FAS-PASS Maths

CSEC ADDITIONAL MATHEMATICS MAY 2024 PAPER 2

SECTION I

ALGEBRA, SEQUENCES AND SERIES

ALL working must be clearly shown.

1. (a) (i) Determine the other linear factors of the polynomial $3x^3+8x^2-20x-16$, given that $x-2$ is a factor.

SOLUTION:
Data: $x-2$ is a factor of $3x^3+8x^2-20x-16$
Required to determine: The other linear factors of $3x^3+8x^2-20x-16$
Solution:

$$\begin{array}{r} 3x^2+14x+8 \\ x-2\overline{)3x^3+8x^2-20x-16} \\ -\ \underline{3x^3-6x^2} \\ 14x^2-20x \\ -\ \underline{14x^2-28x} \\ 8x-16 \\ -\ \underline{8x-16} \\ \underline{0} \end{array}$$

$$3x^2+14x+8=(3x+2)(x+4)$$

Hence, the other linear factors are $(3x+2)$ and $(x+4)$.

(ii) Hence, simplify the multiplication

$$\frac{3x^3+8x^2-10x-16}{x^2-4}\times\frac{x+2}{x+4}$$

SOLUTION:

Required to simplify: $\dfrac{3x^3+8x^2-10x-16}{x^2-4}\times\dfrac{x+2}{x+4}$

Solution:

$$x^2-4=(x)^2-(2)^2 \quad \text{(Difference of 2 squares)}$$
$$=(x-2)(x+2)$$

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So, $\dfrac{3x^3+8x^2-10x-16}{x^2-4} \times \dfrac{x+2}{x+4} = \dfrac{\cancel{(x-2)}(3x+2)\cancel{(x+4)}}{\cancel{(x-2)}\cancel{(x+2)}} \times \dfrac{\cancel{x+2}}{\cancel{x+4}}$

$= 3x+2$

(b) The equation $kx^2+x-15=10$ has roots α and β, where $k \in W$.

(i) Determine expressions for

- $\alpha+\beta$
- $\alpha\beta$

SOLUTION:
Data: The equation $kx^2+x-15=10$ has roots α and β, where $k \in W$.
Required to determine: Expressions for $\alpha+\beta$ and $\alpha\beta$
Solution:

If α and β are the roots of $ax^2+bx+c=0$, then $\alpha+\beta=\dfrac{-b}{a}$ and $\alpha\beta=\dfrac{c}{a}$.

Hence, in the given equation, $kx^2+x-15=10$

$$\Rightarrow kx^2+x-15-10=0$$

$$\Rightarrow kx^2+x-25=0$$

$$\alpha+\beta=\frac{-1}{k}$$

$$\alpha\beta=\frac{-25}{4}$$

(ii) Given that $\alpha^2+\beta^2=\dfrac{61}{4}$, use the expression in (b) (i) to determine the value of k.

SOLUTION:
Data: $\alpha^2+\beta^2=\dfrac{61}{4}$
Required to determine: The value of k
Solution:

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$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$
$$\therefore \frac{61}{4} = \left(-\frac{1}{k}\right)^2 - 2\left(-\frac{25}{k}\right)$$
$$\frac{61}{4} = \frac{1}{k^2} + \frac{50}{k}$$

$(\times\, 4k^2)$

$$61k^2 = 4 + 200k$$
$$61k^2 - 200k - 4 = 0$$

$$k = \frac{-(-200) \pm \sqrt{(-200)^2 - 4(61)(-4)}}{2(61)}$$
$$= \frac{200 \pm \sqrt{39024}}{122}$$
$$= \frac{200 \pm 12\sqrt{271}}{122}$$

$$k = \frac{200 + 12\sqrt{271}}{122} \text{ or } \frac{200 - 12\sqrt{271}}{122}$$

$$= 3.26 \text{ } or \text{ } 0.02$$

According to the data $k \in W$ and so an error lies in the question and NOT in the solution

(c) For the quadratic function $g(x) = -5x^2 - 4x + 2$, determine the value of the maximum point **and** the range using the method of completing the square, or otherwise.

