CSEC MATHEMATICS JANUARY 2024 PAPER 2

SECTION I

Answer ALL questions.

All working MUST be clearly shown.

1. (a) Express, as a single fraction in its simplest form,

$$1-\left(\frac{1}{30}+\frac{4}{15}\right)$$

SOLUTION:

Required to express: $1-\left(\frac{1}{30}+\frac{4}{15}\right)$ as a single fraction in its simplest form

Solution:

$1-\left(\frac{1}{30}+\frac{4}{15}\right)$

First, we consider the part of the computation that is written within brackets:

$\frac{1}{30}+\frac{4}{15}$

$$\frac{1(1)+2(4)}{30}=\frac{1+8}{30}$$
$$=\frac{9}{30}$$

Now, we can complete the entire computation.

We have $1-\left(\frac{1}{30}+\frac{4}{15}\right)=1-\frac{9}{30}$

$$=\frac{30}{30}-\frac{9}{30}$$
$$=\frac{21}{30}$$
$$=\frac{7}{10} \text{ as a single fraction in its simplest form}$$

(b) A two-storey car park has a total of 1 020 parking spaces. At 06:30 hours one morning, $\frac{1}{30}$ of the 1 020 spaces are filled. During the next hour, no cars left the car park but another $\frac{4}{15}$ of 1 020 spaces were filled. Determine the number of parking spaces that were not filled at 07:30 hours.

SOLUTION:
Data: A two-storey car park has a total of 1 020 parking spaces. At 06:30 hours one morning, $\frac{1}{30}$ of the 1 020 spaces were filled. During the next hour, no cars left the car park but another $\frac{4}{15}$ of 1 020 spaces were filled.
Required to determine: The number of parking spaces that were not filled at 07:30 hours.
Solution:
Number of available spaces in the carpark $=1020$

$\frac{1}{30}$ of $1020 = 34$

Hence, at 06:30 hours the number of spaces taken $= 34$

$\frac{4}{15}$ of $1020 = 272$
Therefore, the number of spaces taken at 07:30 hours = 272+34 = 306

Hence, the number of spaces available at 07:30 hours $= 1020 - 306$
$= 714$ spaces

Alternative Method:
In part (a), we were asked to compute $1-\left(\frac{1}{30}+\frac{4}{15}\right)$. We note that to solve the word problem in part (b) we can use the same numerical expression.

Fraction of number of parking spaces that are not filled:

= 1– Fraction of parking spaces that are filled
$= 1 - \left(\frac{1}{30} + \frac{4}{15}\right)$
$= \frac{7}{10}$

This answer was already obtained in part (a).
Total number of parking spaces $= 1020$
Number of spaces that are not filled $= \frac{7}{10} \times 1020$
$= 714$

(c) Of 1 020 parking spaces, 20% are on the top level. How many parking spaces are on the top level?

SOLUTION:
Data: Of 1 020 parking spaces, 20% are on the top level
Required to find: The number of parking spaces that are on the top level.
Solution:

$$\begin{aligned}20\% \text{ of } 1020 &= \frac{20}{100}\times 1020\\ &= 204\end{aligned}$$

Hence, there are 204 parking spaces at the top level.

(d) Some of the spaces are reserved for monthly paying customers. The ratio of reserved spaces to non-reserved spaces is $5:7$.

Calculate the number of non-reserved parking spaces.

SOLUTION:
Data: Some spaces are reserved for monthly paying customers. The ratio of reserved spaces to non-reserved spaces is $5:7$.
Required to calculate: The number of non-reserved parking spaces
Solution:

$$\begin{gathered}\text{Reserved} : \text{Non} - \text{reserved}\\ 5:7\end{gathered}$$

Hence, the number of reserved spaces $= 5x$ and, the number of non-reserved spaces $= 7x$, where $x \in Z^{+}$.

Therefore,
$$\begin{aligned}5x + 7x &= 1020\\ 12x &= 1020\\ x &= \frac{1020}{12}\\ &= 85\end{aligned}$$

$\therefore$ The number of non-reserved spaces = $7x$ $= 7\times 85$
$= 595$

Alternative Method:
The ratio of reserved spaces to non-reserved spaces is $5:7$.

Fraction of spaces that are non-reserved $= \frac{7}{12}$

$$\begin{aligned}\text{Number of non-reserved spaces } &= \frac{7}{12}\times 1020\\ &= 595\end{aligned}$$

(e) The cost for parking at the car park is shown in the table below.

Length of Visit	**Cost ($)**
Under 30 minutes	Free
More than 30 minutes and up to 2 hours	$2.5
More than 2 hours and up to 4 hours	$5.50
More than 4 hours and up to 8 hours	$9.25
More than 8 hours and up to 24 hours	$15.00
One-week ticket	$40.00

(i) Mikayla leaves the car park at 18:30 hours and pays $9.25. Determine the earliest time she could have arrived at the car park.

SOLUTION
Data: Table showing the cost of parking in the car park
Required to determine: The earliest time Mikayla could have arrived in the car park.
Solution:
Mikayla leaves at 18:30 hours and pays $9.25
Hence, according to the table, Mikayla would have had to pay for a stay of between 4 hours and 8 hours.

Hence, the earliest time that Mikayla would have arrived would be 8 hours before 18:30 hours.

$$\begin{array}{r} 18:30 \\ -\quad 8:00 \\ \hline 10:30 \\ \hline \end{array}$$

So, Mikayla could have arrived as early as 10:30 hours.

(ii) Dhanraj bought a weekly parking ticket for $40. That week, he visited the car park five different times. The length of time he parked his car on each occasion is given below.

25 minutes $\quad 7\frac{1}{2}$ hours $\quad$ 11 hours $\quad 8\frac{3}{4}$ hours $\quad$ 8 hours

Show that Dhanraj saves $8.50 by buying the weekly ticket.

SOLUTION:
Data: Dhanraj bought a weekly parking ticket for \$40. He visited the car park on 5 occasions and the length of time he parked his car on each occasion is 25 minutes, $7\frac{1}{2}$hours, 11 hours, $8\frac{3}{4}$hours and 8 hours.
Required to show: Dhanraj saved \$8.50, by buying the weekly ticket.
Proof:
If Dhanraj was paying according to the hourly rate or non-weekly ticket system as shown on the table, his fees would be:

Time	**Cost According to Table**
25 minutes	\$ 0.00
$7\frac{1}{2}$hours	\$ 9.25
11 hours	\$15.00
$8\frac{3}{4}$hours	\$15.00
8 hours	\$ 9.25
Total	\$48.50

The cost of the weekly ticket $= \$40$

Hence, by buying the weekly ticket, Dhanraj would have saved:
\$48.50 – \$8.50
= \$8.50

Q.E.D.

2. (a) Simplify

$$\frac{x^2+7x}{x^2-49}$$

SOLUTION:

Required to simplify: $\frac{x^2+7x}{x^2-49}$

Solution:

$$\frac{x^2+7x}{x^2-49}=\frac{x(x+7)}{(x)^2-(7)^2} \quad \text{Difference of 2 squares}$$

$$=\frac{x\cancel{(x+7)}}{(x-7)\cancel{(x+7)}}$$

$$=\frac{x}{x-7}$$

(b) Find the value of

(i) r when $x^2 \times x^6 = x^r$

SOLUTION:
Required to find: r when $x^2 \times x^6 = x^r$
Solution:

$x^2 \times x^6 = x^r$

$\therefore x^{2+6} = x^r$

$x^8 = x^r$

Equating indices since the bases are equal, we obtain:

$r = 8$

(ii) s when $s^3 = 8$

SOLUTION:
Required to find: s when $s^3 = 8$
Solution:

$s^3 = 8$

$s^3 = 2^3$

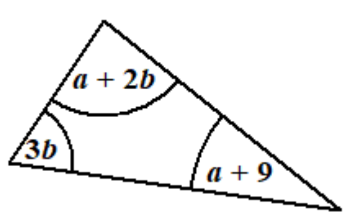
The indices are equal, so the bases are equal.

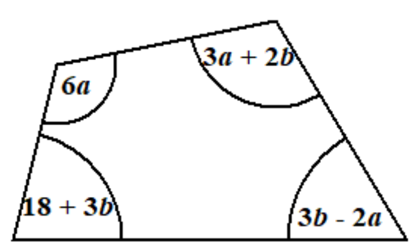
$\therefore s = 2$

OR

We could have said that since $s^3 = 8$ we can take the cube root on both sides of the equation.
So, $s = \sqrt[3]{8}$ or $8^{1/3} = 2$

(c) The diagrams below shows a triangle and a quadrilateral. All angles are in degrees and are written in terms of *a* and/or *b*.





(i) For the triangle, show that $2a + 5b = 171$.

SOLUTION:
Data: Diagrams showing a triangle and a quadrilateral with all interior angles labelled, in terms of *a* and *b*
Required to show: $2a+5b=171$ for the triangle
Proof:
The sum of the three interior angles in a triangle $=180°$
The three angles of the triangle are all written in degrees so we can write

$$(a+2b)+(3b)+(a+9)=180$$
$$a+2b+3b+a+9=180$$
$$\underbrace{a+a}+\underbrace{2b+3b}=180-9$$
$$2a+5b=171$$

Q.E.D.

(ii) For the quadrilateral, show that $7a+8b=342$.

SOLUTION:
Required to show: $7a+8b=342$ for the quadrilateral
Proof:
The sum of the four interior angles of a quadrilateral $=360°$
The four angles of the quadrilateral are all written in degrees so we can write

$$(6a)+(3a+2b)+(3b-2a)+(18+3b)=360°$$
$$6a+3a+2b+3b-2a+18+3b=360°$$
$$\underbrace{6a+3a-2a}+\underbrace{2b+3b+3b}=360-18$$
$$7a+8b=342$$

Q.E.D.

(iii) Solve the pair of simultaneous equations in (i) and (ii) to find the values of *a* and *b*. **Show all working**.

SOLUTION:
Required to solve: The pair of simultaneous equations to find the value of *a* and of *b*.
Solution:
Let $2a+5b=171$ …❶
$7a+8b=342$ …❷

From ❶:
$$2a=171-5b$$
$$a=\frac{171-5b}{2}$$

We substitute this expression for a in equation ❷ to obtain one equation in one unknown. This gives an equation in only b which is solvable.

$$7\left(\frac{171-5b}{2}\right)+8b=342$$

$$(\times 2)$$

$$7(171-5b)+16b=684$$

$$1197-35b+16b=684$$

$$-35b+16b=684-1197$$

$$-19b=-513$$

$$b=\frac{-513}{-19}$$

$$b=27$$

To find a, we can substitute the solved value of b in either of the given equations or the expression for a that we derived from equation ❶.

Hence, $a=\dfrac{171-5(27)}{2}$

$$=\frac{171-135}{2}$$

$$=18$$

Therefore, $a=18$ and $b=27$.

Alternative Method:
Again, we let
$2a+5b=171$ …❶
$7a+8b=342$ …❷

We multiply equations ❶ and ❷ by two different numbers to form two new equations ❸ and ❹
The numbers are so chosen, that when the two new equations❸ and ❹ are added, one of the unknowns is eliminated.
In this case if we plan to eliminate b.

