10. MEASUREMENT

MEASUREMENT ATTRIBUTES

Measurement is one of the most widely used applications of mathematics. It is an essential life skill and is necessary for our very existence. In this chapter, we learn how to measure ***attributes*** of objects. Common attributes are linear measure, area, mass, volume, time and temperature. Each attribute is quantified using a unit of measure.

The metric system of measures

The metric system or SI (International System) is considered as the world standard for measurement. Most nations have adopted it as their standard of measurement and in many countries, it is used alongside the traditional Imperial System

Since the metric system is a denary or base ten system, conversions between units are performed by multiplying and dividing by powers of ten. The basic metric measure for length, volume and mass are the **metre (m)**, the **litre (ℓ)** and the **gram (g)**. Larger or smaller units are derived from the standard units.

The following table shows the prefixes for linear measure. All other prefixes follow the same pattern, the basic units being one metre, one litre or one gram.

Prefix	Meaning	Value	
millimetre (mm)	One thousandth	0.001	10^{-3}
centimetre (cm)	One hundredth	0.01	10^{-2}
decimetre (dm)	One tenth	0.1	10^{-1}
Basic Unit (m)	One metre	1	10^{0}
decametre (dam)	Ten times	10	10^{1}
hectometre (hm)	Hundred times	100	10^{2}
kilometre (km)	Thousand	1000	10^{3}

Metric conversions

The above table is useful when we need to convert measures from one unit to another. When converting a from one unit to another, we first check the table to obtain the conversion factor and then use our knowledge of simple proportion to perform the conversion.

Example 1

(i) Convert 150 grams to milligrams

(ii) Convert 3 600 cm to m.

Solution

(i) To convert grams to milligrams, we consult the table to obtain the conversion factor.

$$1\ gram = 1000\ mg$$

$$150\ g = 1\,50 \times 1000\ mg$$

$$= 150\ 000\ mg$$

(ii) To convert centimetres to metres, we consult the table to obtain the conversion factor.

$$1m = 100\ cm$$

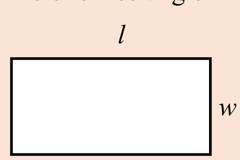
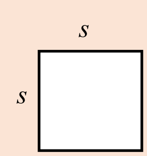
$$1cm = \frac{1}{100}\ m$$

$$3\ 600\ cm = 3\ 600 \times \frac{1}{100}\ m = 36\ m$$

Perimeter of Plane Shapes

The perimeter is a measure of a one-dimensional region and is, therefore, a linear measure. The **perimeter** of a plane shape is the total distance along the boundaries of the plane shape. Hence, the unit for perimeter is the same as the unit for length. The common SI units of length are centimetres, metres and kilometres.

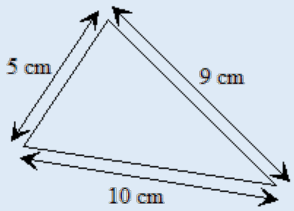
To obtain the perimeter of any plane shape, we simply add the lengths of all the sides. The perimeter of squares and rectangles can be readily found by the use of formulae.

Perimeter of rectangle	Perimeter of square
	
$P = 2(l + w)$	$P = 4s$

With respect to other plane figures, we calculate perimeter by adding up the lengths of the sides.

Example 2

Calculate the perimeter of the triangle shown below.

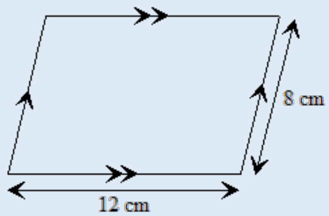


Solution

Perimeter of triangle
= 5 cm+ 9 cm + 10 cm = 24 cm

Example 3

Calculate the perimeter of the parallelogram shown below.



Solution

Perimeter of parallelogram

$$= (12\text{ cm} \times 2) + (8\text{ cm} \times 2)$$
$$= 40\text{ cm}$$

Perimeter of compound shapes

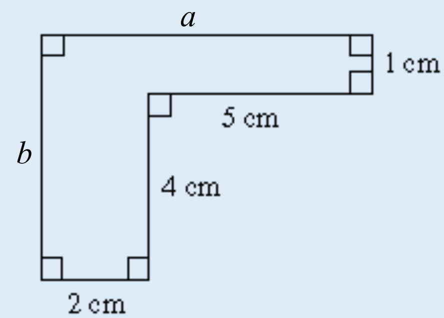
A compound shape is a plane figure made up of two or more basic plane shapes. Basic shapes refer to the simple shapes such as squares, rectangles, triangles, circles and other simple polygons.

To determine the perimeter of compound shapes, we simply add the lengths of the sides that enclose the shape. When solving problems involving compound shapes, it is quite possible to have sides whose lengths are not given. When this is so, we must first calculate any unknown lengths using the properties of basic shapes. Some compound shapes are shown in example 4 below.

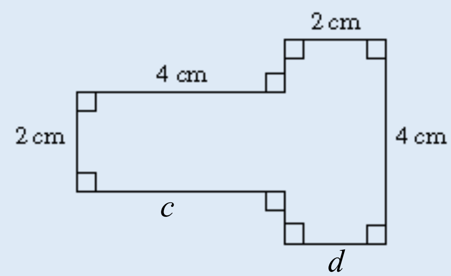
Example 4

Determine the perimeter of the each of the shapes:

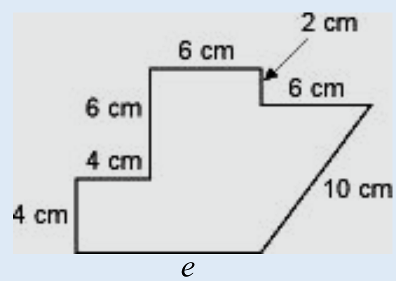
(i)



(ii)



(iii)



Solution

(i) There are two unknown sides in this shape.
The length of side a = (2+5) cm = 7 cm
The length of side b = (1+4) cm = 5 cm
Perimeter of shape = 2+4+5+1+7+5 = 24 cm
(ii) There are two unknown sides in this shape.
The length of side c = 4 cm
The length of side d = 2 cm
Perimeter of shape = 4+2+4+2+4+2 = 24 cm
(iii) There is one unknown side in this shape.
The length of side e = 4 + 6 =10cm
Perimeter of shape = 4+4+6+6+10+10 = 40 cm

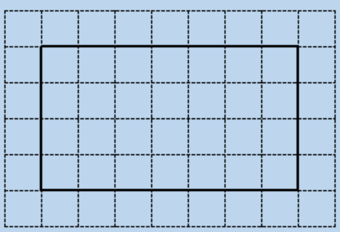
Area of Plane Shapes

The area is a measure of a two-dimensional region and the unit of measure is the square unit. A square

unit has an area of 1(unit)^2 and is represented by a square whose side is 1 unit. The SI units for the area of small regions are square centimetres (cm^2). For larger regions, we use square metres (m^2) or square kilometres (km^2) accordingly.
To measure the area of basic shapes we use formulae. In this section, we will present the formulae but sometimes it is not possible to use formulae and we simply count the number of square units that make up the shape.

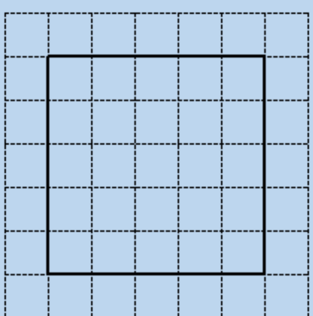
Area of rectangles and squares
The diagrams below show a rectangle and a square drawn on a grid. To measure the area, we count square units inside the shape. Instead of counting in ones, we can multiply the number of square units along the length by the number of square units along width or breadth.

Rectangle



Area
=Number of square units
$= 7 \times 4$
28 sq units

Square



Area
=Number of square units
$= 5 \times 5$
= 25 sq units

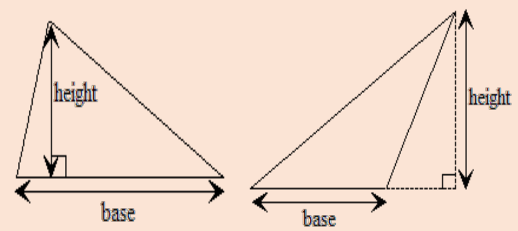
Area of rectangle = Length x Breadth = *lb*
Area of square = Side x Side = s^2

Base and height of plane shapes

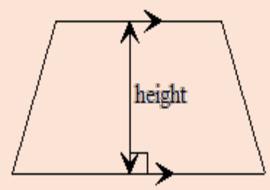
In order to calculate the area of simple, plane figures, we must be familiar with the terms **base and height.**

The base can be any side of the figure (it does not have to be a horizontal line) and the height is the perpendicular distance from the base to the highest point on the figure. In some cases, it is not possible to draw this perpendicular height, ***h*,** to fit inside the figure. In such cases, the base side is extended to afford this accommodation. The length of the base remains the same, though, and does NOT include the length of its extension (shown dotted).

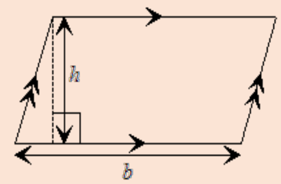
Height of a triangle



Height of a trapezium



Height of a parallelogram

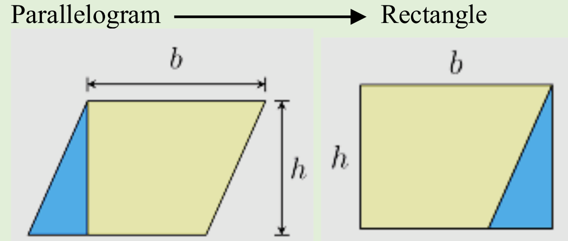


The base of a plane figure is always perpendicular to its height.

Area of parallelogram

We can derive the formula for the area of a parallelogram by transforming a parallelogram into a rectangle. This is done by removing the triangle on the left of the parallelogram and joining it to the right side of the figure. A rectangle of length *l* and width, *w* is formed. This rectangle, whose area is *lb* is the same as the area of the parallelogram. However, the length of the rectangle is the same as the base of the parallelogram and the breadth of the rectangle is the same as the height of the parallelogram.

Parallelogram ⟶ Rectangle

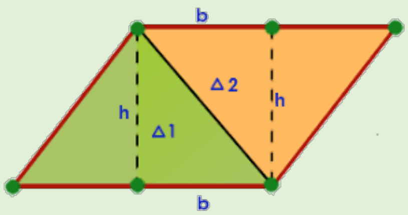


Area of a parallelogram = *bh*

Area of the triangle

If we divide a parallelogram into two equal parts by drawing a diagonal, we obtain two congruent triangles. We can, therefore, think of a triangle as one half of a parallelogram.

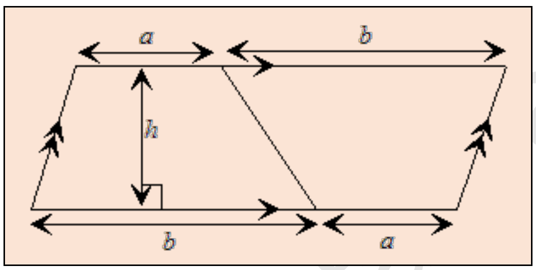
Parallelogram divided into two triangles.



Area of triangle = ½ bh

Area of Trapezium

Consider a trapezium with parallel sides of length a and b units and height h units. If we were to form a parallelogram using two identical trapezia (one flipped), our parallelogram would look like this.



The area of the parallelogram
= Base of the parallelogram × height = $(a + b) \times h$
But, this represents the area of the two congruent trapezia.

Hence, the area of one trapezium is $= \frac{1}{2}(a+b)\times h$

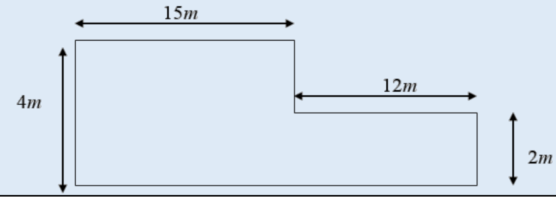
Area of a trapezium
$= \frac{1}{2}$ (sum of parallel sides) × height

Area of Compound shapes

We saw that compound shapes can be divided into two or more basic shapes. The exact area of compound shapes can also be determined by finding the area of each simple shape and adding their areas. Sometimes, there may be more than one way to subdivide the compound shape.

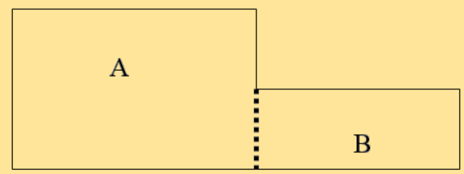
Example 5

Using the dimensions shown in the figure, calculate the area of the shape shown below.



Solution

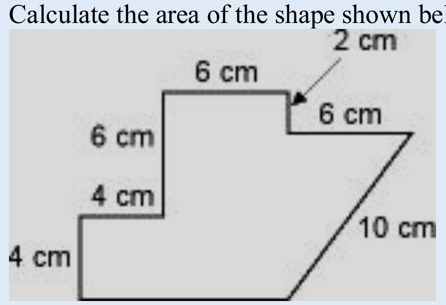
To calculate the area, we can divide the shape into two rectangles, A and B as shown below:



Area of rectangle A = $15 \times 4\ m^2 = 60\ m^2$
Area of rectangle B = $12 \times 2\ m^2 = 24\ m^2$
Area of entire shape = $(60 + 24)\ m^2 = 84 m^2$

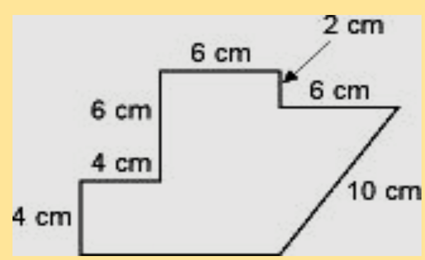
Example 6

Calculate the area of the shape shown below.



Solution

We divide the shape into three regions as shown.



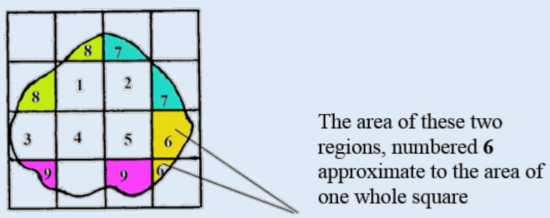
Area of small square = $4 \times 4 = 16cm^2$
Area of rectangle = $10 \times 6 = 60cm^2$
Area of triangle = $\frac{1}{2}(6 \times 8) = 24cm^2$
Area of shape = $(16+60+24)\ cm^2 = 100\ cm^2$

Estimating the area of shapes -curved edges

Many shapes in real life cannot be divided into basic shapes. Land-forms, lakes, ponds, and forests, for example, can take any shape and this makes it difficult to calculate their exact areas. We can, however, estimate their area by placing a grid divided up into unit squares on the shape. The grid serves as a measuring device and we can obtain a reasonable estimate by counting the number of squares that the shape covers.

Example 7

Estimate the area of the shape shown below. The grid drawn over the shape is made up of squares, each one square centimeter in area.



Solution

First count the number of whole squares, numbered 1-5 in the diagram above.
Next, combine part squares to form wholes- these are numbered 6, 7, 8 and 9. They make up 4 wholes.

The total area so far is 5+4 = 9 cm^2 plus one part of a square centimeter that is left. It is reasonable to approximate this part as one-half of a unit square.
Hence the total area is $9\frac{1}{2}\,cm^2$.

For larger regions with more part squares, this method can be quite tedious so we can use an alternative method to estimate the total area of the part-squares. Since some of the parts are greater than one half of a square centimetre and some are larger than one half of a square centimetre, it is reasonable to assume that each is approximately equal to one half of a square centimetre.

For the above shape, the area will be calculated as
$\approx\ 5 + ½\,(9)\ cm^2 = 9\frac{1}{2}\,cm^2$.

Estimated area of shape with curved edges
= Number of whole squares + Half the number of part squares.

Measurement of the circle

A circle is a plane figure whose boundary comprises points that are equidistance from a fixed point, called the center.
For the purpose of this chapter, we will be only interested in calculating the perimeter and area of a circle or parts of a circle. In a further chapter, more detailed information on the geometry of the circle will be presented.

Circumference of a circle

The perimeter of a circle is termed its **circumference**. To calculate the circumference, we use the following formula.

Circumference of a circle, C= $2\pi r$
where r is the radius of the circle and π is a constant, approximately equal to 3.14 or $\frac{22}{7}$

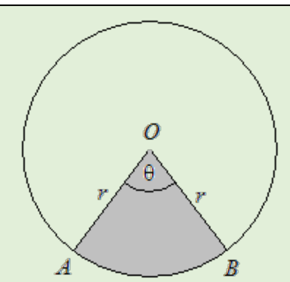
Area of a circle

The area of a circle is calculated using the following formula.

Area of circle, A= πr^2

Arc length and area of sector of a circle

Using the above formulae, we can also calculate the length of an arc of a circle. In the circle shown below, AB is an arc and the region AOB is a sector.



θ is the angle, in degrees, at the center of the circle and r is the radius of the circle.

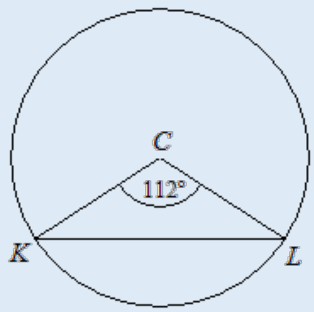
Arc length
$= \dfrac{\theta}{360} \times 2\pi r$

Area of minor sector
$= \dfrac{\theta}{360}\pi r^2$

It should be noted that the above formulae relate to the arc length of the minor arc AB and area of the minor sector AOB. The longer arc AB is called the major arc and the larger area corresponding to this arc is called the major segment. If we were interested in these measures, the value for θ will be $(360 - \theta)$.

Example 8

In the circle shown below, the radius $KC = 10$ cm and $K\hat{C}L = 112°$. (Use $\pi = 3.14$)



Calculate:

i. the circumference of the circle
ii. the area of the circle
iii. the length of the minor arc, KL
iv. the area of the minor sector KCL

Solution

i. Circumference of the circle

$= 2\pi r$

$= 2 \times 3.14 \times 10$ cm

$= 62.8$ cm

ii. Area of the circle $= \pi r^2$

$= 3.14 \times 10 \times 10 \text{ cm}^2$

$= 314 \text{ cm}^2$

iii. To find the length of the arc KL, we use the formula.
Arc length

$$= \frac{\theta}{360} \times 2\pi r$$

iv. To find the area of the sector KCL, we use the formula.
Area of the sector

$$= \frac{\theta}{360} \times \pi r^2$$

$$= \frac{112}{360} \times \left(3.14 \times 10^2\right)$$

$$= \frac{112}{360} \times 314$$

$= 97.7 \text{ cm}^2$ correct to one decimal place

SCALE DRAWINGS

Often times, we may wish to measure distances or areas that cannot be measured directly. For example, we may want to find the distance between two towns or the area of a country. In these instances, we use a method of calculation based on the use of scale drawings.
A scale is usually expressed as a ratio since a comparison is made between two measures of the same types. Linear scale factors compare linear measures with linear measures.

For example, if a map is drawn and the scale is given as **1: 500 000.** We interpret this as a distance of 1 cm on the map is really a distance of 500 000 cm on the ground.

The units in a scale can differ so that we could have expressed the above scale as:
1cm represents 5 km.

Example 9

A map is drawn using a scale of 1:10 000.

i. The distance between two towns on the map is 3.5 cm. What is the actual distance between the two towns in kilometres?
ii. If the actual distance between two towns on the ground is 16 km, what is the distance between the towns on the map?

Solution

i. The actual distance between the towns can be represented as *b km.*
Setting up a proportion, we have:

$$\frac{\text{Distance on the map}}{\text{Actual Distance}}$$

$$\frac{1}{10\ 000} = \frac{3.5}{b}$$

$b = (3.5 \text{ x } 10\ 000)$ cm

$$b = \frac{3.5 \times 10\ 000}{100\ 000}\ km \ [1 \text{ km} = 100\ 000 \text{ cm}]$$

$b =\ 0.35\ km$

ii. Since the distance on the map is in centimetres, we need to convert 16 km to centimeters and set up the proportion as follows:

$$\frac{1}{10\ 000} = \frac{d}{1600\ 000}$$

$$d = \frac{1600\ 000}{10\ 000}\ cm = 160\ cm$$

Errors in a measurement

When a measurement is taken, there is some degree of uncertainty in the value obtained. In mathematics, we refer to this uncertainty as the ***margin error in measurement***. The error is really the difference between the result of the measurement obtained and the true measure.
Measurement errors occur because of limitations in the measuring device, human limitations in reading the instrument and errors caused by faulty procedures in taking the measures. This error will depend on the degree of accuracy of our measurement device.

> The error in a measurement is defined as half the least unit of measure.

If a length of an object is recorded as 13 cm, then it is assumed that the instrument measures to the nearest centimetre. Hence, the measurement error is half of 1 cm or 0.5 cm.

A measurement recorded as 13 cm may, therefore, have an actual value in the range between the lowest possible value and the highest possible value.

The maximum value will be $13 + 0.5 = 13.5$.
The minimum value will be $13 - 0.05 = 12.5$.

If the actual value is r cm, then the **range** of r is given by 13 ± 0.5 **OR** $12.5 \leq r < 13.5$

Note that the minimum value of r includes 12.5. However, the maximum value of r is < 13.5**.** For calculation purposes, this is taken as 13.5. This is because 13.5, rounded to the nearest unit will be rounded to 14 and is out of the range.

Example 10

A thermometer measures temperature in degrees centigrade, correct to the nearest degree. A temperature, *t*, is read as 23.8 ^{0}C. State:
(i) the smallest unit of measure
(ii) the error in the measurement
(iii) the range within which the measurement must lie.

Solution

(i) The smallest unit of measure is 0.1^{0}C.
(ii) The error is one half of the smallest unit of measure or 0.05^{0}C.
(iii) The range is.

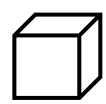
Volume and Surface Area

We learnt earlier that a line has a length only and is **one-dimensional (1D),** while plane shapes have length and breadth (width) and are **two-dimensional (2D)**. Solids, however, differ from lines and plane shapes in that they are **three-dimensional (3D)**, having length, breadth and height.

Solids occupy space and the amount of space occupied is called its ***volume***. A block of wood is an example of a 3D shape, but an empty box, in mathematics, is also considered a 3D shape. Both occupy space, but the empty box is a ***hollow*** solid and is said to have a ***capacity***. The capacity of a container is the measure of substance it can hold.

Common units for capacity are millilitres, litres and kilolitres, while common units for volume are cubic centimetres and cubic metres. It should be noted, however, that both volume and capacity are properties of 3-dimensional regions.

It is important to note that the units for volume and capacity are connected in the metric system. A unit cube with an edge of length one cm has a volume of one cubic centimetre. This space occupied by one cm^3 is the same as the space occupied by one millilitre.

1 cm^3  = 1 ml

Hence, 1000 cm^3 = 1000 ml = 1 litre

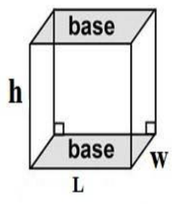
Three-dimensional shapes also have the property of ***surface area***, a measure of the total area of all their surfaces often called faces). Solids such as cubes and cuboids have six polygonal faces while a cylinder has a curved surface and two flat circular faces.

Right Prisms

A prism is any solid with two congruent, parallel surfaces. A prism has a uniform cross-section, that is, if we cut across a prism parallel to its congruent surfaces, all the ‘slices’ will be identical. The uniform cross section of a prism is always in the shape of a polygon. A right prism is one whose vertical sides and faces are perpendicular to its the base. If the joining edges and faces are not perpendicular to the base faces, it is called an oblique **prism**.

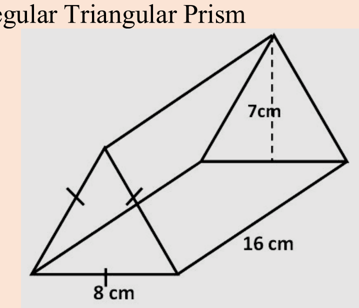
Prisms have no curved edges. Hence, solids with curve edges like cylinders, cannot be classified as prisms even though they can have a pair of congruent, parallel faces. However, these solids have all properties of a prism as we shall soon see when we learn how to calculate their volumes.

Prisms are named according to the shape of their uniform cross-sections. Prisms with triangular cross sections are called triangular prisms. Prisms with rectangular cross sections are called rectangular prisms (cuboids). These congruent faces are called the ***bases of the prism.*** The perpendicular distance between the parallel faces is the ***height of the prism***.



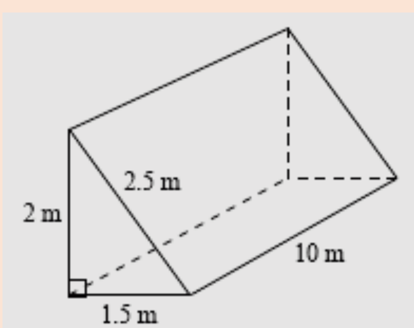
A regular prism has a regular polygon as its cross section. An example of a regular and an irregular triangular prism is shown below. Note that the cross-section of the regular comprises equilateral triangles, while the cross-section of the irregular triangular prism comprises scalene triangles.

A Regular Triangular Prism



The bases comprise equilateral triangles
This prism has a height of 15 cm

This triangular prism is not regular



The bases comprise scalene triangles
This prism has a height of 10 cm

Volume of prisms

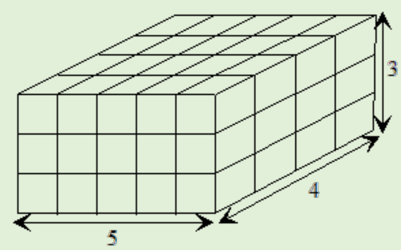
In the same way, we measure the area of a plane figure by counting the number of unit squares inside a two-dimensional region, we can also measure the volume of a solid by counting the number of cubes inside a three-dimensional region. We will now derive the formula for the volume of any prism.

We can illustrate this principle by using one of the simplest solids, a cuboid. The diagram below shows a cuboid made up smaller cubes each measuring one cubic unit in volume. To calculate the volume of the cuboid, we may count the number of cubes one at a time. But, it is always better to use a more efficient method.

A quick way of doing this is to count the number of unit cubes that fill the cross-sectional layer (base) and multiply by the number layers (perpendicular height).

The cuboid, shown below, has 20 unit cubes on the base layer and 3 layers in all. Hence, its volume is $20 \times 3 = 60$ cubic units.

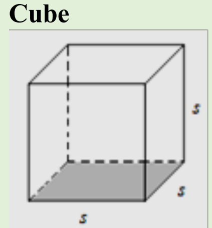
Cuboid



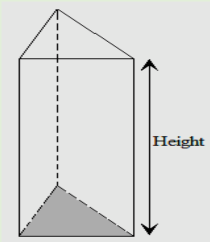
Volume of cuboid = Number of unit cubes
$= 5 \times 4 \times 3 = 60$
$= l \times b \times h$
$= (l \times b) \times h$
= Base Area × Height

The formula, Base area × height, is used to calculate the volume of any prism.

Cube

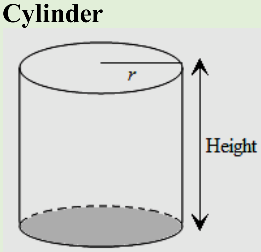


The volume of the cube:
= Base area × Height
$= (s \times s) \times s$
$= s^3$

Triangular prism	
	Volume of Triangular prism = Base area × Height $=$ Area of triangle $\times h$

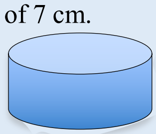
Volume of a cylinder

Although the cylinder is not really a prism, it has a regular cross-sectional area. Hence its volume can be calculated in much the same way as the volume of a prism.

Cylinder	
	The volume of the cylinder: Base area × Height = Area of circle × Height $=\pi r^2 h$

Example 11

A cylindrical nugget made of metal has a height of 3 cm and a radius of 7 cm.



i. Calculate the volume of the nugget.

ii. Five of these nuggets are melted and moulded to make a cube. State, to one decimal place, the dimensions of the cube. **[Use** $\pi=\dfrac{22}{7}$**]**

Solution

i. Volume of nugget:

Base area × height

$\dfrac{22}{7}\times 7\times 7\times 3\ \text{ cm}^2 = 462\ \text{ cm}^3$

ii. Volume of 5 nuggets:

$5\times 462\ \text{cm}^3 = 2310\ \text{cm}^3$

Volume of cube: $s^3 = 2310$

$s=\sqrt[3]{2310}$

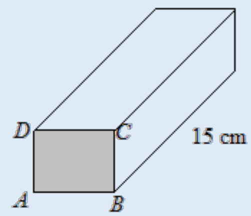
$=13.2$ cm

Example 12

The diagram shows a rectangular prism of length 15 cm, volume 960 cm^3 and has a square cross-section *ABCD*.

(i) Calculate the length of *AB*.

(ii) The total surface area of the prism.



Solution

(i) We know that the volume of a prism

= Cross sectional area × Height

Volume of prism = 960 cm^3

Area of $ABCD \times 15 = 960$

Area of $ABCD = 960 \div 15 = 64$ cm^2

Length of $AB = 8$ cm

(ii) Surface area of the 2 square faces $=64\times 2$

$=128$ cm^2

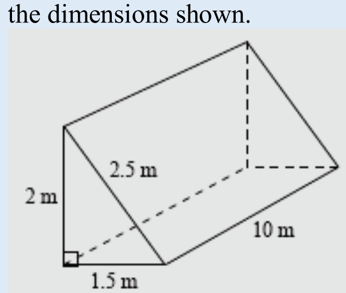
Area of the 4 rectangular faces $=(8\times 15)\times 4$

$=480$ cm^2

$\therefore$ Total surface area $=128+480 = 608$ cm^2

Example 13

Calculate the volume of the triangular prism using the dimensions shown.



Solution

Volume of prism

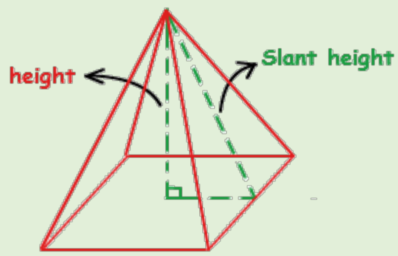
= Area of triangular base × Height

$\frac{1}{2}(2\times 1.5)\times 10 = 150\ cm^3$

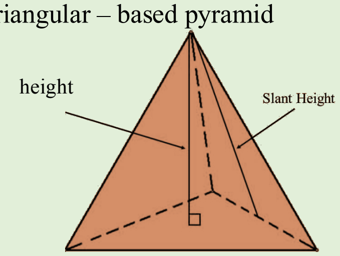
Right Pyramids

A pyramid is a solid with a polygonal base and triangular faces that meet at a common point called its apex. A right prism is one whose apex is directly above the center of its base. Pyramids are named after the shape of its base.

Square– based pyramid

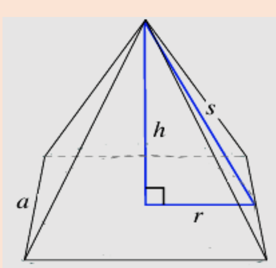


Triangular – based pyramid



The ***height*** of the prism is the perpendicular distance from the apex to the center of the base. A pyramid also has a ***slant height***, which is the height of the triangles that make up the faces. The slant height is the hypotenuse of a right-angled triangle and can be calculated using Pythagoras' Theorem.

If h is the height of the prism and r is the base of the triangle, then we can calculate the slant height as follows:



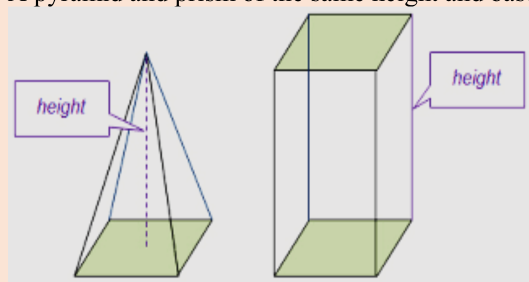
$$s^2 = h^2 + r^2$$

$$s = \sqrt{h^2 + r^2}$$

Volume of pyramid

If we were to compare the volume of a pyramid and a prism of the same base area and height, a relationship emerges. The volume of a pyramid is one-third the volume of a prism of the same base area and height as the prism.

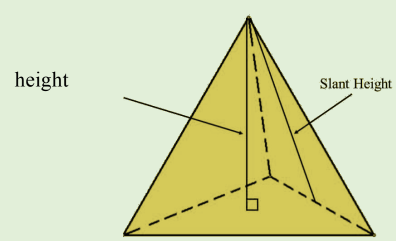
A pyramid and prism of the same height and base



$$\text{Volume of Pyramid} = \frac{1}{3} \times \text{Volume of prism}$$
$$= \frac{1}{3}\ (\text{Base Area} \times \text{Height})$$

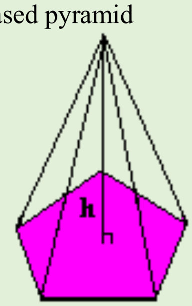
The above relationship holds for any type of pyramid, regardless of the shape of the base. Note that the height in the formula refers to the perpendicular height of the pyramid, not the slant height.

Triangular – based pyramid



Volume of triangular-based pyramid
$$= \frac{1}{3}\ (\text{Area of Triangle} \times \text{Height})$$

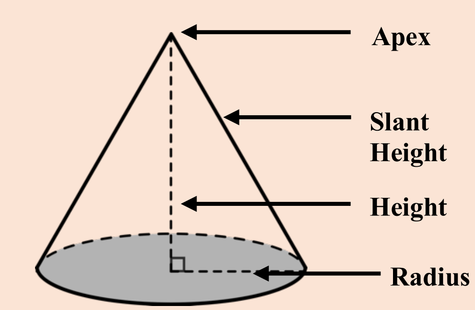
Pentagonal – based pyramid



Volume of pentagonal-based pyramid
$$= \frac{1}{3}\ (\text{Area of Pentagon} \times \text{Height}$$

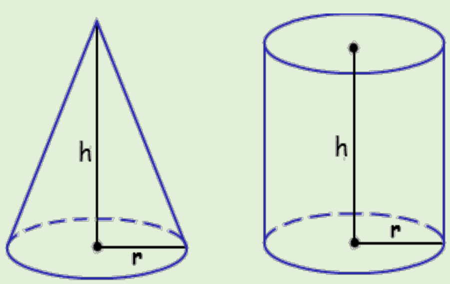
Volume of cone

A cone is a 3D shape with a circular base and an apex. It is not a pyramid because, by definition, a pyramid has a polygonal base. The height of a cone is the perpendicular distance from the apex to the center of the circular base. The slant height is the distance from the apex to a point on the circumference of the circle. The radius of a cone is the radius of the circular base.



If we were to compare the volume of a cone to the volume of a cylinder of the same base area and height, the relationship is the same as that of any pyramid and prism.

A cone and a cylinder of the same height and base.

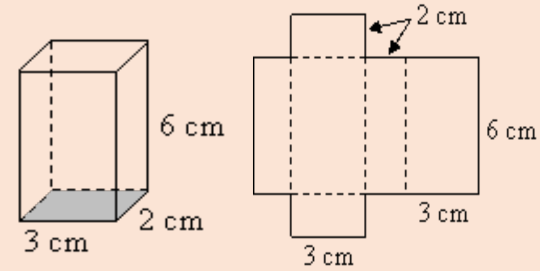


The volume of a cone: $\frac{1}{3} \times \text{Base area} \times \text{height}$

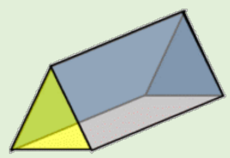
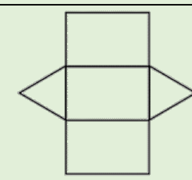
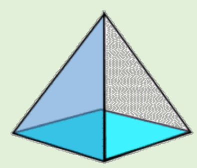
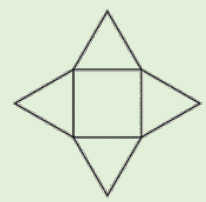
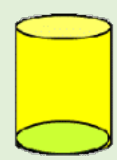
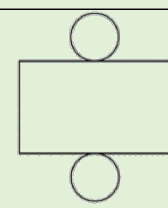
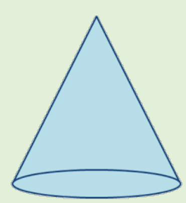
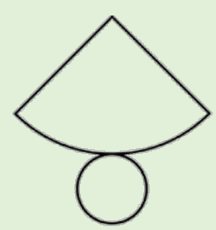
Nets of three-dimensional shapes

Some 3-D shapes, when cut open can be laid out flat. The two-dimensional shape formed is called its net. The net does not only represent all the faces of the solid, but it represents an arrangement of the faces such that, when folded, forms back the solid.

A cuboid and its net



Here are some examples of nets of solids.

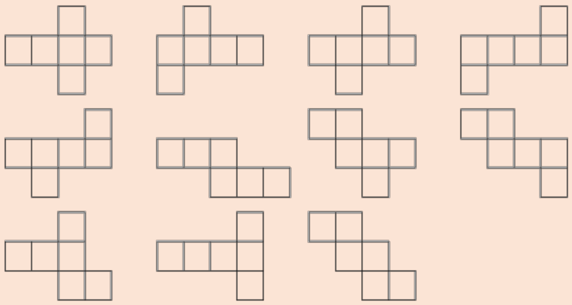
Solid	Net
 Triangular Prism	
 Square-Based Pyramid	
 Cylinder	
 Cone	

The nets of a cube

Some solids have only one net, while others have many different nets. For example, a cube has eleven nets. Each of these nets is different, yet when folded forms a cube. It must be noted too, that not all shapes made up of six squares will form a cube when folded.

The eleven nets of a cube are shown below.

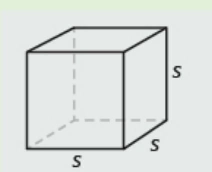
Nets of a cube



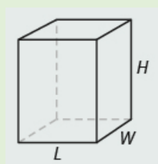
Surface area of solids

When we draw the net of a solid we readily see all the faces that make up the solid. The surface area of a solid is simply the total area of all the faces that form the net of the solid.
We can determine the surface area by observing their nets counting the area of simple shapes (squares or rectangles).

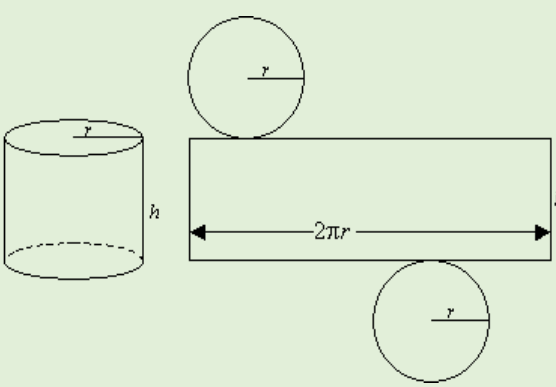
Surface Area of Cube = $6s^2$



Surface Area of Cuboid = 2LW + 2LH + 2WH



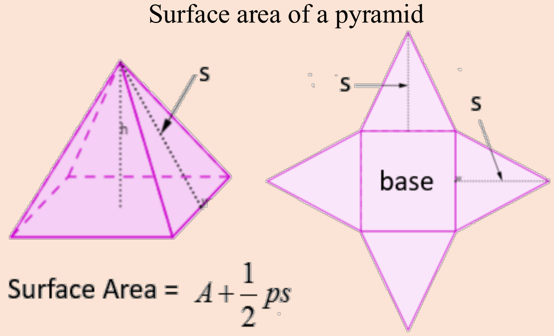
Surface Area of Cylinder = $2\pi r^2 + 2\pi rh$



Surface area of right pyramids

All pyramids have a base and a set of slant faces. For example, the pyramid shown below has a square base and 4 triangles. The sum of the areas of these shapes is the surface area. To derive the formula for the surface area of a pyramid. the following
The area of each triangle is $\frac{1}{2}b \times s,$ where s is the height of the triangle and b is the length of one side of the square.
The area of 4 triangles = $4\left(\frac{1}{2}b \times s\right) = 4b \times \frac{1}{2}\ s$
But the Perimeter of the square base is $4b, p = 4b$
Replacing $4b$ by p in the formula for the area of triangles, we have:
The area of the 4 triangles = $p \times \frac{1}{2}s = \frac{1}{2}ps.$
This area is added to the area of the base to obtain the formula:
Surface Area of pyramid = Base Area + $\frac{1}{2}ps.$
This formula holds for all right pyramids.

Surface area of a pyramid



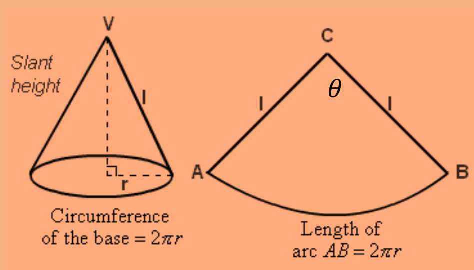
Surface Area = $A + \frac{1}{2}ps$

A = Area of base
p = perimeter of base
s = slant height

Surface area of a cone

A solid cone has two surfaces, a circular base and a curved surface. When a cone is cut open its curved surface has the shape of a sector of a circle. The angle in the sector can vary, but the radius of the sector will always be the length of the slant height of the cone.

To calculate the surface area of a cone, we must add the area of its circular base to the area of its curved surface. The curved surface of a cone takes the shape of a sector of a circle. Hence, to calculate the area of the curved surface we must know the measure of the angle in the sector. The formula for the total surface area of a cone is derived below.



The sector ACB has a radius of, where l is the slant height of the cone.
Let the angle in the sector be θ.

Now the angle at the center, θ is proportional to the arc length. Hence,

$$\frac{\theta}{360} = \frac{Length\ of\ arc\ AB}{Circumference\ of\ circle\ of\ radius,\ l}$$

$$\frac{\theta}{360} = \frac{2\pi r}{2\pi l} = \frac{r}{l}.$$

Area of sector, ACB = $\frac{\theta}{360} \times \pi l^2$,

Replacing $\frac{\theta}{360}$ *by* $\frac{r}{l}$,

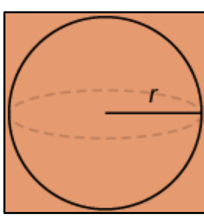
Area of sector, ACB = $\frac{r}{l} \times \pi l^2 = \pi r l$

Measurement of the sphere

A sphere is a three-dimensional shape in which every point on its surface is equidistant from the center. The distance between any point on its surface and the center point is the radius of the sphere.

The sphere is unlike the other solids studied so far with respect to its net. It is impossible to draw an exact net of a sphere because we cannot transform it to a flat shape.

At this level, we need only to be familiar with two measures in relation to a sphere. These measures are volume and surface area. The derivation of these formulae are beyond the scope of this level and so the formulae are presented below.

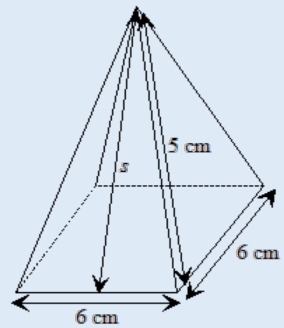


Volume: $V = \frac{4}{3}\pi r^3$

Surface Area: $S = 4\pi r^2$

Example 14

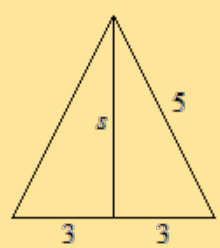
A pyramid has a square base of side 6 cm and a slant edge of 5 cm.



i. Calculate the slant height of the pyramid.
ii. Draw the net of the pyramid.
iii. Calculate the total surface area of the pyramid.

Solution

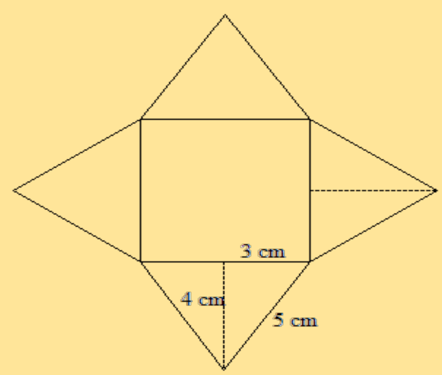
(i)



Using Pythagoras' Theorem

$s^2 = 5^2 - 3^2$
$= 25 - 9$
$= 16$
$s = \sqrt{16}$
$= 4$ cm

(ii)



(iii) Area of one triangle $= \frac{1}{2}bh = \frac{1}{2}(6 \times 4) = 12cm^2$
Area of 4 triangles = 4×12 cm^2 = 48 cm^2
Area of square base = 6×6 = 36 cm^2
Total surface area = $(48 + 36)$ cm^2 = 84 cm^2