FAS-PASS Maths

CSEC MATHEMATICS JUNE 2023 PAPER 2

SECTION I

Answer ALL questions.

All working MUST be clearly shown.

1. (a) Find the EXACT value of

$$\frac{5}{6}+\frac{2}{3}-\frac{12}{35}\times\frac{7}{9}$$

SOLUTION:

Required to calculate: $\frac{5}{6}+\frac{2}{3}-\frac{12}{35}\times\frac{7}{9}$.

Calculation:

The question is better written as $\frac{5}{6}+\frac{2}{3}-\left(\frac{12}{35}\times\frac{7}{9}\right)$

First, we work the part within the brackets: $\frac{{}^{4}\cancel{12}}{{}_{5}\cancel{35}}\times\frac{\cancel{7}^{1}}{\cancel{9}_{3}}=\frac{4}{15}$

The question now becomes:

$$\begin{aligned}\frac{5}{6}+\frac{2}{3}-\frac{12}{35}\times\frac{7}{9}&=\frac{5}{6}+\frac{2}{3}-\frac{4}{15}\\&=\frac{5(5)+2(10)-4(2)}{30}\\&=\frac{25+20-8}{30}\\&=\frac{37}{30}\text{ (in exact form)}\end{aligned}$$

(b) (i) Calculate the value of $\sqrt{1-(\cos 37°)^2}$ correct to 3 decimal places.

SOLUTION:

Required to calculate: $\sqrt{1-(\cos 37°)^2}$

Calculation:

We use the calculator to obtain

$$\begin{aligned}\sqrt{1-(\cos 37°)^2}&=\sqrt{1-(0.79863)^2}\\&=\sqrt{1-0.63781}\\&=\sqrt{0.36218}\\&=0.601\underline{\underline{8}}1\\&\approx 0.602\text{ (correct to 3 decimal places)}\end{aligned}$$

By Fayad W. Ali and Shereen A. Khan.

(ii) Write 0.00527 in standard form.

SOLUTION:
Required to write: 0.00527 in standard form.
Solution:
0 . 0 0 5 2 7

We shift the decimal point 3 places to the right to get.

$\therefore 0.00527 = 5.27 \times 10^{-3}$ in standard form OR scientific notation.

(c) Haresh works at a call centre for 35 hours each week. He is paid an hourly rate of $11.20.

(i) Calculate the amount of money Haresh earns in a **four-week month**.

SOLUTION:

Data: Haresh works at a call centre for 35 hours each week and is paid $11.20 per hour.

Required to calculate: The amount of money that Haresh earns in a four-week month.

Calculation:
Number of hours of work $= 35$
Hourly rate $= \$11.20$

Weekly earnings $= \$11.20 \times 35$

Earnings for a four-week month $= \$11.20 \times 35 \times 4$
$= \$1568.00$

(ii) In a certain week, Haresh works 8 hours of overtime. Overtime hours are paid at $1\frac{1}{2}$ times the usual rate of $11.20 per hour.

Find the TOTAL amount of money Haresh is paid for **that week**.

SOLUTION:
Data: Overtime work is paid for at $1\frac{1}{2}$ times the basic rate of $11.20 per hour and Haresh works 8 hours overtime.

Required to calculate: The total amount of money Haresh earned that week.

Calculation:
Weekly earnings at basic rate $= \$11.20 \times 35$

$$= \$392.00$$

Hourly overtime rate $= \$11.20 \times 1\frac{1}{2}$

$= \$16.80$

Earnings for 8 hours overtime $= \$16.80 \times 8$

$= \$134.40$

Hence, weekly earnings for the week inclusive of 8 hours of overtime

= Basic weekly wage + Overtime earnings $= \$392.00 + \$134.40$

$= \$526.40$

2. (a) Simplify $\frac{4}{5x} \times \frac{15x}{16}$.

SOLUTION:

Required to simplify: $\frac{4}{5x} \times \frac{15x}{16}$

Solution:

$$\frac{{}^{1}\cancel{4}}{{}_{1}\cancel{5}\,\cancel{x}} \times \frac{{}^{3}\cancel{15}\,\cancel{x}}{\cancel{16}_{4}} = \frac{3}{4}$$

Alternative Method:

$$\frac{4}{5x} \times \frac{15x}{16} = \frac{60\cancel{x}}{80\cancel{x}}$$

$$= \frac{{}^{3}\cancel{60}}{{}_{4}\cancel{80}}$$

$$= \frac{3}{4}$$

(b) Solve the inequality $12 - 4m \leq 5 - 8m$.

SOLUTION:

Required to solve: $12 - 4m \leq 5 - 8m$

Solution:

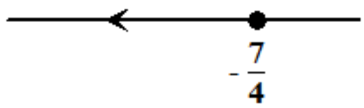
$$12 - 4m \leq 5 - 8m$$

$$-4m + 8m \leq 5 - 12$$

$$4m \leq -7$$

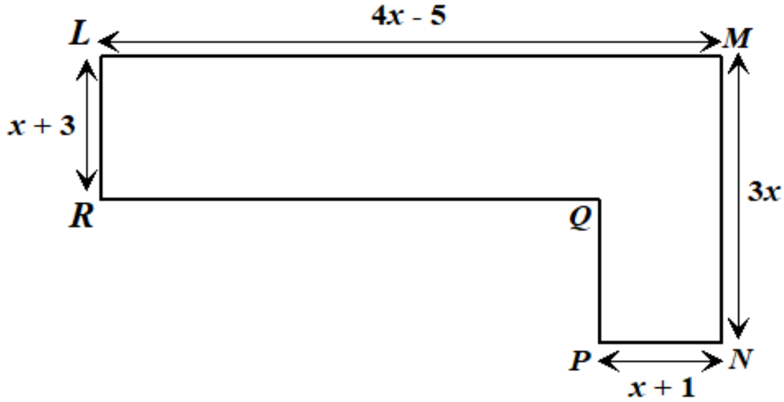
$(\div 4)$

$m \leq -\frac{7}{4}$ or $\left\{ m : m \leq -\frac{7}{4} \right\}$ or



By Fayad W. Ali and Shereen A. Khan.

(c) The diagram below shows a compound shape, *LMNPQR*, made from two rectangles. The lengths in the diagram, which are written terms of x, are in centimetres.



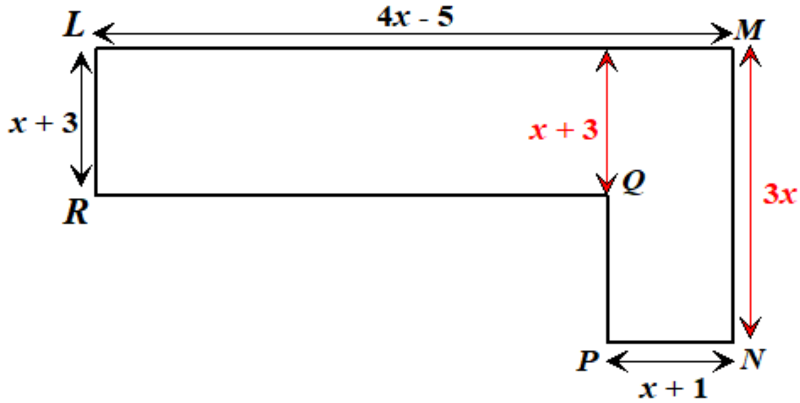
(i) Find an expression, in terms of x, for the length

a) PQ

SOLUTION:
Data: Diagram showing compound shape *LMNPQR*, with the lengths of four sides, in terms of x.