Equation ❶ $\times 8$:	$16a$	$+$	$40b$	$=$	1368	…❸
Equation ❷ $\times -5$:	$-35a$	$-$	$40b$	$=$	-1710	…❹
We add ❸to ❹ to get	$-19a$				$=-342$	

Notice, the terms in b are eliminated, leaving
$-19a=-342$

$$a = \frac{-342}{-19}$$
$$a = 18$$

We can now substitute $a = 18$ in either of the equations to solve for b
We choose equation ❶ to get

$$2(18) + 5(b) = 171$$
$$5b = 171 - 36$$
$$5b = 135$$
$$(\div 5)$$
$$b = 27$$

Hence, $a = 18$ and $b = 27$.

Alternative Method:

The graphical method is yet another alternative but it's not our first choice if we have options. In fairness, it is much longer and a bit tedious when compared to substitution and elimination. However, it is illustrated as a learning experience for readers and candidates.

We consider both equations as graphical and draw both on the same axes.

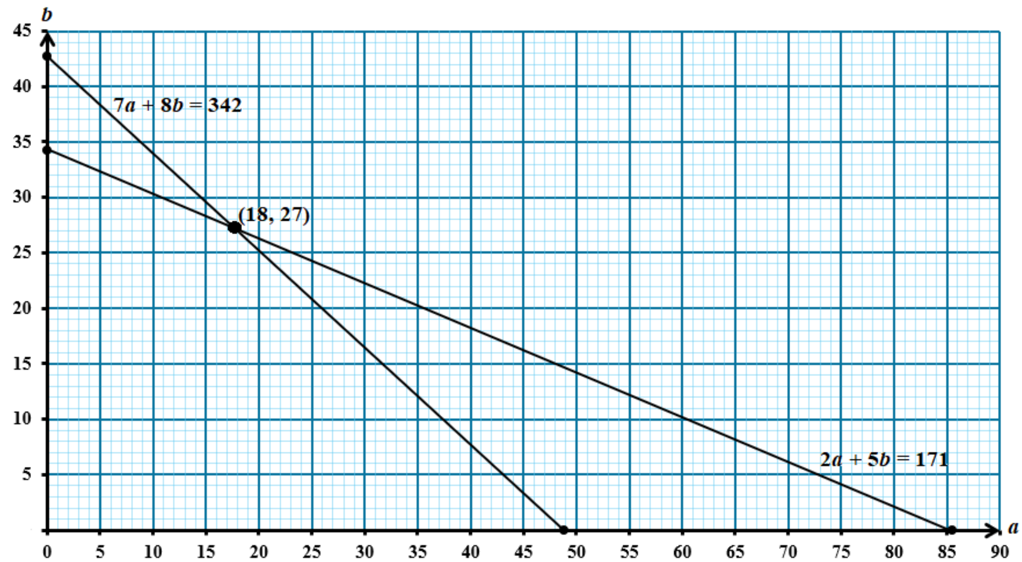
$2a + 5b = 171$

a	***b***
0	$34\frac{1}{5}$
$85\frac{1}{2}$	0

$7a + 8b = 342$

a	***b***
0	$42\frac{3}{4}$
$48\frac{6}{7}$	0

Having obtained two points for each straight line, we plot them and draw the straight lines for each equation. We extend either or both, if necessary, so that they intersect and which will be at the solution point.



The two straight lines intersect at the point with coordinates (18, 27)
Hence, $a = 18$ and $b = 27$.

Alternative Method:

The matrix method is again another alternative. It is a bit longer than substitution or elimination and we added it as a learning exercise for readers and candidates.

Again, we let

$$2a + 5b = 171 \qquad \ldots❶$$
$$7a + 8b = 342 \qquad \ldots❷$$

The linear equations are expressed in a matrix form

$$\begin{pmatrix} 2 & 5 \\ 7 & 8 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 171 \\ 342 \end{pmatrix} \quad \ldots\text{Matrix equation}$$

Let $A = \begin{pmatrix} 2 & 5 \\ 7 & 8 \end{pmatrix}$

We now find the inverse of A

$$\begin{aligned} |A| &= (2 \times 8) - (5 \times 7) \\ &= 16 - 35 \\ &= -19 \end{aligned}$$

$$\therefore A^{-1} = \frac{1}{-19}\begin{pmatrix} 8 & -(5) \\ -(7) & 2 \end{pmatrix}$$
$$= \begin{pmatrix} -\frac{8}{19} & \frac{5}{19} \\ \frac{7}{19} & -\frac{2}{19} \end{pmatrix}$$

We pre-multiply both sides of the matrix equation by A^{-1}:

$$\underbrace{\underbrace{A^{-1} \times A}_{I} \times \begin{pmatrix} a \\ b \end{pmatrix}}_{\begin{pmatrix} a \\ b \end{pmatrix}} = \underbrace{\underset{2\times 2}{\begin{pmatrix} -\frac{8}{19} & \frac{5}{19} \\ \frac{7}{19} & -\frac{2}{19} \end{pmatrix}}\underset{2\times 1}{\begin{pmatrix} 171 \\ 342 \end{pmatrix}}}_{2\times 1}$$
$$= \begin{pmatrix} e_{11} \\ e_{21} \end{pmatrix}$$

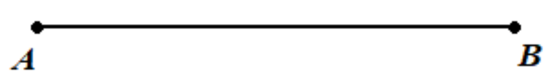
$$e_{11} = \left(-\frac{8}{19}\times 171\right) + \left(\frac{5}{19}\times 342\right)$$
$$= 18$$
$$e_{21} = \left(\frac{7}{19}\times 171\right) + \left(-\frac{2}{19}\times 342\right)$$
$$= 27$$

Hence, $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 18 \\ 27 \end{pmatrix}$

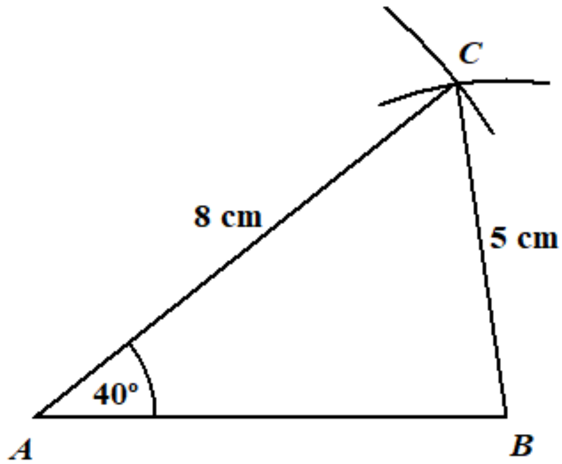
Equating corresponding entries we obtain:
$a = 18$ and $b = 27$

3. (a) In triangle ABC, $AC = 8$ cm and $BC = 5$ cm.

(i) Using a ruler and compasses only, construct triangle ABC. The line AB has been drawn for you.



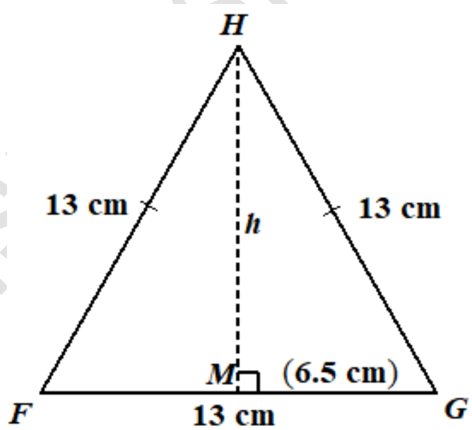
SOLUTION:
Data: The line segment AB
Required to construct: Triangle ABC using AB (given), $AC = 8$ cm and $BC = 5$ cm
Solution:



(ii) Measure and state the value of Angle *BAC*.

SOLUTION:
Required to measure: And state the value of angle *BAC*.
Solution:
By measurement, using the protractor, we obtain $B\hat{A}C = 40^0$

(b) The diagram below shows an equilateral triangle, *FGH*, whose base is 13 cm and its height, *h*.



Calculate the value of *h*.

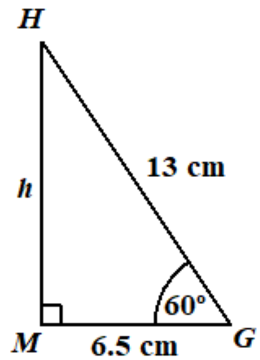
SOLUTION:
Data: Diagram showing triangle *FGH* with base 13 cm and height *h* cm

Required to calculate: *h*
Solution:

Let the midpoint of *FG* be *M*. Hence $MG = \frac{13}{2}\text{cm}$

$= 6.5\,\text{cm}$

HM will be perpendicular to *FG*
Consider ΔHMG



Hence, $\sin 60° = \dfrac{h}{13}$

$h = 11.26$ cm (correct to 2 decimal places)

OR

$\tan 60° = \dfrac{h}{6.5}$

$\therefore h = 6.5 \tan 60°$

$h = 11.26$ cm (correct to 2 decimal places)

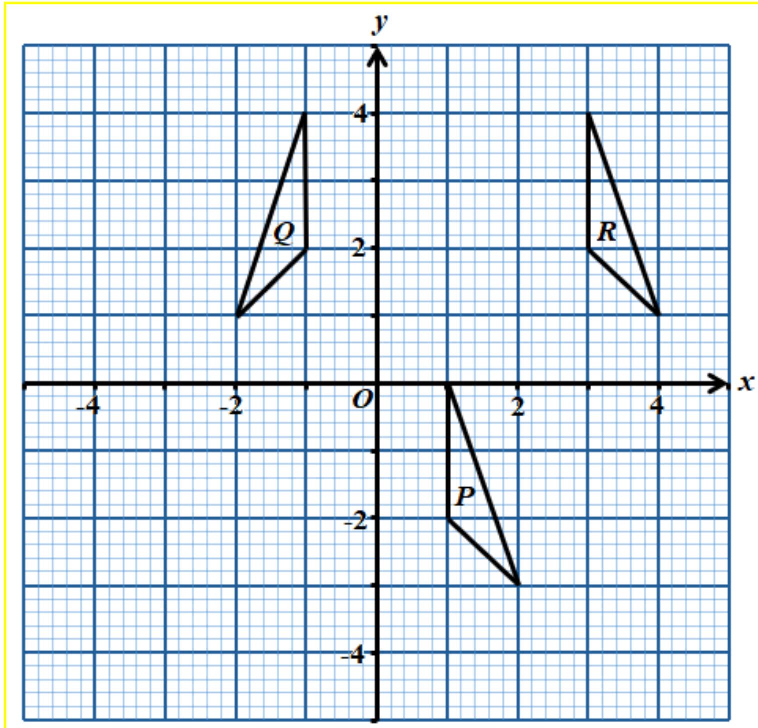
OR

$h^2 + (6.5)^2 = (13)^2$ (Pythagoras' Theorem)

$h = \sqrt{(13)^2 - (6.5)^2}$

$= 11.26$ cm (correct to 2 decimal places)

(c) The diagram below shows 3 triangles, *P*, *Q* and *R*. The triangles *Q* and *R* are images of triangle *P* after it undergoes a double transformation.

