FAS-PASS Maths

NCSE 2016 PAPER 2

Section I

1. (a) **Required to calculate:** The value of $\dfrac{1\frac{1}{3}-\frac{1}{2}}{1\frac{1}{3}}$.

Calculation:

$$\dfrac{1\frac{1}{3}-\frac{1}{2}}{1\frac{1}{3}}$$

Numerator:

$$1\frac{1}{3}-\frac{1}{2}=\frac{4}{3}-\frac{1}{2}$$
$$=\frac{2(4)-3(1)}{6}$$
$$=\frac{8-3}{6}$$
$$=\frac{5}{6}$$

So we now have that,

$$\dfrac{1\frac{1}{3}-\frac{1}{2}}{1\frac{1}{3}}=\dfrac{\frac{5}{6}}{1\frac{1}{3}}$$
$$=\dfrac{\frac{5}{6}}{\frac{4}{3}}$$
$$=\frac{5}{\cancel{6}_2}\times\frac{\cancel{3}}{4}$$
$$=\frac{5}{8}$$

(This is the result in exact form and expressed in its lowest terms)

(b) **(i)** **Required to express:** 17.352 to 1 decimal place.

Solution:

$1\,7.\,3\,\underline{\underline{5}}\,2$

$\uparrow$

Deciding digit $(=5)$

$$\begin{array}{r} 17.3 \\ +\quad\ 1 \\ \hline 17.4 \\ \hline \end{array}$$

$\therefore 17.352 \approx 17.4$ correct to 1 decimal place.

(ii) **Required to express:** 17.352 to 2 significant figures.

Solution:

$1\,7\,.\,\underline{\underline{3}}\,5\,2$

$\uparrow$

Deciding digit (<5)

$\therefore 17.352 \approx 17$ written correct to 2 significant figures.

2. **Data:** The Universal set , U, is such that $U = \{\text{Whole numbers from 2 to 9 inclusive}\}$.

A and B are subsets of U such that $A = \{\text{Multiples of three}\}$ and

$B = \{\text{Even numbers}\}$.

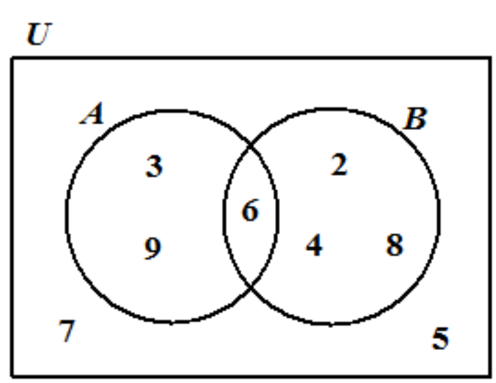
(a) **Required to complete:** The Venn diagram to show the information given.

Solution:

$U = \{2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{3, 6, 9\}$

$B = \{2, 4, 6, 8\}$



(b) **Required to find:** The probability that a number chosen at random is both even and a multiple of three.

Solution:

6 is the only number that is both even and a multiple of three.

$\therefore$ P (A number chosen at random is both even and a multiple of three)

$$= \frac{\text{No. of numbers that are both even and a multiple of three}}{\text{Total no. of numbers}}$$

$$= \frac{1}{8}$$

3. (a) **Required to simplify:** $2a \times 4ab$

Solution:

$$2a \times 4ab = \underbrace{2 \times 4} \times \underbrace{a \times a} \times b$$
$$= 8a^2b$$

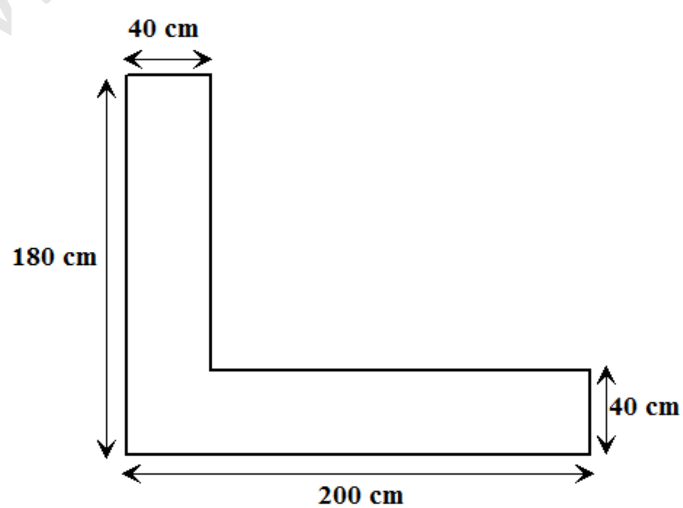
(b) **Data:** $m = 2$ and $n = 3$

Required to calculate: The value of $\frac{4m - 3n}{m \times n}$.