Required to find: The length PQ, in terms of x

Solution:



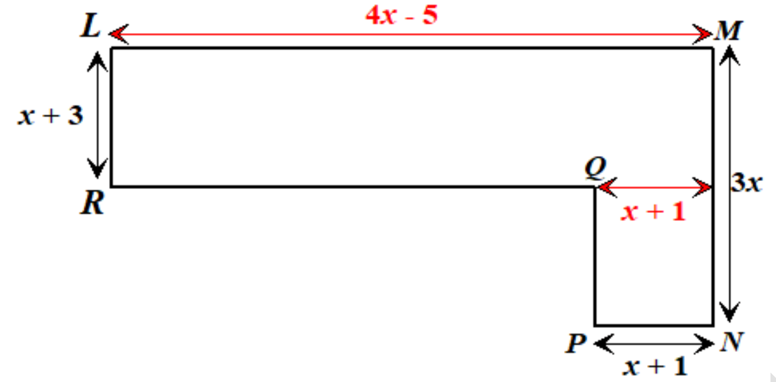
$$PQ = \{3x - (x+3)\} \text{ cm}$$
$$= (2x-3) \text{ cm}$$

b) RQ.

SOLUTION:

Required to find: The length of RQ, in terms of x.

Solution:



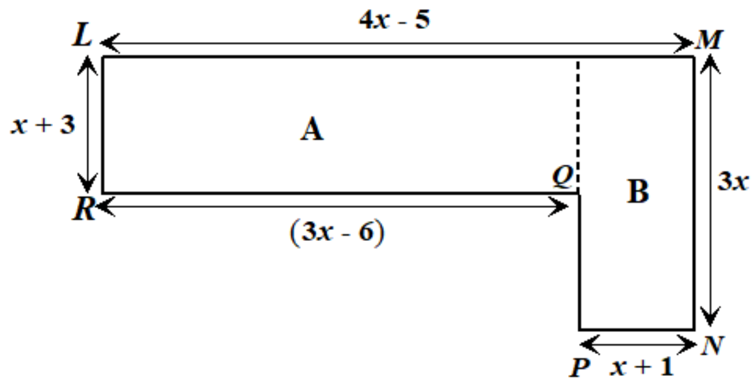
$$RQ = (4x-5)-(x+1)$$
$$= (3x-6) \text{ cm}$$

(ii) Given that the TOTAL area of the shape is 414 cm^2, show that $x^2+x-72=0$.

SOLUTION:
Data: The total area of the shape is 414 cm^2.

Required to show: $x^2+x-72=0$

Proof:
The compound shape is divided into simple shapes marked as regions A and B in the diagram



Area of A $=(x+3)(3x-6)$
Area of B $=(3x)(x+1)$

Area of the entire shape $=(x+3)(3x-6)+(3x)(x+1)$
$$=3x^2+9x-6x-18+3x^2+3x$$
$$=(6x^2+6x-18) \text{ cm}^2$$

By Fayad W. Ali and Shereen A. Khan.

Hence, $6x^2+6x-18=414$ (data)

$6x^2+6x-434=0$

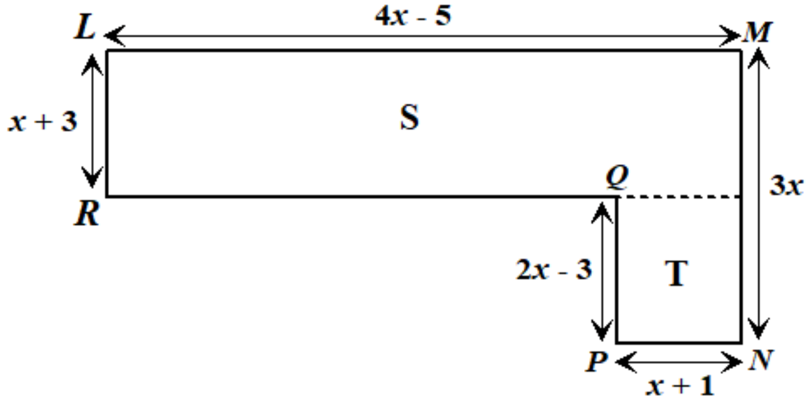
$(\div 6)$

$x^2+x-72=0$

Q.E.D.

Alternative Method:

Alternatively, the figure *LMNPQR* could be divided into simpler shapes S and T as shown:



Area of shape $=\text{Area of } S+\text{Area of } T$

$=(4x-5)(x+3)+(2x-3)(x+1)$

$=4x^2+7x-15+2x^2-x-3$

$=6x^2+6x-18$

Hence, $6x^2+6x-18=414$ (data)

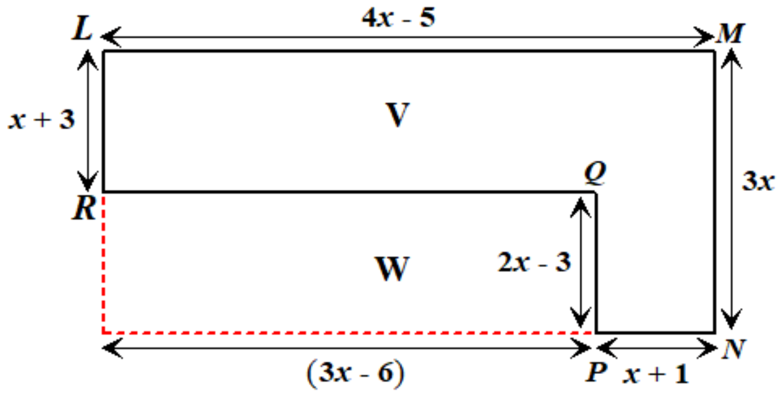
$6x^2+6x-434=0$

$(\div 6)$

$x^2+x-72=0$

Q.E.D.

Alternative Method:



We could complete the given shape by adding the shape W to it so as to form one rectangle composed of V and W.