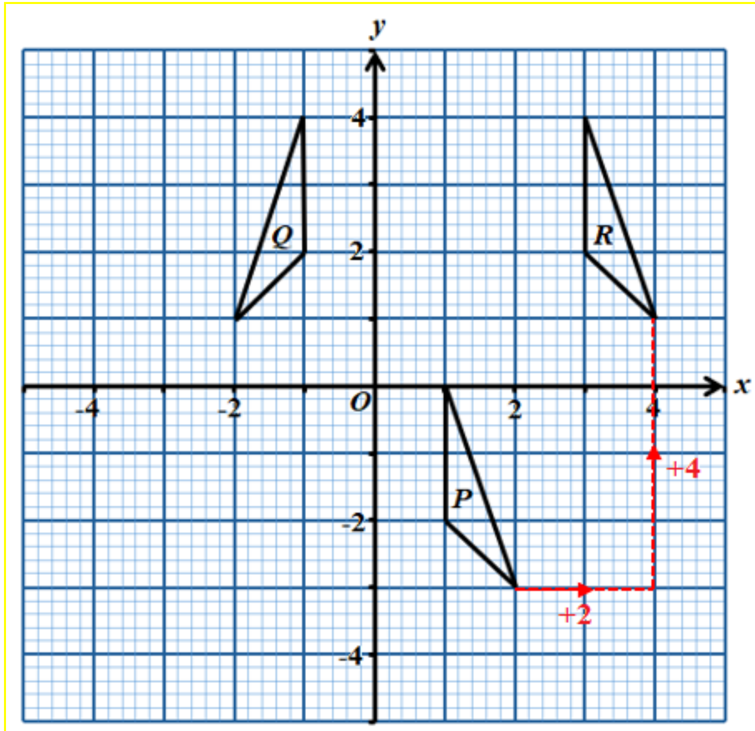


Describe fully the single transformation that maps Triangle

(i) P onto Triangle R

SOLUTION:
Data: Diagram showing triangles P, Q and R, where Q and R are images of P after undergoing a double transformation.
Required to describe: The single transformation that maps triangle P onto triangle R
Solution:
$\Delta P \equiv \Delta R$
The image is congruent to the object but not flipped or re-oriented. The transformation is a translation.

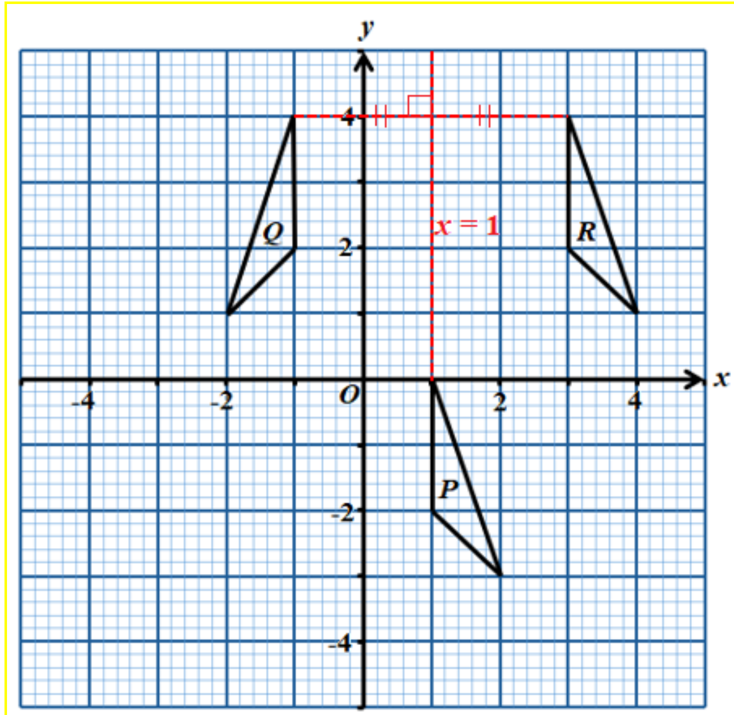
We observe the horizontal and the vertical shift of one object point on P to its corresponding image point on R.



P is shifted 2 units horizontally to the right and 4 units vertically upwards as shown. Hence, the translation vector that maps P onto R is $T = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

(ii) R onto Triangle Q.

SOLUTION:
Required to describe: The single transformation that maps triangle R onto triangle Q.
Solution:
$\Delta R \equiv \Delta Q$



The object and image are congruent and the image Q is flipped or laterally inverted with respect to the object R. Hence, the transformation is a reflection. The perpendicular bisector of the line joining any object point on R to its corresponding image point on Q is the line of reflection. This is the vertical line with equation $x = 1$. Hence, R is mapped onto Q by a reflection in the line $x = 1$, as shown above.

4. Consider the following functions.

$$f(x) = 2 - 5x \text{ and } h(x) = 5^x$$

(a) Calculate the value of

(i) $f(4)$

SOLUTION:
Data: $f(x) = 2 - 5x$ and $h(x) = 5^x$
Required to calculate: $f(4)$
Calculation:

$$f(x) = 2 - 5x$$
$$f(4) = 2 - 5(4)$$
$$= 2 - 20$$
$$= -18$$

(ii) $h(0)$

SOLUTION:

Required to calculate: $h(0)$

Calculation:

$$h(x) = 5^x$$

$$h(0) = 5^0$$
$$= 1$$

(iii) $fh(-2)$

SOLUTION:

Required to calculate: $fh(-2)$

Calculation:

$$fh(x) = 2 - 5(h(x))$$
$$= 2 - 5(5^x)$$

We can now calculate $fh(-2)$to get

Hence, $fh(-2) = 2 - 5(5^{-2})$

$$= 2 - 5\left(\frac{1}{5^2}\right)$$

$$= 2 - \frac{5}{25}$$

$$= 1\frac{4}{5} \text{ or } \frac{9}{5}$$

Alternative Method:

$$h(-2) = 5^{-2}$$

$$= \frac{1}{5^2}$$

$$= \frac{1}{25}$$

$$fh(-2) = f\left(\frac{1}{25}\right)$$

$$= 2 - 5\left(\frac{1}{25}\right)$$

$$= 2 - \frac{1}{5}$$

$$= 1\frac{4}{5} \text{ or } \frac{9}{5}$$

(b) Find $f^{-1}(x)$.

SOLUTION:

Required to find: $f^{-1}(x)$

Solution:

$f(x)=2-5x$

Let $y=5-2x$

$\therefore 5x=2-y$

$$x=\frac{2-y}{5}$$

Replace y by x to get:

$$f^{-1}(x)=\frac{2-x}{5}$$

(c) Given that $ff(x)=a+bx$, determine the values of a and b.

SOLUTION:

Data: $ff(x)=a+bx$

Required to find: The value of a and of b

Solution:

$f(x)=2-5x$

$$\begin{aligned}\therefore ff(x)&=2-5(f(x))\\&=2-5(2-5x)\\&=2-10+25x\\&=-8+25x\end{aligned}$$

This is of the form $a+bx$, where $a=-8$ and $b=25$.

5. The mass, m, in kilograms, of 120 newborns at a hospital is recorded in the table below.

Mass (m kg)	**Frequency (f)**
$2.6<m\le 3.5$	7
$3.5<m\le 4.4$	18
$4.4<m\le 5.3$	30
$5.3<m\le 6.2$	29
$6.2<m\le 7.1$	28
$7.1<m\le 8.0$	8

(a) **(i)** State the modal class.

SOLUTION:

Data: Frequency table showing the mass, *m*, in kilograms, of 120 newborns at a hospital

Required to state: The modal class interval

Solution:

The modal class is the class that occurs most often.

This is $4.4 < m \leq 5.3$ which, as shown on the table, has the greatest frequency.

(ii) Complete the table below and calculate an estimate of the mean mass of the 120 newborns.

Mass (m kg)	**Midpoint (x)**	**Frequency (f)**	**Frequency × Midpoint (fx)**
$2.6 < m \leq 3.5$	3.05	7	21.35
$3.5 < m \leq 4.4$	3.95	18	71.1
$4.4 < m \leq 5.3$	4.85	30	145.5
$5.3 < m \leq 6.2$	________	29	________
$6.2 < m \leq 7.1$	6.65	28	186.2
$7.1 < m \leq 8.0$	________	8	________

SOLUTION:

Data: Incomplete table showing the class midpoint and frequency × class midpoint for each class interval

Required to complete: The table given and estimate the mean mass of the 120 newborns.

Solution:

Mass (m kg)	**Midpoint (x)**	**Frequency (f)**	**Frequency × Midpoint (fx)**
$2.6 < m \leq 3.5$	$\dfrac{2.6+3.5}{2} = 3.05$	7	$7 \times 3.05 = 21.35$
$3.5 < m \leq 4.4$	$\dfrac{3.5+4.4}{2} = 3.95$	18	$18 \times 3.95 = 71.1$
$4.4 < m \leq 5.3$	$\dfrac{4.4+5.3}{2} = 4.85$	30	$30 \times 4.85 = 145.5$
$5.3 < m \leq 6.2$	$\dfrac{5.3+6.2}{2} = 5.75$	29	$39 \times 5.75 = 166.75$
$6.2 < m \leq 7.1$	$\dfrac{6.2+7.1}{2} = 6.65$	28	$28 \times 6.65 = 186.2$
$7.1 < m \leq 8.0$	$\dfrac{7.1+8.0}{2} = 7.55$	8	$8 \times 7.55 = 60.4$
		$\sum f = 120$	$\sum fx = 651.3$

The mean, $\bar{x} = \dfrac{\sum fx}{\sum f}$

$$= \frac{651.3}{120}$$
$$= 5.427$$
$$\approx 5.43 \text{ kg (correct to 2 decimal places)}$$

(iii) One newborn is chosen at random from the hospital. Find the probability that the newborn has a mass greater than 5.3 kg.

SOLUTION:
Required to find: The probability a randomly chosen newborn has a mass greater than 5.3 kg.
Solution:

$$P(\text{mass} > 5.3 \text{ kg}) = \frac{\text{Number of newborns with a mass} > 5.3 \text{ kg}}{\text{Total number of newborns}}$$
$$= \frac{29+28+8}{120}$$
$$= \frac{65}{120}$$
$$= \frac{13}{24}$$

(As a point of interest, if this table refers to the mass of newborn human infants, the figures are very unrealistic)

(b) **(i)** Complete the cumulative frequency table shown below.

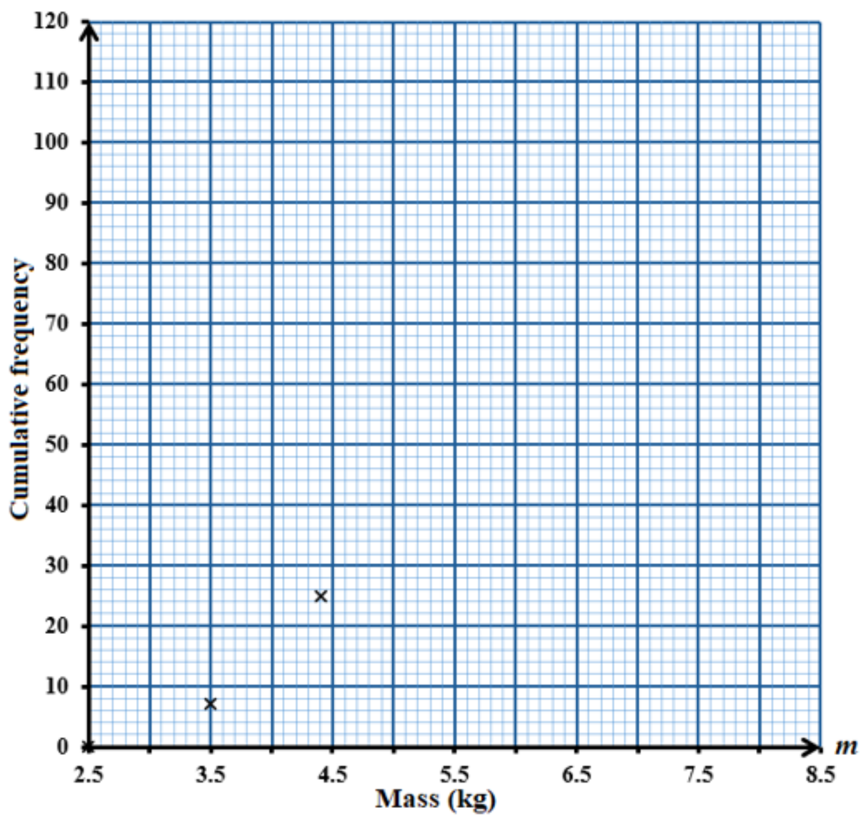
Mass (m kg)	**Cumulative Frequency**
$m \leq 3.5$	7
$m \leq 4.4$	25
$m \leq 5.3$	55
$m \leq 6.2$	___________
$m \leq 7.1$	___________
$m \leq 8.0$	___________

