FAS-PASS Maths

NCSE 2015 PAPER 2

Section I

1. (a) Required to calculate: $6\frac{1}{6}-1\frac{3}{4}$ expressing the answer as a mixed number.

Calculation:

$$
\begin{aligned}
6\frac{1}{6}-1\frac{3}{4} &= \frac{(6\times6)+1}{6}-\frac{(4\times1)+3}{4} \\
&= \frac{37}{6}-\frac{7}{4} \\
&= \frac{2(37)-3(7)}{12} \\
&= \frac{74-21}{12} \\
&= \frac{53}{12} \\
&= 4\frac{5}{12} \text{ as a mixed number.}
\end{aligned}
$$

(b) Required to convert: $\frac{3}{8}$ to a percent.

Solution:

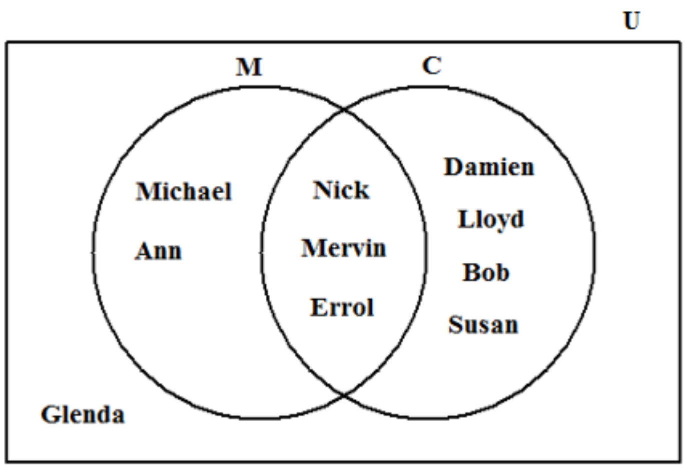
$$
\begin{aligned}
\frac{3}{8} \text{ as a percent } &= \frac{3}{8}\times100 \\
&= \frac{300}{8} \\
&= 37\frac{4}{8}\% \\
&= 37\frac{1}{2}\%
\end{aligned}
$$

(c) **Required to express:** 7.185 correct to 2 significant figures.
Solution:
$7\ .\ 1\ \underline{\underline{8}}\ 5$
$\uparrow$
Deciding digit which is ≥ 5
So we add 1 to the first decimal and ignore all digits to its immediate right

$$\begin{array}{r} 7.1 \\ +\ \ 1 \\ \hline 7.2 \\ \hline \end{array}$$

$\therefore 7.185 \approx 7.2$ correct to 2 significant figures.

2. **Data:** Venn diagram showing friends who are good at Mathematics and friends who play cricket from a group.



(a) **(i)** **Required to list:** All the friends who are good at Mathematics.
Solution:
$M = \{\text{Michael, Ann, Nick, Mervin, Errol}\}$

(ii) **Required to state:** The number of friends who play cricket ONLY.
Solution:
Friends who play cricket only are Damien, Lloyd, Bob and Susan. These number four.
$\therefore$ 4 students play cricket only.

(b) **(i)** **Required to state:** The number of friends who are neither good at Mathematics nor play cricket.
Solution:
Glenda, only, is neither good at Mathematics nor plays cricket.
$\therefore$ 1 friend is neither good at Mathematics nor plays cricket.

(ii) **Required to find:** The probability that a friend is neither good at Mathematics nor plays cricket?
Solution:
P(Friend is neither good at Mathematics nor plays cricket)

$$=\frac{\text{No. of friends who are neither good at Mathematics nor plays cricket}}{\text{Total no. of friends}}$$

$$=\frac{1}{1+2+3+4}$$

$$=\frac{1}{10}$$

It is important to note that, by definition, a set is a clearly defined collection of objects. The description, 'good at mathematics' is a relative term and is not clearly defined. Hence, the statement that defines M, in this question, does not define a set.

