FAS-PASS Maths

NCSE MATHEMATICS PAPER 2
YEAR 2011
Section I

1. (a) **Required to simplify:** $\left(1\frac{1}{4}-\frac{2}{5}\right)\div\frac{1}{10}$

Solution:
Firstly, we work the part of the question that is written within brackets.

$$1\frac{1}{4}-\frac{2}{5}=\frac{5}{4}-\frac{2}{5}$$
$$=\frac{5(5)-4(2)}{20}$$
$$=\frac{25-8}{20}$$
$$=\frac{17}{20}$$

Hence, $\left(1\frac{1}{4}-\frac{2}{5}\right)\div\frac{1}{10}=\frac{17}{20}\div\frac{1}{10}$

Inverting the denominator and multiplying by the inverted form, we get:

$$\frac{17}{20}\div\frac{1}{10}=\frac{17}{\cancel{20}_2}\times\frac{\cancel{10}}{1}$$
$$=\frac{17}{2}\text{ or }8\frac{1}{2}$$
(in exact form)

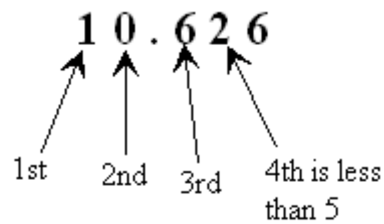
(b) (i) **Required to calculate:** The value of 4.62×2.3 exactly.

Calculation:

$$\begin{array}{r} 4.62\ \times \\ 2.3 \\ \hline 924\quad + \\ 1386 \\ \hline 10.626 \\ \hline \end{array}$$

Answer = 10.626 (exactly)

(ii) **Required to express:** The answer correct to 3 significant figures.
Calculation:



Hence, the third digit remains unaltered and the answer is 10.6 correct to 3 significant figures.

2. **Data:** Lisa owns x CDs. Roger own 3 CDs more than Lisa. Mark owns twice the number of CDs as Roger.

(a) (i) **Required to:** Write an algebraic expression, in terms of x, for the number of CDs that Roger owns.
Solution:
Lisa owns x CDs.
Roger owns 3 CDs more than Lisa.
$\therefore$ Roger owns $(x+3)$ CDs.

(ii) **Required to write:** An algebraic expression, in terms of x, for the number of CDs that Mark owns.
Solution:
Roger owns $(x+3)$ CDs.
Mark owns twice as many as Roger.
$\therefore$ Mark owns $2\times(x+3)$ CDs.
$= 2(x+3)$ or $(2x+6)$ CDs.

(b) **Data:** The number of CDs owned by Roger and Mark is 45.
Required to write: An equation in terms of x to represent this information
Solution:
Number of CDs owned by Roger + Number of CDs owned by Mark

$$= (x+3)+2(x+3)$$
$$= x+3+2x+6$$
$$= 3x+9$$

This total is 45. (data)

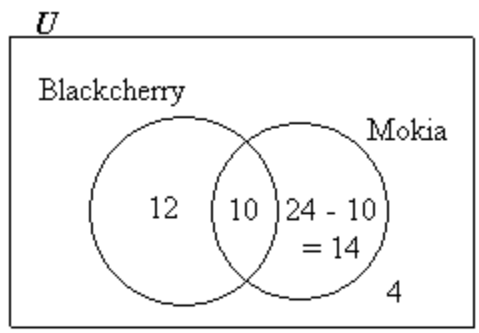
$$\therefore 3x+9=45$$
$$3x = 45-9$$
$$3x = 36$$

(If we were asked to solve for x, we divide by 3 to obtain x = 12).

(c) **Required to factorise:** $2a + ax$
Solution:
$2\underset{=}{a} + \underset{=}{a}x$
Notice, *a* is a common term and can be factored out and brackets introduced to get:
$2\underset{=}{a} + \underset{=}{a}x = \underset{=}{a}(2 + x)$.

3. **Data:** A class consists of 40 students. 12 students own only Blackcherry phones. 10 students own both Blackcherry phones and Mokia phones. 24 students own Mokia phones. 4 students do not own either of these two types of phones.

(a) **Required to complete:** The Venn diagram to show the above information.
Solution:
U = Number of students in the class



(b) **Required to calculate:** Number of students in the class who own only one type of phone.
Calculation:
Number of students who own Blackcherry phones only = 12
Number of students who own Mokia phones only = 14

$\therefore$ Number of students who own only one type of these phones $= 12 + 14$
$= 26$

(c) **Required to calculate:** The probability that a student chosen at random owns both Blackcherry and Mokia phones.
Calculation:
If E = Any event

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total number of possible outcomes}}$$

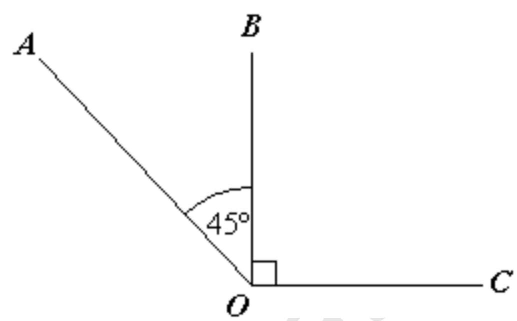
$\therefore$ P(Student chosen at random, owns both types of phones)

$$= \frac{\text{Number of students who own both types of phones}}{\text{Total number of students in the class}}$$

$$= \frac{10}{40}$$

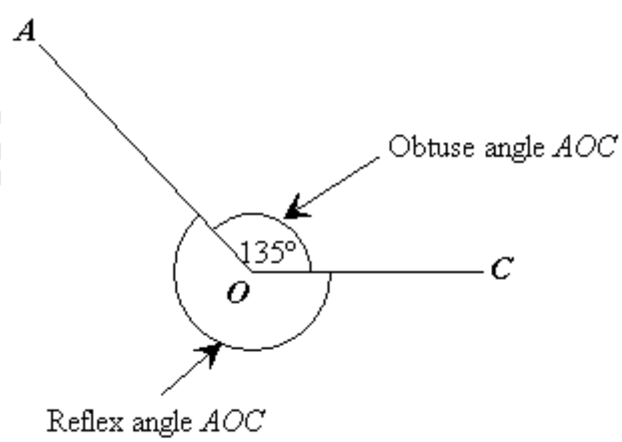
$$= \frac{1}{4} \text{ or } 0.25$$

4. (a) **Data:**



Required to calculate: The size of the reflex angle, AOC
Calculation:

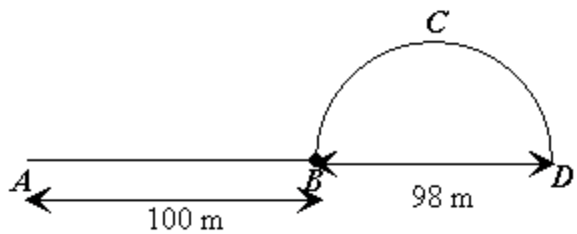
The obtuse angle $A\hat{O}C = 45° + 90°$
$= 135°$



Reflex angle $A\hat{O}C = 360° - 135°$
$= 225°$

(Angle in a circle = 360°)

(b) **Data:** ABCD is a path on a lawn. The straight section $AB = 100$ m. The diameter of the semi-circular section BCD is 98m.



(i) **Required to calculate:** The distance along the semi-circular section BCD.
Calculation:

Radius, r m, of the semi-circle $BCD = \frac{98}{2}$

$$= 49 \text{ m}$$

$\therefore$ Length of the semi-circle $BCD = \frac{1}{2}(2\pi r)$

$$= \frac{1}{2} \times \frac{2}{1} \times \frac{22}{7} \times 49$$

$$= 154 \text{ m}$$

(ii) **Required to calculate:** The distance alone the paved path $ABCD$ on which Maryam journeys.
Calculation:
Distance Maryam journeys along $ABCD$
= Length of AB + Length of BCD
$= 100 \text{ m} + 154 \text{ m}$
$= 254 \text{ m}$

(iii) **Required to calculate:** The distance less, in metres, that Maryann would walk if she did so in the straight line ABD.
Calculation:
The length of the straight line ABD = Length of line AB + Length of line BD

$$= (100 + 98) \text{ m}$$

$$= 198 \text{ m}$$

$\therefore$ The journey along ABD will be $(254 - 198)$ mshorter than the journey along $ABCD$
= 56 m.

5. (a) **Data:** Sticks of the same length are used to make a sequence of squares in a pattern

Required to draw: The next pattern in the sequence
Solution:
The first pattern has one square.
The second pattern has two squares.
The third pattern has three squares.
$\therefore$ The fourth pattern is expected to have four squares.

$\therefore$ The fourth pattern is:

(b) **Required to find:** The number of sticks used to create the 4th pattern.
Solution:
The number of sticks can be found by checking to be 13.

OR

We can create a table of values and observe the sequence.

Pattern number (n)	**Number of sticks used (s)**
1	4
2	7
3	10

The pattern for ***s*** appears to be {3 × pattern number, (***n***)} added to 1, that is, $3n+1$.

We could confirm this as follows:

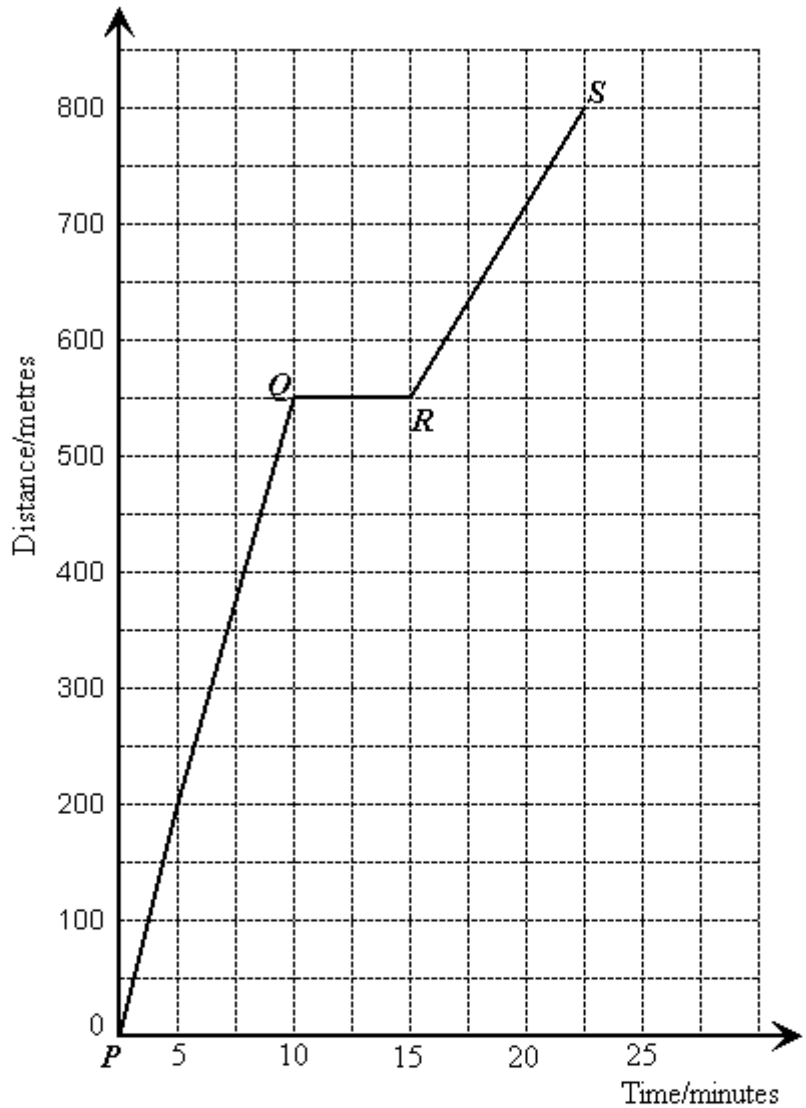
$3(1)+1=4$

$3(2)+1=7$

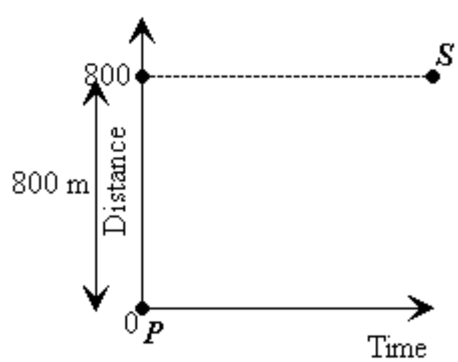
$3(3)+1=10$

$\therefore$ For the 4th pattern, $n=4$ and the number of sticks, $s=3(4)+1$
$=13$

(c) **Data:** Diagram showing a distance time graph, PQRS for Eton's journey from home to school.

