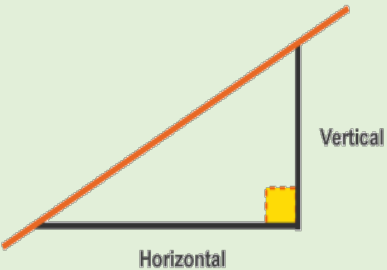# 24. TRAVEL GRAPHS

TYPES OF TRAVEL GRAPHS

Travel graphs display relationships between distance and time or between speed and time. Such relationships enable one to determine unknown quantities using graphical methods. We shall begin our study of this topic by reviewing the concept of the gradient.
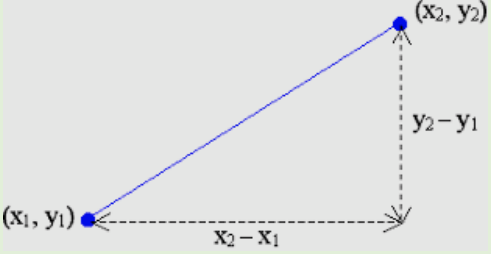
The Gradient of a Straight Line

In our study of coordinate geometry, we have used many different methods to find the gradient of a straight line. Depending on the information given, we usually determine what method is best to use to find the gradient. In this chapter, we will be reading off the gradient of a straight line using mainly two methods.

Gradient of a line

Given a straight-line graph, the gradient is obtained by drawing vertical and horizontal lines as shown and reading off the displacements.

$$Gradient = \frac{Vertical \ (Rise)}{Horizontal \ (Run)}$$

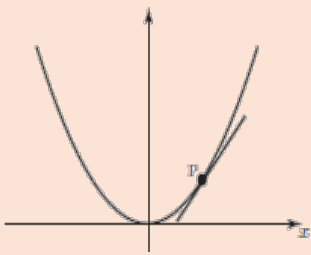Given two points on a graph, the gradient is calculated using the formula:

$$Gradient = \frac{y_2 - y_1}{x_2 - x_1}$$

Gradient of a Curve

As the above examples illustrate, the gradient of a line is constant. It has only one value, and it does not matter where on the line it is measured. However, a curve changes its direction throughout its path, as the value of x changes. And so, the gradient of a curve is different at every point. The term 'gradient of a curve' really means the gradient of the curve at a particular point.

This gradient of a curve at a point is defined as the gradient of the tangent to the curve at that point.

Gradient of a curve

In general, if the gradient of a curve at a point is required, then we are expected to:

(i)   Draw the curve as accurately as possible.
(ii) Draw, as accurately as possible, the tangent to the curve at the point.
(iii) Find by any appropriate method, the gradient of the tangent at that point.

The tangent to a curve is a straight line that just touches the curve at one point only.

Calculating the gradient of a curve at a point

To determine the gradient of the curve $y = x^2$ at the point $x = 3$, we follow the steps outlined above.

(i)   Draw the curve as accurately as possible. By using a set of conveniently chosen values of x, we set up a table of values and find the corresponding values of y. These points are plotted. We draw the curve $y = x^2$ as accurately as possible on carefully labelled axes.

219

x	0	1	2	3	4	5
y	0	1	4	9	16	25

(ii)   Draw as accurately as possible, the tangent at the point $x = 3$.

(iii)   Measure the gradient of the tangent at $x = 3$. This is done by selecting two convenient points on the tangent line and measuring the vertical and horizontal displacements.
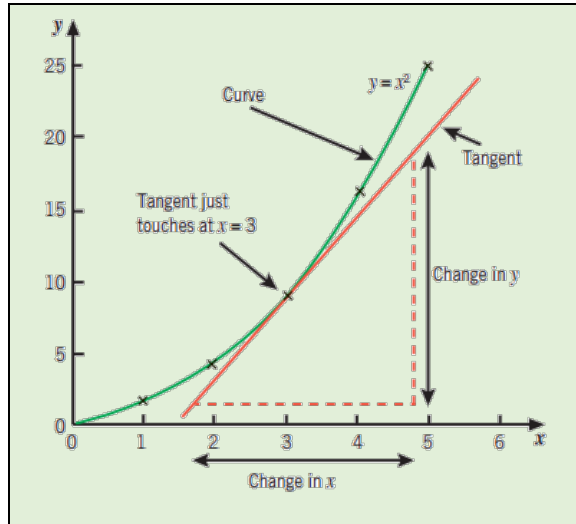
Gradient of the tangent at $x = 3$ is approximated from

$$= \frac{Change\ in\ y}{Change\ in\ x} \approx \frac{19-1}{4.8-1.8} \approx 6$$

Travel Graphs

We will now see how the gradient of straight lines and curves can be applied to solve problems involving speed, distance and time.

Speed, Distance and Time

The following is a basic but important formula which applies when speed is constant (in other words the speed doesn't change):

$$Speed = \frac{Distance}{Time}$$

$$Distance = Speed \times Time$$

When speed is constant, the average (mean) speed can be calculated from the formula:

$$Average\ Speed = \frac{Total\ Distance\ travelled}{Total\ time\ taken}$$

Units for speed, distance and time

In calculations involving speed, distance and time, we sometimes need to change from one unit to another. If distance and time are measured in metres and seconds respectively then the unit for speed (or velocity), is metres per second abbreviated as m/s or preferably ms^{-1} (S.I units of measurement). Another common unit for speed is km/h or kmh^{-1}.

Example 1

Change 180 kmh^{-1} into ms^{-1}.

Solution

Since speed is a rate, we can express a given speed as follows
$$\frac{180\ km}{1\ hr} = \frac{180 \times 1000\ m}{1 \times 60 \times 60\ s} = 50\ ms^{-1}$$

Example 2

If a car travels at a speed of 10ms^{-1} for 3 minutes, calculate the distance it travels.

Solution

Now, 3 minutes = 180 seconds and
Distance = Speed × Time.
Therefore distance travelled
= 10 × 180 = 1800m = 1.8 km.

Velocity and Acceleration

Velocity is the speed of a particle measured in a given direction of motion (therefore velocity is a vector quantity, whereas speed is a scalar quantity).

$$Velocity = \frac{Displacement}{Time}$$

When the velocity (or speed) of a moving object is increasing, we say that the object is *accelerating*. If the velocity decreases it is said to be *decelerating*. Acceleration is, therefore, the rate of change of velocity (change in velocity/time) and is measured in units such as ms^{-2}.

220

$$Acceleration = \frac{Change\ in\ velocity}{Change\ in\ Time}$$

Constant Acceleration and deceleration

The data shown below represents the velocity of an object starting from rest, travelling in a straight line. The object's velocity is changing by $10ms^{-1}$ in each second.

Constant acceleration

Time (s)	0	1	2	3	4	5
Velocity (ms^{-1})	0	10	20	30	40	50

The change in velocity is the same each second and the object is said to have a constant acceleration.

In the table below, the object's velocity is changing by $10ms^{-1}$ but it is decreasing. In this case, the object is slowing down and it has a constant deceleration.

Constant deceleration

Time (s)	0	1	2	3	4	5
Velocity (ms^{-1})	50	40	30	30	10	0

A change in velocity does not have to be constant, for example, if the velocity of the object is changing as shown in the table below, then it is said to have a non-constant acceleration. In this case, the object is increasing its speed at a variable rate.

Non-constant acceleration

Time (s)	0	1	2	3	4	5
Velocity (ms^{-1})	0	10	25	45	70	100

Units for acceleration

Since acceleration is velocity/time, common units for acceleration are ms^{-2} and kms^{-2}.

Travel Graphs

Travel graphs are representations of journeys of objects over a duration of time. Two types of graphs will be discussed in this section, these are distance (or displacement) versus time and velocity (or speed) versus time.

Such graphs may be straight lines or curves and we are reminded at this point that distance is a scalar quantity whilst displacement is a vector quantity (distance in a specified direction). Consequently, speed is a scalar measure whilst velocity, which is speed in a specified direction, is a vector measure.

When drawing travel graphs, time is always on the horizontal axis.

Distance-Time Graphs

We will first look at a straight-line distance-time graph. The graph below shows the journey of a cyclist from his home. Let us examine the graph and interpret the different segments (branches) of the journey. Each straight line on the graph tells us something about the journey.

The first point on the x-axis tells us that at 8:00 a.m. the cyclist began his journey since his distance form home was 0 km.

At 9:00 a.m., the cyclist was 10 km away from home and at 10:00 a.m. the cyclist was 20 km away from home.

Between 10 a.m. and 11 a.m., the cyclist was at rest because his distance from home remained the same for that period (20 km).
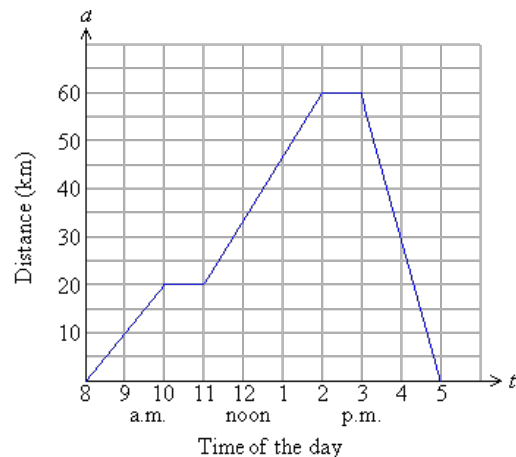
The cyclist continued his journey at 11:00 a.m. took another rest between 2:00 p.m. to 3:00 p.m. and then began his return journey, arriving at home at 5:00 p.m.

The speed can be calculated by finding the gradient of the graph. During the first two hours of the journey, the cyclist travelled a distance of 20 km. The speed during this period is calculated from:

221

$$Speed = \frac{Distance}{Time} = \frac{20}{2} = 10 kmh^{-1}$$

The speed is constant during this period and is really the gradient of the first line segment. The line has a positive gradient because the distance is positive when the object is moving away from a starting point.

During the period from 11:00 a.m. to 2:00 p.m. the speed is also constant and the line has a positive gradient as the cyclist is still on the forward journey. The speed during this period is calculated from:

$$Speed = \frac{Distance}{Time} = \frac{60 - 20}{3} = 13.3 \; kmh^{-1}$$

The speed of the cyclist on the return journey is calculated from

$$Speed = \frac{Distance}{Time} = \frac{60}{3} = 20 \; kmh^{-1}$$

Notice that the gradient of this line is negative indicating that the cyclist is now on his return journey. Since we are only interested in the magnitude of the speed, we ignore the negative sign.

Example 3

The displacement-time graph below describes the journey of a train between two train stations, A and B.
  (i) For how many minutes was the train at rest at B?
  (ii) Determine the average speed of the train, in km/h on its journey from A to B.
  (iii) The train continued its journey away from stations A and B to another station C, which is 50 km from B. The average speed on this journey was 60 km/h. Calculate the time, in minutes, taken for the train to travel from B to C.
  (iv) Draw the line segment which represents the journey of the train from B to C.
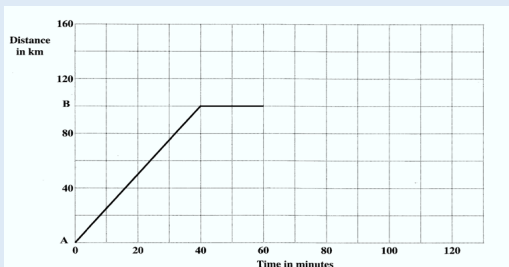
Solution
  (i) Train was at rest at B for
      (60 – 40) minutes = 20 minutes.

  (ii) The average speed in km/h of the train on its journey from A to B is the distance from A to B divided by the time in hours taken to travel from A to B.
       The distance AB = 100 km.
       The time in hours taken to travel from A to B
       $$= \frac{40}{60} = \frac{2}{3} \; hr$$
       $$Average \; Speed = \frac{Distance}{Time} = \frac{100}{2/3} = 100 \times \frac{3}{2}$$
       $$= 150 \; km/h$$

  (iii) To calculate the time taken for the train to travel from B to C, we use the above relationship with time as the subject.
       $$Time = \frac{Distance}{Speed} = \frac{50}{60} = \frac{5}{6} \; hours$$
       $$= \frac{5}{6} \times 60 \; minutes$$
       $$= 50 \; minutes$$

  (iv) To draw the line segment which represents the journey of the train from B to C, we use the following information:
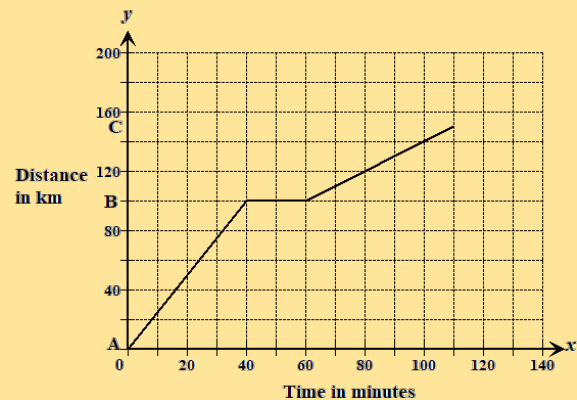
       At B the train is at the point (60, 120) on the graph.
       At C, the train would have moved a distance of 50 km in a time of 50 minutes.

       Hence, the new position will be at 60+50=110 minutes, and at 100+50 =150 km.
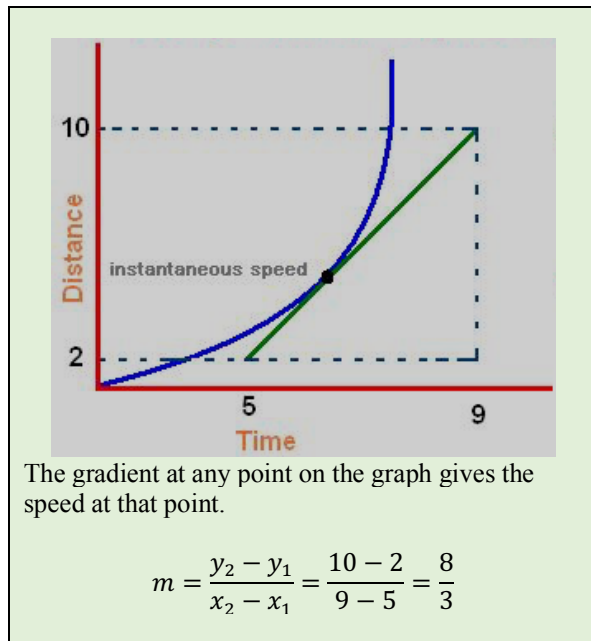   We need to locate the point (110, 150).
       This is shown in the graph below.

222

Curved Distance-Time Graphs

Some distance-time graphs are curves because the speed (or velocity) of the object is not constant. In curved distance-time graphs, we determine the distance travelled at any time by reading off the value at a given time. Conversely, the time taken to cover any distance can also be determined by a read-off.

To obtain the speed at an instant (also called instantaneous speed) requires the drawing the tangent at that point and finding the gradient. This is illustrated in the distance-time graph below.

The gradient at any point on the graph gives the speed at that point.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 2}{9 - 5} = \frac{8}{3}$$

Summary
On a straight-line distance-time graph in which all segments are straight lines, we should note the following:
1. Horizontal lines on the graph indicate that there is no motion or the object is at rest.
2. The speed during a segment is the gradient of the graph for that segment.
3. During the forward journey, the gradient is positive and during the return journey, the gradient is negative.

In a curve distance-time graph,
1. The distance at any time or the time taken to attain any distance can be obtained by a read-off.
2. The gradient at a point gives the velocity at that point.

Example 4
The distance-time graph shows the movement of a vehicle over a period of 10 seconds. Determine the velocity when t = 6 seconds.
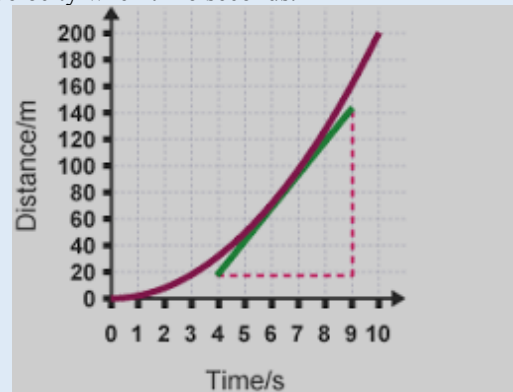

Solution
The velocity at an instant is found by drawing the tangent of the curve. It is the gradient of the tangent drawn at t = 6.
$$Gradient = \frac{140 - 20}{9 - 4} = \frac{120}{5}$$
$$= \mathbf{24} \text{ ms}^{-1}$$

Velocity-Time Graphs/ Speed-Time Graphs

In these graphs, velocity is plotted against time. Velocity or speed goes on the vertical axis and the time on the horizontal axis. We will first look at a straight-line velocity-time graph.

The velocity-time graph below shows the motion of a particle over a period of 30 seconds. There are three segments of the journey, labelled as A, B and C.

223
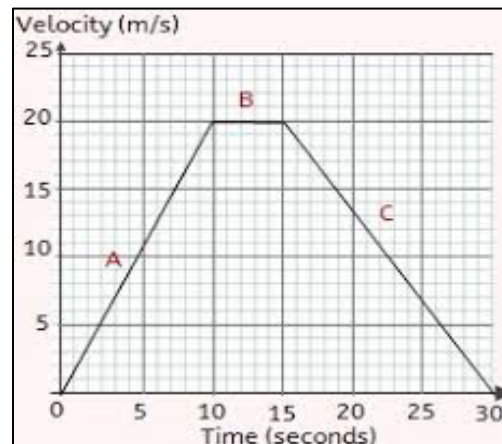
In segment A, the particle travels for 10 seconds starting from rest and steadily increases its velocity to a maximum of 20 m/s.

In segment B, the particle maintains a constant velocity of 20 m/s for a period of 5 seconds.

In segment C, the particle steadily decreases its velocity over a period of 15 seconds coming to rest.

The acceleration can be calculated by finding the gradient of the graph. In segment A, the particle's acceleration is constant and is calculated from the formula:

$$Acceleration = \frac{Change\ in\ velocity}{Change\ in\ Time} = \frac{20 - 0}{10} = 2\ m/s^2$$

In segment C, the particle has a constant deceleration.

$$Deceleration = \frac{Change\ in\ velocity}{Change\ in\ Time} = \frac{0 - 20}{15} = \frac{-20}{15}$$
$$= -\frac{4}{3} m/s^2$$

Notice that in segment A, the gradient is positive, indicating that the velocity was increasing while in segment C, the gradient is negative, indicating that the velocity was decreasing.

It is also possible to determine the distance travelled from a speed/time graph. We know that

$$Velocity = \frac{Distance}{Time}$$
and so, $\quad Distance = Velocity \times Time$

For each segment, we can calculate the distance travelled by examining the area under the graph which is really the same as $(Velocity \times Time)$.

In segment A, the area under the graph is the area of a triangle, whose base is 10 and height is 20.
Distance travelled in segment A $= \frac{10 \times 20}{2} = 100\ m$

In segment B, the area under the graph is the area of the rectangle whose length is 20 and width is 5.
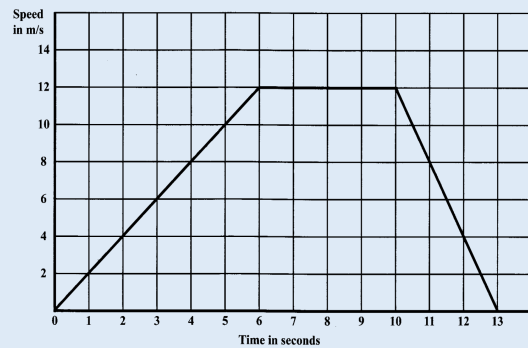Distance travelled in segment B $= 20 \times 5 = 100\ m$

In segment C, the area under the graph is the area of a triangle, whose base is 15 and height is 20.
Distance travelled in segment C $= \frac{15 \times 20}{2} = 150\ m$

The total distance travelled
$= (100 + 100 + 150) = 250$ m

Example 5

The diagram below shows the speed-time graph of the motion of an athlete during a race.

(a) Using the graph, determine
  (i) the maximum speed
  (ii) the number of seconds for which the speed was constant
  (iii) the total distance covered by the athlete during the race.
(b) Calculate
  (i) the acceleration of the athlete in the first 6 seconds.
  (ii) the deceleration of the athlete in the first 6 seconds.

Solution

a) (i) The maximum speed is 12 m/s.

(ii) The speed was constant for $(10 - 6) = 4$ sec

(iii) The total distance covered by the athlete is the area under the graph. This is a trapezium with dimensions as shown below.

Total distance covered $= \frac{1}{2}(12)(4 + 13)$
$= \frac{1}{2} \times 12 \times 17$
$= 102\ m$
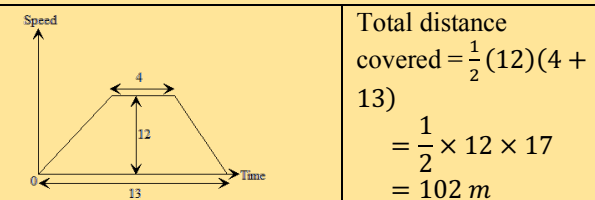
(b) (i) The acceleration of the athlete in the first 6 sec

$$Acceleration = \frac{Change\ in\ velocity}{Change\ in\ Time} = \frac{12 - 0}{6}$$
$$= 2\ m/s^2$$

(ii) The deceleration of the athlete in the last 3 sec

$$Deceleration = \frac{Change\ in\ velocity}{Change\ in\ Time} = \frac{0 - 12}{3} = \frac{-12}{13}$$
$$= -4\ m/s^2$$

224

Curved Velocity Time Graphs

When an object is moving in such a manner that its acceleration is not constant, the graph of velocity against time is not a straight line.
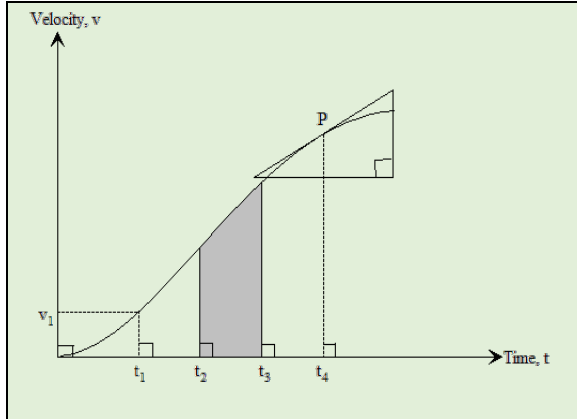
The velocity v_1 occurs at t_1.

The gradient of the tangent at P gives the acceleration at t_4.

An estimate of area of the shaded region gives the total distance travelled between t_2 and t_3

Summary

On a velocity-time graph in which all segments are straight lines, we should note the following:

1. A horizontal line indicates that the object is travelling at constant velocity.
2. The gradient of a line segment gives the constant acceleration or deceleration. during that segment. A positive slope indicates acceleration while a negative slope indicates deceleration
3. The area under the graph is the distance travelled.

On a curved velocity-time graph, we should note the following:

1. The velocity at any time **or** the time taken to attain any velocity is obtained by a read-off.
2. To obtain the acceleration at an instant (also called instantaneous acceleration) requires the drawing the tangent at that point and finding the gradient.
3. The distance covered during any interval of time is the area under the curve during that interval.

Example 6

The velocity-time graph shown below represents the motion of a train. A tangent to the curve is drawn at t = 10 seconds.

(i) State the velocity of the train in the 5th, 15th and the 25th second?
(ii) What is the gradient of the tangent at t = 10 seconds?
(iii) What is the acceleration of the train t = 10 seconds?
(iv) Predict the acceleration of the train at t = 30 seconds.
(v)
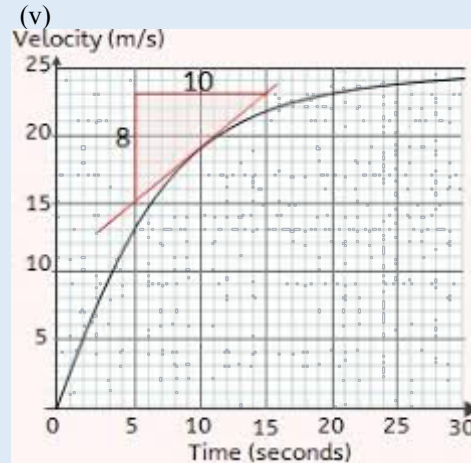
Solution

(i) By reading off the values on the vertical axis:
The velocity of the train in the 5th second is 13 m/s
The velocity of the train in the 15th second is 22 m/s
The velocity of the train in the 25th second is 24 m/s

(ii) The gradient of the tangent at t = 10 seconds is $\frac{8}{10} = 0.8 \, m/s$

(iii) The acceleration of the train t = 10 seconds is 0.8 m/s

(iv) The acceleration of the train t = 30 seconds is close to zero because the tangent to the curve is almost horizontal.

225