Calculation:

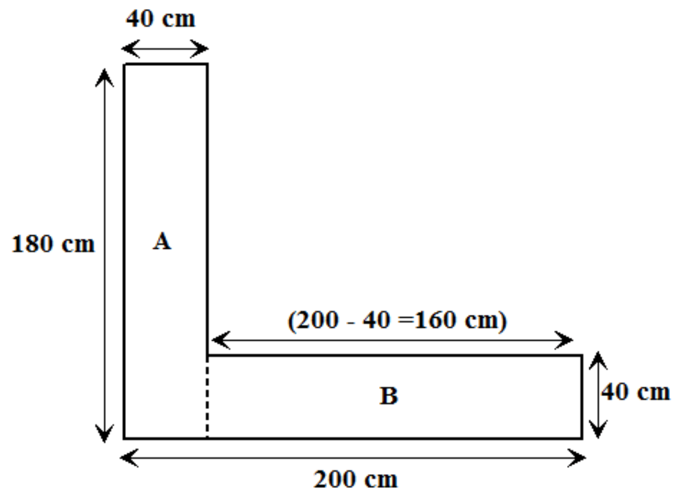
$$\frac{4m - 3n}{m \times n} = \frac{4(2) - 3(3)}{(2) \times (3)}$$
$$= \frac{8 - 9}{6}$$
$$= \frac{-1}{6}$$

4. **Data:** Diagram showing dimensions of a countertop.



(a) **Required to calculate:** The surface area of the countertop.

Calculation:



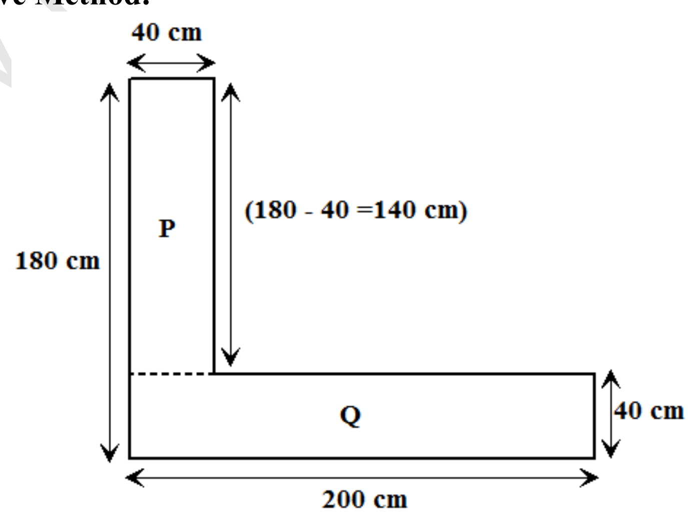
The countertop is a compound shape, so it can be divided into two or more 'simpler' shapes. In this case two and which we call regions, A and B, as shown.

Area of the rectangle A $= (180 \times 40)\ \text{cm}^2$
$= 7200\ \text{cm}^2$

Area of the rectangle B $= (160 \times 40)\ \text{cm}^2$
$= 6400\ \text{cm}^2$

Hence the area of the countertop $= (7200 + 6400)\ \text{cm}^2$
$= 13600\ \text{cm}^2$

Alternative Method:



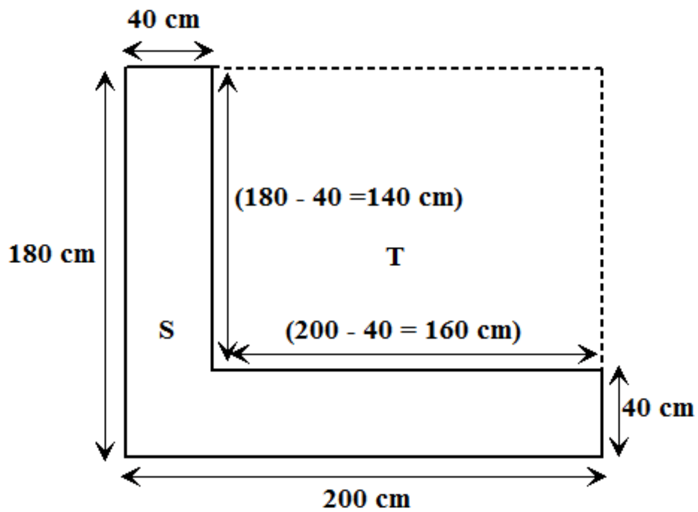
The countertop could be divided into two regions, P and Q, as shown.

Area of rectangle P = $(140 \times 40)\ \text{cm}^2$
$= 5600\ \text{cm}^2$

Area of rectangle Q = $(200 \times 40)\ \text{cm}^2$
$= 8000\ \text{cm}^2$

Hence, area of the countertop $= (5600 + 8000)\ \text{cm}^2$
$= 13600\ \text{cm}^2$

Alternative Method:



The rectangle, T, as shown in the diagram, is added to the countertop S so that together they form a complete rectangle.

Area of the rectangle, composed of S and T together = $(200 \times 180)\ \text{cm}^2$
$= 36000\ \text{cm}^2$

Area of T = $(140 \times 160)\ \text{cm}^2$
$= 22400\ \text{cm}^2$

Hence, area of the countertop = Area of (S and T) – Area of T
$= (36000 - 22400)\ \text{cm}^2$
$= 13600\ \text{cm}^2$

(b) **Data:** The countertop has to be tiled using square tiles of side 5 cm.

Required to find: The number of tiles that are needed to completely cover the countertop

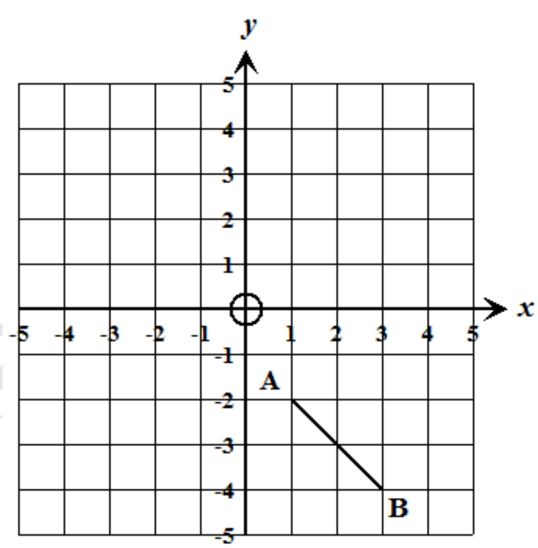
Solution:

$$\text{Area of 1 square tile } = (5 \times 5)\text{ cm}^2$$
$$= 25\text{ cm}^2$$

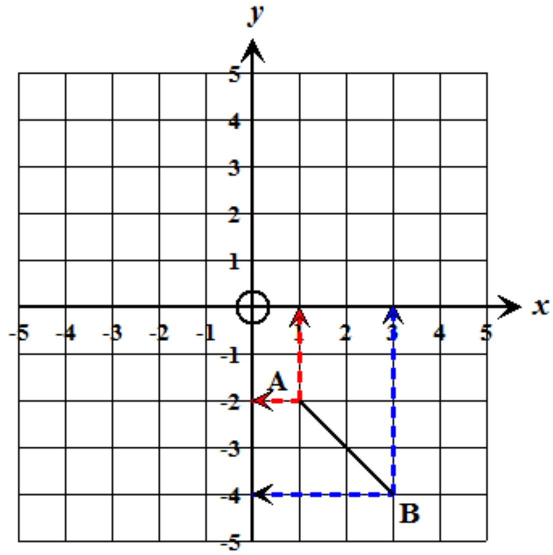
$\therefore$ the number of tiles required to cover the countertop

$$= \frac{\text{Area of countertop}}{\text{Area of 1 tile}}$$
$$= \frac{13\,600}{25}$$
$$= 544\text{ tiles}$$

5. **Data:** Graph showing line segment AB.