SOLUTION:
Data: Incomplete cumulative frequency table for the masses of 120 newborn babies at a hospital.
Required To Complete: The cumulative frequency table given

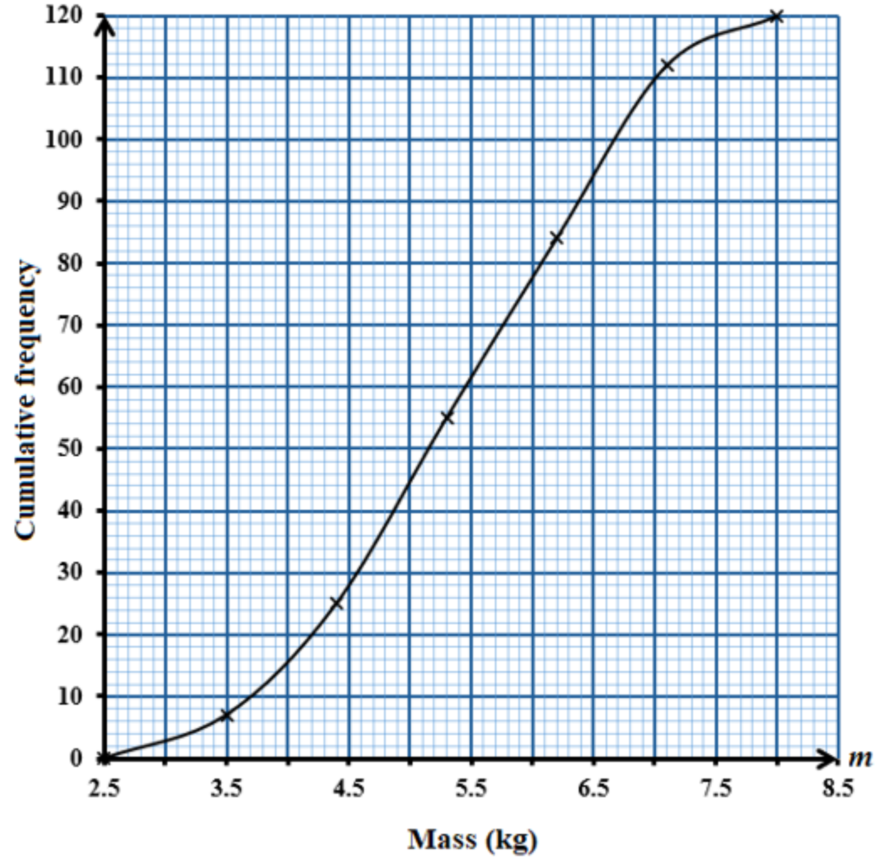
Solution:

Mass (m **kg**)	**Frequency** (f)	**Cumulative Frequency**
$m \leq 3.5$	7	7
$m \leq 4.4$	18	$18 + 7 = 25$
$m \leq 5.3$	30	$30 + 25 = 55$
$m \leq 6.2$	29	$29 + 55 = 84$
$m \leq 7.1$	28	$28 + 84 = 112$
$m \leq 8.0$	8	$8 + 112 = 120$
	$\sum f = 120$	

(ii) On the grid below, draw a cumulative frequency curve to show the information in the table in (b) (i).



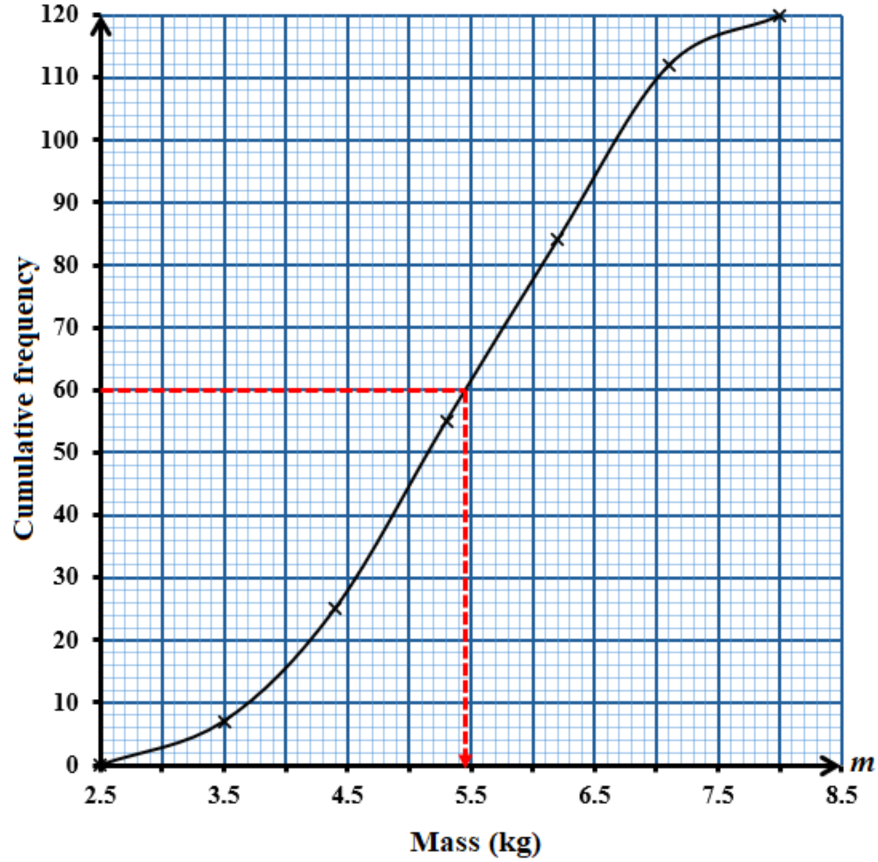
SOLUTION:
Data: Grid with Cumulative frequency vs Mass (kg)
Required to draw: The cumulative frequency curve for the data given in (b) (i)
Solution:



(ii) Use your diagram to find an estimate for the median mass of the newborns.

SOLUTION:
Required to find: The median mass of the newborns, using the graph
Solution:

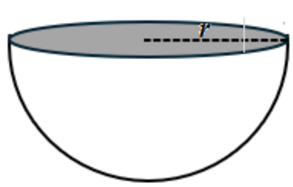
$$\frac{1}{2}\text{Cumulative frequency} = \frac{1}{2}(120)$$
$$= 60$$

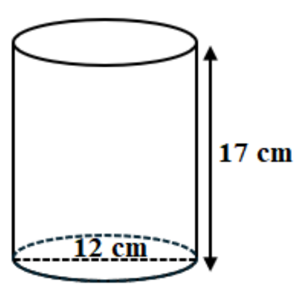


At the point where the horizontal from cumulative frequency = 60 meets the curve, the vertical is drawn to meet the horizontal. This is read off to be at approximately 5.45 kg
Hence, the median mass is taken as 5.45 kg.

6. [In this question, using $\pi = \dfrac{22}{7}$]

The diagrams below show a hemispherical bowl and a cylindrical tin. The diameter of the cylindrical tin is 12 cm, the height is 17 cm and the radius of the bowl is r.





(a) **(i)** Show that the volume of the cylindrical tin is 1 932 cm^3, correct to 2 significant figures.

SOLUTION:
Data: Diagrams showing a hemispherical bowl of radius r and a cylindrical tin of diameter 12 cm and height 17 cm.
Required to show: The volume of the cylindrical tin is 1 923 cm^3, correct to 4 significant figures
Proof:

$$\begin{aligned}\text{Volume of the cylinder} &= \pi r^2 h\\ &= \frac{22}{7}\times\left(\frac{12}{2}\right)^2\times 17\\ &= 1923.4\\ &\approx 1923\text{ cm}^3\ \text{(correct to 4 significant figures)}\end{aligned}$$

Q.E.D.

(ii) The bowl is completely filled with soup. When all the soup is poured into the cylindrical can, 90% of the volume of the tin is filled. Calculate the radius of the bowl.

(The volume, V, of a sphere with radius r is $V=\frac{4}{3}\pi r^3$]

SOLUTION:
Data: The bowl is completely filled with soup, but when this is poured into the cylindrical tin, the tin is 90% filled.
Volume of a sphere is $V=\frac{4}{3}\pi r^3$.
Required to calculate: The radius of the bowl
Calculation:
The bowl is a hemisphere.

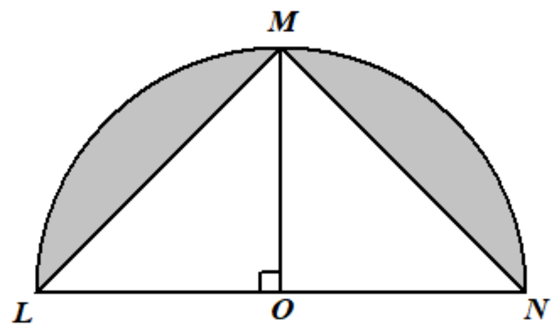
$$\begin{aligned}\text{Volume of the soup in the bowl in cm}^3 &= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)\\ &= \frac{2}{3}\times\frac{22}{7}\times r^3\\ &= \frac{44r^3}{21}\end{aligned}$$

Volume of the bowl $=$90% of the volume of the tin

Hence, $\dfrac{44r^3}{21}=\dfrac{90}{100}\times 1923.4$

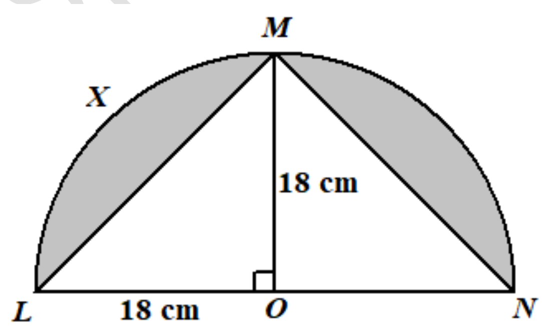
$$r^3 = \frac{0.9 \times 1923.4 \times 21}{44}$$
$$r^3 = 826.187$$
$$r = \sqrt[3]{826.187}$$
$$= 9.383$$
$$\approx 9.38 \text{ cm (correct to 2 decimal places)}$$

(b) In the diagram below, points *L*, *M* and *N* are on the circumference of a semicircle, with centre *O*, and a radius of 18 cm.



Calculate the TOTAL area of the shaded sections in the diagram.

SOLUTION:
Data: Diagram showing the points *L*, *M* and *N* are on the circumference of a semicircle, with centre *O*, and a radius of 18 cm.
Required to calculate: The total area of the shaded sections
Calculation:



The area of the sector, $LMO = \frac{1}{4}\pi(18)^2 \text{ cm}^2$, since this is a quadrant or ¼ of a circle.

