7. SET THEORY

BASIC CONCEPTS

Set Theory is a branch of mathematics that involves collections of objects. When objects have a common characteristic, they form a set.

Defining and describing sets

In describing a set, we use language that is clear so that there is no doubt in identifying its members. A set is a clearly defined collection of objects.

Examples of sets

1. The set of vowels of the English alphabet.
2. The set of planets in our solar system.
3. The set of black cars
4. The set of boys on the school's football team
5. The set of girls in Form 5 at school.

The first two examples are readily identifiable and can be listed. The set of black cars can also be identified even though we do not have information about all its members in terms of their location. What is important is that we can differentiate between the cars that belong to the set and the cars that do not belong to the set. The members of a set may be unknown, but once the description is not vague, it is well defined.

When describing sets, we use curly brackets to replace the words 'the set of'. This is illustrated below.

Description of set in words	Description of the set using set notation
The set of all the whole numbers from one to ten (inclusive).	{Whole numbers from 1 to 10 inclusive}
The set of figures bounded by four straight lines.	{Figures bounded by four straight lines}
The set of multiples of 5.	{5, 10, 15, 20, …}

Example 1

Does each of the following describe sets?
(A) Tall girls in Form 6.
(B) Girls in the school choir at Valley Government.
(C) Men who can sing well.
(D) Boys who are good at sports.

Solution

Only (B) is clearly defined and hence only (B) describes a set.
(A), (C) and (D) that describe 'tall' or 'who can sing well' or 'good at sports' are all relative terms and **not** clearly defined.

The statements, (A), (C) and (D), can be re-worded to describe sets. For example,

(A) Girls who are over 160 cm tall.
(C) Men who have auditioned for the Church Choir.
(D) Boys who are members of their school cricket team

Each of the above statements is now clearly defined and describes a set. A set must be defined in such a way that there is no hesitation in deducing whether or not an object belongs to the set **or** does not belong to the set. Every collection is not a set

Naming sets

A set is usually named using a capital letter. If we are speaking of the set of students in a school who study Spanish, then we may choose to name the set, S where S is described as follows:
$S = \{\text{Students at our school who study Spanish}\}$.
The following are examples of sets named by letters of the alphabet.

$A = \{\text{Multiples of 10}\}$

$B = \{\text{Words that begin with the letter 'p'}\}$

$C = \{\text{Members of the school's football team}\}$

$D = \{\text{Red sports cars}\}$

Elements of a set

The constituents of a set are called members or elements. When listing the members, we use a comma to separate them. When a set is written, the order of the elements is of no consequence. All elements must be listed only once.
For example, the set $P = \{a, b, c\}$ can be written as:

$P = \{a, b, c\}$ or $\{b, a, c\}$ or $\{a, c, b\}$
The symbol $\in$ is used to replace 'is an element of' or 'belongs to'
The symbol $\notin$ is used to replace 'is not a member of' or 'does not belong to'. To illustrate the use of these symbols, consider the sets A and B defined below.

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$A = \{a, e, i, o, u\}$ $\quad B = \{2,4,6,8,10\}$

Words	Symbols
u is a member of A	$u \in A$
8 is an element of B	$8 \in B$
p is not a member of A	$p \notin A$
3 is not a member of B	$3 \notin B$.

Number of elements in a set

The number of elements in a set, A, is called the cardinality of the set and is expressed as $n(A)$.

Set	Number of elements
$A = \{m, n, p\}$	$n(A) = 3$
B ={cherries, plums, mangoes, oranges}	$n(B) = 4$
C = {Odd numbers from 1 to 10 inclusive}={1,3,5,7,9}	$n(C) = 5$
D = {Multiples of 10 between 39 and 99}={40,50,60,70,80,90}	. $n(B) = 6$

Empty sets

If a set has no elements, it is called the empty set or null set and is denoted by $\{\ \}$ or φ, only.

For example, if all the students in a class are taller than 120 cm, then we can define an empty set, G, as:
G = {students in the class whose heights are less than120 cm}.
Since there are no members in set G, we say that set G is an empty set. The sets P, Q, R and S are examples of empty sets.

$P = \left\{\text{Whole numbers between } \frac{1}{4} \text{ and } \frac{1}{2}\right\}$,

$Q = \{\text{Prime numbers from 8 to 10}\}$.

R = {days of the week, which start with the letter L}

J = {Integers that lie between 10 and 11},

Finite and infinite sets

If a set is composed of a countable number of elements, it is called finite.

If a set has an infinite number of elements, then it is called an infinite set. It is not possible to count the number of elements in an infinite set. Hence, it is not possible to state the cardinality of an infinite set.

Even though we may think of the cardinality of an infinite set as being infinity, infinity is not really defined as a countable number. The null set or the set with no elements is considered to be a finite set and has a cardinality of zero.

Finite sets

The set of prime numbers between 1 and 10 is a finite set. Let us name this set, A. $A = \left\{\begin{matrix}\text{Prime numbers} \\ \text{from 1 to 10}\end{matrix}\right\}$ $A = \{2, 3, 5, 7\}$ $n(A) = 4$	The set of vowels of the alphabet is a finite set. Let us name this set, B. $B = \left\{\begin{matrix}\text{Vowels of} \\ \text{the alphabet}\end{matrix}\right\}$ $B = \{a, e, i, o, u\}$ $n(B) = 5$

Infinite sets

The set of multiples of 5 is an infinite set. Let us name this set, Q. $Q = \{5,10,15,20, \ldots\}$ There are countless members of this set	The set of multiples of 3 is an infinite set. We name this set, P. $P = \{3, 6, 9, 12, 15\ldots\}$ There are countless members of this set

Notice that in Q and P, the three dots at the end indicates that the elements of the set continue forever.

Set builder notation

'Set Builder Notation' is a simple and convenient way of describing the elements of a set, without actually listing all its members. This is especially useful for a set that has a large or infinite number of elements.

The set builder notation is also useful to describe an interval on the number line, comprising real numbers. Sets comprising real numbers are examples of infinite set that are impossible to list and thus the set builder notation is absolutely necessary in such situations.

Using the set builder notation, we can describe sets as follows:

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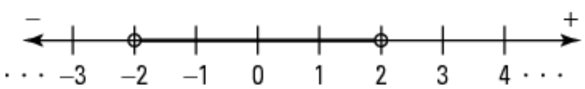
Description of set	Set Builder Notation
The set of all values of x such that x is greater than 5 and x is a member of the set of integers. $\{6, 7, 8, 9, 10, \ldots\}$.	$\{x : x > 5, x \in Z\}$ Since this is an infinite set, the interval, $x > 5$ is open.
The set of all values of x such that x is less than 5 and greater than 1, and x is a member of the set of Natural Numbers' $\{2, 3, 4\}$.	$\{x : 1 < x < 5, x \in N\}$ Since this is a finite set the interval, $1 < x < 5$ is closed
The set of all values of x such that x is less than 2 and greater than -2, and x is a member of the set of Real Numbers.	$\{x: -2 < x < 2\}$ This is a closed infinite set. It is best represented on a number line, shown below.

The set $\{x: -2 < x < 2\}$ is described below.



Summary

We can describe a set in three ways:

- We may use words, for example, the set of multiples of 2.
- We may list its members, for example, $\{2, 4, 6, 8, \ldots\}$.
- We can use set builder notation, for example, $\{x : x \text{ is a multiple of } 2\}$

Universal set

Sometimes it is necessary to classify a set as belonging to a larger or more inclusive set. For example, the set of capital cities in the Caribbean islands is an example of a universal set when considering the capital cities of countries in the Caribbean. If the entire world were considered then the set of the capital cities of all the countries in the world is also a Universal set for the set of capital cities in the Caribbean region.

Universal sets are not unique for a given set but in all cases, members of a Universal set must contain all the members of the set we are making reference to.

The Universal set, usually represented by U or Є, Sometimes, we may be asked to construct a universal set for a given set.

Set	Universal set
A={rectangles, squares, trapezoids, kites}	U={Quadrilaterals}
D ={2, 3, 5, 7,11}	U={Prime Numbers}
$E = \{\propto, \beta, \theta\}$	U={Letters of the Greek Alphabet}

Subset

There are many situations in which there are smaller sets within a larger set. For example, if E is the set of letters of the English alphabet, then the set E contains the set of vowels, $V = \{a, e, i, o, u\}$. We note that all the members of the set V are members of the set, E. Therefore, V is a part of E or V and is said to be a **subset** of E.

If A is the set of numbers from 1 to 20, then set A has some odd numbers, some even numbers, some multiples of 5 and other groups of numbers with a common property. All of these, considered separately, are sets themselves and form part of the set A. Since each is a part of A then each is called a **subset** of the set A.

Notation for Subset

If all the members of A are members of B, then A is contained in B, A is a part of B or A is a subset of B.
This is written as $A \subset B$.
The phrase 'is a subset' is symbolised as $\subset$.
The symbol $\not\subset$ is the symbol for 'is not a subset of'.

Some examples of subsets of a set are shown below.

Set	Subset
Q ={letters of the English alphabet}	$P = \{a, b, c, d\}, P \subset Q$
$U = \{\text{Integers}\}$	$S = \{2,4,6 \ldots\},\ S \subset U$ $T = \{5, 10,15, \ldots\}, T \subset U$ $R = \{-1, -2, -3, \ldots\}\ R \subset U$
G ={Names of girls that begin with A}	$N = \{\text{Anah, Ann, Avril}\}$ $N \subset G$

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The number of subsets of a set with n elements

When listing all the subsets of a set, we must note the following
- the empty set is a subset of every set
- a set is a subset of itself

If the number of elements in a subset is less than the number of elements in the set, then we say the subset is a **proper subset** of the set.

Using the above principle, we can derive a rule to calculate the number of subsets in a set with n elements. Starting with a set with one element, we list the subsets as shown below.

Set	Subsets
X ={a}	{a} { }
Y ={a, b}	{a} {b} {a, b} {b, c} { }
Z ={a, b, c}	{a} {b} {c} {a, b} {b, c} {a, c} {a, b, c} { }
P={a, b, c, d}	{a} {b} {c} {d} {a, b} {a, c} {a, d} {b, c} {b, d} {c, d} {a, b, c} {a, b, d} {a, c, d} {b, c, d} {a, b, c, d} { }

We now count the subsets and tabulate the above results in the table below. This allows us to predict the rule for n elements in a set.

Set	Number of elements	Number of subsets
$X=\{a\}$	1	$2=2^1$
$Y=\{a, b\}$	2	$4=2^2$
$Z=\{a, b, c\}$	3	$8=2^3$
$P=\{a, b, c, d\}$	4	$16=2^4$
If a set R has n elements	n	2^n

The number of subsets of a set with n elements
If a set has n elements then it has 2^n subsets.

Complement of a set

Sometimes we may be interested in the members that remain in a set when we remove a subset. For example, we can separate the letters of the alphabet, E into two subsets, the set of vowels, say V and the set of consonants, say C. If we remove the set of vowels, V, then the set of consonants remain and we can say that the complement of the set of vowels, V is the set of consonants, C. This is written as, $V'=C$.

The complement of a set is the set of all the elements that belong to the Universal set, except the members of the set itself. If the set is denoted by A, the complement of A is denoted by A' or A^c .

We may also speak of the complement of a set in relation to another set that is not necessarily a Universal set. The relative **complement** of **sets** A and B denoted A – B is the **set** of all elements in A that are not in B. Some examples are shown below.

If $U=\{2,4,6,8,10\}$ and $A=\{2,6\}$
then $A'=\{4,8,10\}$

If $P=\{a,b,c,d,e,f,g,h\}$ and $Q=\{a,b,c\}$
then $Q'=\{d,e,f,g,h\}$

Equivalent sets

We have met the term 'equivalent' in mathematics, before when we studied fractions. We saw that a pair of equivalent fractions represents the same quantity, but they comprised different numbers. When we use the term 'equivalent' in relation to two sets, the meaning is quite similar. Equivalent sets have the same cardinality, but they comprise different elements.

Two sets are equivalent if they have the same number of elements.
If set A and set B are equivalent, then n(A) = n(B).

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$X=\{4, 2, 7, 3, 1\}$ $Y=\{a, b, c, d, e\}$ $n(X)=n(Y)=5$	X and Y are equivalent sets. They have the same number of elements but the elements are different.
$P=\begin{Bmatrix}\text{Mars, Saturn,}\\ \text{Jupiter}\end{Bmatrix}$ $Q=\begin{Bmatrix}\text{Jupiter, ,Earth,}\\ \text{Neptune}\end{Bmatrix}$ $n(P)=n(Q)=3$	P and Q are equivalent sets. They have the same number of elements but the elements are different.

Equal sets

For two sets to be equal, they must have the same number of elements and also the same elements.

Two sets, A and B are equal only if
1. All the elements of A are also elements of B.
   AND
2. All the elements of B are also elements of A.

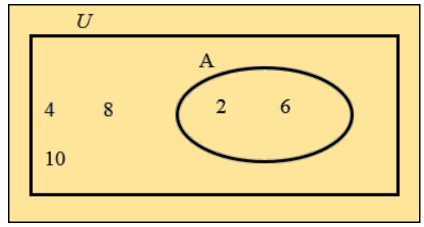
Equal sets

$X=\{4,5,6\}$ $Y=\{6,4,5\}$ $X=Y$	X and Y are equal sets. They have the same number of elements and the elements are the same.
$P=\{a, m, n, t\}$ $Q=\{m, t, a, n\}$ $P=Q$	X and Y are equal sets. They have the same number of elements and the elements are the same.

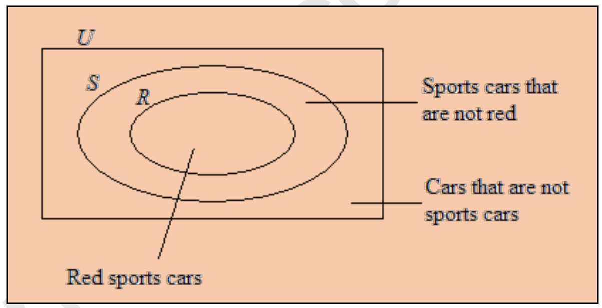
Venn diagrams

Diagrams used to represent sets are called Venn diagrams. In drawing Venn Diagrams, any enclosed shape is used to represent a set. The universal set is usually represented by a rectangle. Venn diagrams show the relationships among sets and one has to observe the relationships between them before drawing the sets.

If $U=\{2, 4, 6, 8, 10\}$ and $A=\{2, 6\}$, we can represent this information by drawing a rectangle for the universal set, U, and a circle (or oval) for the set A. The shape representing A is drawn inside of U because all the members of A belong to U. Next, we insert the elements to complete the diagram, ensuring that no element is listed twice.



If U = {Cars}, S = {Sports Cars}and R = {Red Sports Cars} then R is a subset of S. The Venn Diagram must show the relationship between R and S when $R\subset S$.
The Venn Diagram must show two sets, R and S, in a universal set with R drawn inside of S since R is a subset of S.



In order to continue our study of Venn Diagrams, it is necessary to become familiar with two basic operations on sets.

Union of two sets

We may combine two sets to form another set, called the union of the sets. Let us assume that in a class, there is a set of students who study Spanish (S) and a set of students who study Geography (G). In this class, we shall probably find some students who study Spanish only, others who study Geography only and there may even be some who study both Spanish and Geography. This combined group of students is called the union of the sets S and G. For example, if

$$S=\{a,b,c,d,e,f,g,h\}$$
$$G=\{g,h,i,j,k,l\}$$

then $S\cup G=\{a,b,c,d,e,f,g,h,i,j,k,l\}$

To list the members of the union, written as $S\cup G$ we will include all members of S and all members of G, making sure we do not list any member twice.

The union of two sets, P and Q is the set of all the elements when both sets are joined or combined. The union of P and Q is denoted by $P\cup Q$.
If $x\in P\cup Q$, then $x\in P$ **or** $x\in Q$.

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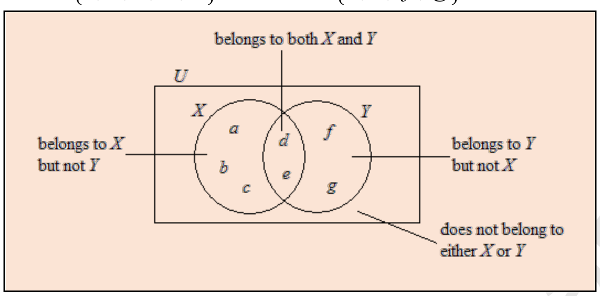
Intersection of two sets

As illustrated in the example above, sometimes when we combine two sets, we notice that there are elements common to both. Observe that S and G both contain the set {g, h}. This means that two children in the class study both Spanish and Geography. The set which contains all the elements that are common to both sets is called the intersection of the sets.

> If A and B are two sets, then the set of elements that are common to both sets is called the intersection of the sets and is denoted by $A \cap B$.
> If $x \in A \cap B$, then $x \in A$ **and** $x \in B$.

When intersecting sets are represented on a Venn diagram, their regions must overlap so as to show the common elements.
This is shown in a Venn diagram below by the overlapping of the circles representing X and Y.

$X = \{a, b, c, d, e\}$ $\qquad$ $Y = \{d, e, f, g\}$



Disjoint sets

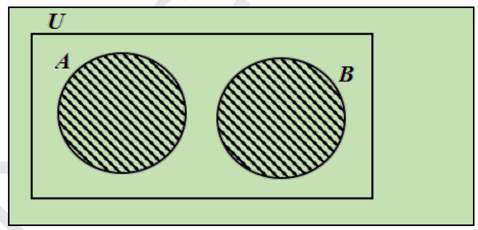
The set of odd numbers and the set of even numbers are disjoint sets as they have no members in common.

> If A and B are two sets and they have no elements in common, they are called disjoint sets. Their intersection is empty, so $A \cap B = \{\ \}$

Some more examples of disjoint sets are shown below.

1. The set of vowels and the set of consonants.
2. The set of 12-year-olds in a class and the set of 13-year-olds in the same class are disjoint sets.
3. The set of prime numbers and the set of multiples of 10 are disjoint sets.

When disjoint sets are represented on a Venn diagram, they must be drawn separately. The sets must not overlap, as shown below to illustrate that $A \cap B = \{\ \}$.



Number of elements in the union of two sets

There is a fundamental law of set theory which states that if A and B are two sets, then
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

This law holds regardless of the relationship between the two sets. as we shall illustrate below.

Relationship between A and B		
A and B are disjoint	$A = \{f, g, h\}$ $B = \{k, l\}$ $n(A) = 3 \quad n(B) = 2$ $A \cap B = \{\ \}, \quad n(A \cap B) = 0$	$A \cup B = \{f, g, h, k, l\}$ $n(A \cup B) = 5$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ $= 3 + 2 + 0$ $= 5$
A and B intersect	$A = \{a, b, c, d, e\}$ $B = \{d, e, f, g\}$ $n(A) = 5 \quad n(B) = 4$ $A \cap B = \{d, e\} \quad n(A \cap B) = 2$	$A \cup B = \{a, b, c, d, e, f, g\}$ $n(A \cup B) = 7$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ $= 5 + 4 - 2$ $= 7$
B is a subset of A	$A = \{p, q, r, s, t\}$ $B = \{s, t\}$ $n(A) = 5 \quad n(B) = 2$ $A \cap B = \{s, t\} \quad n(A \cap B) = 2$	$A \cup B = \{p, q, r, s, t\}$ $n(A \cup B) = 5$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ $= 5 + 2 - 2$ $= 5$

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Solving problems on sets –Venn Diagrams

Example 1

Draw a Venn Diagram to show the sets, P and Q, given

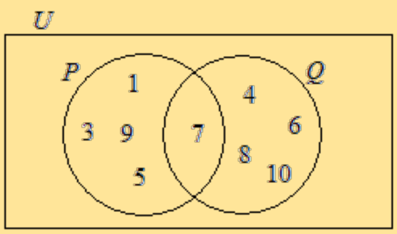
$$P = \{1, 3, 5, 7, 9\} \qquad Q = \{4, 6, 7, 8, 10\}$$

Hence, list the sets $P \cap Q$ and $P \cup Q$.

Solution

$P \cap Q = \{7\}$

$P \cup Q = \{1, 3, 4, 5, 6, 7, 8, 9, 10\}$



Example 2

Draw a Venn Diagram to show the sets S and T, where

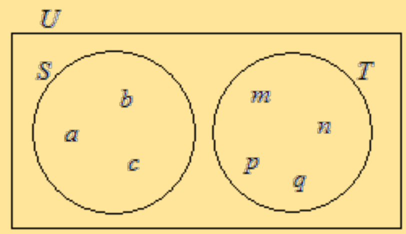
$S = \{a, b, c\} \qquad T = \{m, n, p, q\}$.

Hence, list the sets $S \cap T$ and $S \cup T$.

Solution

$S \cap T = \{\ \}$

$S \cup T = \{a, b, c, m, n, p, q\}$

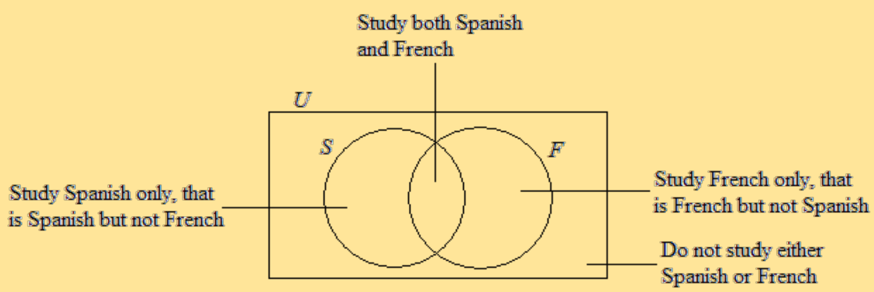


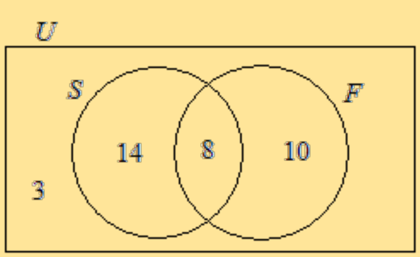
Example 3

In a class of 35 students, 18 study French and 22 study Spanish. If 8 study both languages. Find the number of students who do not study either French or Spanish.

Solution

The following Venn Diagram will be used to represent the information given.
U – Represents the entire class with total 35.
S – Students who study Spanish
F – Students who study French





$n(S \cap F) = 8$ (given)

The number of students who study Spanish only $= 22 - 8 = 14$
The number of students who study French only $= 18 - 8 = 10$.
The number of students who do not study either French or Spanish
$= 35 - (14 + 8 + 10) = 3$

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Example 4

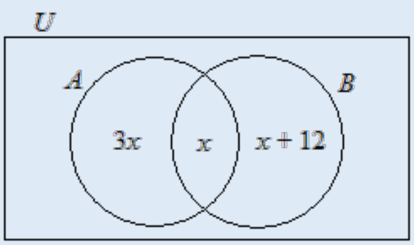
State the number of subsets in the set $P = \{m, n, o, p, q\}$

Solution

$n(P) = 5$, $\therefore$ The number of subsets of $P = 2^5 = 32$

Example 5

In the diagram below, A and B are two sets such that $n(A) = n(B)$ and $n(U) = 50$.



Determine

i. the value of x

ii. $n(A \cup B)'$

Solution

i. $n(A) = 3x + x = 4x$

$n(A) = x + (x+12) = 2x + 12$

Since $n(A) = n(B)$

$\therefore 4x = 2x + 12$

$2x = 12$

$x = 6$

ii. $n(A \cup B) = 3(6) + 6 + (6+12)$

$= 18 + 6 + 18$

$= 42$

$\therefore n(A \cup B)' = 50 - 42$

$= 8$

Example 6

Given that

$A = \{a, b, c, d, e\}$ and $B = \{x, y, z, a, c\}$

Determine

$(i) A \cap B$ $(ii) A \cup B$ $(iii) n(A \cup B)$

Solution

$(i) A \cap B = \{a, c\}$

$(ii) A \cup B = \{a, b, c, d, e, x, y, z\}$

$(iii) n(A \cup B) = 8$

Example 7

$U = \{1, 2, 3, 4, \ldots 12\}$

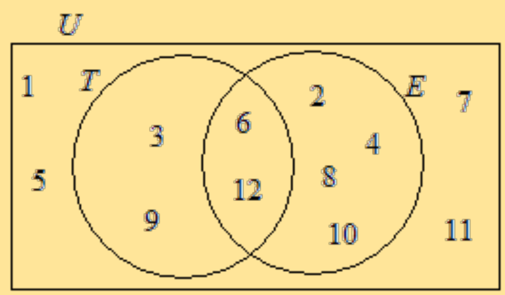
$T = \{\text{multiples of } 3\}$ and $E = \{\text{even numbers}\}$

(i) Draw a Venn diagram to represent the information given.

(ii) List the members of a) $T \cap E$ b) $T \cup E'$

Solution

(i)



(ii) a) $T \cap E = \{6, 12\}$

b) $T \cup E' = \{1, 5, 7, 11\}$

Example 8

A survey conducted on 40 tourists was done to find out who visited Anguilla and/or Barbuda.

28 visited Anguilla (A)

30 visited Barbuda (B)

$3x$ visited both Anguilla and Barbuda

x visited did not visit either Anguilla nor Barbuda

(i) Draw a Venn diagram given to represent the information given.

(ii) Write an equation in x to represent the information

(iii) Calculate the value of x.

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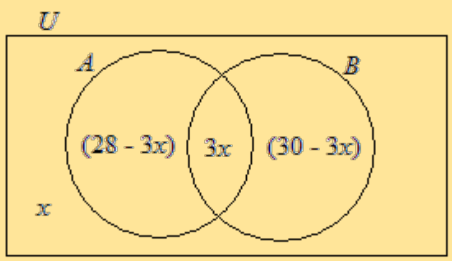
Solution

(i)



(ii) Total number of tourist

$= 28 - 3x + 3x + 30 - 3x + x$

$= 58 - 2x$

Since there were 40 tourists in all

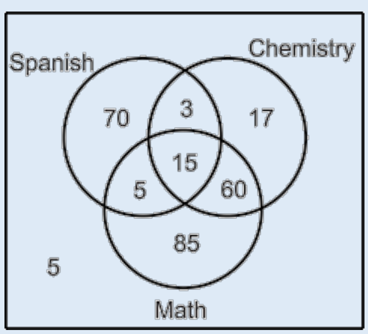
$58 - 2x = 40$

$2x = 19$

$x = 9$

Example 9

A survey was conducted in a school among a group of students. The Venn Diagram below shows the results of the survey.



(a) How many students took part in the survey?
(b) Determine the number of students who study
(i) all three subjects.
(ii) none of the three subjects.
(iii) Chemistry.
(iv) only Spanish.
(iv) any two of the three subjects.
(c) How many students do not study mathematics?