SOLUTION:
Data: $g(x) = -5x^2 - 4x + 2$
Required to determine: The value of the maximum point and the range of $g(x)$.
Solution:
$g(x) = 2 - (5x^2 + 4x)$

$$= 2 - 5\left(x^2 + \frac{4}{5}x\right)$$
$$= * -5\left(x + \frac{2}{5}\right)^2$$

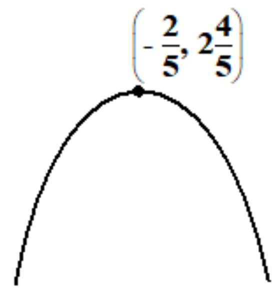
$$g(x) = * -5\left(x^2 + \frac{4}{5}x + \frac{4}{25}\right)$$

$$g(x) = * - 5x^2 - 4x - \frac{4}{5}$$
$$g(x) = -5x^2 - 4x - \frac{4}{5} + 2\frac{4}{5}$$
$$\text{So, } * = 2\frac{4}{5}$$
$$\therefore g(x) = 2\frac{4}{5} - 5\left(x + \frac{2}{5}\right)^2$$
$$\downarrow$$
$$\geq 0 \forall x$$

$$\therefore g(x)_{\text{maximum}} = 2\frac{4}{5} \text{ and this occurs when } \left(x + \frac{2}{5}\right)^2 = 0$$
$$\text{i.e when } x = -\frac{2}{5}$$

Hence, the maximum point is $\left(-\frac{2}{5}, 2\frac{4}{5}\right)$

and the range is $g(x) \leq 2\frac{4}{5}$.



Alternative Method:

Coefficient of $x^2 = -$ve in a quadratic function, so $g(x)$ has a maximum point.

Axis of symmetry is $x = \dfrac{-(-4)}{2(-5)}$

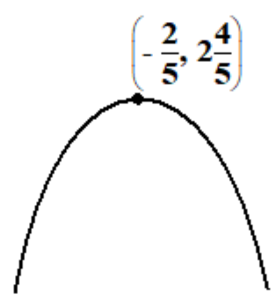
$$x = -\frac{2}{5}$$

This is the x-coordinate of the maximum point.

$$g\left(-\frac{2}{5}\right) = -5\left(-\frac{2}{5}\right)^2 - 4\left(-\frac{2}{5}\right) + 2$$
$$= 2\frac{4}{5}$$

Hence, the maximum point is $\left(-\frac{2}{5}, 2\frac{4}{5}\right)$

and the range is $g(x) \leq 2\frac{4}{5}$.



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Alternative Method:

$g(x) = -5x^2 - 4x + 2$

$g'(x) = -5(2x) - 4(1) + 0$

$= -10x - 4$

Let $g'(x) = 0$

$10x - 4 = 0$

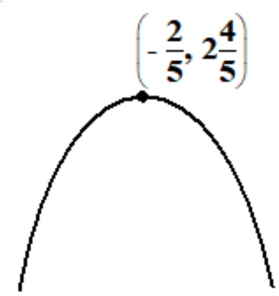
$x = -\frac{2}{5}$

$g''(x) = -10$

$\therefore g(x)$ has a maximum value at $x = -\frac{2}{5}$ since $g''(x) < 0$.

$g\left(-\frac{2}{5}\right) = -5\left(-\frac{2}{5}\right)^2 - 4\left(-\frac{2}{5}\right) + 2$

$= 2\frac{4}{5}$



Hence, the maximum point is $\left(-\frac{2}{5}, 2\frac{4}{5}\right)$

and the range is $g(x) \leq 2\frac{4}{5}$.

2. (a) Write the expression $2\log_3 x + 2 - \log_3 y$ as a single term.

SOLUTION:
Required to write: $2\log_3 x + 2 - \log_3 y$ as a single term
Solution:

$$2\log_3 x + 2 - \log_3 y = 2\log_3 x + 2\log_3 3 - \log_3 y$$
$$= \log_3 x^2 + \log_3 3^2 - \log_3 y$$
$$= \log_3 9 \times x^2 - \log_3 y$$
$$= \log_3\left(\frac{9x^2}{y}\right)$$

(b) (i) By using logarithms, express the relationship $V = 7 \times 5^t$ in linear form.