Area of $\Delta LOM = \frac{18 \times 18}{2} \text{ cm}^2$

Hence, the area of the shaded segment shown as *LXM*
$= \text{Area of sector } LOM - \text{Area of } \Delta LOM$

$= \frac{1}{4}\pi(18)^2 - \frac{1}{2}(18)(18)$

$= 254.502 - 162$

$= 92.502 \text{ cm}^2$

Hence, the area of the entire shaded region $= 2(92.502) \text{ cm}^2$

$= 185.0 \text{ cm}^2$

Alternative Method:

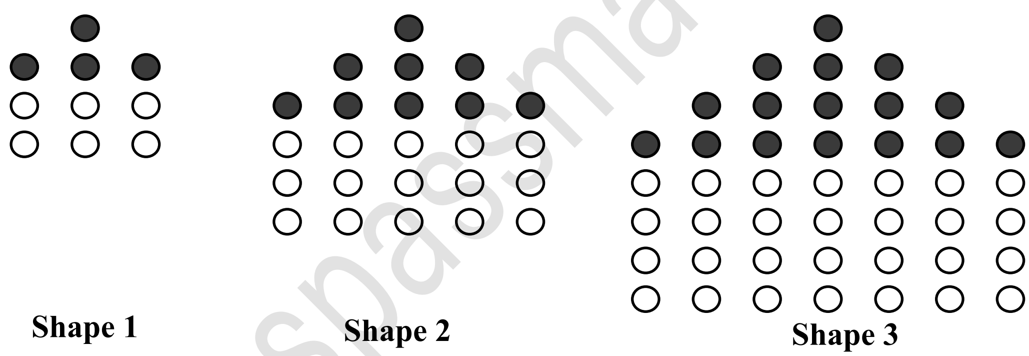
Area of the shaded region $=$ Area of the semicircle $LMN -$ Area of ΔMLN

$= \frac{1}{2}(\pi \times 18^2) - \frac{(18+18)\times 18}{2}$

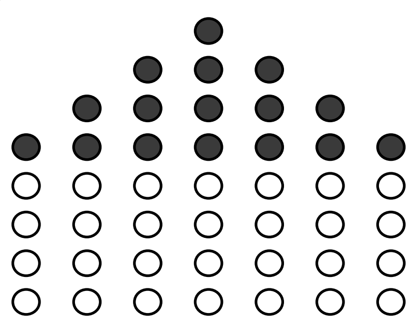
$= 509.004 - 324$

$= 185.0 \text{ cm}^2$

7. The diagram below shows the first 3 shapes in a sequence, which forms a pattern. Each shape is made using a set of small white counters and black counters.



(a) Complete the diagram below to represent Shape 4.



SOLUTION:

Data: Diagrams showing a pattern of shapes made using small white counters and small black counters.

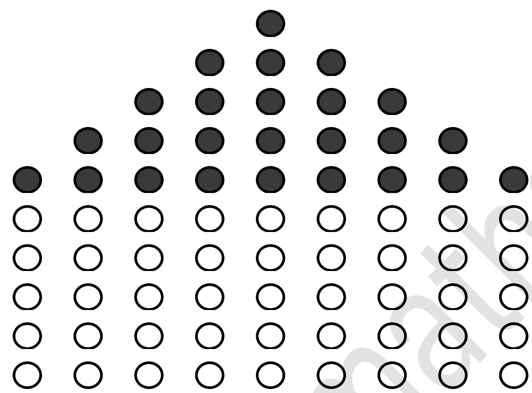
Required to complete: The diagram given to represent Shape 4 in the pattern.

Solution:

The black counters form an isosceles triangular with rows of 1+3, 1+3+5, and 1+3+5+7 counters in the first three figures. Hence, in the fourth figure the pattern would extend to rows of 1 + 3 + 5 + 7 + 9 counters as shown.

The white counters of each figure are in rows made up of the same number of counters as the last row of black counters. These rows number 2, 3, and 4 in the first three figures. Hence, they would number 5 in the fourth figure.

The completed 4th figure is shown below.



(b) The number of white counters, W, the number of black counters, B, and the total number of counters, T, that form each shape follow a pattern. The values for W, B and T for the first 3 shapes are shown in the table below. Study the pattern of numbers in each row of the table and answer the questions that follow.\

We have to complete the rows marked (i), (ii) and (iii) in the table below.

There may be more than one approach to answering this part of the question. A convenient one is to observe the pattern of obtaining the values for columns W, B and T. In other words, we can look at answering (iii) first and then work out the answers to rows (i) and (ii).

We take each column separately.

	Shape Number (S)	Number of White Counters (W)	Number of Black Counters (B)	Total Number of Counters (T)
	1	$(1+1)[2(1)+1]=6$	$(1+1)^2=4$	10
	2	$(2+1)[2(2)+1]=15$	$(2+1)^2=9$	24
	3	$(3+1)[2(3)+1]=28$	$(3+1)^2=16$	44
(i)	4	45	________	________
	⋮	⋮	⋮	⋮
(ii)	________	____________	144	420
	⋮	⋮	⋮	⋮
(iii)	n	$(\_\_\_+\_\_\_)(\_\_\_+\_\_\_)$	$(\_\_\_+\_\_\_)^2$	$3n^2+5n+2$

SOLUTION:
Data: Incomplete table showing the number of white counters (W), the number of black counters (B) and the total number of counters (T) used to make each shape in the pattern.
Required to complete: The table given
Solution:
(i)
The column for B appears to be $(\text{Shape number}+1)^2$

Hence, when the shape number is n, the value of $B=(n+1)^2$

For part (i):
Shape number, $n = 4$

	S	W	B	T
(i)	4	$(4+1)[2(4)+1]$ $=5(9)$ $=45$	$(4+1)^2=25$	$T=W+B$ $45+25=70$ Or $3(4)^2+5(4)+2$ $=70$

(ii)

Number of White Counters + Number of Black Counters = Total Number of Counters

Number of White Counters = Total Number of Counters – Number of Black Counters

= 420 – 144 = 276

Hence, $W = 276$

We can deduce the value of n by using 144 as the number of Black Counters in Column B as follows:

$$144 = (12)^2$$
$$= (11+1)^2$$
$$n = 11$$

Alternative Method

The column for W appears to be found by:

$$(\text{Shape number } +1)\left[2(\text{Shape number})+1\right]$$

When the shape number is n

$$W = (n+1)\left[2(n)+1\right]$$
$$= (n+1)(2n+1)$$

Substituting $n = 11$ (see above)

$$(11+1)\left[2(11)+1\right]$$
$$= 12(23)$$
$$= 276$$

	S	W	B	T
(ii)	11	$(11+1)\left[2(11)+1\right]$ $= 12(23)$ $= 276$	$144 = (12)^2$ $= (11+1)^2$ $n = 11$	$T = W + B$ $276 + 144 = 420$

For part (iii) we now have the completed row to read :

	S	W	B	T
(iii)	n	$(n+1)[2(n)+1]$ $=(n+1)(2n+1)$	$(n+1)^2$	$(n+1)(2n+1)+(n+1)^2$ $=2n^2+n+2n+1+n^2+n+n+1$ $=3n^2+5n+2$

The completed table now looks like:

	Shape Number (*S*)	**Number of White Counters (*W*)**	**Number of Black Counters (*B*)**	**Total Number of Counters (*T*)**
	1	$(1+1)[2(1)+1]=6$	$(1+1)^2=4$	10
	2	$(2+1)[2(2)+1]=15$	$(2+1)^2=9$	24
	3	$(3+1)[2(3)+1]=28$	$(3+1)^2=16$	44
(i)	4	45	$(4+1)^2=25$	$45+25=70$ OR $3(4)^2+5(4)+2$ $=70$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(ii)	11	$(11+1)(2(11)+1)$ $=12(23)$ $=276$	144 $(11+1)^2=144$	420
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(iii)	n	$(n+1)[2(n)+1]$ $=(n+1)(2n+1)$	$(n+1)^2$	$3n^2+5n+2$

(c) The expression for the total number of counters, $T = W + B$, in Shape S is given by $T = aS^2 + bS + 2$, where a and b are both positive integers.

By substituting suitable values for S, show that the total number of counters in Shape 1 and Shape 3, in terms of a and b, is represented by the equations

$$a + b = 8$$
$$3a + b = 14 \text{ respectively}$$

SOLUTION:
Data: $T = aS^2 + bS + 2$
Required to show: the total number of counters in Shape 1 and Shape 3, in terms of a and b, is represented by the equations

$$a + b = 8$$
$$3a + b = 14 \text{ respectively}$$

Proof:

$$T = W + B$$
$$= (n+1)(2n+1) + (n+1)^2$$
$$= 3n^2 + 5n + 2$$

When $n = S$

$$T = aS^2 + bS + 2$$
$$= 3S^2 + 5S + 2, \text{ where } a = 3 \text{ and } b = 5$$

In Shape 1:
When $S = 1, T = 10$

$$a(1)^2 + b(1) + 2 = 10$$
$$a + b + 2 = 10$$
$$a + b = 10 - 2$$
$$a + b = 8$$

When $S = 3, T = 44$

$$a(3)^2 + b(3) + 2 = 44$$
$$9a + 3b + 2 = 44$$
$$9a + 3b = 44 - 2$$
$$9a + 3b = 42$$

$(\div 3)$

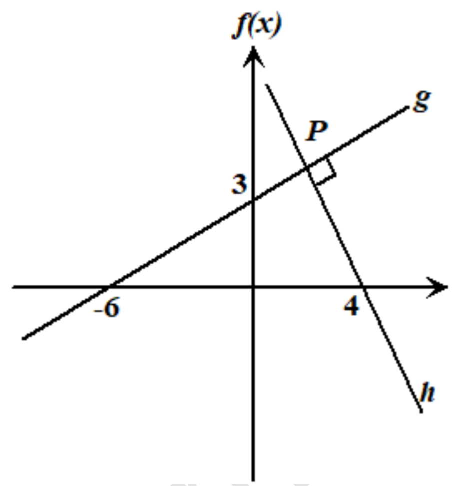
$$3a + b = 14$$

Hence, $a + b = 8$ and $3a + b = 14$
(If solved, we ought to get $a = 3$ and $b = 5$ as seen in the above equation.) **Q.E.D.**

SECTION II
Answer ALL questions.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

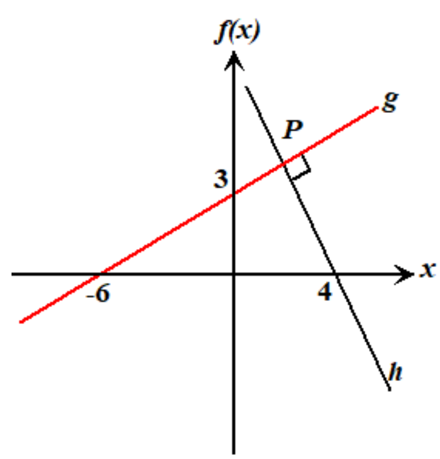
8. (a) The diagram below shows the graphs of two functions on the same pair of axes. The lines g and h are perpendicular.



Determine the:

(i) equation that represents the line g

SOLUTION:
Data: Diagram showing the graphs of two straight lines, g and h, on the same axes. The lines g and h are perpendicular (to each other).
Required To Determine: The equation of line g
Solution:



Two points on g are $(0, 3)$, the intercept on the y – axis and $(-6, 0)$, the intercept on the x- axis. We use these two points to obtain the gradient.