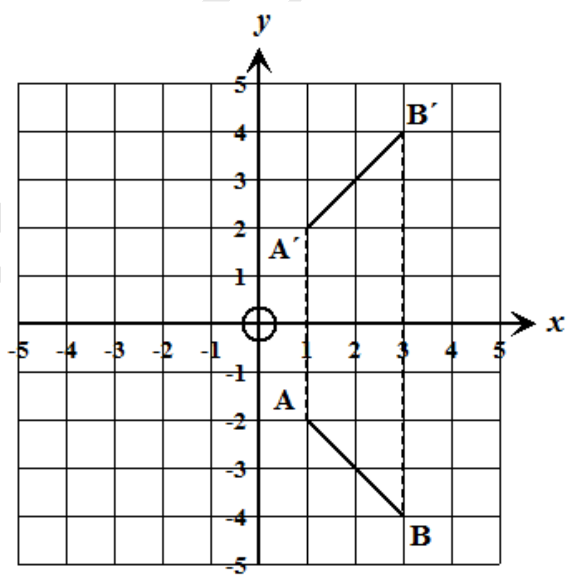


(a) **(i)** **Required to state:** The coordinates of A and of B.
Solution:

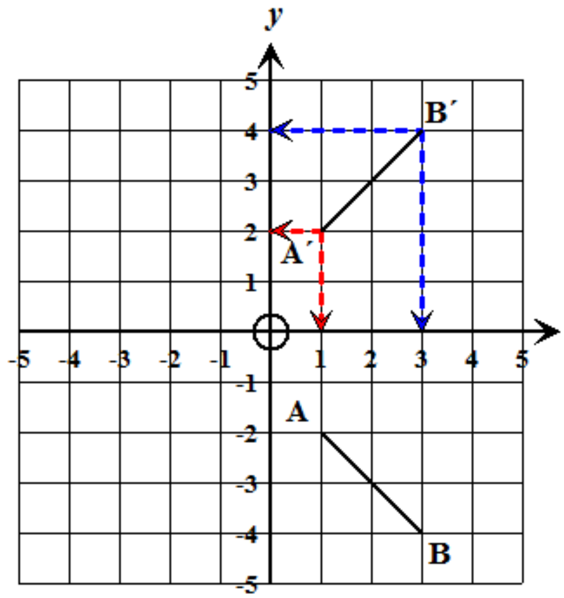


$A = (1, -2)$
$B = (3, -4)$

(ii) **Required to draw:** The line segment $A'B'$ which represents the image of AB after a reflection in the x – axis.
Solution:



(iii) **Required to state:** The coordinates of A' and of B'.
Solution:



$A' = (1, 2)$
$B' = (3, 4)$
Notice that under a reflection in the *x*-axis, the *x* coordinates of the points remain the same, but the *y* coordinates are negated.

6. Data: The marks obtained by 30 students on a Mental Mathematics Test.

2	4	5	1	2	6	2	5	5	3
4	5	6	3	2	3	1	1	1	3
4	3	4	2	3	6	4	5	4	4

(a) **Required to complete:** The frequency table given.
Solution:

Marks	Tally	Frequency
1	||||	4
2	~~||||~~	5
3	~~||||~~ |	6
4	~~||||~~ ||	7
5	~~||||~~	5
6	|||	3

$\sum f = 30$

(b) **(i)** **Required to determine:** The modal score.
Solution:
The modal score is 4 since this score occurred most often (7 times).

(ii) **Required to determine:** The median score.
Solution:
There is an even number of scores (30) in the distribution. Hence, there is no single middle score that is the median. There are two middle scores in this case, the 15th and the 16th. The scores are arranged in ascending (or descending order) of magnitude.
In ascending ordering the scores are:
1, 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, **3**, **4**, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6

The median score will therefore be $\dfrac{15^{\text{th}}\ \text{score} + 16^{\text{th}}\ \text{score}}{2}$

$$\begin{aligned}\text{Median } &= \frac{3+4}{2}\\ &= \frac{7}{2}\\ &= 3.5 \text{ or } 3\frac{1}{2}\end{aligned}$$