Solution

From the Venn Diagram, we deduce:

$(a)\ 5 + 85 + 60 + 15 + 5 + 70 + 3 + 17 = 260$

$(b)(i) 15 \quad (ii)\ 5 \quad (iii) 3 + 15 + 60 + 17 = 95$

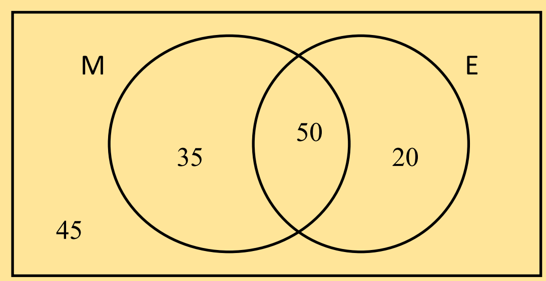
$(c)\ 70 + 3 + 17 + 5 = 95$

Example 10

150 Form 5 students were interviewed.
85 signed up for a Math class
70 signed up for an English class
50 signed up for both Math and English
Some did not sign up for either Maths or English.
(i) How many signed up only for a math class?
(ii) How many signed up only for an English class?
(iii) How many signed up for math or English?
(iv) How many did not sign up for either Math nor English?

Solution

We draw the Venn Diagram to include the number who signed up for both, 50 in the intersection.
(i) Since 85 signed up for Maths, 85 – 50 = 35 signed up for Maths only.
(ii) Since 70 signed up for English, 70 – 50 = 20 signed up for English only.



(iii) Subtracting the number in the union from the number interviewed gives 150 – (35+50+20) = 45
Hence 45 did not sign up for either Maths or English.

Example 11

100 were students interviewed
28 took PE
31 took Biology
42 took Science
9 took PE and Biology
10 took PE and Science
6 took Biology and Science
4 took all three subjects
(i) How many students took none of the three subjects?
(ii) How many students took PE, but not Biology or Science?
(ii) How many students took Biology and PE but not Science?

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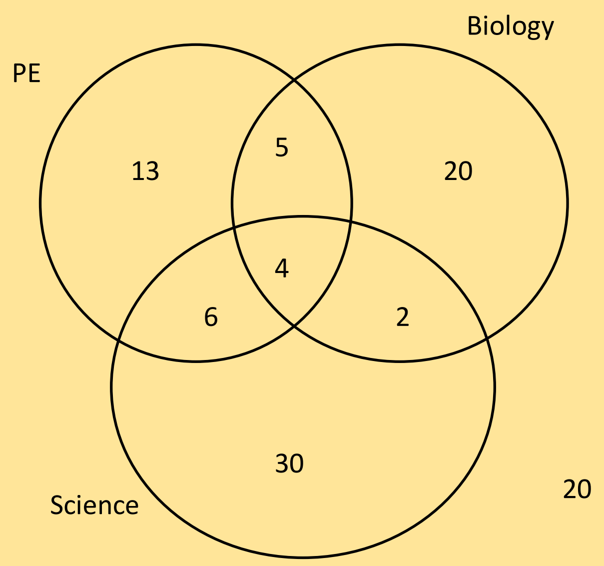
Solution



Using the principle of starting to fill the diagram from the intersection of the three sets first, we first insert the 4, then move outwards using the given information in a reverse order to how it was presented.

Next, we fill the regions with the intersection of two sets
6 took Biology and Science, so 6 – 4 = 2, hence, 2 students took Biology and Science only.
10 took PE and Science, so 10 – 4 = 6, hence 6 students took PE and Science only.
9 took PE and Biology, so 9 – 4 = 5, hence 5 students took PE and Biology only.

Next, we fill the remaining regions within each circle.
42 took Science, so 42 – (6+4+2) =30 took Science only.
31 took Biology, so 31 – (5+4+2) =20 took Biology only.
28 took PE, so 28 – (6+4+5) =13 took Science only.

(i) The number of students who took no subjects
= 100 – (13+5+4+6+20+2+30) = 20

(ii) The number of students who took PE, but not Biology or Science = 13

(iii) The number of students who took Biology and PE but not Science
=13 + 5 + 20 = 38

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