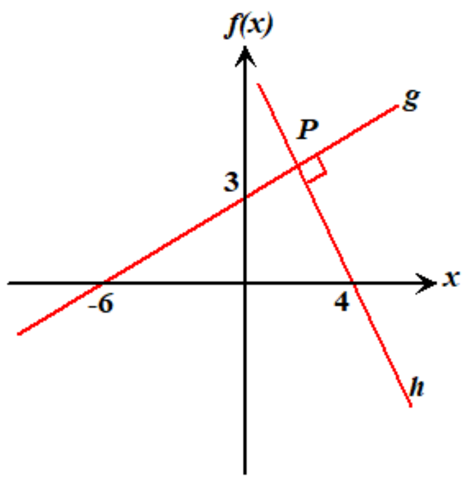
The gradient of g is $\dfrac{3-0}{0-(-6)}=\dfrac{1}{2}$

The equation of the line g is $y=mx+c$ where m is the gradient and c is the intercept on the vertical axis.

Hence, the equation of g is $y=\dfrac{1}{2}x+3$.

(ii) The equation that represents the line h

SOLUTION:
Required to determine: The equation of the line h
Solution:



We recall that the gradient of $g=\dfrac{1}{2}$

$\therefore$ Gradient of h is $=\dfrac{-1}{\frac{1}{2}}$
$=-2$
(The product of the gradients of perpendicular lines $=-1$)

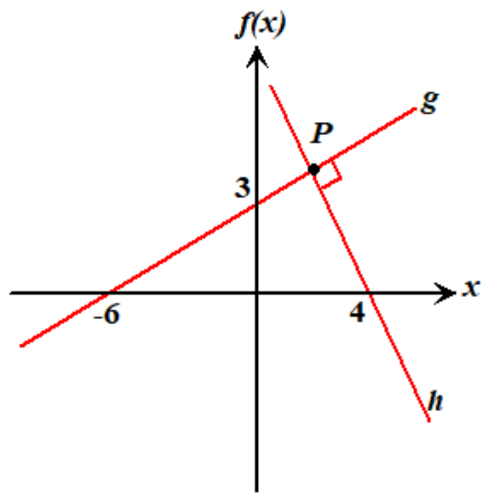
Using the point $(4,\ 0)$ on h:

The equation of h is $\dfrac{y-0}{x-4}=-2$

$$y=-2(x-4)$$
$$y=-2x+8$$

(iii) the coordinates of the point P. Show all working.

SOLUTION:
Required to determine: The coordinates of the point P.
Solution:



P is the point of intersection of the lines h and g.
To obtain P, we solve simultaneously.

Let $y = \frac{1}{2}x + 3$ ...❶
and $y = -2x + 8$ ...❷

We can equate ❶ and ❷ to get

$$\frac{1}{2}x + 3 = -2x + 8$$
$$3 - 8 = -2x - \frac{1}{2}x$$
$$-5 = -2\frac{1}{2}x$$
$$\therefore x = \frac{-5}{-2\frac{1}{2}}$$
$$x = 2$$

When $x = 2$ we substitute in either equation to solve for y. In this case we choose equation ❶ to get $y = \frac{1}{2}(2) + 3 = 4$

$$y = 4$$

Hence, the point P is (2, 4)

(b) **(i)** Write $4x^2-24x+31$ in the form $a(x+h)^2+k$.

SOLUTION:
Required to write: $4x^2-24x+31$ in the form $a(x+h)^2+k$
Solution:

$4x^2-24x+31$
$=4(x^2-6x)+31$
$=4[(x-3)^2-9]+31$
$=4(x-3)^2-36+31$
$=4(x-3)^2-5$

Hence, $4x^2-24x+31\equiv 4(x-3)^2-5$, which is of the form $a(x+h)^2+k$ where $a=4$, $h=-3$ and $k=-5$.

OR

$4x^2-24x+31$
$=4(x^2-6x)+31$
$\frac{1}{2}$ coefficient of x is $\frac{1}{2}(-6)=-3$
The equation is now rewritten as
$4(x^2-6x)+31$
$=4(x-3)^2+?$
$\uparrow$
$4(x-3)(x-3)$
$4(x^2-6x+9)$
$4x^2-24x+36$
$4x^2-24x+36$
-5
$4x^2-24x+31$
Hence, $-5=?$

Hence, $4x^2-24x+31\equiv 4(x-3)^2-5$, which is of the form $a(x+h)^2+k$ where $a=4$, $h=-3$ and $k=-5$.

OR

$$a(x+h)^2+k=a(x^2+2hx+h^2)+k$$
$$\equiv ax^2+2ahx+ah^2+k$$

Equating coefficients of x^2

$a=4$

Equating coefficients of x

$2(4)h=-24$

$h=-3$

and equating the constant terms

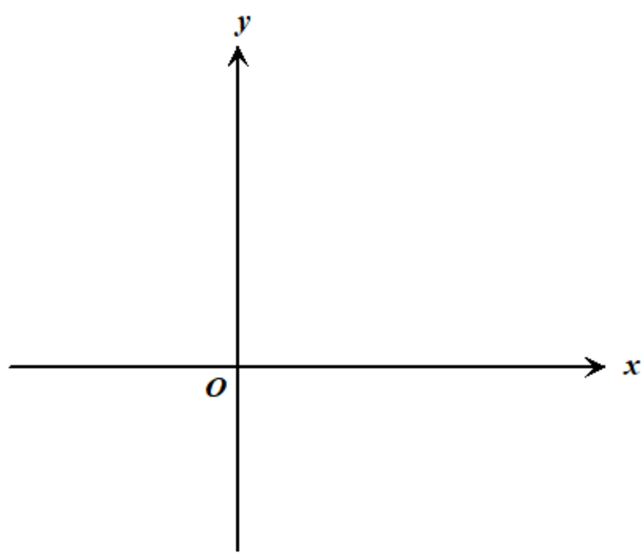
$4(-3)^2+k=31$

$k=31-36$

$=-5$

Hence, $4x^2-24x+31=4(x-3)^2-5$, which is of the form $a(x+h)^2+k$ where $a=4$, $h=-3$ and $k=-5$.

(ii) On the axes below, sketch the graph of $4x^2-24x+31$, indicating the coordinates of the maximum/minimum point and the y – intercept.



SOLUTION:

Required to sketch: The graph of $y=4x^2-24x+31$, indicating the coordinates of the maximum/minimum points and the y – intercept.

Solution:

Let $y = 4x^2 - 24x + 31$

$$= 4\underbrace{(x-3)^2}_{\geq 0\ \forall x} - 5$$

$$\therefore y_{\min} = 4(0) - 5$$
$$= -5$$

$y_{\min}$ occurs at $(x-3)^2 = 0$

$x = 3$

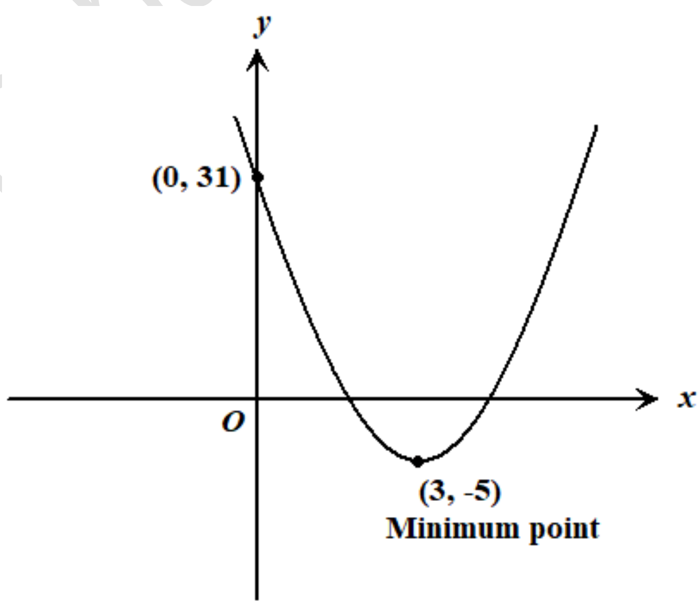
$\therefore$ Minimum point is $(3, -5)$.

When $x = 0$ $\quad y = 4(0)^2 - 24(0) + 31$

$y = 31$

$\therefore$ The graph cuts the $y-$axis at $(0, 31)$.

The coefficient of $x^2 > 0$, therefore, the graph has a minimum turning point (as established before).
The completed sketch looks like:



(iii) State the equation of the axis of symmetry.

SOLUTION:
Required to state: The equation of the axis of symmetry
Solution:

The axis of symmetry in $y = ax^2 + bx + c$ is $x = \frac{-b}{2a}$

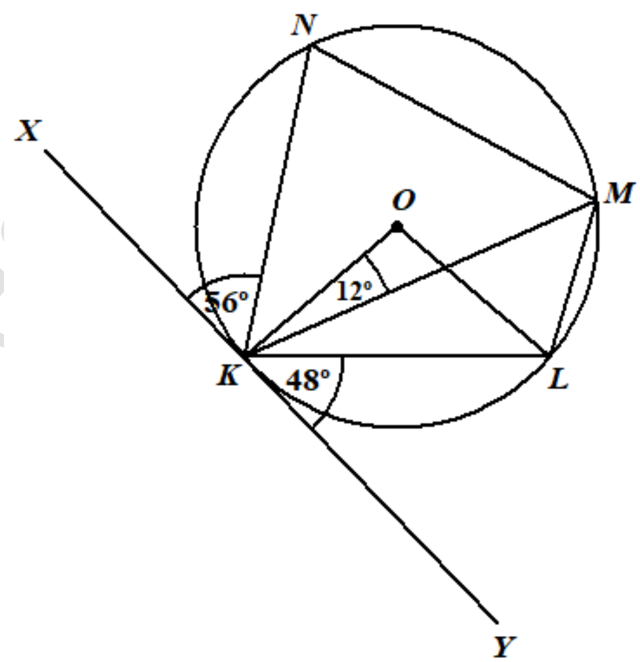
Hence, the equation of the axis of symmetry in $y = 4x^2 - 24x + 31$ is

$$x = \frac{-(-24)}{2(4)}$$

$$x = 3$$

GEOMETRY AND TRIGONOMETRY

9. (a) K, L, M and N are points on the circumference of a circle with centre O. XY is a tangent to the circle at K. Angle $LKY = 48°$, $MKO = 12°$ and angle $XKN = 56°$.



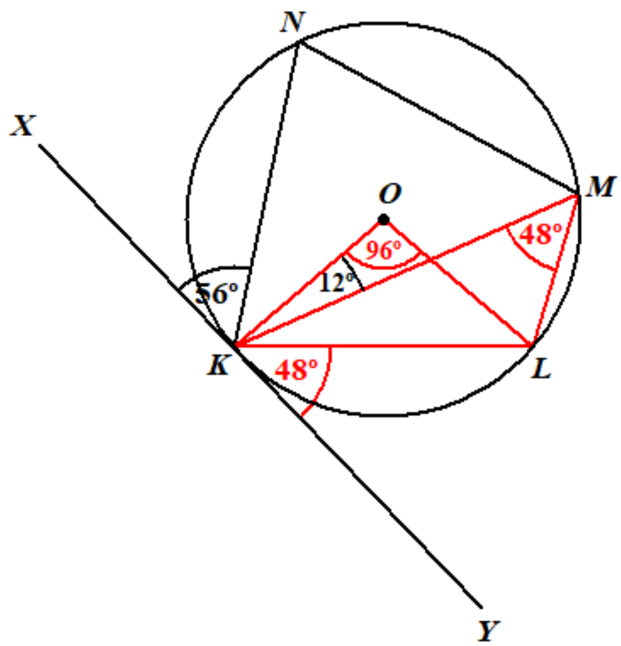
(i) Find the size of angle KOL, giving reasons for EACH step of your work.