SOLUTION:
Required to express: $V = 7 \times 5^t$ in linear form

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Solution:

$V = 7 \times 5^t$

Take $\lg$:

$\lg V = \lg\left(7 \times 5^t\right)$

$\lg V = \lg 7 + \lg 5^t$

$\lg V = \lg 7 + t\lg 5$

$\lg V = t\lg 5 + \lg 7$ which is in the linear form
(lg V and t are variables and lg 5 and lg 7 are constants).

(ii) Hence, state the value of the gradient of the line which represents the relationship in (b) (i).

SOLUTION:
Required to state: The value of the gradient of the line which represents the relationship in (b) (i).
Solution:
Recall when $y = mx + c$, where m and c are constants and x and y are variables, $m =$ gradient and $c =$ the intercept on the vertical axis

$\lg V = t\lg 5 + \lg 7$

$\lg V = (\lg 5)t + \lg 7$

This is of the form $y = mx + c$, where $m = \lg 5$ is the gradient.

(c) Rationalise the denominator of the expression $\dfrac{1+\sqrt{2}}{3-\sqrt{2}}$.

SOLUTION:

Required to Rationalise: The denominator in the expression $\dfrac{1+\sqrt{2}}{3-\sqrt{2}}$

Solution:
To rationalise the denominator, we multiply both the numerator and the denominator by the conjugate of the denominator.
This gives

$$\frac{1+\sqrt{2}}{3-\sqrt{2}} = \frac{1+\sqrt{2}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$$

$$= \frac{3+3\sqrt{2}+\sqrt{2}+2}{9-3\sqrt{2}+3\sqrt{2}-2}$$

$$= \frac{5+4\sqrt{2}}{7}$$

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(d) Evaluate $\sum_{i=0}^{4} 5^{i-2}$.

SOLUTION:

Required to evaluate: $\sum_{i=0}^{4} 5^{i-2}$

Solution:

The sum of 5^{i-2} from $i=0$ to $i=4$ inclusive is

$$\sum_{i=0}^{4} 5^{i-2} = 5^{-2} + 5^{-1} + 5^{0} + 5^{1} + 5^{2}$$

$$= \frac{1}{25} + \frac{1}{5} + 1 + 5 + 25$$

$$= 26\frac{6}{25}$$

(e) Determine whether the following the sequence is divergent or convergent. Justify your response.

$$1, \quad -\frac{2}{3}, \quad \frac{4}{9}, \quad -\frac{8}{27}, \ldots$$

SOLUTION:

Required to determine: Whether the sequence $1, \quad -\frac{2}{3}, \quad \frac{4}{9}, \quad -\frac{8}{27}, \ldots$ is divergent or convergent.

Solution:

$$1 \times -\frac{2}{3} = -\frac{2}{3}$$

$$-\frac{2}{3} \times -\frac{2}{3} = \frac{4}{9}$$

$$\frac{4}{9} \times -\frac{2}{3} = -\frac{8}{27}$$

The n th term of the series can be expressed in the form ar^n, where $a = 1$ and $r = \frac{-2}{3}$

The series is a geometric progression with the first term $a = 1$ and common ratio $r = -\frac{2}{3}$,

We notice that $|r| < 1$.

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Hence, $S_\infty = \dfrac{a}{1-r}$

$= \dfrac{1}{1-\left(-\frac{2}{3}\right)}$

$= \dfrac{1}{\frac{5}{3}}$ that

$= \dfrac{3}{5}$

$\therefore$ The sequence converges to a limit of $\dfrac{3}{5}$.

This means that the more terms we add, the closer and closer the sum approaches $\dfrac{3}{5}$.

SECTION II

COORDINATE GEOMETRY, VECTORS AND TRIGONOMETRY

ALL working must be clearly shown.

3. (a) Determine the points of intersection of the circle $x^2 + y^2 - 4x + 6y + 8 = 0$ and the line $y = x - 6$.