(c) **Required to find:** The probability that a friend is good at Mathematics and plays cricket.
Solution:
P(Friend is good at Mathematics and plays cricket)

$$=\frac{\text{No. of friends who are good at Mathematics and play cricket}}{\text{Total no. of friends}}$$

$$=\frac{3}{1+2+3+4}$$

$$=\frac{3}{10}$$

3. (a) **(i)** **Required to simplify:** $6m-2p-3m+4p$
Solution:

$$\begin{aligned}6m-2p-3m+4p &= 6m-3m-2p+4p\\ &= 3m+2p\end{aligned}$$

(ii) **Required to simplify:** $3x(5x-5)-4x^2$
Solution:

$$\begin{aligned}3x(5x-5)-4x^2 &= 15x^2-15x-4x^2\\ &= 11x^2-15x\\ &= x(11x-15)\end{aligned}$$

(b) **(i)** **Required to factorise:** $30+24p$

Solution:

$30 + 24p = \underline{6} \text{ x } 5 + \underline{6} \text{ x } 4 \text{ x } p$

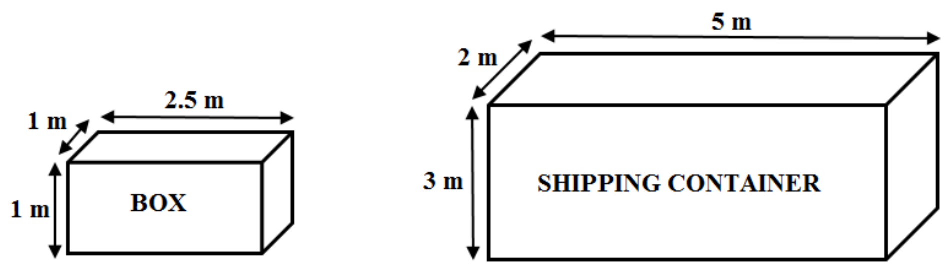
$= 6\,(5+4p)$

(ii) **Required to factorise:** $6p^2+19p+15$

Solution:

$6p^2+19p+15=(3p+5)(2p+3)$

4. **Data:** Diagrams of cuboid shaped containers, a box and a shipping container. The box has dimensions 2.5 m by 1 m by 1 m and the shipping container has dimensions 3 m by 2 m by 5 m.



(a) **Required to find:** The volume of a box, in cubic metres.

Solution:

Volume of a box $=(2.5\times1\times1)\text{ m}^3$

$=2.5\text{ m}^3$

(b) **Required to determine:** The number of boxes that will completely fill the shipping container.

Solution:

Number of boxes that will completely fill the shipping container will be

$$=\frac{\text{Volume of shipping container}}{\text{Volume of a box}}$$

$$=\frac{5\times2\times3}{2.5}$$

$=12$ boxes

(c) **Required to convert:** The volume of a box from cubic metres to cubic centimetres.
Solution:

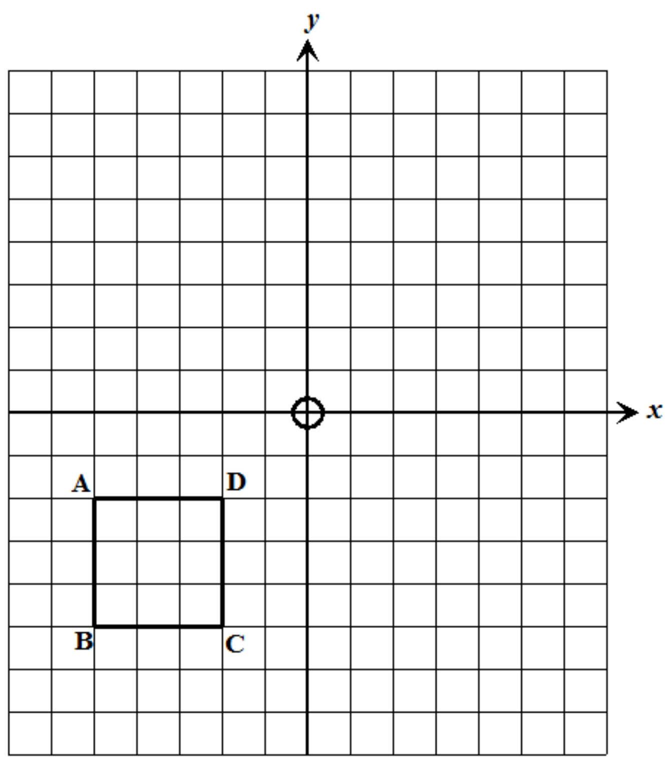
$1\text{ m} = 100\text{ cm}$

$\therefore 1\text{ m}^3 = (100 \times 100 \times 100)\text{ cm}^3$

$1\text{ m}^3 = 1000\,000\text{ cm}^3$

Volume of a box $= 2.5\text{ m}^3$
$= (2.5 \times 100 \times 100 \times 100)\text{ cm}^3$
$= 2\,500\,000\text{ cm}^3$
$= 2.5 \times 10^6\text{ cm}^3$

5. **Data:** Diagram showing a quadrilateral ABCD on a grid.