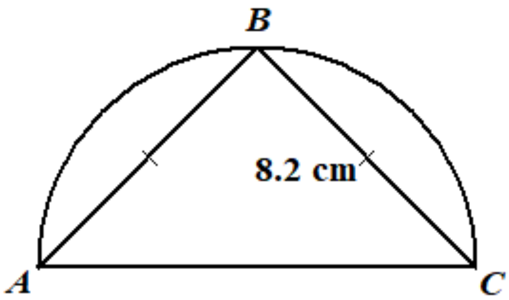
Area of the given shape $= \text{Area of } (V+W) - \text{Area of } W$

$$= (4x-5)(3x) - (3x-6)(2x-3)$$
$$= 12x^2 - 15x - 6x^2 + 21x - 18$$
$$= 6x^2 + 6x - 18$$

Hence, $6x^2 + 6x - 18 = 414$ (data)

$$6x^2 + 6x - 434 = 0$$
$$(\div 6)$$
$$x^2 + x - 72 = 0$$

3. (a) The diagram below shows a semi-circle with diameter *AC*. *B* is a point on the circumference and $AB = BC = 8.2$ cm.



(i) State the geometrical name of the line *AB*.

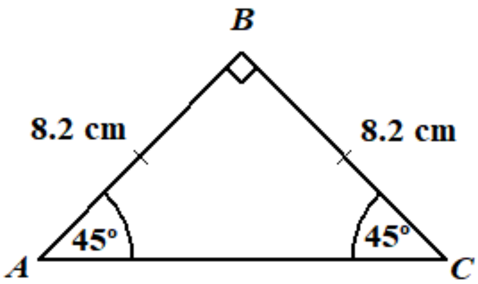
SOLUTION:
Data: Diagram showing a semi-circle with *AC* and $AB = BC = 8.2$ cm, where *B* is a point on the circumference.

Required to state: The geometrical name of the line *AB*

Solution:
The line *AB* joins two points on the circle, namely *A* and *B*. Hence, *AB* is a chord.

(ii) Find the value of the radius of the circle.

SOLUTION:
Required to find: The value of the radius of the circle.
Solution:



$A\hat{B}C = 90°$
(The angle in a semi-circle is a right angle.)

Hence, $B\hat{A}C = B\hat{C}A = \dfrac{180° - 90°}{2} = 45°$
(the sum of all the angles in a triangle = 180^0 **AND** the two base angles of an isosceles triangle are equal)

By Pythagoras' Theorem:

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = (8.2)^2 + (8.2)^2$$
$$= 67.24 + 67.24$$
$$= 134.48$$
$$\therefore AC = \sqrt{134.48}$$
$$= 11.596 \text{ cm}$$

AC is a diameter $= \text{radius} \times 2$

Hence, the length of the radius $= \dfrac{11.596}{2}$
$= 5.80\,\text{cm}$

Alternative Method:
Consider the right-angled triangle ABC

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$
$$\cos 45° = \frac{8.2}{AC}$$
$$AC = \frac{8.2}{\cos 45°}$$
$$\text{Radius } = \frac{1}{2}\left(\frac{8.2}{\cos 45°}\right)$$
$$= \frac{4.1}{\cos 45°}$$
$$\approx 5.80 \text{ cm}$$

Alternative Method:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 45° = \frac{8.2}{AC}$$

$$AC = \frac{8.2}{\sin 45°}$$

$$\text{Radius } = \frac{1}{2}\left(\frac{8.2}{\sin 45°}\right)$$

$$= \frac{4.1}{\sin 45°}$$

$$\approx 5.80 \text{ cm}$$

(b) Each interior angle of a regular polygon is 160°. Calculate the number of sides of the polygon.

SOLUTION:
Data: A regular polygon has interior angles of 160° each.

Required to calculate: The number of sides of the polygon.

Calculation:
Let the number of sides of the polygon be n.
The sum of the interior angles $=(2n-4)\times 90°$
If each angle is 160°, then the sum of all n interior angles is $160°\times n$.

Hence, $(2n-4)\times 90° = 160°\times n$

$$180n - 360 = 160n$$

$$20n = 360$$

$$n = \frac{360}{20}$$

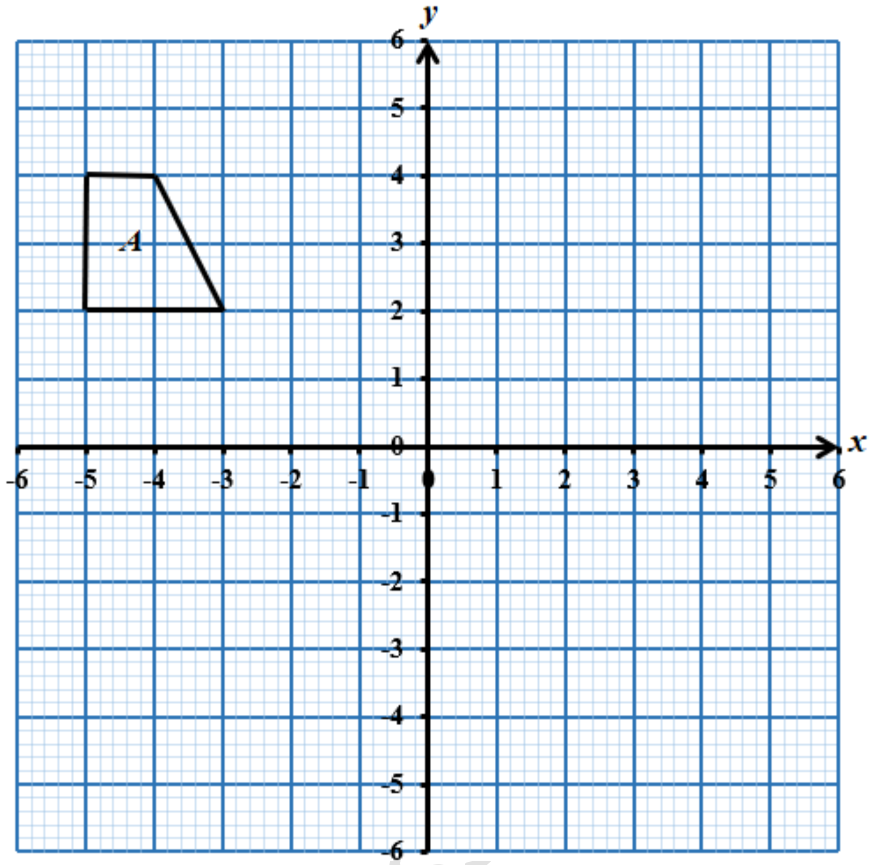
$$n = 18$$

Hence, the polygon has 18 sides and is called an octadecagon or an octakaidecagon or an 18-gon.

Alternative method:

If each interior angle is 160^0 then each exterior angle is $180°-160°=20°$ (the interior and the exterior angle lie on a straight line and total 180^0)
The sum of all the exterior angles of any polygon is always 360°.
Hence, the number of exterior angles is $\frac{360°}{20}=18$.
So, the polygon has 18 sides.
(A polygon of n sides has n interior angles and n exterior angles)

(c) The diagram below shows a trapezium, A, drawn on a square grid.