SOLUTION:
Required to determine: The points of intersection of the circle $x^2 + y^2 - 4x + 6y + 8 = 0$ and the line $y = x - 6$
Solution:
Let $y = x - 6$ …❶
And $x^2 + y^2 - 4x + 6y + 8 = 0$ …❷

Substitute ❶ into ❷:

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$$x^2+(x-6)^2-4x+6(x-6)+8=0$$
$$x^2+x^2-12x+36-4x+6x-36+8=0$$
$$2x^2-10x+8=0$$
$$x^2-5x+4=0$$
$$(x-1)(x-4)=0$$
$$\therefore x=1 \quad \text{and} \quad x=4$$

When $x=1$ $\qquad y=1-6$
$y=-5$

When $x=4$ $\qquad y=4-6$
$=-2$

$\therefore$ The points of intersection of the line and the circle are $(1,-5)$ and $(4,-2)$.

(b) **(i)** Given the coordinates of the points A and B are $(7,-3)$ and $(2, 1)$ respectively, state the position vectors corresponding to the points A and B in the form $x\text{i}+y\text{j}$.

SOLUTION:
Data: The coordinates of the points A and B are $(7,-3)$ and $(2, 1)$
Required to state: The position vectors corresponding to the points A and B in the form $x\text{i}+y\text{j}$.
Solution:
$A=(7,-3)$
$\overrightarrow{OA}=7\text{i}-3\text{j}$ which is of the form $x\text{i}+y\text{j}$, where $x=7$ and $y=-3$.

$B=(2, 1)$
$\overrightarrow{OB}=2\text{i}+\text{j}$, which is of the form $x\text{i}+y\text{j}$, where $x=2$ and $y=1$.

(ii) Determine $\overrightarrow{AB}$.

SOLUTION:
Required to determine: $\overrightarrow{AB}$
Solution:

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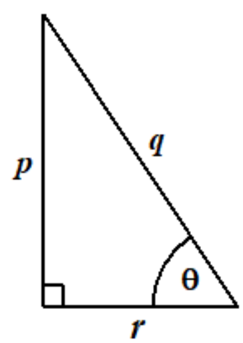
$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
$= -(7\text{i} - 3\text{j}) + 2\text{i} + \text{j}$
$= -5\text{i} + 4\text{j}$ is of the form $x\text{i} + y\text{j}$, $x = -5$ and $y = 4$.

(iii) Calculate the value of the scalar product $OA.\ OB$.

SOLUTION:
Required to calculate: The value $OA.\ OB$
Calculation:

$$\overrightarrow{OA}.\overrightarrow{OB} = (7i - 3j).(2i + j)$$
$$= (7 \times 2) + (-3 \times 1)$$
$$= 14 - 3$$
$$= 11$$

(c) **(i)** Using the letters *p*, *q* or *r*, write an expression for EACH of the following trigonometric ratios.



$\sin\theta$

$\cos\theta$

SOLUTION:
Data: Diagram of a right-angled triangle with the sides labelled *p*, *q* and *r* and one interior angle θ.
Required to write: An expression for $\sin\theta$ and $\cos\theta$, in terms of *p*, *q* and *r*
Solution:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{p}{q} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{r}{q}$$

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(ii) Using your answers in (c) (i), determine the value of $\sin^2\theta+\cos^2\theta$. Recall that $\sin^2\theta=(\sin\theta)^2$.

SOLUTION:

Data: $\sin^2\theta=(\sin\theta)^2$

Required to determine: The value of $\sin^2\theta+\cos^2\theta$

Solution:

$$\sin^2\theta+\cos^2\theta=\left(\frac{p}{q}\right)^2+\left(\frac{r}{q}\right)^2$$

$$=\frac{p^2}{q^2}+\frac{r^2}{q^2}$$

$$=\frac{p^2+r^2}{q^2}$$

Recall: $p^2+r^2=q^2$ (Pythagoras' Theorem)

So, $\sin^2\theta+\cos^2\theta=\frac{q^2}{q^2}=1$

(d) Prove the identity $\tan A+\frac{1}{\tan A}=\frac{1}{\sin A\cos A}$.