(a) **Data:** ABCD is translated under $\begin{pmatrix} 6 \\ 9 \end{pmatrix}$ to produce its image, $A'B'C'D'$.

Required to draw: $A'B'C'D'$ on the same diagram.
Solution:

$\text{ABCD} \xrightarrow{\begin{pmatrix} 6 \\ 9 \end{pmatrix}} A'B'C'D'$
$A = (-5, -2)$

$$\begin{pmatrix}-5\\-2\end{pmatrix}\xrightarrow{\binom{6}{9}}\begin{pmatrix}-5+6\\-2+9\end{pmatrix}=\begin{pmatrix}1\\7\end{pmatrix}$$

$\therefore A'=(1,\ 7)$

$B=(-5,\ -5)$

$$\begin{pmatrix}-5\\-5\end{pmatrix}\xrightarrow{\binom{6}{9}}\begin{pmatrix}-5+6\\-5+9\end{pmatrix}=\begin{pmatrix}1\\4\end{pmatrix}$$

$\therefore B'=(1,\ 4)$

$C=(-2,\ -5)$

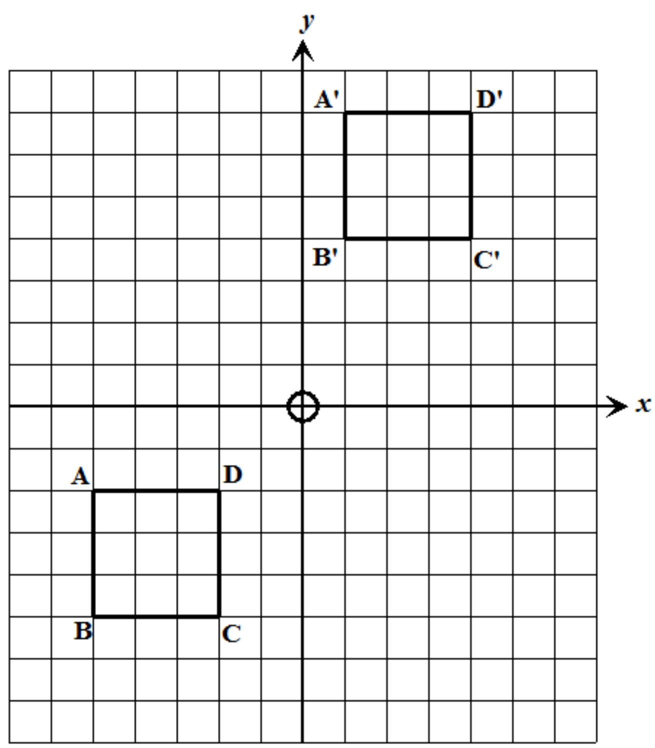
$$\begin{pmatrix}-2\\-5\end{pmatrix}\xrightarrow{\binom{6}{9}}\begin{pmatrix}-2+6\\-5+9\end{pmatrix}=\begin{pmatrix}4\\4\end{pmatrix}$$