(iii) **Required to determine:** The mean score.
Solution:

$$\begin{aligned}\text{The mean score } &= \frac{\sum fx}{\sum f} \text{ where } \sum = \text{sum of, } f = \text{frequency, } x = \text{score}\\ &= \frac{(4\times1)+(5\times2)+(6\times3)+(7\times4)+(5\times5)+(3\times6)}{30}\\ &= \frac{4+10+18+28+25+18}{30}\\ &= \frac{103}{30}\\ &= 3.43\underline{\underline{3}}\\ &= 3.4\underline{\underline{3}} \text{ correct to 2 decimal places}\\ &= 3.4 \text{ correct to 1 decimal place}\end{aligned}$$

Section II

7. (a) **Data:** The marked price of a stove is \$3 600.00. It can be bought using Plan A, cash with a 20% discount on the marked price or using Plan B, on hire purchase with no down payment and 24 equal monthly installments of \$200.00 each.

(i) **Required to calculate:** The discount using Plan A.
Calculation:

The discount using Plan A is 20% of the marked price $= \frac{20}{100} \times \$3600$
$= \$720$

(ii) **Required to calculate:** The price paid using Plan A.
Calculation:
If Plan A is used, the price that would be paid $= \text{Marked price} - \text{Discount}$
$= \$3600 - \$720$
$= \$2880$

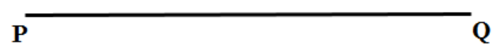
OR

We could find (100-20) % = 80% of the marked price.
This would be $\frac{80}{100} \times \$3\,600$
$= \$\,2\,880$

(iii) **Required to calculate:** The hire purchase price using Plan B.
Calculation:
The hire purchase price using Plan B
$= \text{Down payment} + \text{Installments over 24 months}$
$= \$0 + (24 \times \$200)$
$= \$4800$

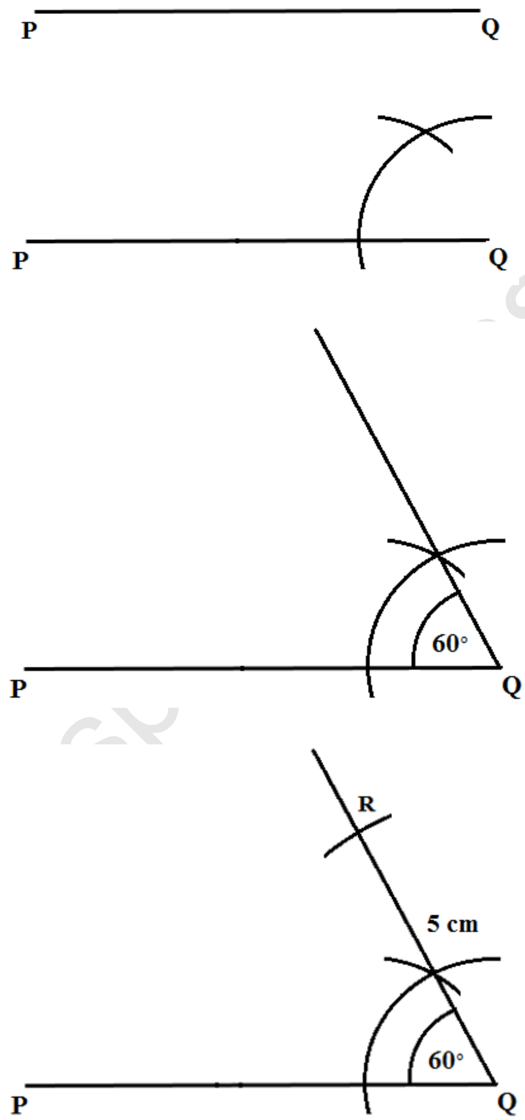
(iv) **Required to calculate:** The difference in price between Plan A and Plan B.
Calculation:
The difference in price between Plan B and Plan A $= \$4800 - \$2880$
$= \$1920$
That is, the hire purchase plan costs \$ 1 920 more than the cash payment plan.

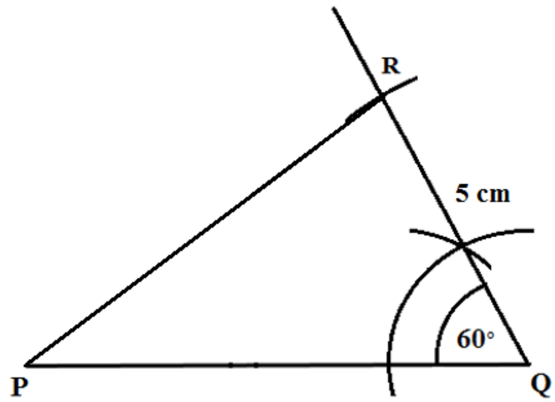
(b) **Data:** Line segment PQ.