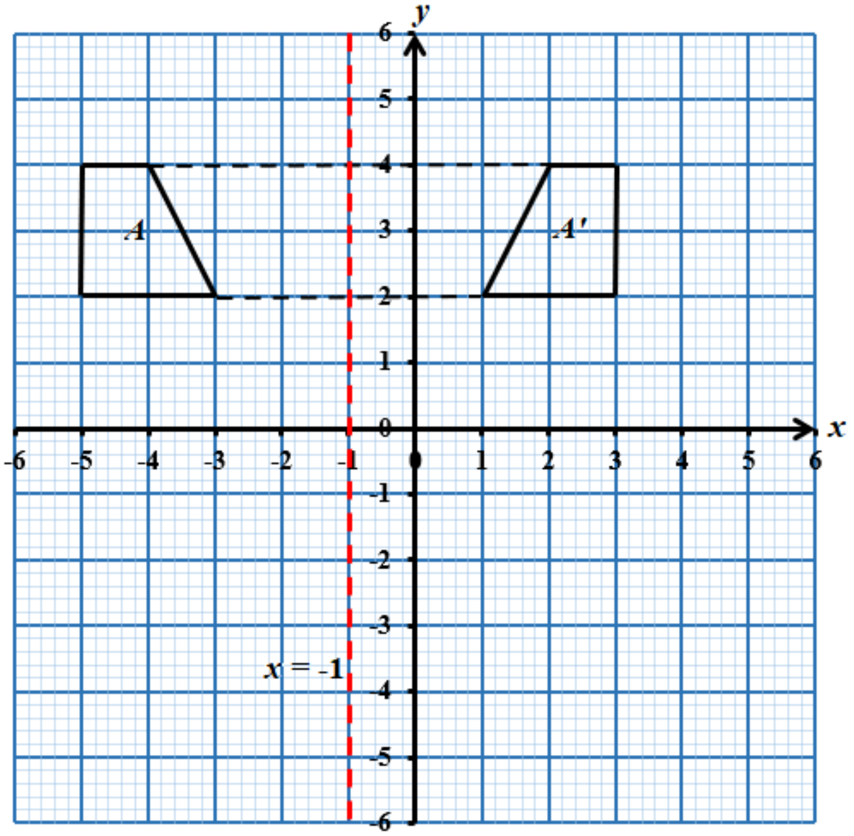
On the diagram above, draw the image of A after it undergoes a

(i) reflection in the line $x = -1$ and label this image A'.

SOLUTION:
Data: Diagram showing a trapezium, A, drawn on a square grid.

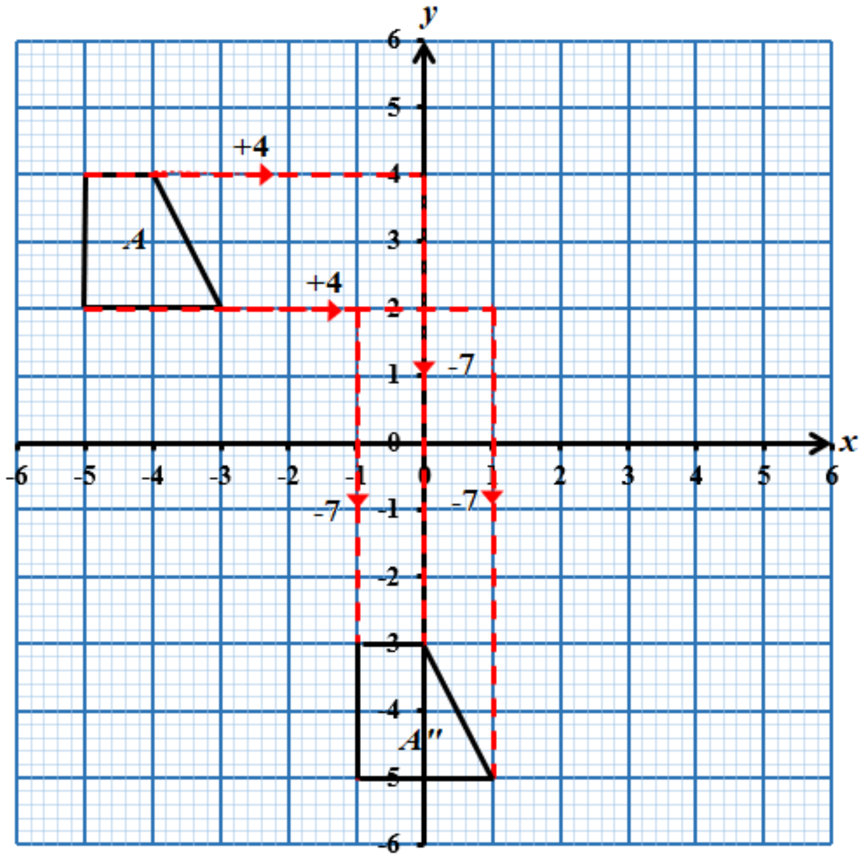
Required to draw: The image, A', of A after it undergoes a reflection in the line $x = -1$.

Solution:
The image A' lies at the same perpendicular distance on the opposite side of the line $x = -1$ as the object A.
We take each vertex of A and obtain the corresponding vertices of A'. This is easily done by counting blocks or squares on either side of the reflection line.
We join the vertices of the image of the object A to obtain the image A'.



(ii) translation with vector $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$ and label this image A''.

SOLUTION:
Required to draw: The image A'', of A after it undergoes a translation with vector $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$.
Solution:
$T = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$ indicates a shift of 4 units horizontally to the right followed by 7 units vertically down. This shift is done on each vertex of A to obtain the vertices of the image A''. We join the vertices after translation to obtain A''.



4. Consider the following functions:

$$f(x)=\frac{3}{x+2},\ g(x)=4x-5 \text{ and } h(x)=x^2+1$$

(a) **(i)** For what value of x is $f(x)$ undefined?

SOLUTION:

Data: $f(x)=\frac{3}{x+2},\ g(x)=4x-5,\ h(x)=x^2+1$

Required to find: The value of x for which $f(x)$ is undefined.

Solution:

$f(x)=\frac{3}{x+2}$

The function is undefined when $x+2=0$

$$\text{As } x\to -2, f(-2)\to \frac{3}{0}$$

which is undefined and expressed mathematically as ∞.

Hence, $f(x)$ is said to be undefined OR not defined OR discontinuous OR not continuous OR does not exist OR is not valid OR invalid at $x=-2$.

(ii) Find the value of

a) $g\left(\frac{1}{4}\right)$

SOLUTION:

Required to find: $g\left(\frac{1}{4}\right)$

Solution:

$$g(x) = 4x - 5$$

$$\therefore g\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right) - 5$$

$$= 1 - 5$$

$$= -4$$

b) $h(-3)$

SOLUTION:

Required to find: $h(-3)$

Solution:

$$h(x) = x^2 + 1$$

$$\therefore h(-3) = (-3)^2 + 1$$

$$= 9 + 1$$

$$= 10$$

c) $ff(0)$

SOLUTION:

Required to find: $ff(0)$

Solution:

$ff(0)$ or $f^2(0)$ can be computed by first finding

$$f(0) = \frac{3}{0+2}$$

$$= \frac{3}{2}$$

So, $ff(0) = f(f(0))$

$$\therefore ff(0) = f\left(\frac{3}{2}\right)$$
$$= \frac{3}{\frac{3}{2}+2}$$
$$= \frac{3}{3\frac{1}{2}}$$
$$= \frac{3}{1} \times \frac{2}{7}$$
$$= \frac{6}{7}$$

Alternative Method:

$$ff(x) = \frac{3}{\frac{3}{x+2}+2}$$
$$ff(0) = \frac{3}{\frac{3}{0+2}+2}$$
$$= \frac{3}{3\frac{1}{2}}$$
$$= \frac{6}{7}$$

(b) Write an expression, in its **simplest form**, for $gh(x)$.