SOLUTION:
Data: Diagram showing the points K, L, M and N are four points along the circumference of a circle with centre O, such that XY is a

tangent to the circle at *K*. Angle $LKY = 48°$, $MKO = 12°$ and angle $XKN = 56°$.

Required to find: Angle *KOL*

Solution:



$K\hat{M}L = 48°$

(The angle made by a tangent (*KY*) to a circle and a chord (*KM*) at the point of contact (*K*) is equal to the angle in the alternate segment $\left(K\hat{M}L\right)$).

$K\hat{O}L = 2\left(48°\right)$
$\quad = 96°$

(The angle subtended by a chord (*KL*) at the centre of a circle ($K\hat{O}L$) is twice the angle that chord subtends at the circumference, standing on the same arc ($K\hat{M}L = 48°$)).

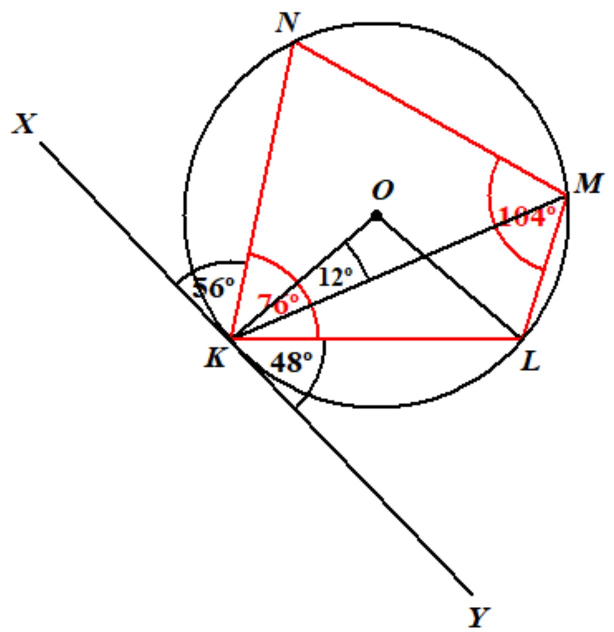
(ii) Find the value of EACH of the following angles:

a) Angle *LMN*

SOLUTION:

Required to find: The value of angle *LMN*

Solution:

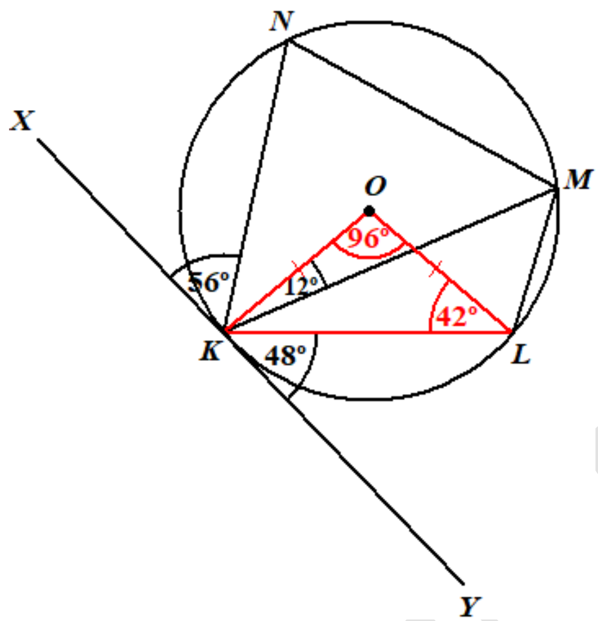


$N\hat{K}L = 180° - (56° + 48°)$ (The angles in a straight line total 180°.)
$= 76°$

$L\hat{M}N = 180° - 76°$
$= 104°$
(Opposite angles in a cyclic quadrilateral (*KLMN*) are supplementary).

b) Angle *KLO*

SOLUTION:
Required to find: Angle *KLO*
Solution:



$OK = OL$ (radii)
Hence the triangle KLO is isosceles.

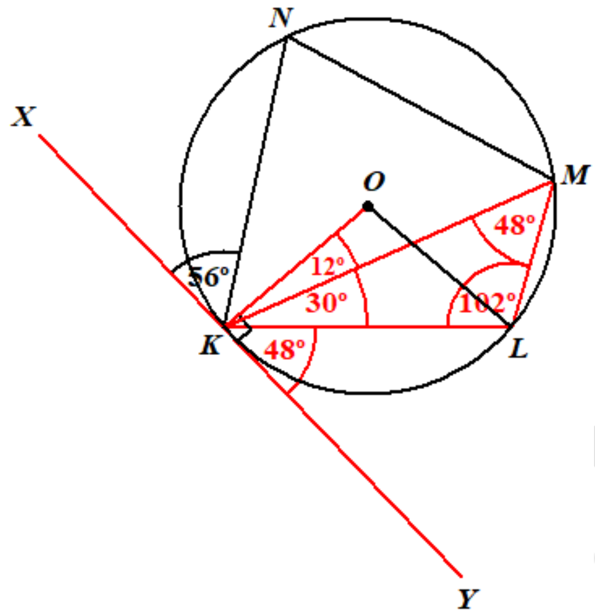
$O\hat{K}L = K\hat{L}O$
(Base angles in an isosceles triangle are equal)

$$\therefore K\hat{L}O = \frac{180° - 96°}{2}$$
$$= 42°$$

(Sum of the interior angles in a triangle (KOL) is equal to 180°.)

c) Angle MLK

SOLUTION:
Required to find: Angle MLK
Solution:



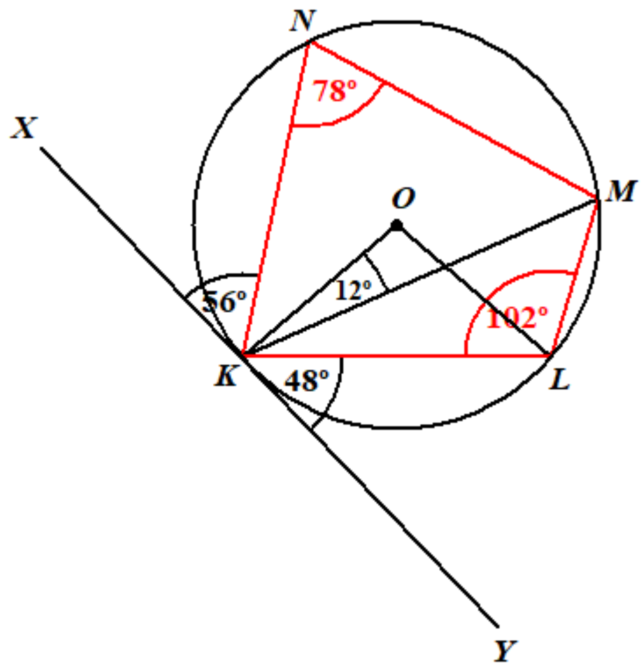
$O\hat{K}Y = 90°$
(Angle made a tangent to a circle (*KY*) and a radius (*OK*), at the point of contact, is equal to 90°.)

$\therefore M\hat{K}L = 90° - (12° + 48°)$
$= 30°$

$\therefore M\hat{L}K = 180° - (48° + 30°)$
$= 102°$
(Sum of the interior angles in a triangle (*MOL*) is equal to 180°.)

d) Angle *KNM*

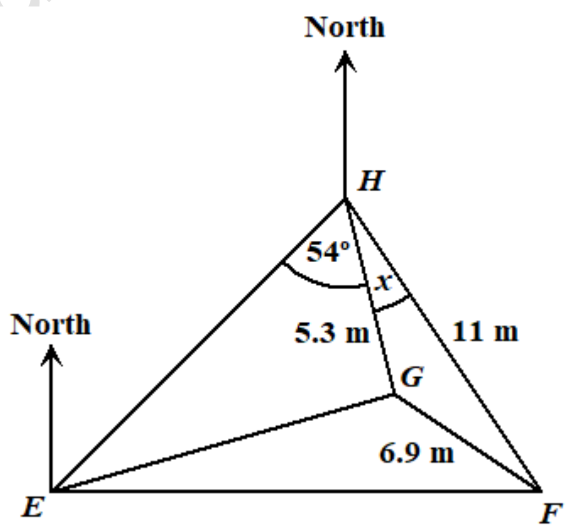
SOLUTION:
Required to find: Angle *KNM*
Solution:



$$K\hat{N}M = 180° - 102°$$
$$= 78°$$

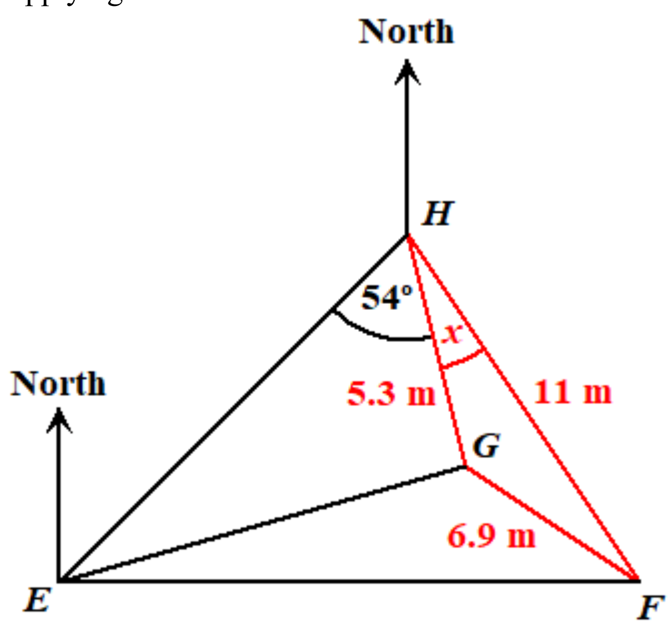
(Opposite angles in a cyclic quadrilateral (*KLMN*) are supplementary.)

(b) *E*, *F*, *G* and *H* are 4 points on the level ground. The diagram below gives information on the distances and angles between the points.



(i) Show that the x is 29.5°, correct to 1 decimal place.

SOLUTION:
Data: Diagram showing E, F, G and H as 4 points on the level ground.
$E\hat{H}G = 54°$, $F\hat{H}G = x°$, $GH = 5.3$ m, $GF = 6.9$ m and $FH = 11$ m.
Required to show: x is 29.5°, correct to 1 decimal place
Solution:
Consider triangle GHF:
Applying the cosine law:



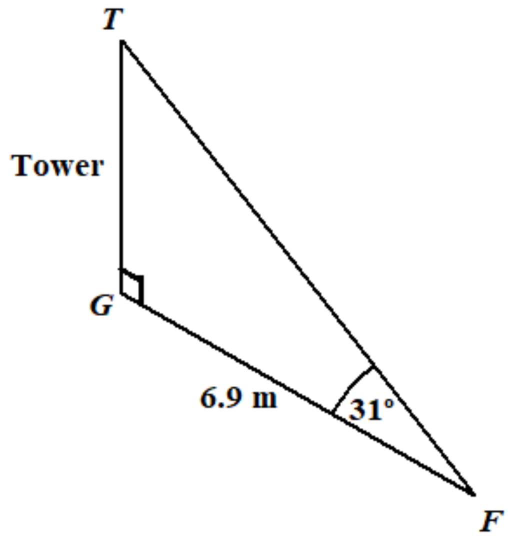
$$(6.9)^2 = (5.3)^2 + (11)^2 - 2(5.3)(11)\cos x°$$
$$\therefore 2(5.3)(11)\cos x° = (5.3)^2 + (11)^2 - (6.9)^2$$
$$\cos x = \frac{(5.3)^2 + (11)^2 - (6.9)^2}{2(5.3)(11)}$$
$$= \frac{101.48}{116.6}$$
$$= 0.8703$$
$$\therefore x = \cos^{-1}(0.8703)$$
$$= 29.5\underline{\underline{0}}°$$
$$\approx 29.5° \text{ (correct to 1 decimal place)}$$

Q.E.D.