(i) **Required to construct:** Triangle PQR with angle $PQR = 60°$ and $QR = 5\text{cm}$.

Solution:



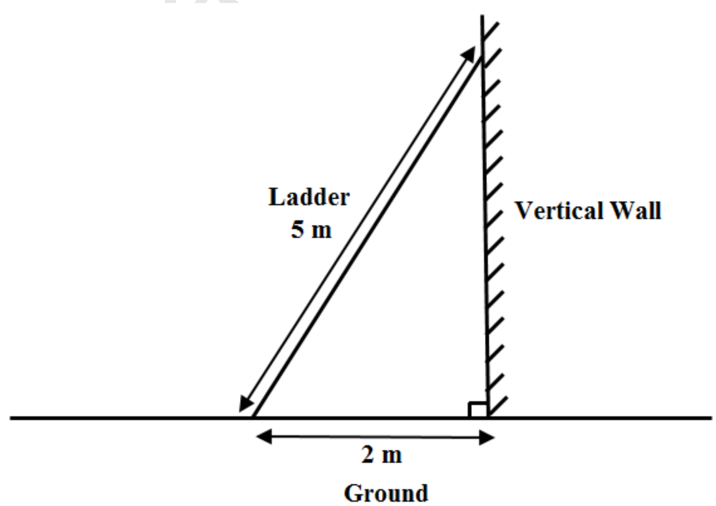


(ii) **Required to measure:** And state the length of the line segment PQ.
Solution:
Length of PQ $= 8.5$ cm
(by measurement using a pair of dividers and the ruler)

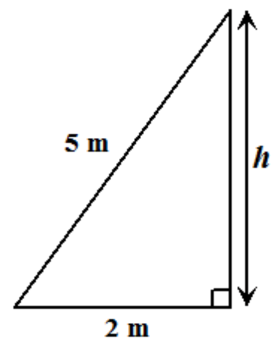
(iii) **Required to measure:** And state the size of the angle QPR.
Solution:
The angle QPR = 37^0 (by measurement, using the protractor)

8. (a) **Data:** A 5 m long ladder is placed against a vertical wall. The foot of the ladder is 2 m from the wall.

(i) **Required to draw:** A labelled diagram to represent the information given.
Solution:



(ii) **Required to calculate:** The vertical distance of the top of the ladder to the floor, correct to 2 decimal places.
Calculation:



Let h be the vertical distance of the top of the ladder to the floor.

$$(2)^2 + (h)^2 = (5)^2 \qquad \text{Pythagoras' Theorem}$$
$$h^2 = (5)^2 - (2)^2$$
$$h = \sqrt{25-4}$$
$$= \sqrt{21}$$
$$= 4.58\underline{\underline{2}} \text{ m}$$
$$= 4.58 \text{ m correct to 2 decimal places}$$

(b) **Data:** Susan took 5 minutes to walk along a straight road 600 m long.

(i) **Required to find:** The number of centimetres that would be used to represent 600 m on a map with scale of 1 cm representing 100 m.

Solution:

Scale $1 \text{ cm} \equiv 100 \text{ m}$

$$\therefore 100 \text{ m} \equiv 1 \text{ cm on the map}$$
$$\text{So, } 1 \text{ m} \equiv \frac{1}{100} \text{ cm}$$
$$600 \text{ m} \equiv \frac{1}{100} \times 600 \text{ cm}$$
$$= 6 \text{ cm on the map}$$

(ii) **Required to find:** The distance Susan walked in kilometres.

Solution:

$1000 \text{ m} \equiv 1 \text{ km}$

$$\therefore 1 \text{ m} = \frac{1}{1000} \text{ km}$$

$$600 \text{ m} = \frac{1}{1000} \times 600$$

$$= \frac{600}{1000}$$

$$= \frac{3}{5} \text{ km}$$

Hence, Susan walked $\frac{3}{5}$ km or 0.6 km

(iii) **Required to convert:** 5 minutes to hours, expressing the answer as a fraction in its lowest terms.