SOLUTION:

Required to prove: $\tan A+\frac{1}{\tan A}=\frac{1}{\sin A\cos A}$

Proof:

Consider the L.H.S.:

and recall: $\tan A=\frac{\sin A}{\cos A}$

$$\tan A+\frac{1}{\tan A}=\frac{\sin A}{\cos A}+\frac{1}{\frac{\sin A}{\cos A}}$$

$$=\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}$$

$$=\frac{(\sin A)(\sin A)+(\cos A)(\cos A)}{\cos A\sin A}$$

$$=\frac{\sin^2 A+\cos^2 A}{\cos A\sin A}$$

Recall: $\sin^2 A+\cos^2 A=1$the LHS reduces to $\frac{1}{\sin A+\cos A}=\text{RHS}$

Hence LHS $=$ RHS and $\tan A+\frac{1}{\tan A}=\frac{1}{\sin A\cos A}$ **Q.E.D.**

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SECTION III

INTRODUCTORY CALCULUS

ALL working must be clearly shown.

4. (a) (i) Use the definition of the derivative as a limit to find $f'(x)$ for the function $f(x)=x^2-3$.

SOLUTION:
Data: $f(x)=x^2-3$
Required to find: $f'(x)$ using the definition of the derivative as a limit
Solution:

$$f'(x)=\lim_{h\to 0}\frac{\{(x+h)^2-3\}-(x^2-3)}{(x+h)-x}$$

$$=\frac{x^2+2hx+h^2-3-x^2+3}{h}$$

$$=\frac{2hx+h^2}{h}$$

$$=2x+h$$

$$\lim_{h\to 0}(x^2-3)=2x+0$$

$$\therefore f'(x)=2x$$

(ii) Hence, determine the value of $f''(5)$.

SOLUTION:
Required to determine: The value of $f''(5)$.
Solution:
Since $f'(x)=2x$, $f''(x)=2$, which is a constant.
Hence, $f''(5)$ is not feasible.

(b) Given that $y=\frac{\sin x}{\cos x}$, show that $\frac{dy}{dx}=\frac{1}{\cos^2 x}$.

SOLUTION:
Data: $y=\frac{\sin x}{\cos x}$
Required To Show: $\frac{dy}{dx}=\frac{1}{\cos^2 x}$

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Proof:

$y=\frac{\sin x}{\cos x}$ is of the form $\frac{u}{v}$, where $u=\sin x$, $\frac{du}{dx}=\cos x$

and $v=\cos x$, $\frac{dv}{dx}=-\sin x$

$$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2} \quad \text{(Quotient law)}$$

$$=\frac{(\cos x)(\cos x)-(\sin x)(-\sin x)}{(\cos x)^2}$$

$$=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}$$

Recall: $\sin^2 x+\cos^2 x=1$

$$\frac{dy}{dx}=\frac{1}{\cos^2 x}$$

Q.E.D.

(c) The equation of a curve is given by $y=\left(5x^2-7\right)^4$. Determine the equation of the gradient of the curve.

SOLUTION:

Data: The curve has equation $y=\left(5x^2-7\right)^4$

Required to determine: The equation of the gradient of the curve

Solution:

$$y=\left(5x^2-7\right)^4$$

Let $t=5x^2-7$ $\Rightarrow$ $\therefore y=t^4$

$$\frac{dt}{dx}=5\left(2x^{2-1}\right)$$
$$=10x$$

$$\frac{dy}{dt}=4t^{4-1}$$
$$=4t^3$$

$$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx} \quad \text{(Chain rule)}$$

$$=4t^3\times 10x$$

$$=40x\left(5x^2-7\right)^3$$

Hence, the gradient function of the curve is $\frac{dy}{dx}=40x\left(5x^2-7\right)^3$.

