FAS-PASS Maths

CSEC MATHEMATICS PAPER 3 JUNE 2018

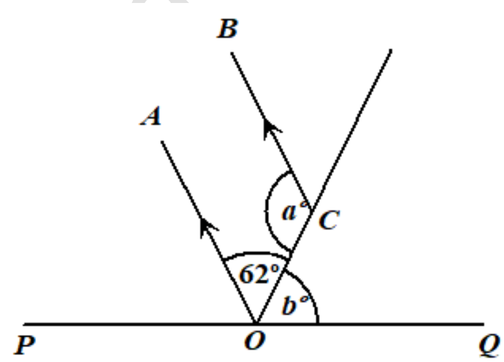
1. (a) The internal angles, in degrees, of a quadrilateral are $12x+15$, $17x-10$, $x+50$ and $15x+35$. Find the value of x.

SOLUTION:
Data: The internal angles, in degrees, of a quadrilateral are $12x+15$, $17x-10$, $x+50$ and $15x+35$.
Required to find: x
Solution:
The sum of the four interior angles of a quadrilateral is 360°.

Hence, $(12x+15)+(17x-10)+(x+50)+(15x+35)=360$

$$12x+17x+x+5x+15-10+50+35=360$$
$$45x+90=360$$
$$45x=360-90$$
$$45x=270$$
$$x=\frac{270}{45}$$
$$x=6$$

(b) The diagram below, **not drawn to scale**, shows four straight lines *POQ*, *OA*, *OC* and *CB*. *OA* is parallel to *CB*.



Given that *OA* bisects $P\hat{O}C$ and $A\hat{O}C=62°$, find the value of:

(i) a

SOLUTION:
Data: Diagram showing four straight lines *POQ*, *OA*, *OC* and *CB*, where *OA* is parallel to *CB*. *OA* bisects $P\hat{O}C$ and $A\hat{O}C=62°$.
Required to find: a

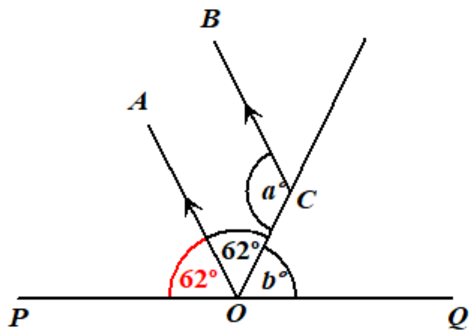
Solution:

$a° + 62° = 180°$ (Co-interior angles are supplementary)

$$a° = 180° - 62°$$
$$= 118°$$
$$a = 118$$

(ii) b

SOLUTION:
Required to find: b
Solution:



Since OA bisects $P\hat{O}C$, then $A\hat{O}P = 62°$.

$b° + 62° + 62° = 180°$ (angles in a straight line total 180°.)

$$b° = 180° - (62° + 62°)$$
$$b° = 56°$$
$$b = 56$$

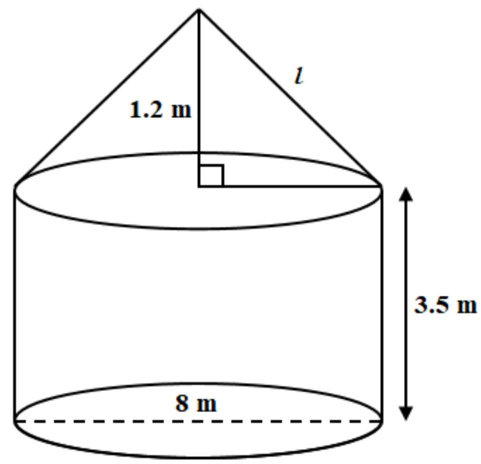
(c) For this question, you may find the following formulas, related to a cone, useful:

Volume $= \frac{1}{3}(\text{base area}) \times (\text{perpendicular height})$

Curved surface area $= \pi r \times (\text{slant height})$

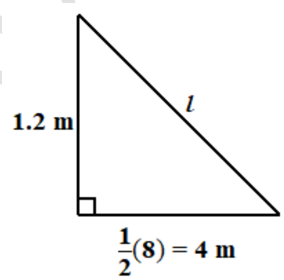
A farmer constructs a storage tank as shown in the diagram below. The shape of the tank consists of a cylindrical body of height 3.5 m and diameter 8 m, and a conical roof of vertical height 1.2 m.

Use $\pi = 3.14$



(i) Calculate the slant height, l, of the roof of the tank.

SOLUTION:
Data: Diagram showing the storage tank constructed by a farmer consisting of a cylindrical body of height 3.5 m and diameter 8 m, and a conical roof of vertical height 1.2 m.
Required to calculate: l
Calculation:



$$l = \sqrt{(1.2)^2 + (4)^2}$$
$$= \sqrt{17.44}$$
$$l = 4.17\underline{\underline{6}}$$
$$= 4.18 \text{ m (correct to 2 decimal places)}$$

(ii) Assume that the storage tank is completely sealed and is to be filled with diesel from an opening at the top. Find the capacity, in m^3, of the tank inclusive of the conical roof.