(ii) A vertical tower, GT, is constructed at point G and is pivoted to the ground at the points E, F and H using pieces of wire. The angle of elevation of the top of the tower, T, from the point F is 31°.

What length of wire was used to secure Point T to Point F?

SOLUTION:
Data: A vertical tower, GT, is constructed at point G and is pivoted to the ground at the points E, F and H using pieces of wire. The angle of elevation of the top of the tower, T, from the point F is 31°.
Required to find: The length of wire used to secure Point T to Point F
Solution:



TF is the wire used to secure T to F.
Triangle TGF is a right-angled triangle

$$\cos 31° = \frac{6.9}{TF}$$

$$\therefore TF = \frac{6.9}{\cos 31°}$$

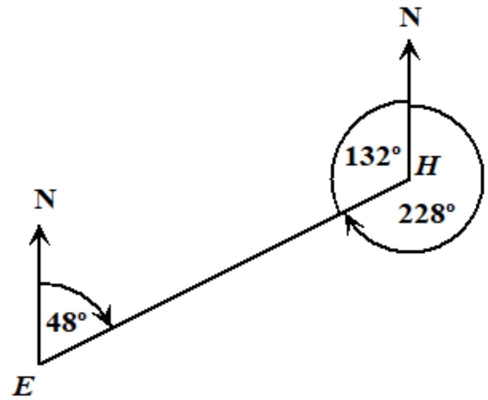
$$= 8.04\underline{\underline{9}}$$

$$\approx 8.05 \text{ m (correct to 2 decimal places)}$$

(iii) The bearing of E from H is 228°. Find the bearing of:

a) H from K

SOLUTION:
Data: The bearing of E from H is 228°.
Required to find: The bearing of H from E
Solution:



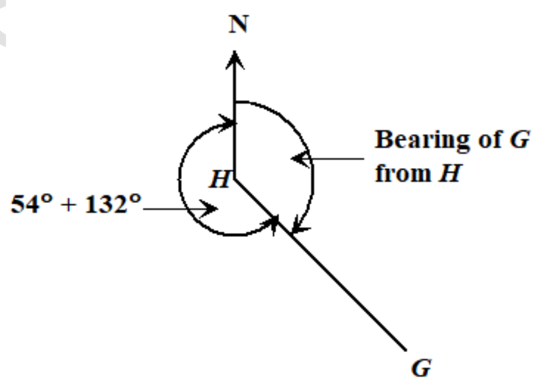
$N\hat{H}E = 360° - 228°$ (angle in a circle)
$= 132°$

$N\hat{E}H = 180° - 132°$
$= 48°$
(co-interior angles are supplementary.)

Hence, the bearing of *H* from *E* is 048°.

b) the bearing of *G* from *H*.

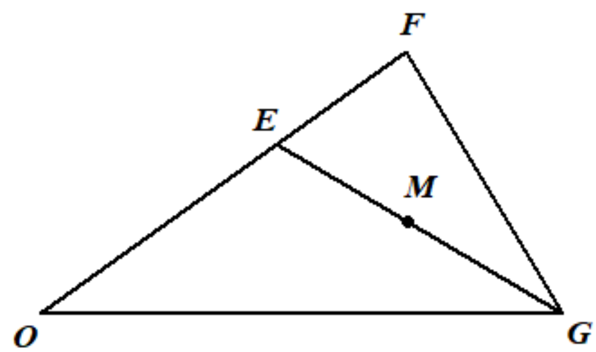
SOLUTION:
Required to find: The bearing of *G* from *H*
Solution:



The bearing of *G* from *H* is shown by the obtuse angle in the diagram $= 360° - (54° + 132°)$ (sum of angles in a circle = 360^0)
$= 360° - 186°$
$= 174°$

VECTORS AND MATRICES

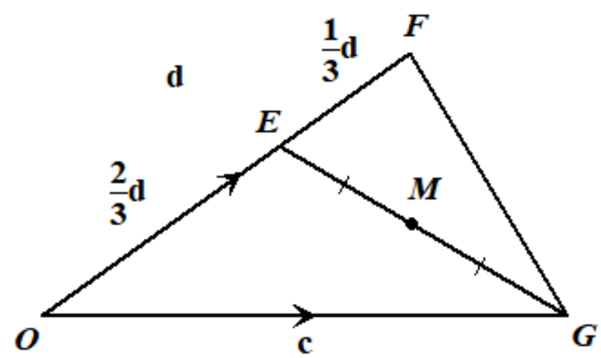
10. (a) In the diagram below, O is the origin, $OE = 2EF$ and M is the midpoint of EG. $\overrightarrow{OG} = \mathbf{c}$ and $\overrightarrow{OF} = \mathbf{d}$.



Find in terms of **c** and **d**, in its simplest form:

(i) $\overrightarrow{FG}$

Data: Diagram showing O as the origin with $OE = 2EF$ and M is the midpoint of EG. $\overrightarrow{OG} = \mathbf{c}$ and $\overrightarrow{OF} = \mathbf{d}$.
Required to find: $\overrightarrow{FG}$, in terms of **c** and **d**
Solution:



First, based on the data it is best to establish that

$\overrightarrow{OF} = \mathbf{d}$

$OE = 2EF$

$\therefore \overrightarrow{OE} = \frac{2}{3}\mathbf{d}$ and $\overrightarrow{EF} = \frac{1}{3}\mathbf{d}$

We consider the triangle FOG

$$\overrightarrow{FG} = \overrightarrow{FO} + \overrightarrow{OG}$$
$$= -(\mathbf{d}) + \mathbf{c}$$
$$= -\mathbf{d} + \mathbf{c} \text{ or } \mathbf{c} - \mathbf{d}$$

(ii) $\overrightarrow{EG}$

SOLUTION:
Required to find: $\overrightarrow{EG}$ in terms of **c** and **d**
Solution:
We consider the triangle *EOG*

$$\overrightarrow{EG} = \overrightarrow{EO} + \overrightarrow{OG}$$
$$= -\left(\frac{2}{3}\mathbf{d}\right) + \mathbf{c}$$
$$= \mathbf{c} - \frac{2}{3}\mathbf{d}$$

(iii) $\overrightarrow{OM}$

SOLUTION:
Required to find: $\overrightarrow{OM}$ in terms of **c** and **d**
Solution:
We consider the triangle *EOM*

$$\overrightarrow{OM} = \overrightarrow{OE} + \overrightarrow{EM} \qquad EM = \frac{1}{2}EG$$
$$= \frac{2}{3}\mathbf{d} + \frac{1}{2}\overrightarrow{EG}$$
$$= \frac{2}{3}\mathbf{d} + \frac{1}{2}\left(\mathbf{c} - \frac{2}{3}\mathbf{d}\right)$$
$$= \frac{2}{3}\mathbf{d} + \frac{1}{2}\mathbf{c} - \frac{1}{3}\mathbf{d}$$
$$= \frac{1}{2}\mathbf{c} + \frac{1}{3}\mathbf{d}$$

(b) The matrices P, Q and R are given below, in terms of the scalar constants a, b and c as

$$P = \begin{pmatrix} 3 & -9 \\ a & 7 \end{pmatrix}, \quad Q = \begin{pmatrix} -1 & b \\ -4 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} c & -3 \\ -4 & 8 \end{pmatrix}$$

Given that $P + Q = R$, find the values of a, b and c.

SOLUTION:
Data: $P = \begin{pmatrix} 3 & -9 \\ a & 7 \end{pmatrix}, Q = \begin{pmatrix} -1 & b \\ -4 & 1 \end{pmatrix}, R = \begin{pmatrix} c & -3 \\ -4 & 8 \end{pmatrix}$ and $P + Q = R$
Required to find: The values of a, b and c

Solution:

$$P+Q=R$$

$$\begin{pmatrix}3 & -9\\ a & 7\end{pmatrix}+\begin{pmatrix}-1 & b\\ -4 & 1\end{pmatrix}=\begin{pmatrix}c & -3\\ -4 & 8\end{pmatrix}$$

$$\begin{pmatrix}3+(-1) & -9+b\\ a+(-4) & 7+1\end{pmatrix}=\begin{pmatrix}c & -3\\ -4 & 8\end{pmatrix}$$

$$\begin{pmatrix}2 & -9+b\\ a-4 & 8\end{pmatrix}=\begin{pmatrix}c & -3\\ -4 & 8\end{pmatrix}$$

Equating corresponding entries we obtain

$c=2$

$a-4=-4$

$a=0$

$-9+b=-3$

$b=-3+9$

$b=6$

$\therefore a=0,\ b=6$ and $c=2$

(c) Solve the following pair of simultaneous equations using a matrix method.

$$5x-2y=44$$
$$2x+3y=10$$

SOLUTION:
Data: $5x-2y=44,\ 2x+3y=10$
Required to solve: For *x* and *y*, using a matrix method
Solution:
Let $5x-2y=44 \quad \ldots❶$
$2x+3y=10 \quad \ldots❷$

Expressing the equations in a matrix form we obtain

$$\begin{pmatrix}5 & -2\\ 2 & 3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}44\\ 10\end{pmatrix}$$

$$A=\begin{pmatrix}5 & -2\\ 2 & 3\end{pmatrix}$$

We first find $|A|=(5\times 3)-(-2\times 2)$

$$=15+4$$

$$=19$$

$$\therefore A^{-1}=\frac{1}{19}\begin{pmatrix}3 & -(-2)\\ -(2) & 5\end{pmatrix}$$

$$=\begin{pmatrix}\frac{3}{19} & \frac{2}{19}\\ -\frac{2}{19} & \frac{5}{19}\end{pmatrix}$$

Per-multiply both sides of the matrix equation by A^{-1}:

$$\underbrace{\underbrace{A^{-1}\times A}_{I}\times\begin{pmatrix}x\\ y\end{pmatrix}}_{\begin{pmatrix}x\\ y\end{pmatrix}}=\underbrace{\underset{2\times 2}{\begin{pmatrix}\frac{3}{19} & \frac{2}{19}\\ -\frac{2}{19} & \frac{5}{19}\end{pmatrix}}\underset{2\times 1}{\begin{pmatrix}44\\ 10\end{pmatrix}}}_{2\times 1}$$

$$=\begin{pmatrix}e_{11}\\ e_{21}\end{pmatrix}$$

$$e_{11}=\left(\frac{3}{19}\times 44\right)+\left(\frac{2}{19}\times 10\right)$$

$$=\frac{132+20}{19}$$

$$=\frac{152}{19}$$

$$=8$$

$$e_{21}=\left(-\frac{2}{19}\times 44\right)+\left(\frac{5}{19}\times 10\right)$$

$$=-\frac{88}{19}+\frac{20}{19}$$

$$=-\frac{38}{19}$$

$$=-2$$

$$\therefore\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}8\\ -2\end{pmatrix}$$

Equating corresponding entries we obtain

$x=8$ and $y=-2$