SOLUTION:
Required to write: $gh(x)$ in its simplest form.
Solution:
$g(x) = 4x - 5$
$h(x) = x^2 + 1$

$$\therefore gh(x) = 4\left[h(x)\right] - 5$$
$$= 4\left(x^2 + 1\right) - 5$$
$$= 4x^2 + 4 - 5$$
$$= 4x^2 - 1$$

(c) Find $g^{-1}(-2)$.

SOLUTION:
Required to find: $g^{-1}(-2)$
Solution:

$g(x)=4x-5$

Let $y=4x-5$

$\therefore y+5=4x$

$$x=\frac{y+5}{4}$$

Replace y by x to get:

$$g^{-1}(x)=\frac{x+5}{4}$$

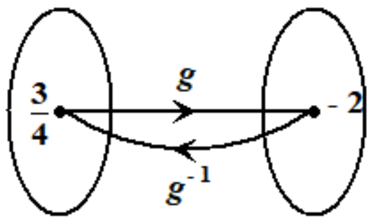
Hence, $g^{-1}(-2)=\dfrac{-2+5}{4}$

$$=\frac{3}{4}$$

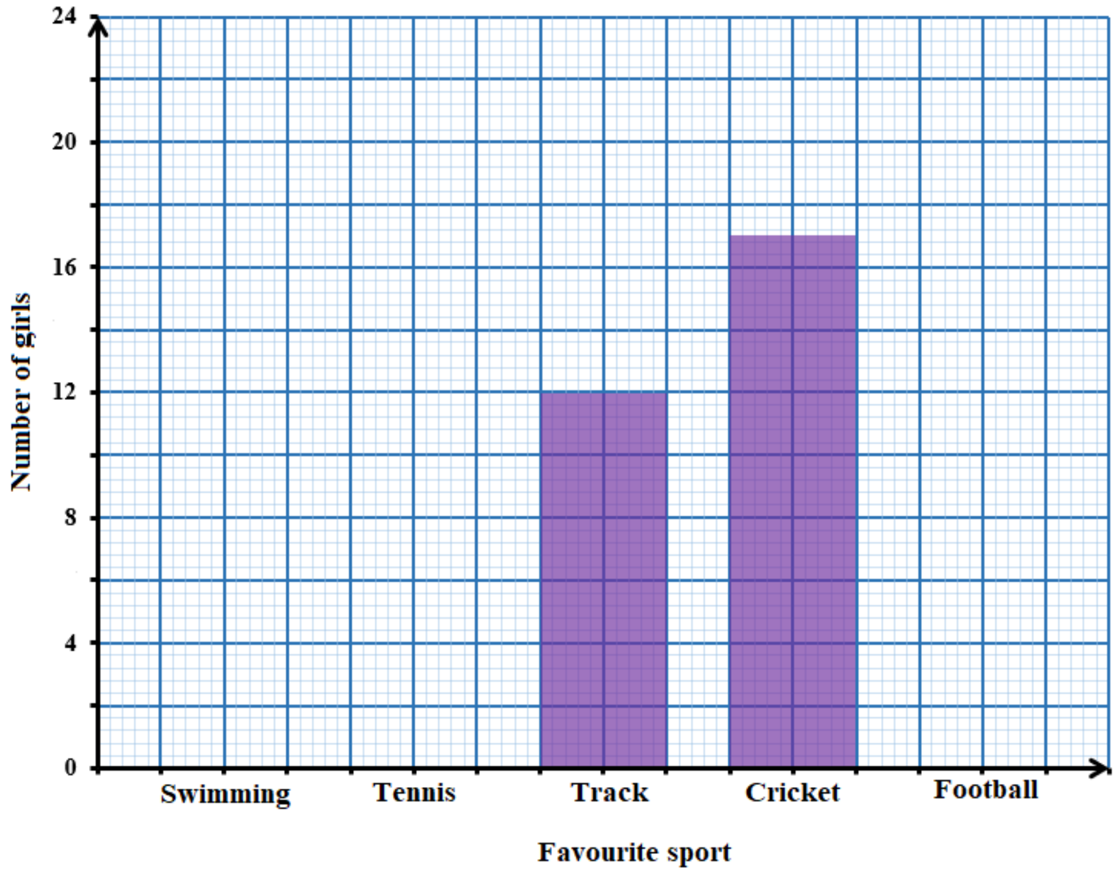
Alternative Method:

Let $g(x)=-2$

$$4x-5=-2$$
$$4x=3$$
$$x=\frac{3}{4}$$
$$\therefore g^{-1}(-2)=\frac{3}{4}$$



5. Each of 75 girls recorded the name of her favourite sport. The number of girls who chose track and cricket are shown on the bar chart below.



(a) How many **more** girls chose cricket than track as their favourite sport?

SOLUTION:
Data: Incomplete bar chart showing the number of girls out of a group of 75 who chose track and cricket as their favourite sport.

Required to state: How many more girls chose cricket than track as their favourite sport.
Solution:
Number of girls who chose cricket $=17$
Number of girls who chose track $=12$

Hence, $17-12=5$ more girls chose cricket than track as their favourite sport.

(b) Eleven girls recorded tennis as their favourite sport. For the remaining girls, the number who chose swimming compared to the number who chose football was in the ratio $2:3$.

Use this information to complete the bar chart above.

SOLUTION:

Data: 11 girls chose tennis as their favourite sport. Of the remaining girls, the number who chose swimming to the number who chose football was in the ratio $2:3$.