$\therefore C'=(4,\ 4)$

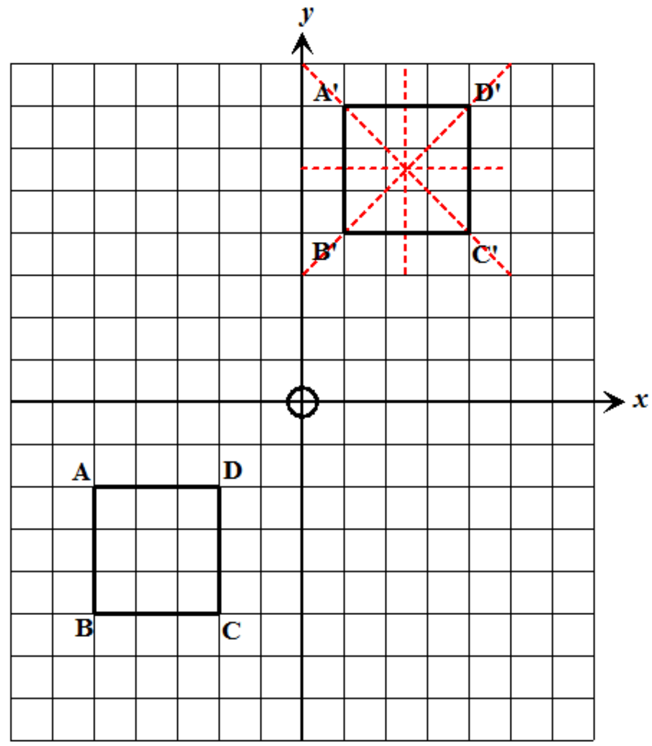
$D=(-2,\ -2)$

$$\begin{pmatrix}-2\\-2\end{pmatrix}\xrightarrow{\binom{6}{9}}\begin{pmatrix}-2+6\\-2+9\end{pmatrix}=\begin{pmatrix}4\\7\end{pmatrix}$$

$\therefore D'=(4,\ 7)$



(b) **Required to draw:** Lines of symmetry for $A'B'C'D'$ on the diagram.
Solution:



$A'B'C'D'$ is a square.

There are 4 lines of symmetry, shown broken on the diagram.

6. **Data:** List of the numbers of occupants in each of 25 cars passing the lighthouse travelling towards Port of Spain during a 2 minute period.

2	2	5	4	2
5	2	3	1	5
2	1	2	3	3
3	2	4	2	6
3	4	3	2	4

(a) **Required to complete:** The frequency table given using the data.
Solution:

Number of occupants per car	Tally	Frequency
1	$\|\|$	2
2	$\cancel{\|\|\|\|}$ $\|\|\|\|$	9
3	$\cancel{\|\|\|\|}$ $\|$	6
4	$\|\|\|\|$	4
5	$\|\|\|$	3
6	$\|$	1

$$\sum f = 25$$

(b) **Required to find:** The modal number of occupants per car.
Solution:
The modal number of occupants per car is 2, since this number of occupants in a car occurs most often in accordance with the data.

(c) **Required to calculate:** The mean number of occupants per car.
Calculation:
The mean number of occupants per car is $\bar{x}$, where

$$\bar{x} = \frac{\sum fx}{\sum f} \qquad \sum = \text{sum of},\ f = \text{frequency},\ x = \text{number of occupants per car}$$

$$= \frac{(2\times1)+(9\times2)+(6\times3)+(4\times4)+(3\times5)+(1\times6)}{25}$$

$$= \frac{2+18+18+16+15+6}{25}$$

$$= \frac{75}{25}$$

$$= 3$$

Section II

7. (a) **Data:** The exchange rate between TT dollars and Euros is $\text{TT}\$1.00 = €0.125$.

(i) **Required to convert:** €1 to TT dollars.
Solution:

$$\text{TT}\$1.00 = €0.125$$
$$€0.125 = \text{TT}\$1.00$$
$$\therefore €1.00 = \text{TT}\$\frac{1.00}{0.125}$$
$$= \text{TT}\$8.00$$

(ii) **Data:** A painting costs € 250 000.
Required to find: The cost of the painting in TT dollars
Solution:
Painting costs € 250 000

$$\therefore \text{Cost in TT dollars} = 250\,000 \times 8$$
$$= \text{TT}\,\$2\,000\,000$$

(b) **Data:** Mary invests \$12 000 at a bank which pays 3% simple interest. After a certain number of years, the total amount she receives is \$15 240.

(i) **Required to find:** The amount of interest Mary received.
Solution:

Principal $= \$12\,000$
Rate $= 3\%$ per annum
Total received $= \$15\,240$

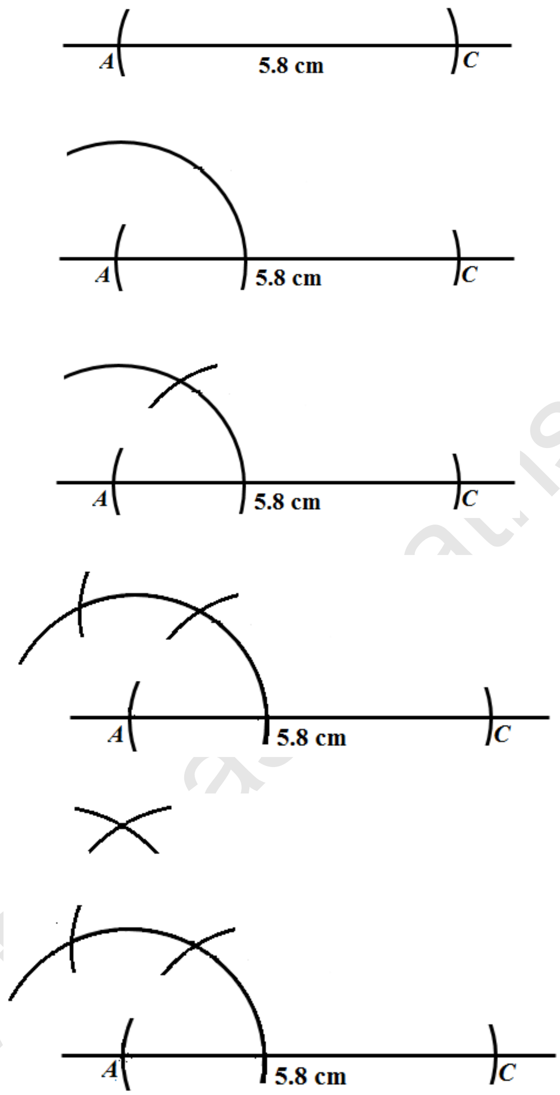
$$\therefore \text{Interest received} = \text{Amount received} - \text{Principal}$$
$$= \$15\,240 - \$12\,000$$
$$= \$3\,240$$

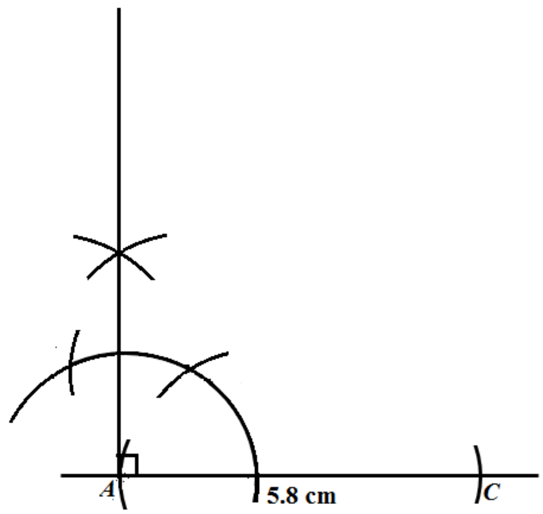
(ii) **Required to calculate:** The number of years in which Mary invested her money.
Calculation:
Let the number of years of investment be *T*.

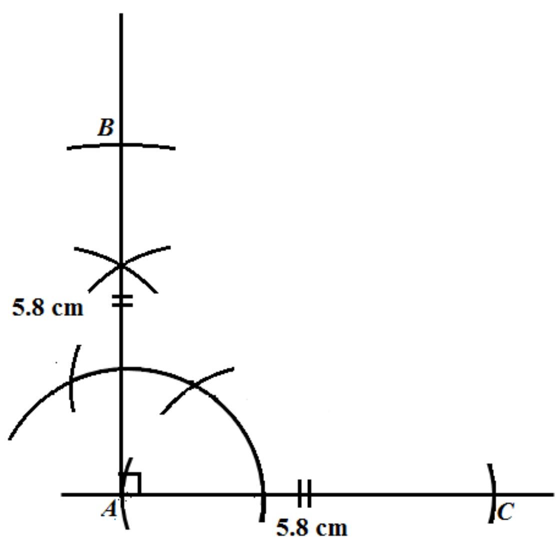
$$\text{Recall: Simple Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$
$$\therefore 3\,240 = \frac{12\,000 \times 3 \times T}{100}$$
$$T = \frac{3\,240 \times 100}{12\,000 \times 3} \quad = 9$$
$$= 9 \text{ years}$$