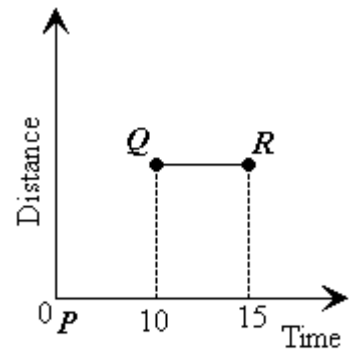


(i) **Required to find:** The total distance from Eton's home to school.
Solution:



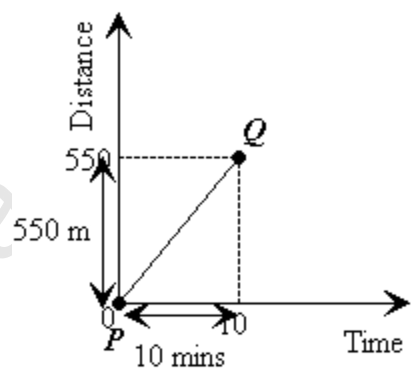
The distance is 800 m when read along the vertical or distance axis, as shown.

(ii) **Required to find:** The time that Eton rested.
Solution:
Eton rested during the period when there is no change in distance covered. This is shown by the horizontal branch *QR*.



This occurred for a period of, $15-10=5$ minutes.

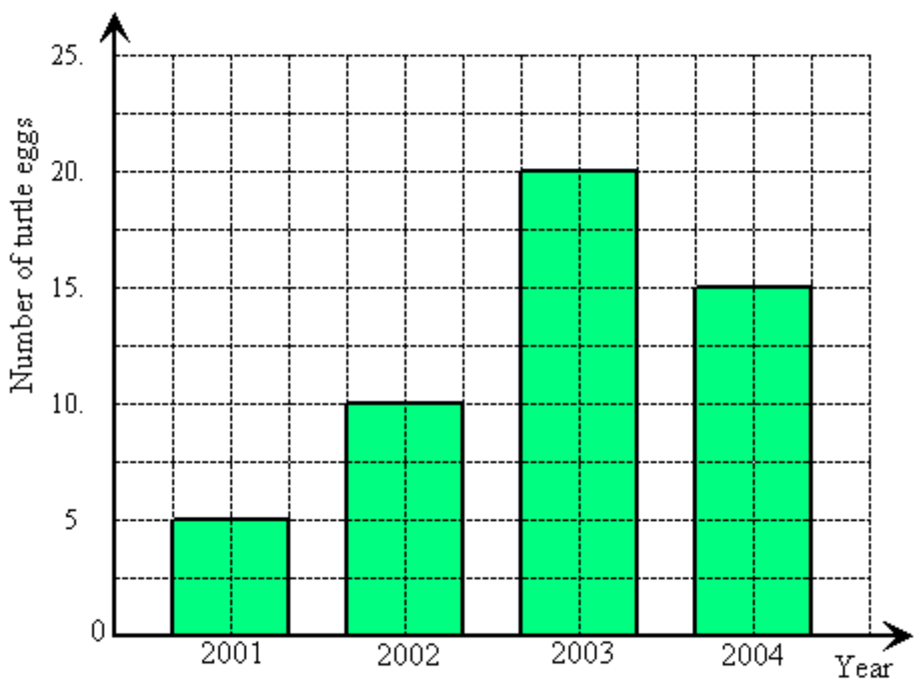
(iii) **Required to calculate:** Eton's average speed in ms^{-1} for the period *PQ*.
Calculation:



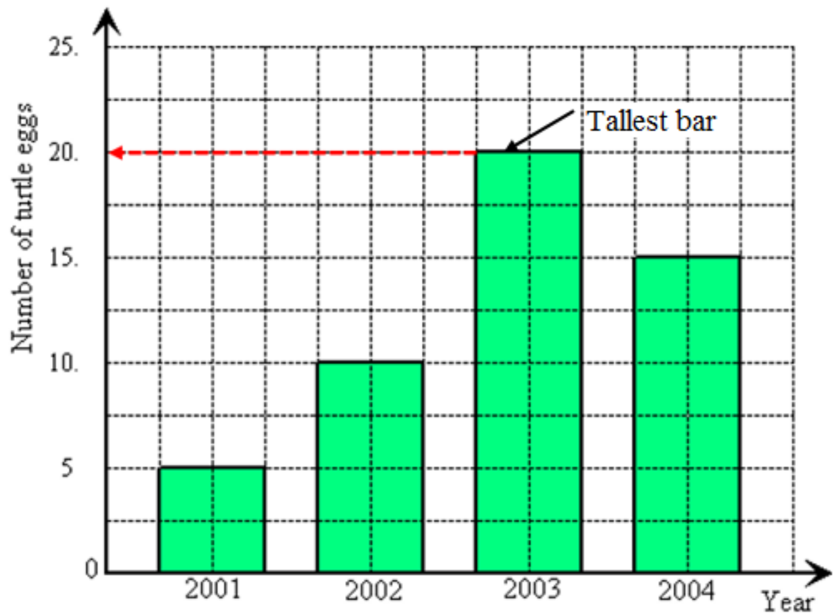
The distance $PQ=550$ m
Time taken to cover *PQ* = (60 x 10) = 600 s

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$
$$= \frac{550 \text{ m}}{600 \text{ s}}$$
$$= \frac{11}{12} \text{ ms}^{-1} \text{ (exact)}$$
$$\approx 0.91\underline{\underline{6}} \text{ ms}^{-1}$$
$$\approx 0.92 \text{ ms}^{-1} \text{ to 2 decimal places}$$

6. **Data:** Bar chart showing the number of turtle eggs in the sand on a beach from 2001 to 2004.

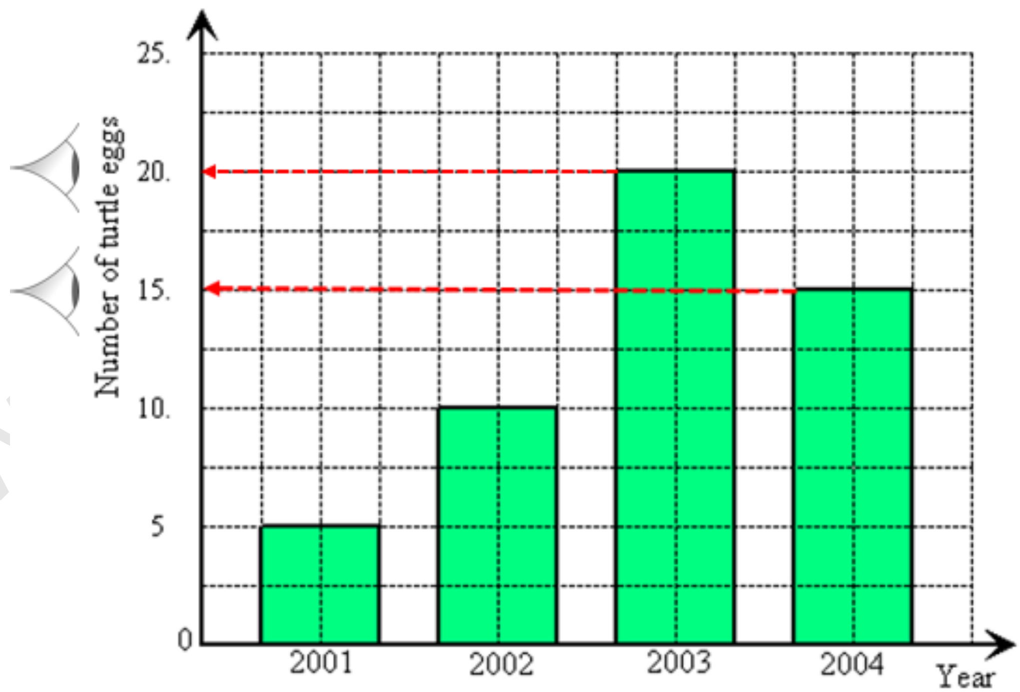


(a) (i) **Required to find:** The year in which the greatest number of eggs was found
Solution:
The highest or tallest bar corresponds to the year 2003. Hence, the greatest number of turtle eggs was found in the year 2003.



(ii) **Required to find:** The difference in the number of turtle eggs found in 2003 and 2004.
Solution:

The number of eggs found in 2003 is 20 (as read off from diagram).



The number of eggs found in 2004 is 15 (as read off from diagram).

$\therefore$ The difference between the number of eggs found in 2003 and 2004

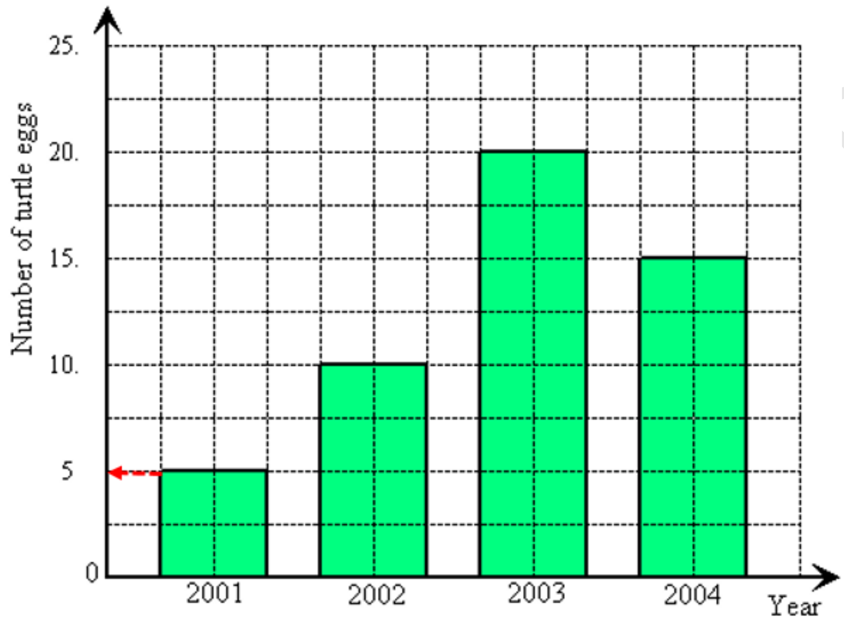
$= 20 - 15$

$= 5$ eggs

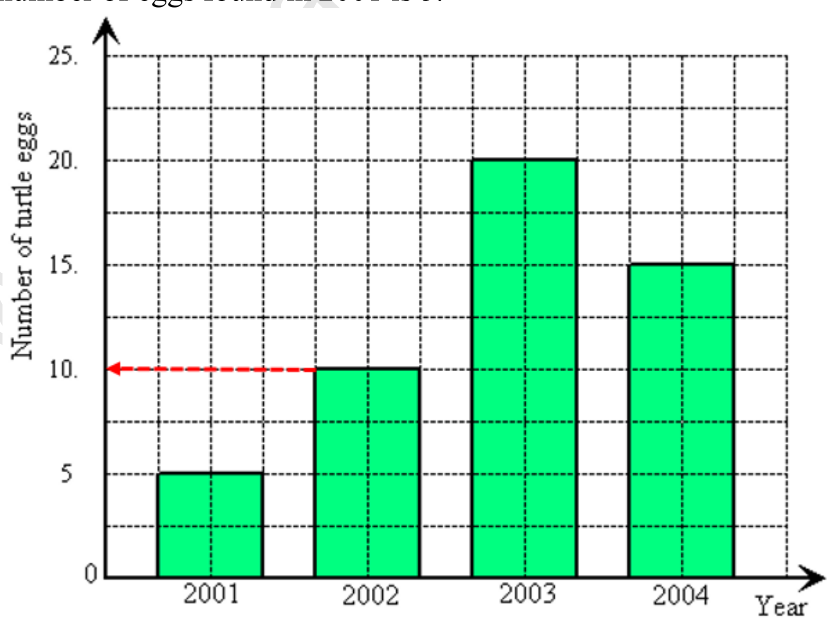
(b) **Required to calculate:** The mean number of eggs found during the period 2001 to 2004.

Calculation:

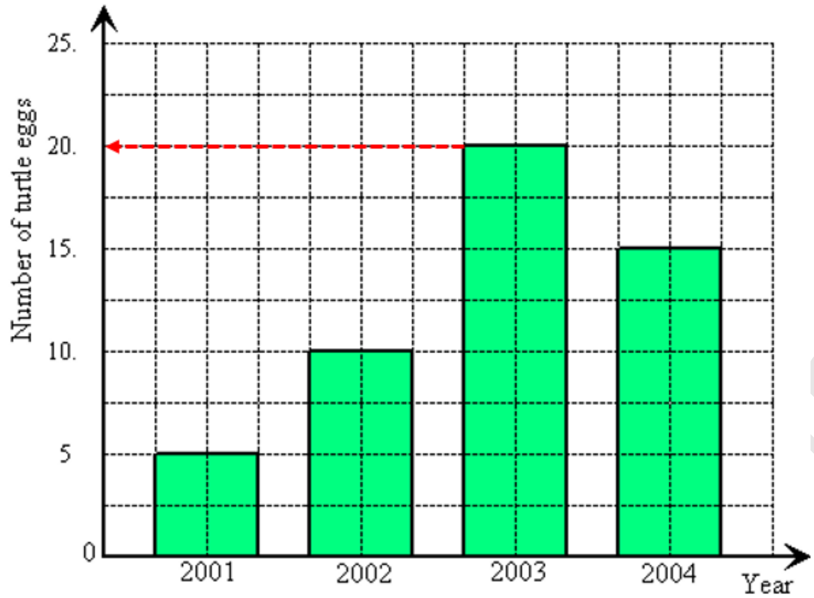
The heights of each bar was read off from the diagram.



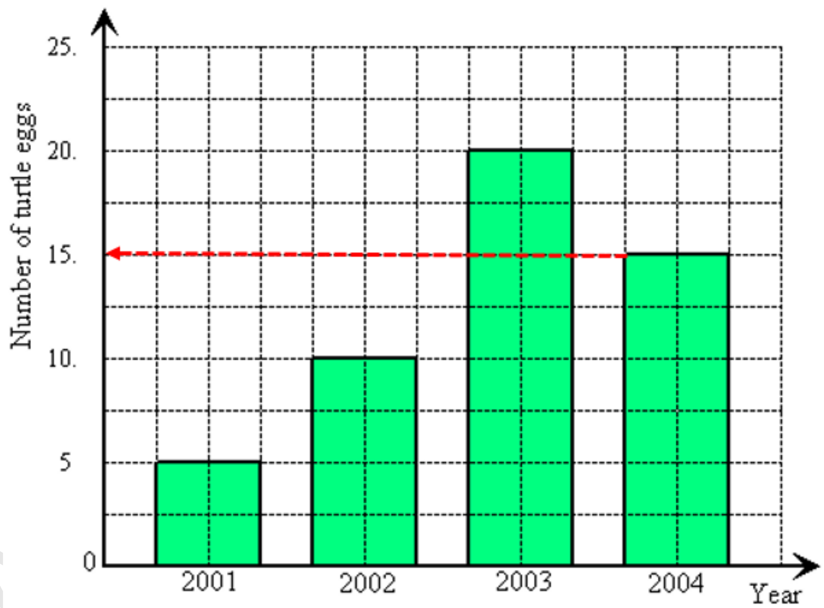
The number of eggs found in 2001 is 5.



The number of eggs found in 2002 is 10.



The number of eggs found in 2003 is 20.



The number of eggs found in 2004 is 15.

$\therefore$ Total number of eggs found from 2001 to 2004 $= 5 + 10 + 20 + 15$
$= 50$ eggs

Mean number of eggs found in the period 2001 to 2004, a period of 4 years

$$= \frac{\text{Total number of eggs found}}{\text{Total number of years}}$$

$$= \frac{50}{4} \text{ eggs}$$

$$= 12.\underline{\underline{5}} \text{ eggs}$$

$$= 13 \text{ eggs when expressed to the nearest whole number}$$

Section II

7. (a) **Data:**

iPod Sale **USA MART** **US $400 each**	**iPod Sale** **TRINI MART** **TT $2200 plus** **15% VAT**
Foreign Exchange Rates US $1.00 = TT $6.30 BDS $1.00 = TT $3.00	

(i) **Required to calculate:** The cost of the iPod in TT dollars when purchased by Chris at USA Mart.
Calculation:
Cost of the iPod at USA MART = US $400

$$= \text{TT}\$\left(400 \times 6.30\right)$$

$$= \text{TT}\$2520$$

(ii) **Data:** Mark purchases the iPod in USA MART.
Required to calculate: The cost in BDS dollars when bought at USA MART
Calculation:

$$\text{Cost of the iPod in BDS dollars} = \frac{\text{Cost in TT}}{3.00}$$

$$= \text{BDS}\$\frac{2520.00}{3.00}$$

$$= \text{BDS}\$840.00$$

(iii) **Required to calculate:** The cost of the iPod at TRINI MART.
Calculation:
Cost of the iPod at TRINI MART = \$ 2 200 + VAT of 15% of \$2 200

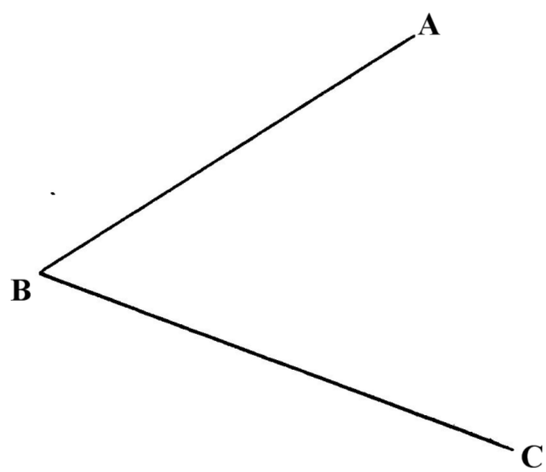
$$= \left(\$2200 + \frac{15}{100} \times \$2200 \right) \text{TT}$$
$$= \$2200 + \$330$$
$$= \text{TT}\,\$2530$$

(iv) **Required to calculate:** The difference between the prices of the iPod at USA MART and TRINI MART in TT dollars.
Calculation:
Cost of the iPod at TRINI MART = \$2530
Cost of the iPod at USA MART = \$2520

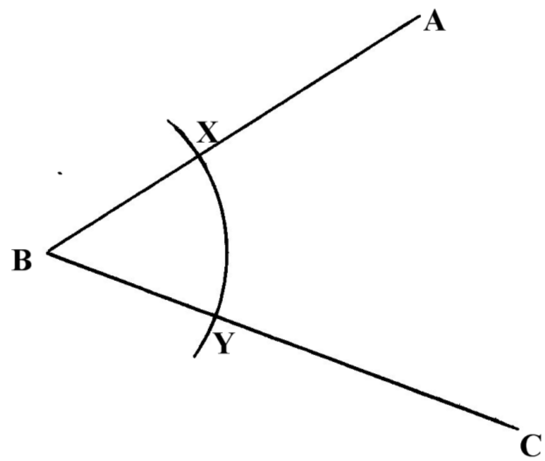
$$\text{Difference in price } = \$2530 - \$2520$$
$$= \text{TT}\,\$10$$

That is, the price is TT \$10 more if purchased at TRINI MART.

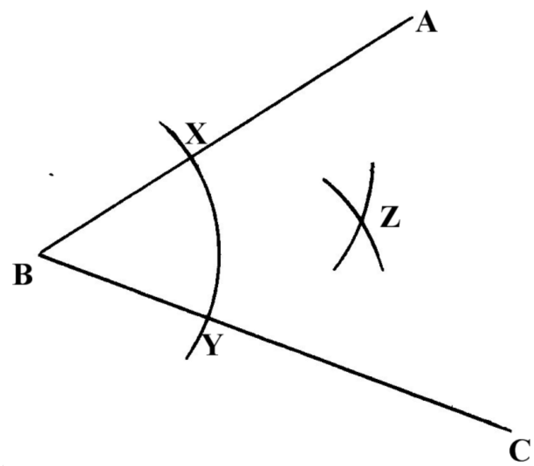
(b) **Data:** Given:



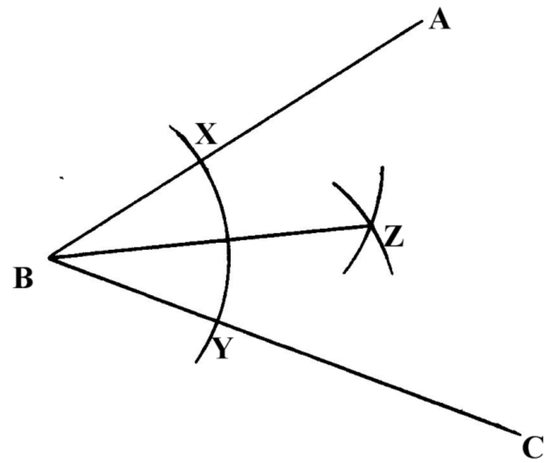
Required to bisect: The angle *ABC*
Construction:
With center B, an arc is drawn to cut the arms BA and BC at X and Y.



With center X and afterwards Y and the same radii, two arcs are drawn to cut at Z.



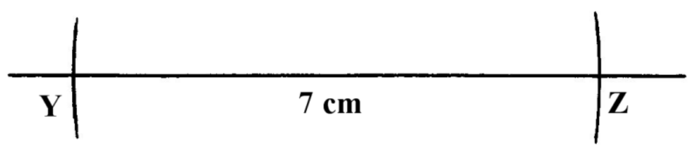
Join B to Z.
The line BZ bisects $\hat{B}$, that is, $A\hat{B}Z = C\hat{B}Z$.



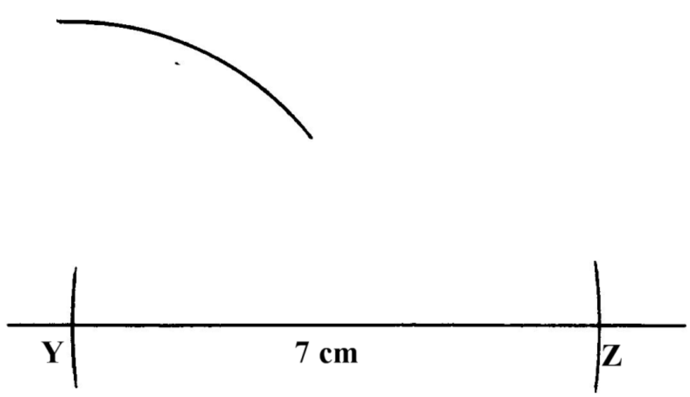
(c) (i) **Required to construct:** ΔXYZ with, XY = 4 cm, XZ = 5 cm and YZ = 7 cm.

Construction:

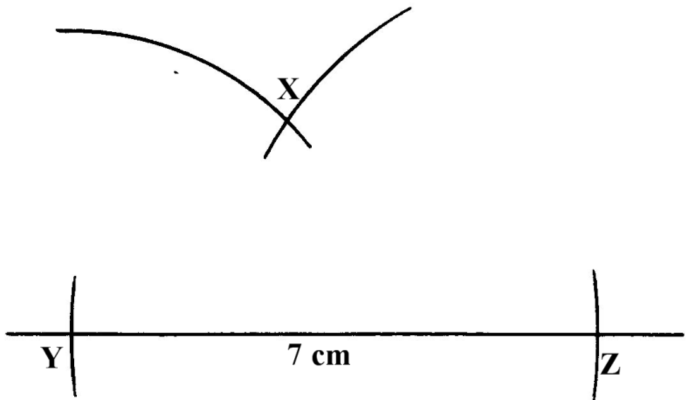
We can draw a straight line and cut off YZ = 7 cm, using the pair of compasses.



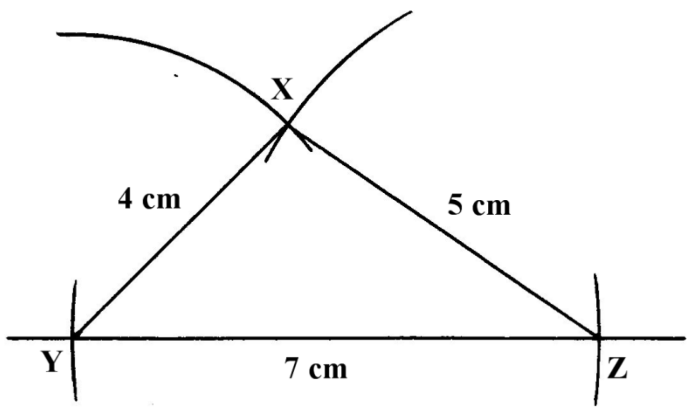
With center Y and radius 4 cm, an arc is drawn.



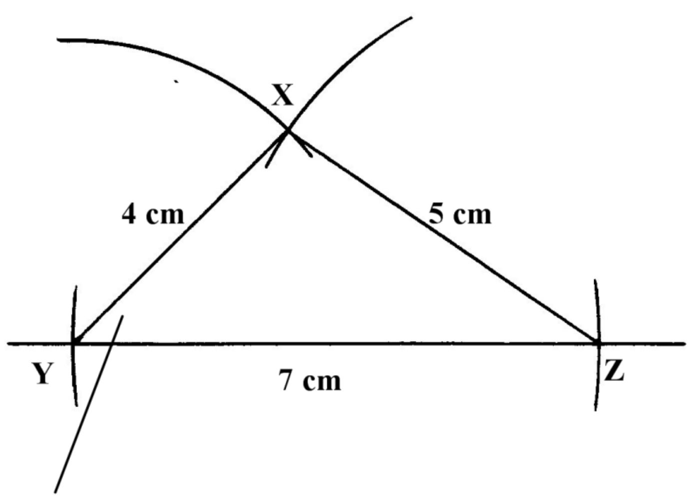
With center Z and radius 5 cm, an arc is drawn to cut the arc drawn in the above step. These two arcs will cut at X.



Join X to Y and X to Z to complete the triangle XYZ.



(ii) **Required to measure:** The size of angle, $X\hat{Y}Z$.
Solution:



The angle XYZ appears to be between 44° and 45°when measured with the protractor and so we may take the measurement as $44\frac{1}{2}°$.

8. (a) **Data:** 800 people attended a football game between two schools. $\frac{2}{5}$ of the people were teenagers.

(i) **Required to calculate:** The number of teenagers who attended the game.
Calculation:

The number of teenagers who attended the game $= \frac{2}{5}$ of 800

$$= \frac{2}{5} \times 800$$
$$= 320 \text{ teenagers}$$

(ii) **Data:** 60% of the remainder (who were not teenagers) were male adults.
Required to calculate: The number of male adults present at the match
Calculation:
The number of people who attended the match and were not teenagers
$= 800 - 320$
$= 480$

OR

$= \left(1 - \frac{2}{5}\right)$ of 800
$= \frac{3}{5}$ of 800
$= 480$

$\therefore$ 60% of 480 were male adults.

$\therefore$ Number of male adults $= \frac{60}{100} \times 480$
$= 288$ male adults

(b) **Data:** Freeport High School has 240 students and 12 teachers.

(i) **Required to write:** The ratio of the number of teachers to the number of students.
Solution:
The number of teachers: The number of students $= 12 : 240$
$= 1 : 20$

(ii) **Data:** The student population is increased to 360.
Required to calculate: The number of teachers needed to maintain the same ratio
Calculation:
Let the number of teachers required for the new school population be N.
$\therefore N : 360 = 1 : 20$

$$\frac{N}{360} = \frac{1}{20}$$
$$20 \times N = 360 \times 1$$
$$N = \frac{360}{20}$$
$$= 18$$

The school requires 18 teachers for 360 students.
Since the school already has 12 teachers then 18 – 12 = 6 more would be required.
Answer =6

(c) **Data:** A tourist bought 4 handbags and 5 bandanas for a total cost of \$50. Another tourist bought 3 handbags and 1 bandana for a total cost of \$21.
1 handbag costs $\$x$
1 bandana costs $\$y$

(i) **Required to express:** A pair of simultaneous equations to represent the above information.
Solution:

$$\underbrace{4 \text{ handbags at } \$x}_{4\times x} \text{ and } \underbrace{5 \text{ bandanas at } \$y}_{5\times y} \underbrace{\text{at a total of } \$50}_{=50}.$$

$4x + 5y = 50$ **…(1)**

$$\underbrace{3 \text{ handbags at } \$x \text{ each}}_{3\times x} \text{ and } \underbrace{1 \text{ bandana at } \$y}_{y} \underbrace{\text{cost a total of } \$21}_{=21}.$$

$3x + y = 21$ **…(2)**

The pair of equations is:
$4x + 5y = 50$ **…(1)**
$3x + y = 21$ **…(2)**

(ii) **Required to calculate:** The value of x and of y.
Calculation:
$4x + 5y = 50$ **…(1)**

$3x + y = 21$ **...(2)**

From (2)
$3x + y = 21$
$\therefore y = 21 - 3x$

Substituting $y = 21 - 3x$ into equation (1)

$$4x + 5(21 - 3x) = 50$$
$$4x + 105 - 15x = 50$$
$$4x - 15x = 50 - 105$$
$$-11x = -55$$
$$\therefore x = \frac{-55}{-11}$$
$$= 5$$

When $x = 5$

$$y = 21 - 3(5)$$
$$= 6$$

Hence, $x = 5$ and $y = 6$.

ALTERNATIVE METHOD:

$4x + 5y = 50$ **...(1)**
$3x + y = 21$ **...(2)**

Equation (2) $\times -5$

$$-5(3x) - 5(y) = -5(21)$$
$$-15x - 5y = -105 \text{ ...(3)}$$

Equation (1) + Equation (3)

$$\begin{array}{l} 4x + 5y = 50 \quad + \\ \underline{-15x - 5y = -105} \\ -11x = -55 \end{array}$$

$$x = \frac{-55}{-11}$$
$$= 5$$

Substituting $x = 5$ in equation (2) $3x + y = 21$**,** we get:

$$3(5)+y=21$$
$$y=21-15$$
$$=6$$

Hence, $x=5$ and $y=6$.

Also, if we draw the graph of $4x + 5y = 50$ and $3x+y=21$ on the same axes, we would find that these two lines intersect at the point with coordinates $(5,\ 6)$. Hence $x=5$ and $y=6$.

9. **Data:** Linear relation between x and y is given by $y=2x+1$.

x	1	5	6
y		11	

(a) **Required to complete:** The table given.

Solution:

When $x=1$

$$y=2(1)+1$$
$$=2+1$$
$$=3$$

When $x=6$

$$y=2(6)+1$$
$$=12+1$$
$$=13$$

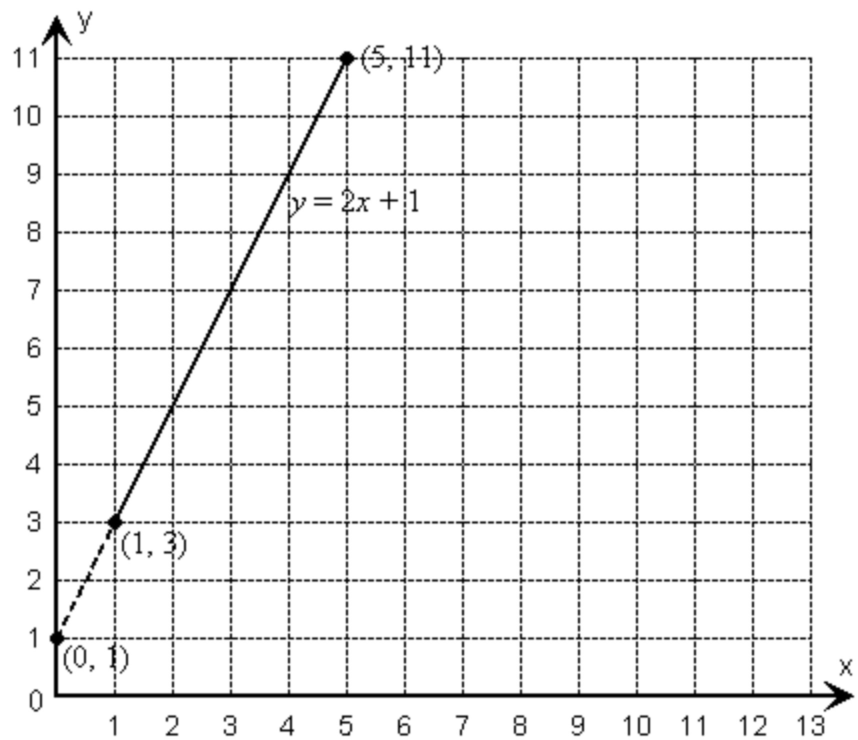
$\therefore$ The completed table is:

x	1	5	6
y	3	11	13

(b) (i) **Required to draw:** The graph of the linear equation represented by $y=2x+1$.

Solution:

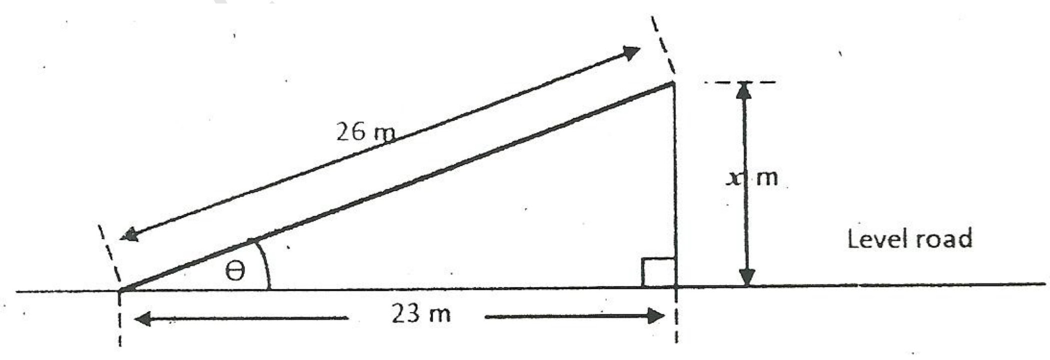
Using the table of values that is completed and the labelled axes given on the answer sheet provided, we plot the points with coordinates $(1,\ 3)$, $(5,\ 11)$ and $(6,\ 13)$.



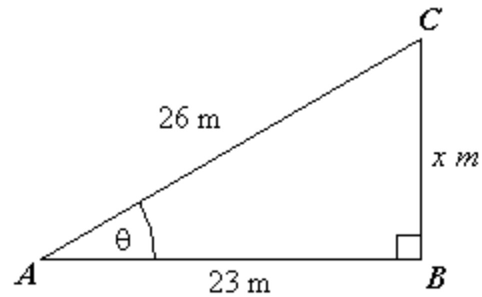
The point $(6, 13)$ cannot be plotted on the given axes and is ignored. However, this third point is not necessary to obtain the graph of the straight line $y = 2x + 1$ as two points are sufficient to draw a straight line.

The line is extended so as to cut the y – axis. The line with equation $y = 2x + 1$ cuts the y – axis at $(0, 1)$ as shown.

(c) **Data:** A stunt cyclist speeds up the slant section of a ramp before landing on the level road.



(i) **Required to calculate:** The height of the ramp, x, in metres.
Calculation:
Label the triangle made by the ramp and the level road as ABC for convenience.



$$AC^2 = AB^2 + BC^2 \quad \text{(Pythagoras' Theorem)}$$

$$(26)^2 = (23)^2 + (x)^2$$

$$x^2 = (26)^2 - (23)^2$$

$$= 676 - 529$$

$$= 147$$

$$x = \sqrt{147}$$

$$= 12.1\underline{\underline{2}} \text{ m}$$

$$= 12.1 \text{ m correct to 1 decimal place}$$

$\therefore$ The height of the ramp is 12.1 metres.

(ii) **Required to calculate:** The value of θ
Calculation:

$$\sin\theta = \frac{BC}{AC}$$

$$= \frac{12.12}{26}$$

$$\therefore \theta = \sin^{-1}\left(\frac{12.12}{26}\right)$$

$$= 27.7\underline{\underline{8}}^\circ$$

$$= 27.8^\circ \text{ to the nearest } 0.1^\circ$$

OR

$$\cos\theta = \frac{AB}{AC}$$
$$= \frac{23}{26}$$
$$\therefore \theta = \cos^{-1}\left(\frac{23}{26}\right)$$
$$= 27.7\underline{\underline{9}}^\circ$$
$$= 27.8^\circ \text{ to the nearest } 0.1^\circ$$

OR

$$\tan\theta = \frac{BC}{AB}$$
$$= \frac{12.12}{23}$$
$$\therefore \theta = \tan^{-1}\left(\frac{12.12}{23}\right)$$
$$= 27.7\underline{\underline{8}}^\circ$$
$$= 27.8^\circ \text{ to the nearest } 0.1^\circ$$