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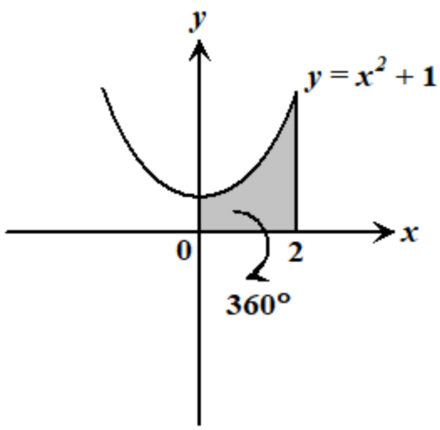
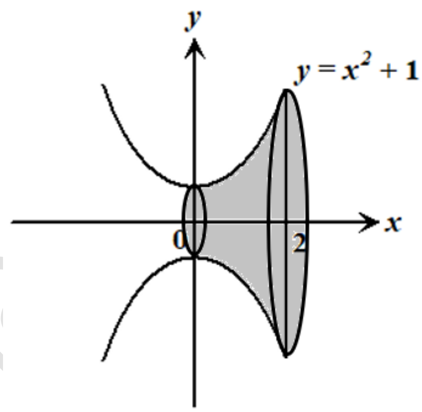
5. (a) The region enclosed between the curve $y = x^2 + 1$, the *x-axis*, the y – axis and the line $x = 2$ is rotated about the *x–axis* through an angle of 360°.

Calculate the volume of the solid of revolution that is formed.

SOLUTION:
Data: The region enclosed between the curve $y = x^2 + 1$, the *x–axis*, the y – axis and the line $x = 2$ is rotated about the x – *axis* through an angle of 360°.
Required to calculate: The volume of the solid of revolution formed
Calculation:





Volume of the solid generated $= \pi \int_0^2 y^2 \, dx$

$$= \pi \int_0^2 \left(x^2 + 1\right)^2 \, dx$$

$$= \pi \int_0^2 \left(x^4 + 2x^2 + 1\right) dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x + C \right]_0^2, \text{ where } C \text{ is a constant}$$

$$= \pi \left[\left(\frac{(2)^5}{5} + \frac{2(2)^3}{3} + 2 \right) - (0) \right]$$

$$= 13\frac{11}{15}\pi \text{ units}^3$$

(b) As a particle moves along a straight line, its displacement is measured from a fixed point, O, on the line. At time t seconds, the acceleration, a, is given by $a = 24t - 14$.

(i) Find the expression for the velocity, v, of the particle, given that when $t = 1$, $v = 3$ ms^{-1}.

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SOLUTION:
Data: The displacement of a particle is measured from a fixed point, *O*, as it moves along a straight line. Its acceleration is, $a = 24t - 14$ after t seconds.
Required to find: An expression for the velocity, *v*, of the particle given that $t = 1,\ v = 3 \text{ ms}^{-1}$
Solution:

$$v = \int a\,dt$$
$$= \int (24t - 14)\,dt$$
$$v = \frac{24t^2}{2} - 14t + C, \text{ where } C \text{ is } a \text{ constant}$$
$$v = 12t^2 - 14t + C$$

When $t = 1,\ v = 3 \text{ ms}^{-1}$:

$$\therefore 3 = 12(1)^2 - 14(1) + C$$
$$3 = -2 + C$$
$$C = 5$$

Hence, $v = 12t^2 - 14t + 5$.

(ii) Using your answer in (b) (i), find an expression for the displacement, *s*, of the particle, given that when $t = 1,\ s = 10$ m.

SOLUTION:
Required to find: An expression for the displacement, *s*, of the particle, given that when $t = 1,\ s = 10$ m
Solution:

$$s = \int v\ dt$$
$$= \int (12t^2 - 14t + 5)\ dt$$
$$= \frac{12t^3}{3} - \frac{14t^2}{2} + 5t + K, \text{ where } K \text{ is a constant}$$
$$s = 4t^3 - 7t^2 + 5t + K$$

When $t = 1,\ s = 10$ m:

$$\therefore 10 = 4(1)^3 - 7(1)^2 + 5(1) + K$$
$$K = 8$$

Hence, $s = 4t^3 - 7t^2 + 5t + 8$.

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(c) The gradient function of the curve C is given by $7-2x$. If the curve passes through the point $(3,\ 8)$, find the equation of the curve.

SOLUTION:

Data: For a curve, C, that passes through the point $(3,\ 8)$, $\frac{dy}{dx}=7-2x$

Required to find: The equation of the curve
Solution:

Let the gradient function be $\frac{dy}{dx}$.