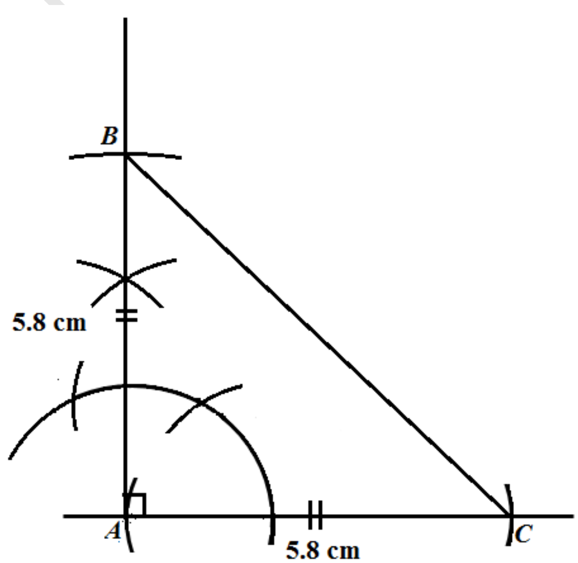
(c) **(i)** **Required to construct:** An isosceles triangle with $AB = AC = 5.8$ cm and angle $BAC = 90°$ and state the length of BC correct to 1 decimal place.

Solution:




A
5.8 cm
C

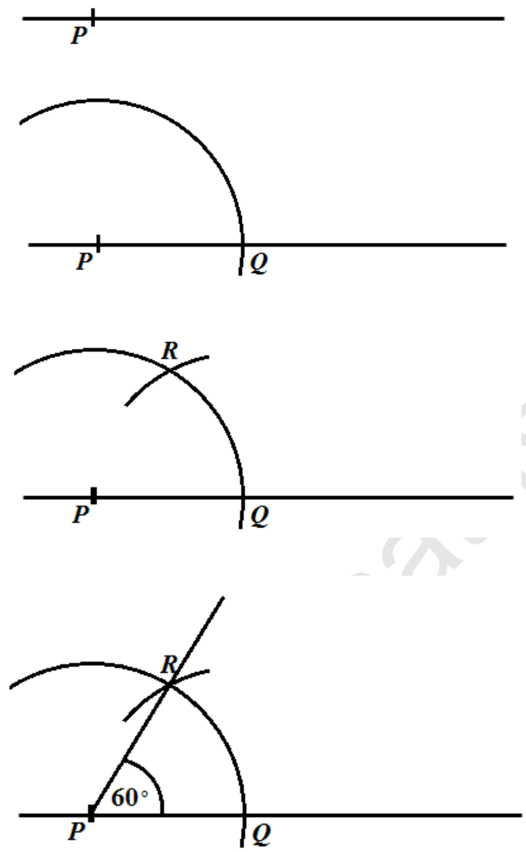

B
5.8 cm
A
5.8 cm
C


B
5.8 cm
A
5.8 cm
C

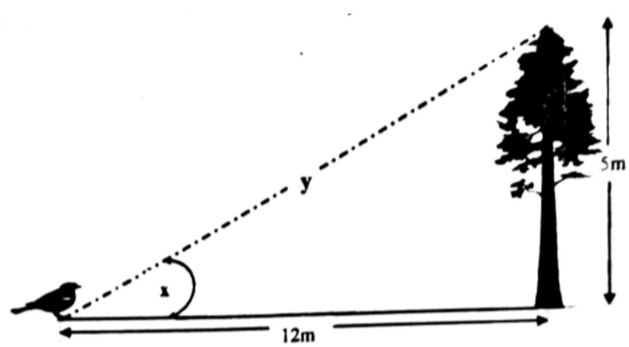
$BC = 8.2$ cm measured correct to 1 decimal place.

(ii) **Required To Construct:** An angle $PQR = 60°$.

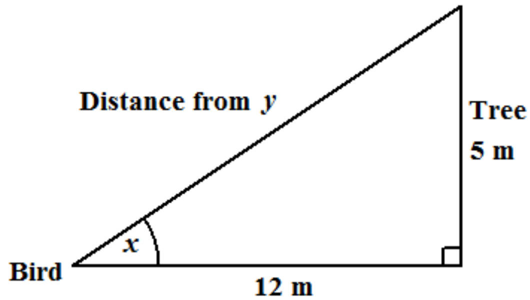
Solution:



8. (a) **Data:** Diagram showing a bird on the ground 12 m away from the base of a tree flying directly to the top of the tree that is 5 m tall.