SOLUTION:
Data: The tank is sealed and is to be filled with diesel.
Required to find: The capacity of the tank, in m^2
Solution:
The volume of the storage tank
$= \text{Volume of the base cylinder} + \text{Volume of the conical top}$

$$= \pi(4)^2 \times 3.5 \text{ m}^3 + \frac{1}{3}\pi(4)^2 \times 1.2 \text{ m}^3$$

$$= 175.84 + 20.096$$

$$= 195.936 \text{ m}^3$$

(iii) Show that the TOTAL curved surface area of the tank, to the nearest square metre, is 140 m^2.

SOLUTION:
Required to show: The curved surface area of the tank is 140 m^2, correct to the nearest square metre
Proof:
Area of the figure
= Area of conical top + Area of curved surface of the cylinder
$= [\pi(4) \times 4.176] + [2\pi(4) \times 3.5]$
= 52.450 + 87.92
= 140.37 m^2
= 140 m^2 correct to the nearest metre

Q.E.D.

(iv) The farmer wishes to paint the **exterior** of the tank. Given that 1 gallon (1 gallon ≈ 3.79 litres) of paint covers 30 m^2 of the surface, determine the amount of paint needed, in litres, to completely paint the exterior of the tank.

SOLUTION:
Data: One gallon of paint covers 30 m^2 of the surface of the tank and 1 gallon ≈ 3.79 litres.
Required to determine: The amount of paint, in litres, needed to paint the exterior of the tank
Solution:
30 m^2 is covered by 3.79 litres.

1 m^2 will be covered by $\frac{3.79}{30}$ l.

140.37 m^2 will be covered by $\frac{3.79}{30} \times 140.37 \text{ l} = 17.73 \text{ l}$

The farmer will likely have to buy 18 litres of paint.

However, this assumes that the base of tank is not considered to be painted.
Area of base of the tank = $\pi \times (4)^2 = 50.24\ m^2$

Total area to be painted = 140.37 + 50.24 = 190.611

If it is, the amount of paint required will be $\frac{3.79}{30} \times 190.61 \text{ l} = 24.08 \text{ l}$.
The farmer will likely have to buy 25 litres of paint.

2. (a) A toy rocket is projected upwards from a point, *O*, on level ground and the vertical height it travels can be modelled by the quadratic function

$$h(x) = 40x - 8x^2$$

where *x* is the horizontal distance travelled from the point *O*.

(i) Find the vertical height of the rocket when it is 1 metre away from *O*.

SOLUTION:
Data: The horizontal distance of a rocket projected upwards from a point *O* is modelled by $h(x) = 40x - 8x^2$, where *x* is the horizontal distance travelled from *O*.
Required to find: The vertical height of the rocket when it is 1 metre away from *O*
Solution:
When $x = 1$

$$h(x) = 40x - 8x^2$$
$$h(1) = 40(1) - 8(1)^2$$
$$= 40 - 8$$
$$= 32$$

So the rocket is 32 metres upwards, that is, at a vertical height of 32 m when it is 1 metre from *O*

(ii) Determine the distance from *O* when the rocket returns to the ground.

SOLUTION:
Required to determine: The distance from *O* when the rocket returns to the ground
Solution:

When the rocket returns to O, $h(x)=0$.

So $40x-8x^2=0$

$8x(5-x)=0$

$x=0$ or 5

$x=0$ at the start, so the rocket returns to the ground when $x=5$.

(iii) What is the value of x when h is greatest? Calculate the value of this greatest height.

SOLUTION:
Required to find: The value of x when h is greatest and the greatest value of h
Solution:
The path of the rocket will be symmetrical about the highest point. The graph representing its path is a parabola as is the shape of a quadratic graph. The vertical through the highest point and which is the line $x = 2.5$ is the axis of symmetry

At the greatest height $x=\frac{1}{2}(5)=2\frac{1}{2}$

$$h(x)=40\left(2\frac{1}{2}\right)-8\left(2\frac{1}{2}\right)^2$$
$$=100-50$$
$$=50 \text{ m}$$

(b) P is the point with coordinates $(-2, 3)$ and Q is the point with co-ordinates $(-5, -1)$.

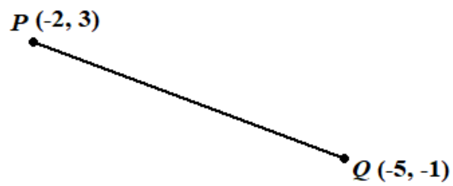
(i) Show that the gradient (slope), m, of the straight line that passes through the points P and Q is $m=\frac{4}{3}$.

SOLUTION:

Data: $P=(-2, 3)$ and $Q=(-5, -1)$

Required To Show: The gradient of the line through P and Q is $\frac{4}{3}$.

Proof:



Gradient of the line passing through P and Q is $\dfrac{3-(-1)}{-2-(-5)}=\dfrac{4}{3}$.

Q.E.D.