$\therefore$ Equation of the curve is $y=\int\frac{dy}{dx}\,dx$

$$y=\int(7-2x)\,dx$$

$$y=7x-\frac{2x^2}{2}+C, \text{ where } C \text{ is a constant}$$

$$y=7x-x^2+C$$

$(3,\ 8)$ lies on the curve.

$$\therefore 8=7(3)-(3)^2+C$$
$$8=21-9+C$$
$$C=-4$$

$\therefore$ The equation of the curve is $y=7x-x^2-4$.

SECTION IV

PROBABILITY AND STATISTICS

ALL working must be clearly shown.

6. (a) The heights of 35 children at a nursery were recorded to the nearest centimetre. The data is shown below.

61	118	79	90	83	70	80
95	75	76	92	62	115	79
71	103	109	84	65	86	92
111	78	94	74	99	81	108
108	67	109	92	79	116	62

(i) Display the data shown on a stem-and-leaf diagram.

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SOLUTION:
Data: Raw data showing the heights of 35 children at a nursery, recorded to the nearest centimetre
Required to display: The data on a stem-and-leaf diagram
Solution:
Organising the data in ascending order:

61	62	62	65	67				
70	71	74	75	76	78	79	79	79
80	81	83	84	86				
90	92	92	92	94	95	99		
103	108	108	109	109	111	115	116	118

The completed stem-and-leaf looks like:

Stem	**Leaf**								
6	1	2	2	5	7				
7	0	1	4	5	6	8	9	9	9
8	0	1	3	4	6				
9	0	2	2	2	4	5	9		
10	3	8	8	9	9				
11	1	5	6	8					

Key:
Stem $= 10\text{'s}$
Leaf $= 1\text{'s}$
Example: $6|8 = 68$

(ii) From the stem-and-leaf diagram in (a) (i), determine the value of the

- median

SOLUTION:
Required to determine: The value of the median
Solution:
The middle value of 35, which is the 18$^{\text{th}}$ value, is the median.

Stem	**Leaf**								
6	1	2	2	5	7				
7	0	1	4	5	6	8	9	9	9
8	0	1	3	4	6				
9	0	2	2	2	4	5	9		
10	3	8	8	9	9				
11	1	5	6	8					

$\therefore$ Median $= 84$

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- lower quartile

SOLUTION:
Required to determine: The value of the lower quartile
Solution:
The middle of the first 17 values is the 9^{th} value.

Stem	Leaf								
6	1	2	2	5	7				
7	0	1	4	5	6	8	9	9	9
8	0	1	3	4	6				
9	0	2	2	2	4	5	9		
10	3	8	8	9	9				
11	1	5	6	8					

$\therefore$ Lower quartile $= 75$

- upper quartile.

SOLUTION:
Required to determine: The value of the upper quartile
Solution:

The middle value for the 19^{th} to the 35^{th} value is the 27^{th} value.

Stem	Leaf								
6	1	2	2	5	7				
7	0	1	4	5	6	8	9	9	9
8	0	1	3	4	6				
9	0	2	2	2	4	5	9		
10	3	8	8	9	9				
11	1	5	6	8					

$\therefore$ Upper quartile $= 103$

(iii) At another nearby nursery, the heights of children of the same age were recorded. The median height was 79 cm and the interquartile range was 24 cm.

Compare the characteristics of the two groups of children at the two nurseries and describe ONE distinct observation about the distributions.

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SOLUTION:
Data: At another nearby nursery, the heights of children of the same age were recorded. The median height was 79 cm and the interquartile range was 24 cm.
Required to compare: The characteristics of the two groups of children and describe one distinct observation about the distributions.
Solution:
From (a), the median $= 84$ and the interquartile range $= 103 - 75$
$= 28$
Consider the groups as Group A and Group B.

	Group A	Group B
Median	84	79
I.Q.R.	28	24

Group A has a greater interquartile range than Group B. This indicates that the data in group A has a greater spread than the data in Group B.