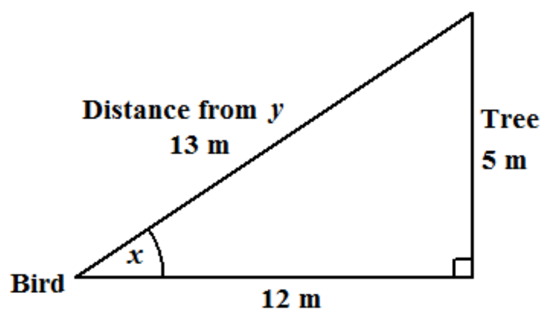
(i) **Required to calculate:** The distance, y, flown by the bird.
Calculation:



$$y^2 = (12)^2 + (5)^2 \qquad \text{(Pythagoras' Theorem)}$$

$$\begin{aligned} y &= \sqrt{(12)^2 + (5)^2} \\ &= \sqrt{144 + 25} \\ &= \sqrt{169} \\ &= 13 \text{ m} \end{aligned}$$

(ii) **Required to calculate:** x correct to 2 decimal places.
Calculation:



$$\begin{aligned} x &= \tan^{-1}\left(\frac{5}{12}\right) \\ &= 22.61\underline{\underline{9}}^\circ \\ &= 22.62^\circ \text{ correct to 2 decimal places} \end{aligned}$$

Alternative Method:

$$\begin{aligned} x &= \sin^{-1}\left(\frac{5}{13}\right) \\ &= 22.61\underline{\underline{9}}^\circ \\ &= 22.62^\circ \text{ correct to 2 decimal places} \end{aligned}$$

Alternative Method:

$$x = \cos^{-1}\left(\frac{12}{13}\right)$$
$$= 22.61\underset{=}{9}^\circ$$
$$= 22.62^\circ \text{ correct to 2 decimal places}$$

(b) **Data:** Airplane departs Piarco International Airport at 2:35 p.m. and arrives at the Arthur Napoleon Raymond Robinson International Airport at 3:05 p.m. The distance between the airports is 83 km.

(i) **Required to calculate:** The actual distance between the airports in centimetres.

Calculation:

Distance between the two airports $= 83$ km

$$1 \text{ km} = 100\,000 \text{ cm}$$
$$\therefore 83 \text{ km} = (83 \times 100\,000) \text{ cm}$$
$$= 8\,300\,000 \text{ cm}$$
$$= 8.3 \times 10^6 \text{ cm}$$

(ii) **Required to find:** The distance between the two airports, in metres.

Solution:

$$1 \text{ km} = 1000 \text{ m}$$
$$\therefore 83 \text{ km} = (83 \times 1000) \text{ m}$$
$$= 83\,000 \text{ m}$$
$$= 8.3 \times 10^4 \text{ m}$$

(iii) **Required to find:** The time taken to arrive at the Arthur Napoleon Raymond Robinson International Airport, in minutes.

Solution:

$$\begin{array}{lcl} \text{Arrival time} & = & 3:05 \text{ p.m.} \\ \text{Departure time} & = & \underline{2:35 \text{ p.m.}} \\ \text{Time of flight} & = & \underline{:30 \text{ minutes}} \end{array}$$

$\therefore$ Time taken to arrive at the Arthur Napoleon Raymond Robinson International Airport is 30 minutes.

(iv) **Required to find:** The time taken to arrive at the Arthur Napoleon Raymond Robinson International Airport, in hours.
Solution:

$$1 \text{ minute } = \frac{1}{60} \text{ hour}$$

$$\text{Hence, the time taken, in hours } = \frac{30}{60}$$
$$= \frac{1}{2} \text{ hour}$$

(v) **Required to find:** The average speed of travel for the journey in kilometres per hour.
Solution:

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Time taken}}$$
$$= \frac{83 \text{ km}}{\frac{1}{2} \text{ hour}}$$
$$= 166 \text{ kmh}^{-1}$$

9. (a) **Data:** Indra bought 3 cups of coffee and 2 pieces of cake for a total of \$12.00 and Carol bought 2 cups of coffee and 3 pieces of cake for a total of \$13.00.
x represents the cost of one cup of coffee and y represents the cost of a piece of cake.

(i) **Required to write:** An equation using x and y to represent the total cost of cups of coffee and pieces of cake that Indra bought.
Solution:
Cost of 3 cups of coffee at \$$x$ each and 2 pieces of cake at \$$y$ dollars each

$$= (3 \times x) + (2 \times y)$$
$$= 3x + 2y$$

Hence, $3x + 2y = 12 \quad \ldots❶$

(ii) **Required To Find:** The cost of one cup of coffee and one piece of cake.
Solution:
Cost of 2 cups of coffee at \$$x$ each and 3 pieces of cake at \$$y$ dollars each

$$= (2 \times x) + (3 \times y)$$
$$= 2x + 3y$$

Hence, $2x + 3y = 13 \quad \ldots❷$

$3x + 2y = 12 \quad \ldots❶$
$2x + 3y = 13 \quad \ldots❷$

Equation ❶ $\times 3$ $\quad 9x+6y=36 \quad \ldots$❸

Equation ❷ $\times -2$ $\quad -4x-6y=-26 \quad \ldots$❹

Equation ❸ + Equation ❹ $\quad 5x = 10$

$$\therefore x=2$$

Substitute $x=2$ into equation ❶:

$$3(2)+2y=12$$
$$2y=6$$
$$y=3$$

Hence, 1 cup of coffee costs \$2 and 1 piece of cake costs \$3.

Alternative Method:

$3x+2y=12 \quad \ldots$❶

$2x+3y=13 \quad \ldots$❷

Make y the subject in equation ❶:

$$2y=12-3x$$
$$y=\frac{12-3x}{2}$$

Substitute in equation ❷:

$$2x+3\left(\frac{12-3x}{2}\right)=13$$

$\times 2$

$$4x+3(12-3x)=26$$
$$4x+36-9x=26$$
$$-5x=-10$$
$$x=\frac{-10}{-5}$$
$$=2$$

Substitute $x=2$ in the expression, $y=\frac{12-3x}{2}$:

$$y=\frac{12-3(2)}{2}$$
$$=\frac{6}{2}$$
$$=3$$

Hence, 1 cup of coffee costs \$2 and 1 piece of cake costs \$3.

(b) **Data:** The $y = 2x + 1$ gives the relationship between x and y.

(i) **Required to complete:** The table of values given for this equation.

Solution:

$y = 2x + 1$

When $x = 2$ $\quad y = 2(2) + 1 = 5$

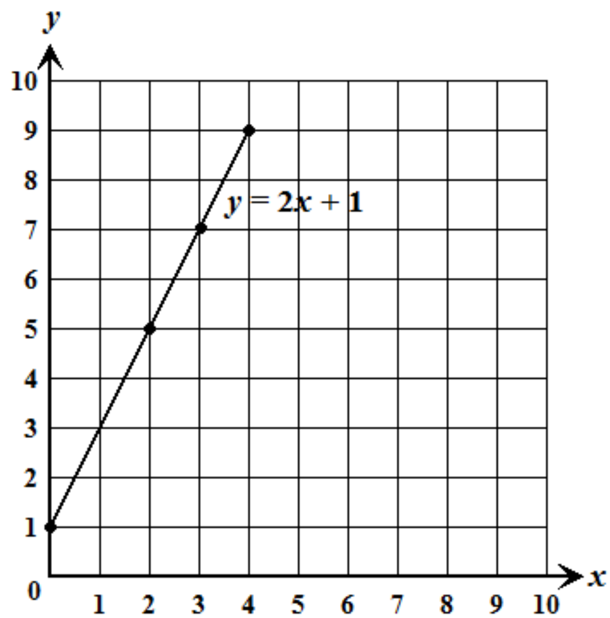
When $x = 4$ $\quad y = 2(4) + 1 = 9$

The completed table is

x	0	2	3	4
y	1	5	7	9

(ii) **Required to draw:** The graph of $y = 2x + 1$ on the grid provided.

Solution:



(iii) **Required to state:** The $y-$intercept for the graph of $y=2x+1$.

Solution:

From the graph the intercept on the $y-$axis can be read off as $(0,\ 1)$.

From the table, the $y-$intercept can be seen as $(0,\ 1)$, that is, when $x=0,\ y=1$.

Also, we may substitute $x=0$ to find

$$y=2(0)+1$$
$$=1$$

$\therefore$ $y-$intercept is 1 and the line cuts the vertical axis at $(0,\ 1)$.