Solution:

$60 \text{ minutes} \equiv 1 \text{ hour}$

$$\therefore 1 \text{ minutes} = \frac{1}{60} \text{ hour}$$

$$5 \text{ minutes} = \frac{1}{60} \times 5$$

$$= \frac{5}{60}$$

$$= \frac{1}{12} \text{ hour}$$

(iv) **Required to find:** Susan’s speed in kilometres per hour.

Solution:

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time taken}}$$

$$= \frac{\frac{3}{5} \text{ km}}{\frac{1}{12} \text{ hour}}$$

$$= \frac{3}{5} \times \frac{12}{1}$$

$$= 7\frac{1}{5} \text{ kmh}^{-1}$$

$$= 7.2 \text{ kmh}^{-1}$$

9 (a) (i) **Data:** The cost of four pencils and one ruler is \$11.00. The cost of two pencils and one ruler is \$7.00. The cost of one pencil is \$$p$ and the cost of one ruler is \$$r$.

Required to write: Two equations to the represent the information given

Solution:

Cost of 4 pencils at \$$p$ each and 1 ruler at \$$r$ each is \$11.00

$$4\times p \quad + \quad 1\times r \quad = \quad 11$$

$\therefore 4p+r=11 \quad \ldots$ ❶

Cost of 2 pencils at \$$p$ each and 1 ruler at \$$r$ each is \$7.00

$$2\times p \quad + \quad 1\times r \quad = \quad 7$$

$\therefore 2p+r=7 \quad \ldots$ ❷

(ii) **Required to solve:** The pair of equations simultaneously to find the value of p and of r.

Solution:

$4p+r=11 \quad \ldots$ ❶

$2p+r=7 \quad \ldots$ ❷

Equation ❶ − Equation ❷:

$$\begin{array}{rl} & 4p+r=11 \\ - & \underline{2p+r=\ 7} \\ & \underline{2p \quad\ =4} \\ & \quad p=2 \end{array}$$

Substitute $p=2$ into equation ❶:

$4(2)+r=11$

$\therefore r=11-8$

$=3$

$\therefore p=2$ and $r=3$

Alternative Method:

$4p+r=11 \quad \ldots$ ❶

$2p+r=7 \quad \ldots$ ❷

From equation ❶:

$r=11-4p \quad \ldots$ ❸

Substitute equation ❸ into equation ❷:

$$2p+(11-4p)=7$$
$$-2p+11=7$$
$$-2p=7-11$$
$$-2p=-4$$
$$p=\frac{-4}{-2}$$
$$p=2$$

Substitute $p=2$ into equation ❸:

$$r=11-4(2)$$
$$=11-8$$
$$=3$$

$\therefore\ p=2$ and $r=3$

(b) **(i)** **Required to complete:** The arrow diagram for the relation $f:x\to 2x+3$ for the domain $\{0, 1, 2, 3\}$.

Solution:

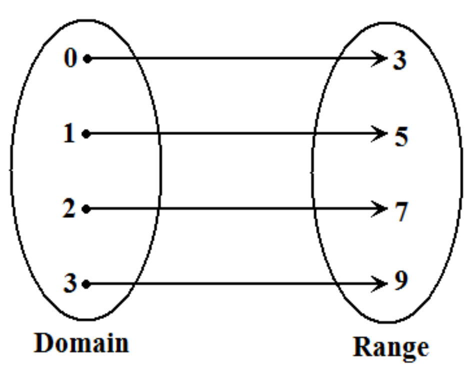
When $x=2$

$$f(2)=2(2)+3$$
$$=7$$

When $x=3$

$$f(3)=2(3)+3$$
$$=9$$

Hence, the completed arrow diagram is



(ii) **Required to write:** The list of ordered pairs for the function $f: x \to 2x+3$ for the domain $\{0, 1, 2, 3\}$.

Solution:

$f(0)=3$

$f(1)=5$

$f(2)=7$

$f(3)=9$

$\therefore$ The list of ordered pairs is $(0, 3)$, $(1, 5)$, $(2, 7)$ and $(3, 9)$.

(iii) **Required to plot:** The ordered pairs in (b) (ii) above.
Solution: