11. GEOMETRIC CONSTRUCTIONS

GEOMETRIC INSTRUMENTS

In this chapter, we will learn how to construct plane figures. A construction is an accurate drawing, the accuracy of which depends on the geometrical instruments used to create the drawing. In geometry, when we are asked to construct a plane figure, we are expected to use the appropriate geometrical instruments. A pair of compasses, a ruler, a set-square and a protractor are common instruments used in drawing and constructing plane figures.

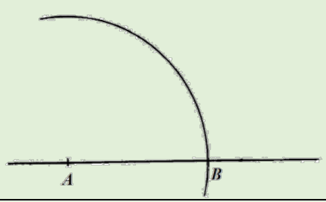
Constructing Angles

Before we can construct figures we must learn to construct angles using only a pair of compasses, a pencil and a ruler.

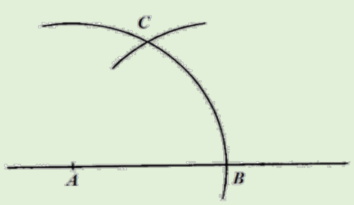
Constructing an angle of 60°

We shall construct the angle at the point *A*, on the straight line shown below.

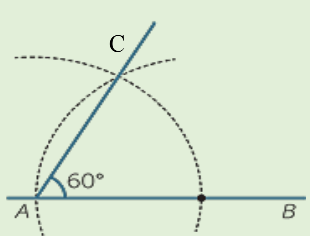
1. With center *A*, draw an arc, cutting the straight line at *B*.



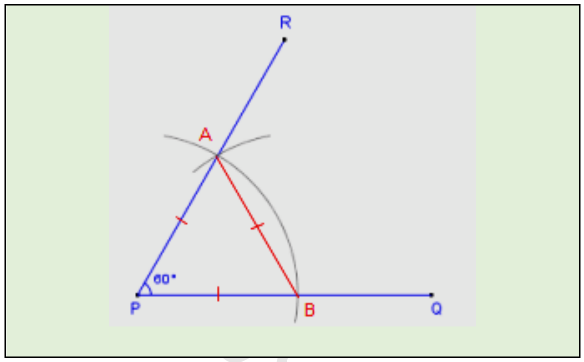
2. With center *B* and the same radius as before, draw another arc as to cut the first arc at *C*.



3. Join *A* to *C*. The angle *CAB* = 60^0



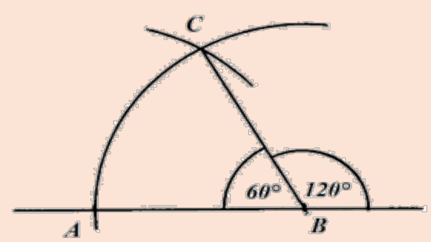
We may confirm this by measurement with the protractor. We can also show that the triangle *ABC* is equilateral and all its interior angles are equal to 60^0.



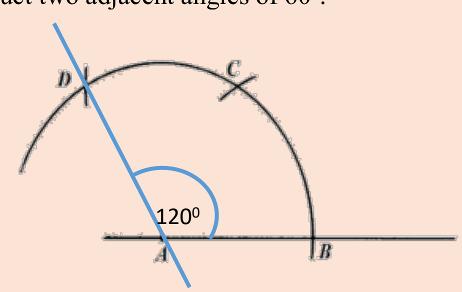
Constructing an angle of 120°

To construct an angle of 120^0, we may construct an angle of 60^0 and use the adjacent angle at the point of construction. This is because the angle in a straight line is 180^0. Alternatively, we may follow the above steps for constructing a 60^0 angle then mark off another 60^0 with the pair of compasses using the same radii. Both methods are shown below.

Construct 60^0 and use the adjacent angle.



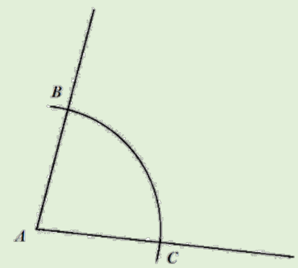
Construct two adjacent angles of 60^0.



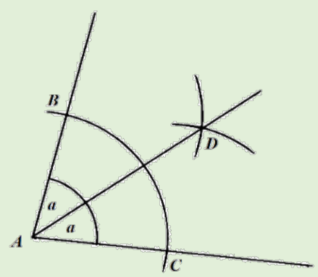
Constructing the bisector of an angle

We wish to bisect the angle at A.

1. With center A, draw an arc to cut the arms of the angle, say, at B and at C.



2. With center B and afterwards C and the same radii, draw two arcs to cut each other at D. Join A to D.

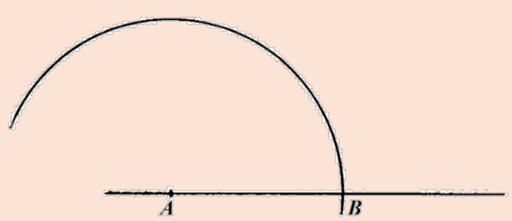


AD will be the bisector of $\hat{A}$, that is $B\hat{A}D = C\hat{A}D$.
It is advisable to confirm this by measuring the two angles with the protractor.

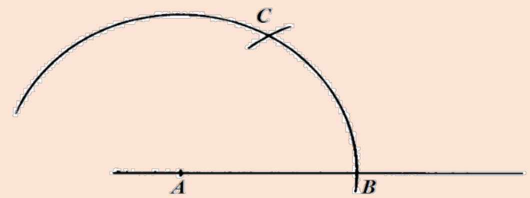
Constructing an angle of 90°

To construct an angle of 90^0 at A, we carry out the following steps.

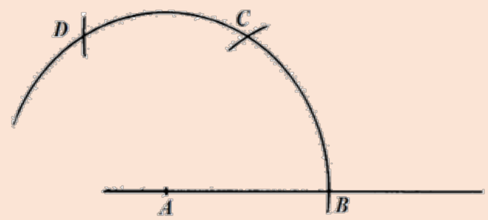
1. Place the compass point at A and draw an almost semi-circular arc so as to cut the straight line at B.



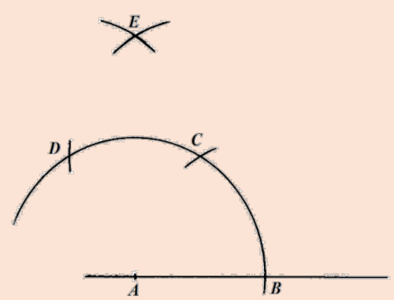
2. With center B and the same radius, draw an arc to cut the first arc at C.



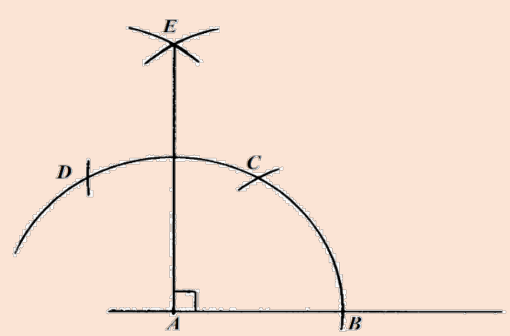
3. With center C and again the same radius, draw another arc to cut the first arc at D.



4. With center C and afterwards D and the same radii, draw two arcs to cut at E.



Join E to A. EA is the bisector of the 60^0 angle DAC. The angle $EAB = 60^0 + 30^0 = 90^0$.

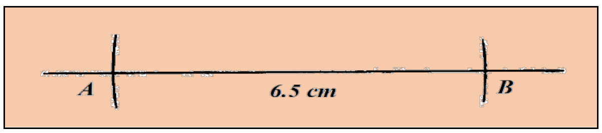


Constructing angles of 45° and 30^0

If we wish to construct an angle of 45^0 we first construct a 90^0 angle and then bisect it. Similar, if we wish to construct an angle of 30^0, we first construct a 60^0 angle and then bisect it.

Drawing a line of a given length

During construction, if we have to draw a line, $AB = 6.5\,\text{cm}$ long, we are expected to draw a line longer than 6.5 cm. Then with our ruler and using the pair of compasses, we would cut off 6.5 cm, clearly showing the arcs. This is illustrated in the diagram, shown below.

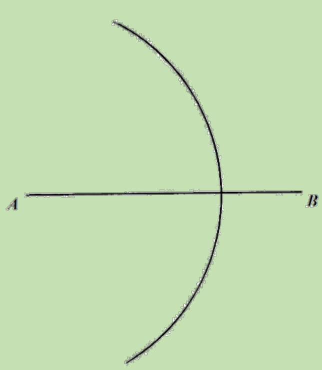


Constructing the perpendicular bisector of a straight line

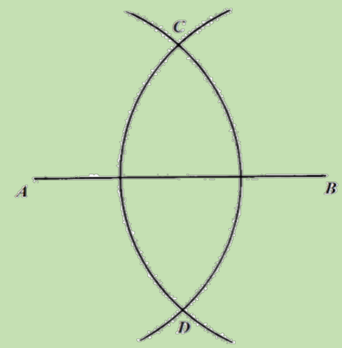
If AB is a straight line and M is the midpoint of AB, then an infinite number of straight lines that may pass through M and all are bisectors of AB. However, only one of these lines will cut AB at right angles and this is called the perpendicular bisector of AB. Hence, there is only one perpendicular bisector of a straight line.

We wish to construct the perpendicular bisector of the straight line, AB.

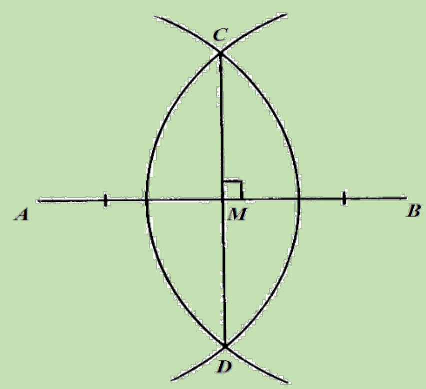
1. With center, A, and a radius of more than half the length of AB, draw an almost semi-circular arc to cut AB.



2. With center B and the same radius, we draw another arc to cut the first arc at C and at D.



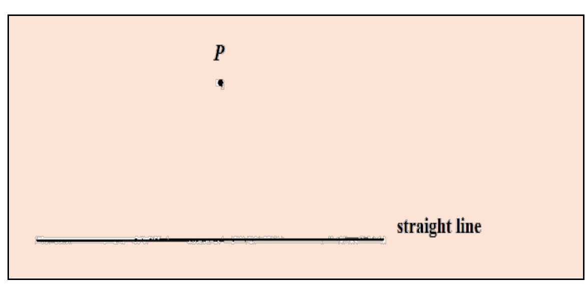
3. Join C to D. Let CD meet AB at M. CD is the perpendicular bisector of AB.



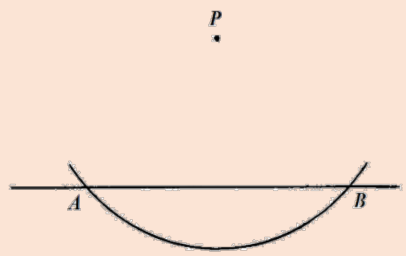
We may confirm all of the above by simple measurements using our geometrical apparatus.

Constructing the perpendicular to a line from a point outside the line

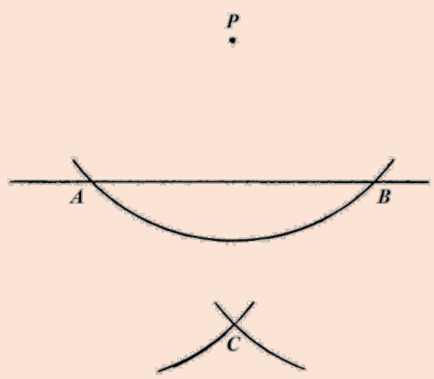
We are given a straight line and a point, P, that is not on the line. We wish to construct a perpendicular to the straight line, passing through P.



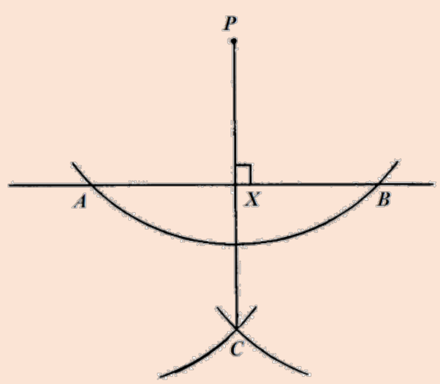
1. With center P, draw an arc to cut the given line at two points A and B.



2. With center A and afterwards B and the same radii draw two arcs to cut at C.



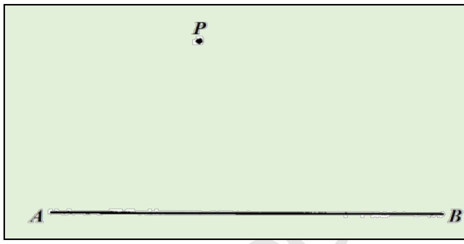
3. Join P to C and let the line PC meet the line AB at X.



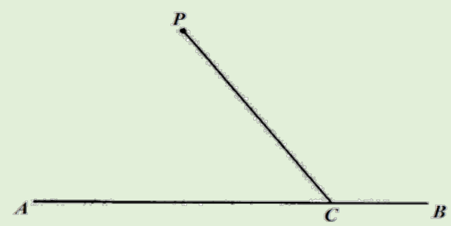
The angle at X is 90º, and so PX is the perpendicular from P to AB, meeting AB at X.
We may confirm this by measurement.

Constructing a line passing through a given point and parallel to a given line

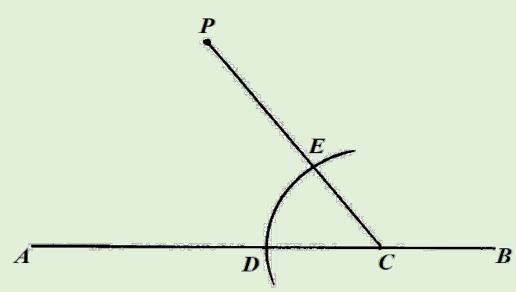
The diagram below shows a straight line, AB and a point P, not on the line. We wish to construct a line passing through P, parallel to AB.



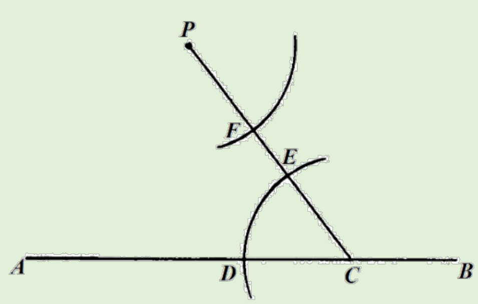
1. Draw a line from P to any point on the line AB, meeting AB at say, C.



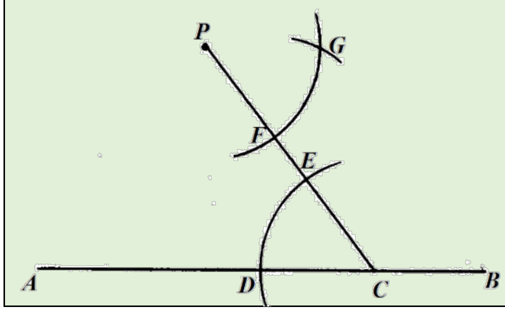
2. At C, we draw an arc to cut CP and CA at D and at E.



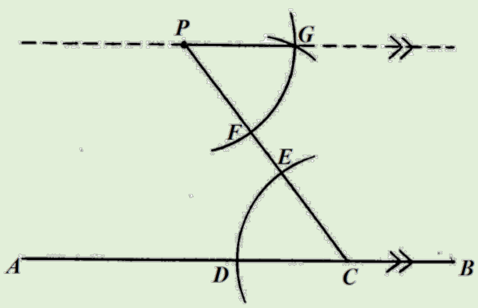
3. With center P, we draw an arc with the same radius as that of CE (or CD). This arc cuts PC at F



4. With center F, we draw an arc with the same radius as that of DE. Let the arc cut the previous arc at G.



5. We join P to G. The line PG is parallel to AB.



Constructing plane figures

We are now in a position to construct any figure given basic information about it. It is good practice to draw a sketch and plan the sequence of steps that are required to produce the figure.

Constructing triangles

To construct a triangle, we must be given three out of its six elements. They can be any of the following:

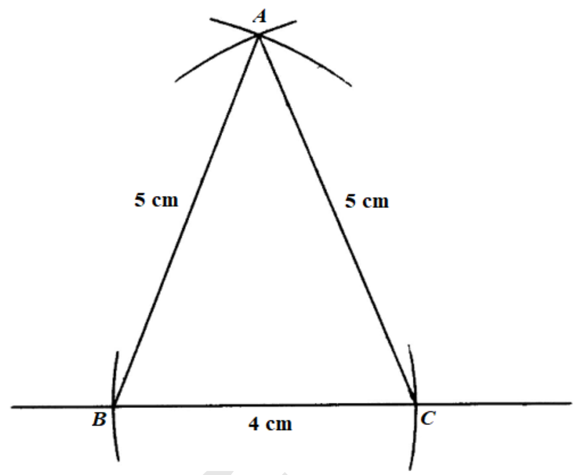
1. Three sides
2. Two sides and the included angle
3. Two angles and the side containing the angles and which is called the corresponding side

Example 1

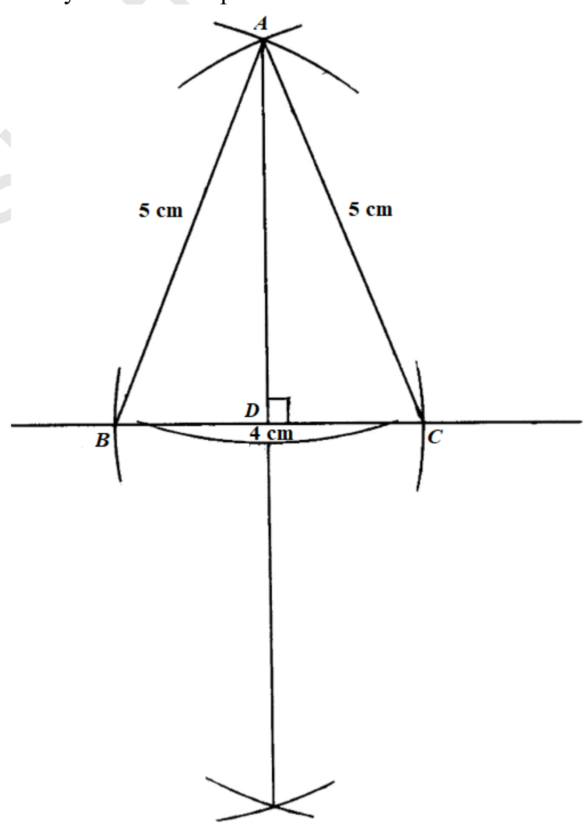
Construct ΔABC with $BC = 4$ cm and $AB = AC =$ 5cm. Construct AD such that AD meets BC at D and is perpendicular to BC.
Measure and state
(i) the length of AD
(ii) the size of $A\hat{B}C$.

Solution

Construct the line $BC = 4$ cm. With center B and then C and a radius of 5 cm, draw two arcs to cut at A.



With center A, draw an arc to cut BC, then bisect this arc by to locate the point F. Join AF.



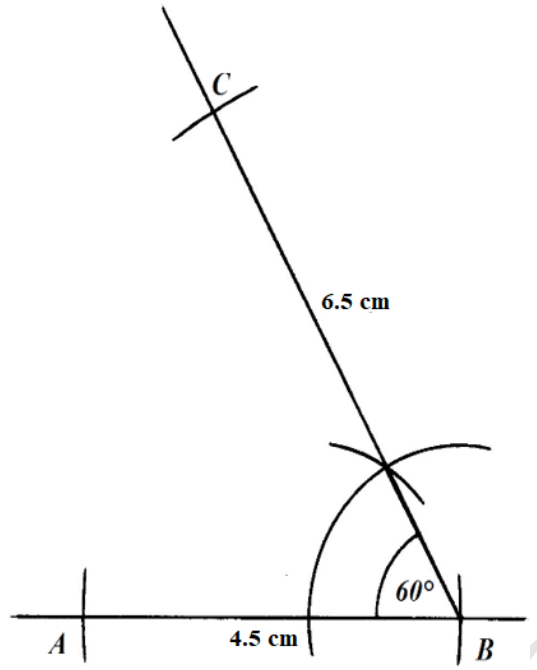
(i) $AD = 5$ cm (ii) angle ABC = 68^0

Example 2

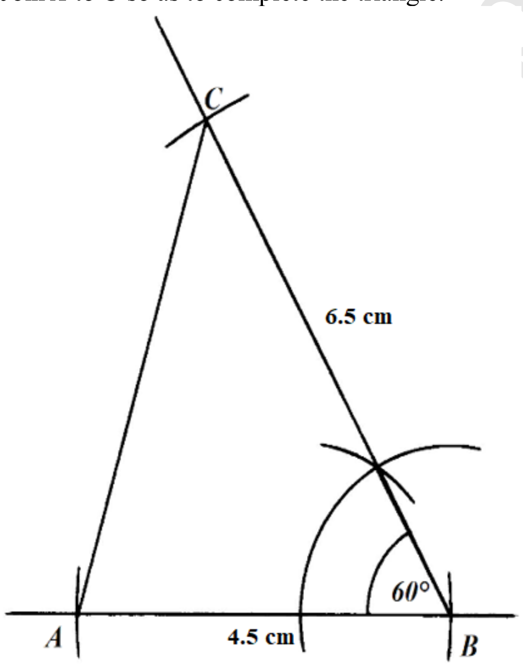
Construct a triangle ABC with AB = 4.5cm, BC= 6.5 cm and $A\hat{B}C = 60°$.
Measure and state the length of AC.

Solution

Draw AB = 4.5 cm. At B, construct $A\hat{B}C = 60°$.
Cut off BC = 6.5cm.



Join A to C so as to complete the triangle.

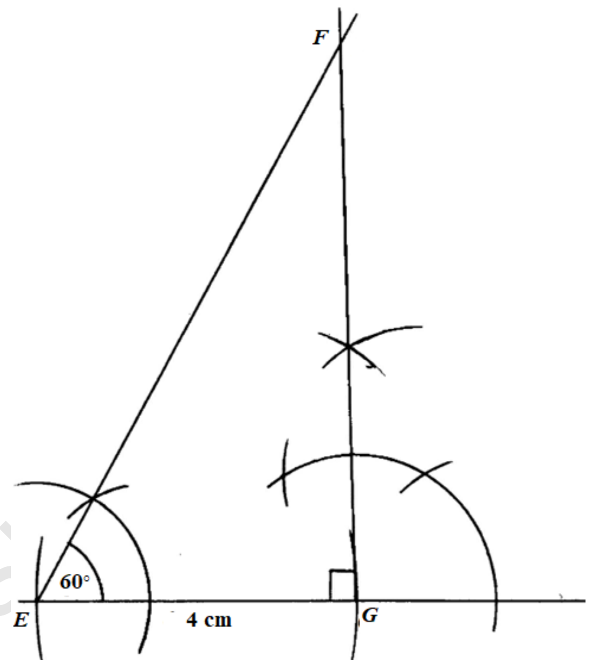


AC = 6 cm

Example 3

Construct triangle EFG, in which, EG = 4 cm, $F\hat{E}G = 60°$ and $E\hat{G}F = 90°$. Measure and state
(i) the length of EF
(ii) the length of FG.

Solution



(i) EF = 8 cm (by measurement)
(ii) FG = 7 (by measurement)

Constructing a parallelogram

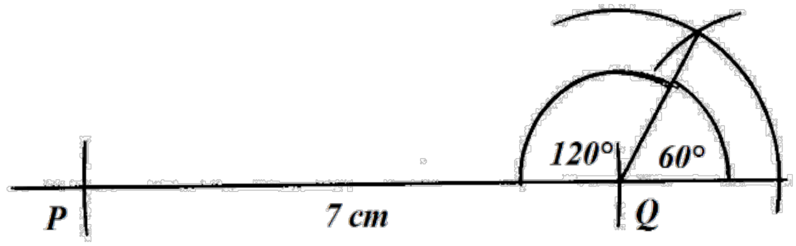
A parallelogram has opposite sides parallel and equal. Once two alternate sides are given we do not need any more information on the sides. The opposite angles of a parallelogram are also equal, so we need to know only one interior angle to construct the parallelogram.

Example 4

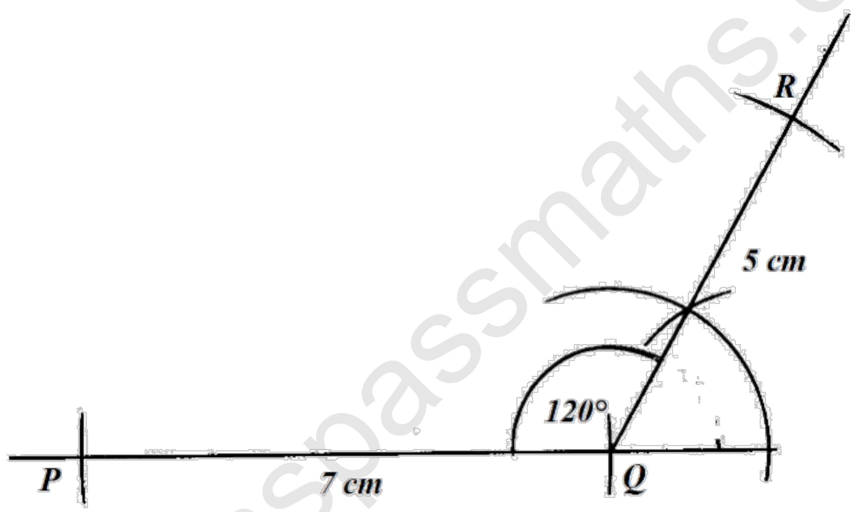
Construct the parallelogram $PQRS$ in which $PQ = 7$ cm, $QR = 5$ cm and $\hat{Q} = 120°$. Measure and state the lengths of both diagonals of $PQRS$.

Solution

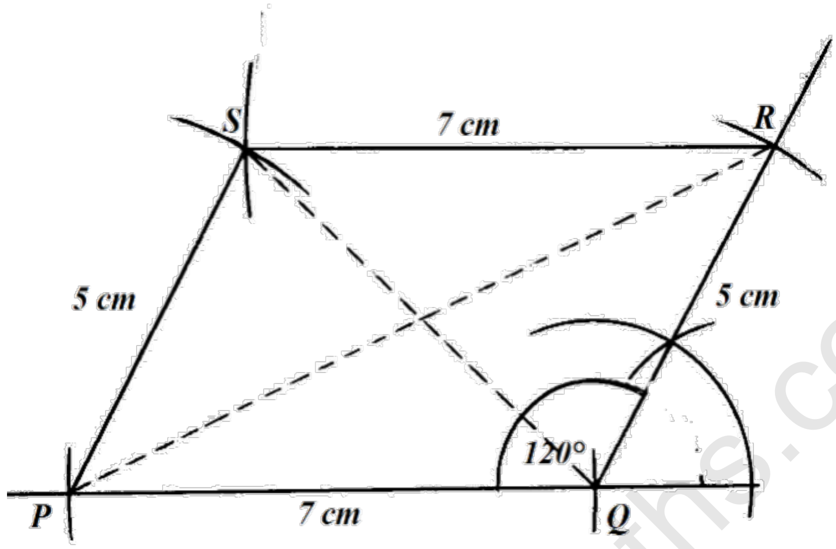
Construct PQ = 7 cm. At Q, construct an angle of 120^0.



Extend the constructed line at Q (if necessary) and cut off $QR = 5$ cm.



Draw an arc with center *P*, 5 cm long and from *R* draw an arc 7 cm long. The two arcs will then intersect at *S*.
[The opposite sides of a parallelogram are both parallel and equal in length.]



PR = 10.3cm, (correct to 1 decimal place), by measurement.

QS = 6.4 cm, (correct, to 1 decimal place), by measurement.