Required to complete: The bar chart given
Solution:
The number of girls who chose swimming and football $=75-(17+12+11)$
$=35$

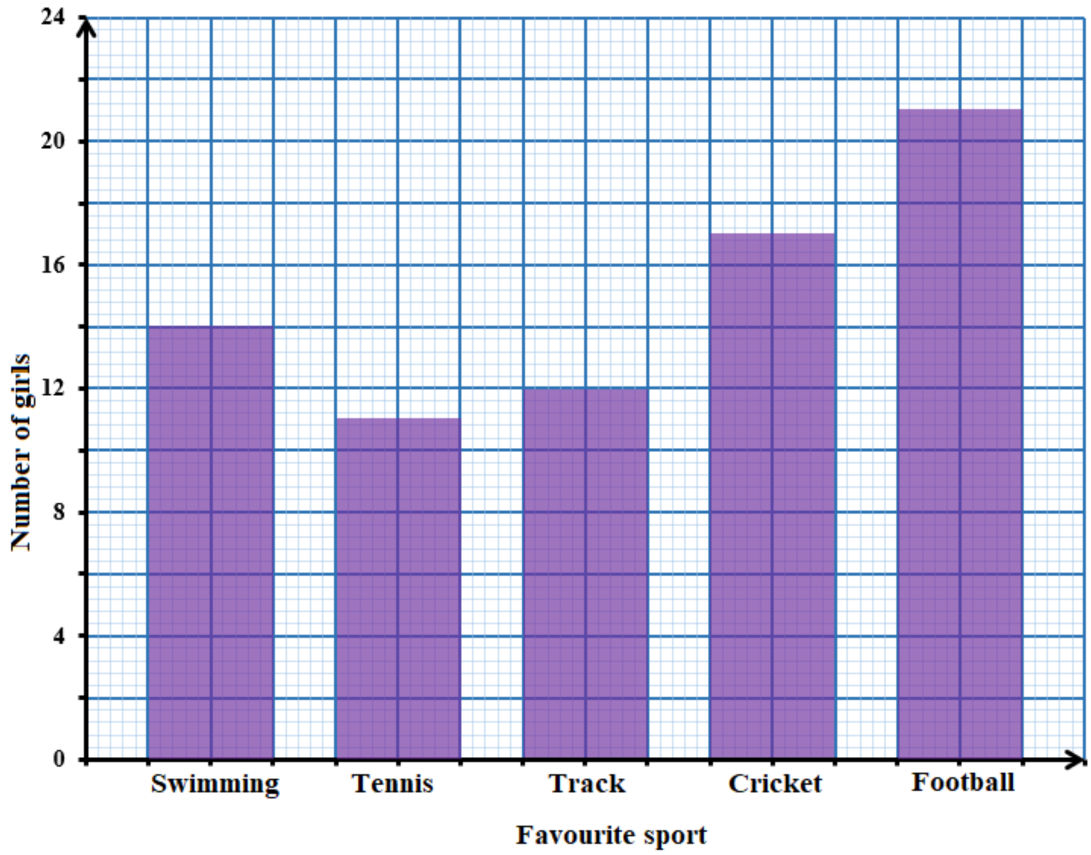
Ratio of the number of girls who chose swimming to the number who chose football $=2:3$

Hence, the number who chose swimming $=2x$ and the number who chose football $=3x$ where x is a positive integer

$$2x+3x=35$$
$$5x=35$$
$$x=7$$

$\therefore$ Number of girls who chose swimming $=2\times 7$
$=14$
$\therefore$ Number of girls who chose football $=3\times 7$
$=21$

Using these figures, we can complete the bar chart
The completed bar chart looks like:



By Fayad W. Ali and Shereen A. Khan.

(c) Determine the modal sport.

SOLUTION:
Required to determine: The modal sport
Solution:
The modal sport is the one that was most chosen. This is football, since more girls chose football than any other sport.

(d) One of the girls is selected at random. What is the probability that she chose NEITHER track NOR cricket as her favourite sport?
(It is better to say- "did NOT choose either Track or Cricket" as opposed to "chose neither track nor cricket")

SOLUTION:
Required to determine: The probability that a girl, chosen at random, did not choose either track or cricket as her favourite sport.

Solution:
Number of girls who did not choose either track or cricket
=Number who chose swimming, tennis or football
$= 75 - 29$
$= 46$
So, P(girl did not choose track or cricket)
$$= \frac{\text{Number of girls who did not choose track or cricket}}{\text{Total number of girls}}$$
$$= \frac{46}{75}$$

Alternative Method:
P(girl chooses track or cricket)
$$= \frac{\text{Number of girls who chose track or cricket}}{\text{Total number of girls}}$$
$$= \frac{12+17}{75}$$
$$= \frac{29}{75}$$
P(girl did not choose track or cricket) $= 1 - \frac{29}{75}$ (Law of total probability)
$$= \frac{46}{75}$$

(e) The information on the favourite sport of the 75 girls is to be shown in a pie chart. Calculate the sector angle for football.

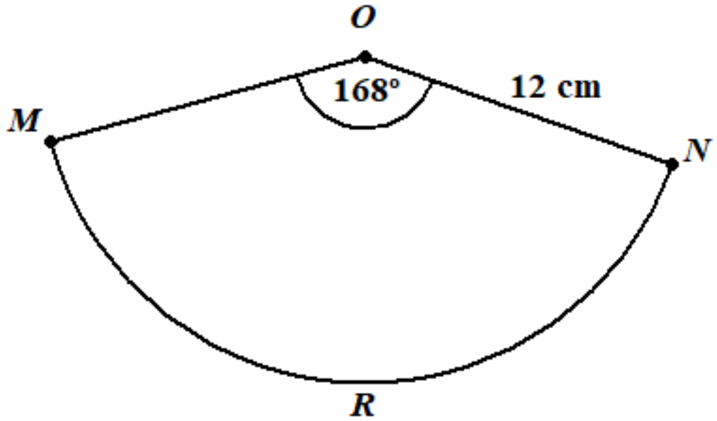
SOLUTION:
Data: The information of the favourite sport of 75 girls is to be represented on a pie chart.

Required to calculate: The angle of the sector representing football.
Calculation:

$$\text{Angle of the sector for football} = \frac{\text{Number of girls who chose football}}{\text{Total number of girls}} \times 360^\circ$$
$$= \frac{21}{75} \times 360^\circ$$
$$= 100.8^\circ$$

6. [In this question, take $\pi = \frac{22}{7}$ and the volume, V, of a cone with radius r and height h as $V = \frac{1}{3}\pi r^2 h$.]

The diagram below shows a sector, *OMRN*, of a circle with centre *O*, radius 12 cm and sector angle 168°, which was formed using a thin sheet of metal.



(a) Calculate the perimeter of the sector above, made from the thin sheet of metal.

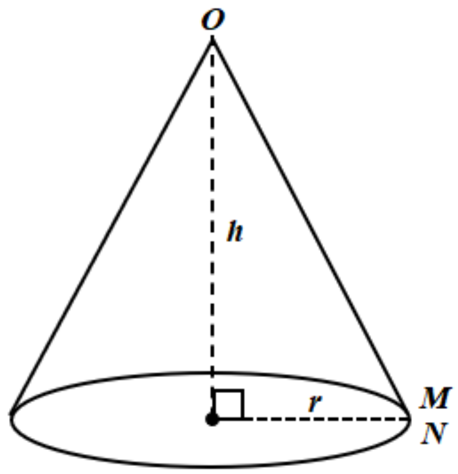
SOLUTION:
Data: Diagram showing a sector of a circle, *OMRN*, made from a thin sheet of metal, with centre *O*, radius 12 cm and sector angle 168°.

Required to calculate: The perimeter of the sector.

