi\ (15)^2\ h$
Volume of Pizza $A = \pi\ (7.5)^2\ h$

Ratio of the size of Pizza *B* : Size of Pizza *A*
$= \pi\ (15)^2\ h : \pi\ (7.5)^2\ h$
$= 4 : 1$

Assuming that the pizzas are truly similar in the mathematical context

Let h = thickness of Pizza *A*
Then the thickness of Pizza *B* will be $2h$.

Ratio of size of Pizza *B* : Size of Pizza *A*
$= \pi\ (15)^2\ 2\ h : \pi\ (7.5)^2\ h$
$= 8 : 1$

Conclusion

Regardless, Pizza *B* is either 4 times or 8 times the size of Pizza *A* and the conclusion is that Pizza *B* is not two times the size of pizza *A*.

(ii) Pizza *B* is cut into 5 equal slices and is sold at \$6.95 per slice, while Pizza *A* (is sold as a whole) at \$9.95. Determine, with reason, which of the two options (a slice of Pizza *B* or Pizza *A*), is the better buy for the customer.

SOLUTION:
Data: Pizza *B* is cut into 5 slices and sold at \$6.95 per slice and Pizza *A* is sold as a whole for \$9.95.
Required to determine: If Pizza *B* or Pizza *A* is the better buy

Solution:
For this comparison we assume that the thickness of the pizzas are the same.
The volume of pizza in one slice of Pizza *B* is $\frac{1}{5}\pi\ (15)\ (15)h = 45\ \pi h$ cubic units.
The volume of pizza in the whole of Pizza *A* is $\pi\ (7.5)\ (7.5)h = 56.25\ \pi h$ cubic units.

So, 45 πh cubic units of Pizza *B* costs \$6.95.
πh cubic units costs $\frac{\$\ 6.95}{45}$ = **15.44** cents correct to 2 decimal places.

And 56.25 πh cubic units of Pizza *A* costs \$9.95.
πh cubic units costs $\frac{\$\ 9.95}{56.25}$ = **17.69** cents correct to 2 decimal places.

A unit price of Pizza *A* costs more than a unit price of Pizza *B*. So, if better is to mean cheaper, then buying a slice of **Pizza *B* is a better buy than buying Pizza *A*.**

(Point to note, though, is that one slice of Pizza *B* is less in quantity than the whole of Pizza *A*.)

Alternative Approach:
However, we may take the word similar in the real mathematical context.

Volume of a slice of Pizza $B = \frac{1}{5}\pi(15)^2\ 2h$
$= 90\pi h$ cubic units

$90\pi h$ cubic units cost \$6.95.
πh cubic units cost $\frac{\$6.95}{90} = \mathbf{7.72}$ cents correct to 2 decimal places

Volume of a whole Pizza *A* cost \$9.95.
Volume of a whole Pizza $A = \pi(7.5)^2\ h$
$= 56.25\pi h$ cubic units

$56.25\pi h$ cubic units of Pizza *A* costs \$9.95.
πh cubic units cost $\frac{\$9.95}{56.25} = \mathbf{17.69}$ cents correct to 2 decimal places

So, if better means cheaper, then a slice of **Pizza *B* is a better buy than the whole of Pizza *A*.**

Also, the amount of pizza in $\frac{1}{5}$ slice of Pizza *B* is more than the amount of pizza in Pizza *A*.

So, regardless, buying $\frac{1}{5}$ slice of Pizza *B* is better than buying the whole of Pizza *A*.

2. (a) In a football tournament, points are awarded as follows: 3 points for a win, 1 point for a draw and 0 points for a loss.

(i) Write a 3×1 matrix, *P*, to represent the information.

SOLUTION:

Data: In a football tournament, 3 points are awarded for a win, 1 point for a draw and 0 points for a loss.

Required to write: A 3×1 matrix *P* for this information

Solution:

$$P=\underset{3\times1}{\begin{pmatrix}3\\1\\0\end{pmatrix}}$$

(ii) During the tournament, Team Alpha recorded 5 wins, 1 draw and 3 losses, while Team Beta recorded 3 wins, 4 draws and 2 losses.

Write a 2×3 matrix, *R*, to represent this information.

SOLUTION:

Data: Team Alpha won 5 games, drew 1 game and lost 3 games. Team Beta won 3 games, drew 4 games and lost 2 games.

Required to write: A 2×3 matrix, *R*, for this information

Solution:

$$R=\underset{2\times3}{\begin{pmatrix}5&1&3\\3&4&2\end{pmatrix}}$$

(iii) Calculate the matrix product *RP*.

SOLUTION:

Required to calculate: *RP*

Calculation:

$$RP = \begin{pmatrix} 5 & 1 & 3 \\ 3 & 4 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} e_{11} \\ e_{12} \end{pmatrix}$$
$$2 \times 3 \qquad 3 \times 1 \rightarrow \quad 2 \times 1$$

$$e_{11} = (5 \times 3) + (1 \times 1) + (3 \times 0) = 15 + 1 + 0 = 16$$

$$e_{12} = (3 \times 3) + (4 \times 1) + (2 \times 0) = 9 + 4 + 0 = 13$$

$$RP = \begin{pmatrix} 16 \\ 13 \end{pmatrix}$$

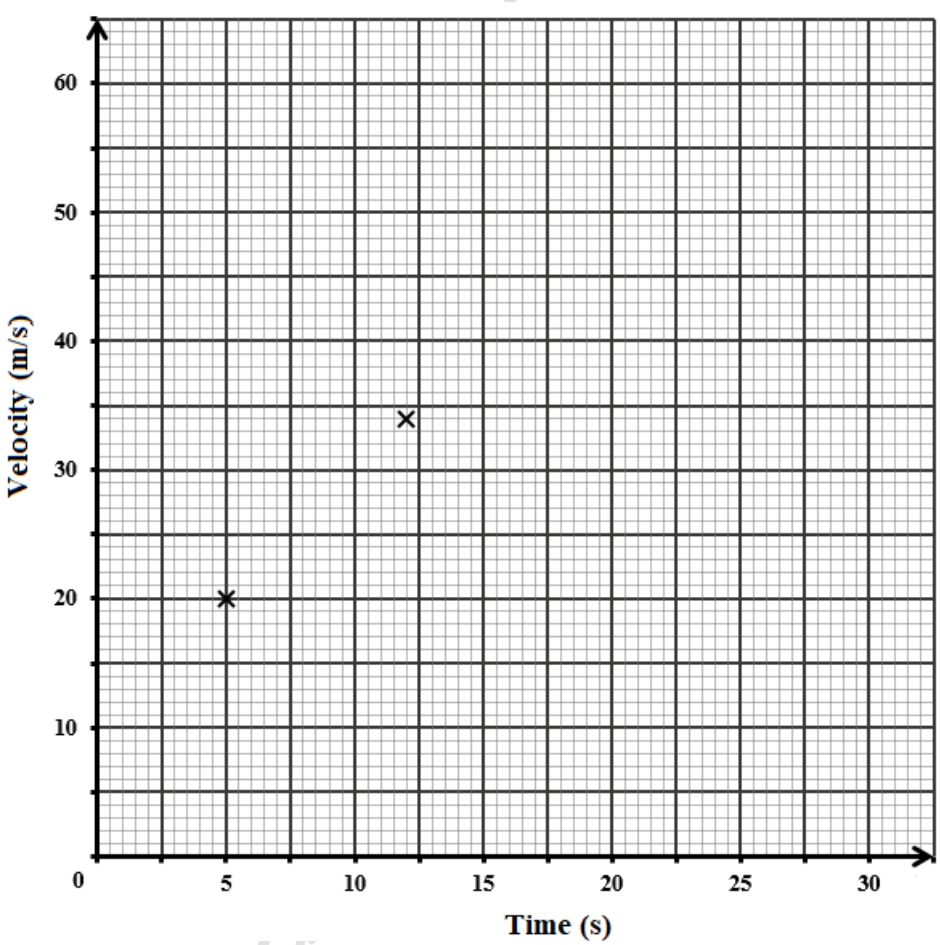
(iv) What does the matrix product RP represent?

SOLUTION:
Required to state: The meaning of the matrix product RP
Solution:
RP represents the total number of points scored by each team in the tournament. So, $RP = \begin{pmatrix} 16 \\ 13 \end{pmatrix}$ indicates that Team Alpha has 16 points after 9 matches and Team Beta has 13 points after 9 matches.

(b) The values recorded in the table below represent the velocity of an object over a period of time.

Velocity, v (m/s)	20	34	46	60
Time, t (s)	5	12	18	25

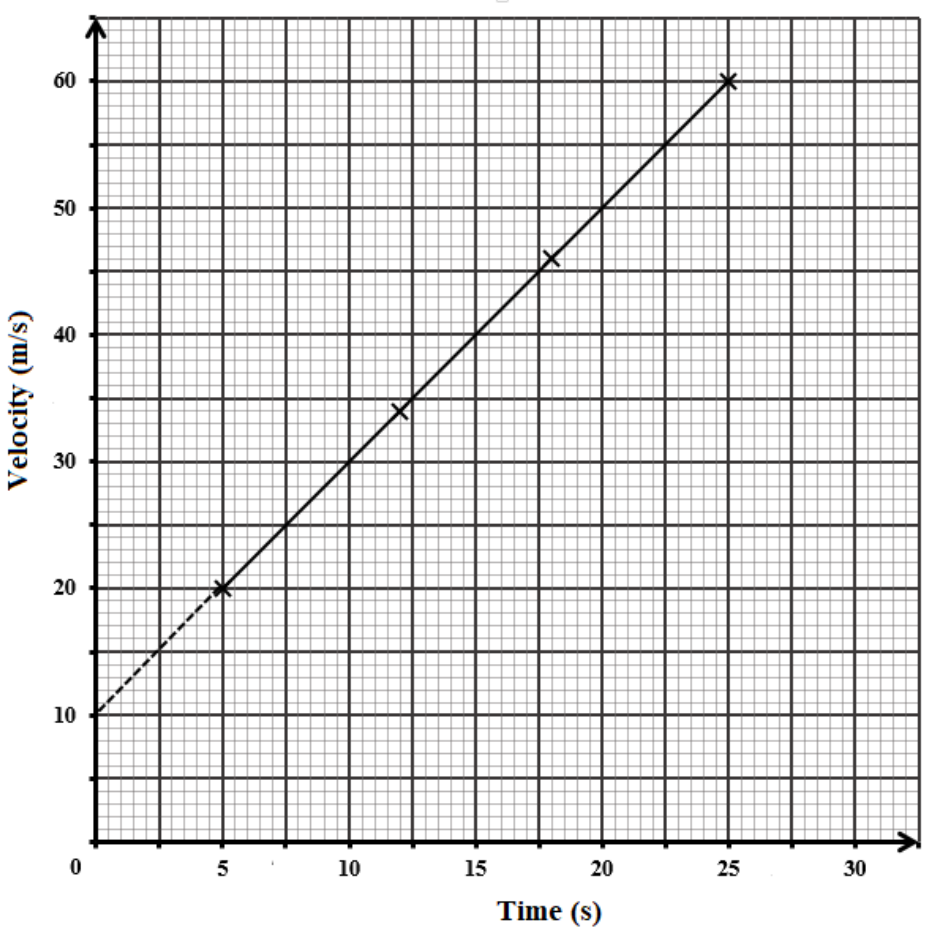
(i) On the grid below, two points are plotted. Plot the remaining two points on the grid and draw a line of best fit through the points.



SOLUTION:
Data: Table showing the velocity, v(m/s) of an object over a period of time, t(s). Graph of velocity vs time showing the first two points plotted.

Required to plot: The remaining two points and draw the best fit line

Solution:

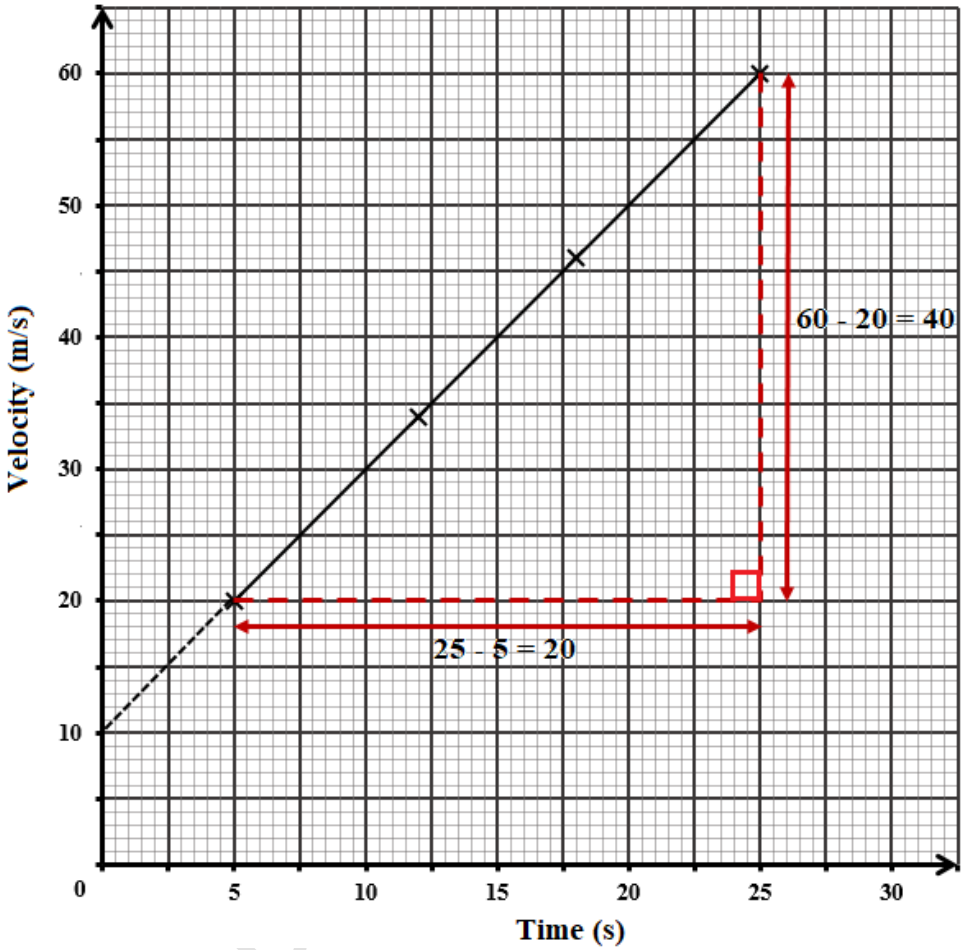


(ii) Given that the linear motion of the object can be expressed in the form $v = at + u$, where a and u are constants, use your graph to determine the values of a and u.

SOLUTION:
Data: The linear motion of the object is of the form $v = at + u$, where a and u are constants.
Required to find: The value of a and of u.
Solution:
In $v = at + u$, v and t are variables and a and u are constants. A linear equation is of the general form $y = mx + c$, where y and x are variables and m and c are constants. In the case of $y = mx + c$, m is the gradient of the line and c is the intercept on the vertical axis. We compare the following:

$y = mx + c$

$v = at + u$



The intercept on the vertical axis is $c = 10$, hence $u = 10$.

The gradient of the graph is $\frac{40}{20} = 2$, Hence, $a = 2$.