When the medians are compared, the data indicates that the median of Group A is greater than the median of Group B. This implies:

In Group A, 50% of the students have heights greater than 84 cm while 50% of the students have heights lower than 84 cm.

In Group B, 50% of the students have heights greater than 79 cm while 50% of the students have heights lower than 79 cm.

We can therefore conclude that the children in Group A are taller when compared to the children in Group B.

(b) **(i)** A bag contains 6 red marbles and 5 black marbles. During 2 rounds of a marble game, a student is required to randomly draw 1 marble without replacement for each round. Construct a probability tree diagram to represent this information.

SOLUTION:
Data: A bag contains 6 red marbles and 5 black marbles. During 2 rounds of a marble game, a student is required to randomly draw 1 marble without replacement for each round.
Required to construct: A probability tree diagram to represent this information
Solution:
Let R represent the event of choosing a red marble
Let B represent the event of choosing a black marble

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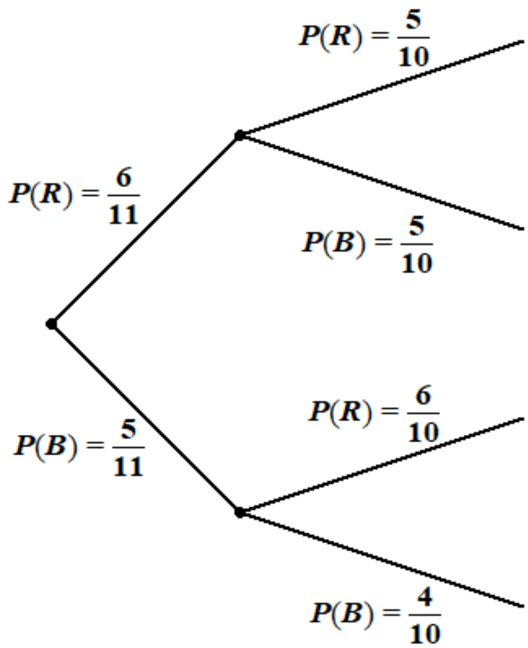


(ii) Using your answer in (b) (i), find the probability that the marbles drawn are ALL the SAME colour.

SOLUTION:
Required to find: The probability that the marbles drawn are all the same colour
Solution:

$$P(\text{same colour}) = P(R \text{ and } R) \text{ or } P(B \text{ and } B)$$
$$= P((R \cap R) \cup (B \cap B))$$
$$= \left(\frac{6}{11} \times \frac{5}{10}\right) + \left(\frac{5}{11} \times \frac{4}{10}\right)$$
$$= \frac{30 + 20}{110}$$
$$= \frac{50}{110}$$
$$= \frac{5}{11}$$

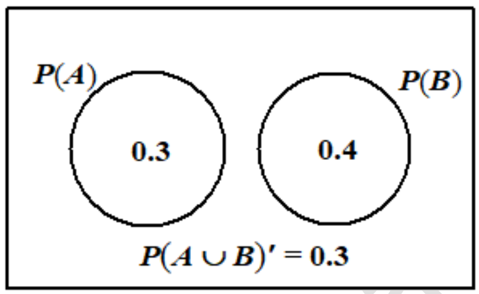
(c) Given two events, A and B, $P(A \cup B) = 0.7$, $P(A) = 0.3$ and $P(B') = 0.6$.

(i) Calculate $P(A \cap B)$

SOLUTION:
Data: Two events A and B are such that $P(A \cup B) = 0.7$, $P(A) = 0.3$ and $P(B') = 0.6$.

Required to calculate: $P(A \cap B)$
Calculation:
If $P(B') = 0.6$, then $P(B) = 1 - 0.6$
$= 0.4$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\therefore 0.7 = 0.3 + 0.4 - P(A \cap B)$$
$$P(A \cap B) = 0$$



$\therefore A$ and B are disjoint sets and $P(A \cap B) = 0$.

(ii) What is the relationship between Events A and B?

SOLUTION:
Required To State: The relationship between Events A and B
Solution:
$$P(A \cup B) = P(A) + P(B)$$
$$0.7 = 0.3 + 0.4$$

Hence, A and B are mutually exclusive events.

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