Calculation:
The perimeter is the sum of the lengths of the two radii, *OM* and *ON* and the length of the arc *MRN*.

$$\text{Perimeter of the sector} = 12 + 12 + \text{length of arc } MRN$$
$$= 12 + 12 + \left\{\frac{168^\circ}{360^\circ} \times (2 \times \pi \times 12)\right\}$$
$$= 24 + \left\{\frac{168^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 12\right\}$$
$$= 24 + 35.2$$
$$= 59.2 \text{ cm}$$

(b) A cone is made from the sector in (a) by joining *OM* to *ON*, as shown below.



(i) Calculate the

a) radius, *r*, of the cone

SOLUTION:
Data: Diagram showing how sector *OMRN* is folded to form a cone of height, *h* and radius *r*.

Required to calculate: The radius, *r*, of the cone.

Calculation:
When the sector is folded to form the cone, the radius becomes the slant height and the length of the arc is the circumference of the circular base of the cone.

Hence, $2\pi r = \frac{168°}{360°} \times 2\pi(12)$

$\therefore r = \frac{168°}{360°} \times 12$

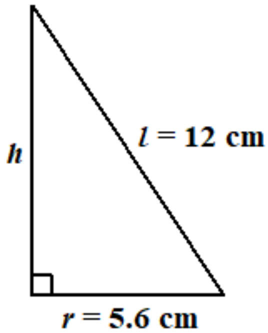
$= 5.6$ cm

b) height, *h*, of the cone.

SOLUTION:
Required to calculate: The height, *h*, of the cone.
Calculation:

Consider the right-angled triangle with sides *r*, *h* and *l*

By Pythagoras' Theorem:



$$
\begin{aligned}
h^2 + r^2 &= l^2 \\
h^2 &= (12)^2 - (5.6)^2 \\
&= 144 - 31.36 \\
h^2 &= 112.64 \\
h &= \sqrt{112.64} \\
&= 10.613 \\
&\approx 10.61 \text{ cm (correct to 2 decimal places)}
\end{aligned}
$$

$\therefore$ The perpendicular height, h, of the cone is 10.61 cm (correct to 2 decimal places.)

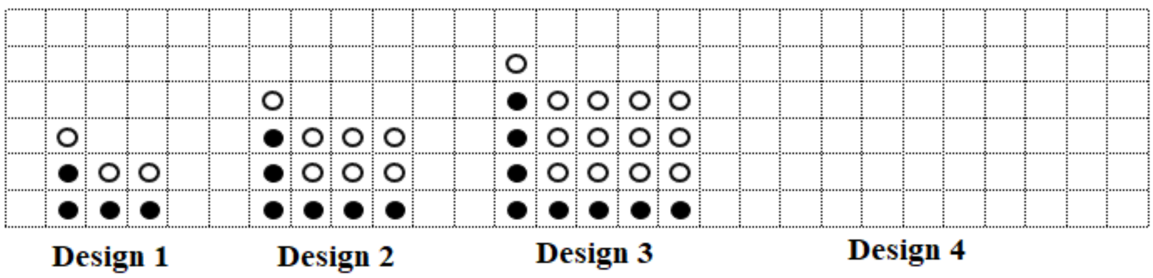
(ii) Calculate the capacity of the cone, **in litres**.

SOLUTION:
Required to calculate: The capacity of the cone, in litres.
Calculation:

Capacity of the cone, $V = \frac{1}{3}\pi r^2 h$

$$
\begin{aligned}
&= \frac{1}{3} \times \frac{22}{7} \times 5.6 \times 5.6 \times 10.61 \text{ cm}^3 \\
&= 348.5738667 \text{ cm}^3 \\
&= \frac{348.5738667}{1000} \text{ litres} \\
&\approx 0.349 \text{ litres (correct to 3 decimal places)}
\end{aligned}
$$

7. A sequence of designs is made using black discs and white discs. The first 3 designs in the sequence are shown below.



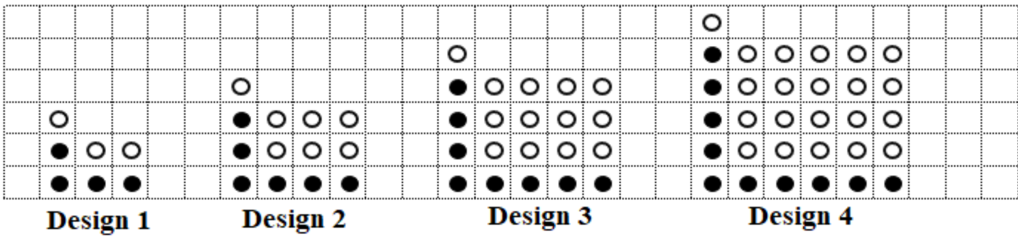
(a) In the space provided on the grid above, draw Design 4.

SOLUTION:
Data: The first 3 designs in a sequence made using black discs and white discs.

Required to draw: The 4th design in the sequence.

By Fayad W. Ali and Shereen A. Khan.

Solution:
The 4th design in the sequence looks like:



(b) The number of white discs, W, the number of black discs, B, and the total number of discs, T, that form each design follow a pattern. The values for W, B and T for the first 3 designs are shown in the table below. Study the pattern of numbers in the table.

Complete rows (i), (ii) and (iii) in the table below.

	Design Number (P)	**Number of White Discs (W)**	**Number of Black Discs (B)**	**Total Number of Discs (T)**
	1	$(1\times1)+1+1=3$	4	7
	2	$(2\times2)+2+1=7$	6	13
	3	$(3\times3)+3+1=13$	8	21
	⋮	⋮	⋮	⋮
(i)	9	$(.....\times.....)+.....+.....=.....$	…………..	111
	⋮	⋮	⋮	⋮
(ii)	……….	$(20\times20)+20+1=421$	…………..	…………..
	⋮	⋮	⋮	⋮
(iii)	n	………………………..	…………..	…………..

SOLUTION:
Data: Incomplete table showing the pattern formed by the number of white discs, W, the number of black discs, B, and the total number of discs, T.
Required to complete: Rows (i), (ii) and (iii) of the above table
Solution: Number of white discs, W
Data:

Design Number (P)	Number of White Discs (W)
1	$(1\times1)+1+1=3$
2	$(2\times2)+2+1=7$
3	$(3\times3)+3+1=13$

We observe the pattern for designs 1, 2 and 3
This pattern appears to be:
(design number $\times$design number) + design number + 1

Design Number (P)	Number of White Discs (W)
n	(design number, n $\times$design number, n) + (design number, n) + 1 $(n\times n)+n+1=n^2+n+1$

Hence, solution for column 1- number of white discs:

Design Number (P)	Number of White Discs (W)
9	$(9\times9)+9+1=91$
20	$(20\times20)+20+1=421$
n	$(n\times n)+n+1$ or n^2+n+1

Solution: Number of black discs, B
Data:

Design Number (P)	Number of Black Discs (B)
1	4
2	6
3	8

The number of blank discs in each successive column increases by 2.

Hence, for design number, n, the value of B can be expressed as.
$2n + k$, where k is a constant.
When $n = 1, B = 4$
$4 = 2(1) + k$
$\therefore k = 2$
$B = 2n + 2$
Testing the formula:
When $n = 2$: $B = 2(2) + 2 = 6$
When $n = 3$: $B = 2(3) + 2 = 8$
Hence: $B = 2n + 2$
Alternative Method

Design Number (*P*)	Number of Black Discs (*B*)
1	$2 \times 2 = 4$
2	$3 \times 2 = 6$
3	$4 \times 2 = 8$
n	$(n + 1) \times 2 = 2n + 2$

Hence: $B = 2n + 2$

Design Number (*P*)	Number of Black Discs (*B*)
9	$2(9) + 2 = 20$
20	$2(20) + 2 = 42$
n	$2n + 2$

Solution: Total Number of discs, T

$T =$ No. of white discs + No. of black discs
$T = W + B$
$= (n^2 + n + 1) + (2n + 2)$
$= n^2 + 3n + 3$
Testing:
$n = 1$: $T = (1)^2 + 3(1) + 3 = 7$
$n = 2$: $T = (2)^2 + 3(2) + 3 = 13$
$n = 3$: $T = (3)^2 + 3(3) + 3 = 21$
$n = 9$ $T = (9)^2 + 3(9) + 3 = 111$

Design Number (*P*)	Total Number of Discs (*T*)
1	7
2	13
3	21

Hence:

Design Number (P)	Total Number of Discs (T)
20	$(20)^2+3(20)+3=463$
n	n^2+3n+3

The completed table looks like:

	Design Number (P)	Number of White Discs (W)	Number of Black Discs (B)	Total Number of Discs (T)
	1	$(1\times1)+1+1=3$	4	7
	2	$(2\times2)+2+1=7$	6	13
	3	$(3\times3)+3+1=13$	8	21
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(i)	9	$(9\times9)+9+1=91$	20	111
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(ii)	20	$(20\times20)+20+1=421$	42	463
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(iii)	n	n^2+n+1	$2n+2$	n^2+3n+3

(c) Stephen has 28 black discs and 154 white discs, and wants to make Design 12. Explain why it is NOT possible for him to make Design 12.

SOLUTION:
Data: Stephen has 28 black discs and 154 white discs, and wants to make Design 12.
Required to explain: Why it is not possible for him to make Design 12.
Solution:
When $n=12$, the number of black discs required will be $2(12)+2=26$.
Stephen has 28 black discs and which is sufficient and with 2 extra.
Number of white discs required $=(12)^2+(12)+1$

$$=144+12+1$$
$$=157$$

Stephen has 154 white discs, which means that he will be 3 short of white discs to form Design 12. Hence, it is not possible for Stephen to create Design 12.

SECTION II

Answer ALL questions.

ALL working MUST be clearly shown.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) Complete the table for the function $y = -x^2 + x + 7$.

x	-3	-2	-1	0	1	2	3	4
y		1		7		5		-5

SOLUTION:
Data: Incomplete table of values for the function $y = -x^2 + x + 7$
Required to complete: The table of values
Solution:

When $x = -3$
$$\begin{aligned} y &= -(-3)^2 + (-3) + 7 \\ &= -9 - 3 + 7 \\ &= -5 \end{aligned}$$

When $x = -1$
$$\begin{aligned} y &= -(-1)^2 + (-1) + 7 \\ &= -1 - 1 + 7 \\ &= 5 \end{aligned}$$

When $x = 1$
$$\begin{aligned} y &= -(1)^2 + (1) + 7 \\ &= -1 + 1 + 7 \\ &= 7 \end{aligned}$$

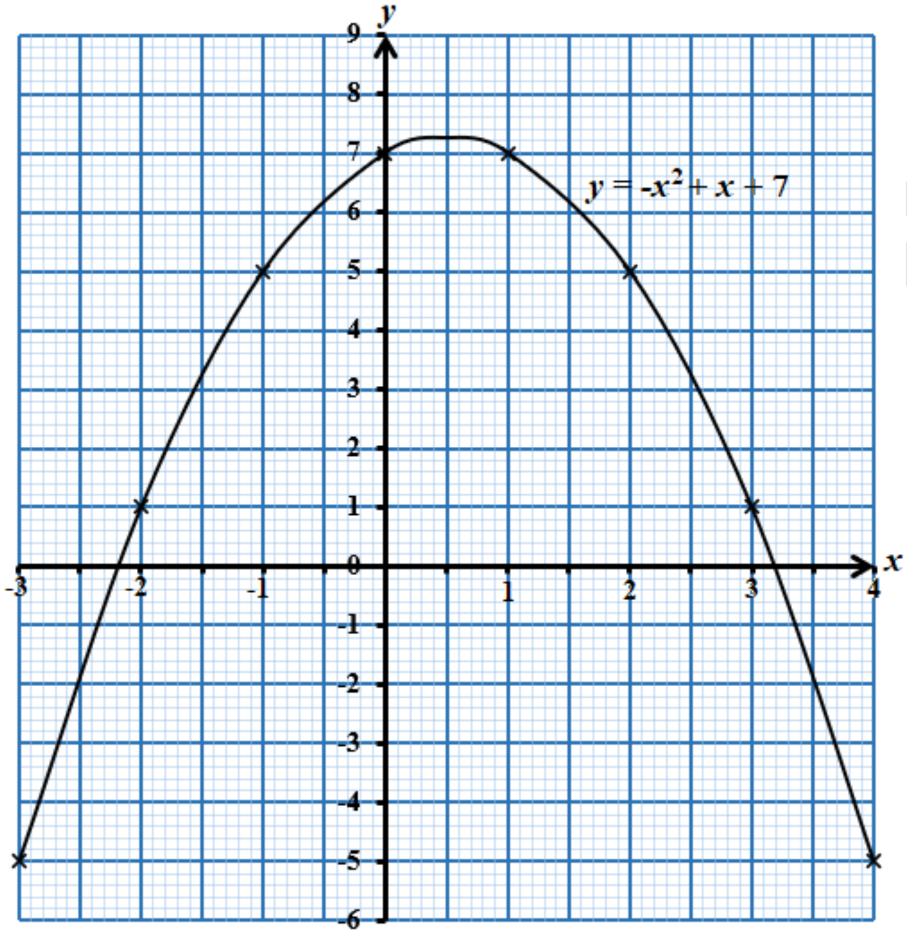
When $x = 3$
$$\begin{aligned} y &= -(3)^2 + (3) + 7 \\ &= -9 + 3 + 7 \\ &= 1 \end{aligned}$$

The completed table looks like:

x	-3	-2	-1	0	1	2	3	4
y	-5	1	5	7	7	5	1	-5

(b) On the grid below, draw the graph of $y = -x^2 + x + 7$ for $-3 \le x \le 4$.

SOLUTION:
Required to draw: The graph of $y = -x^2 + x + 7$ for the domain $-3 \le x \le 4$.
Solution:

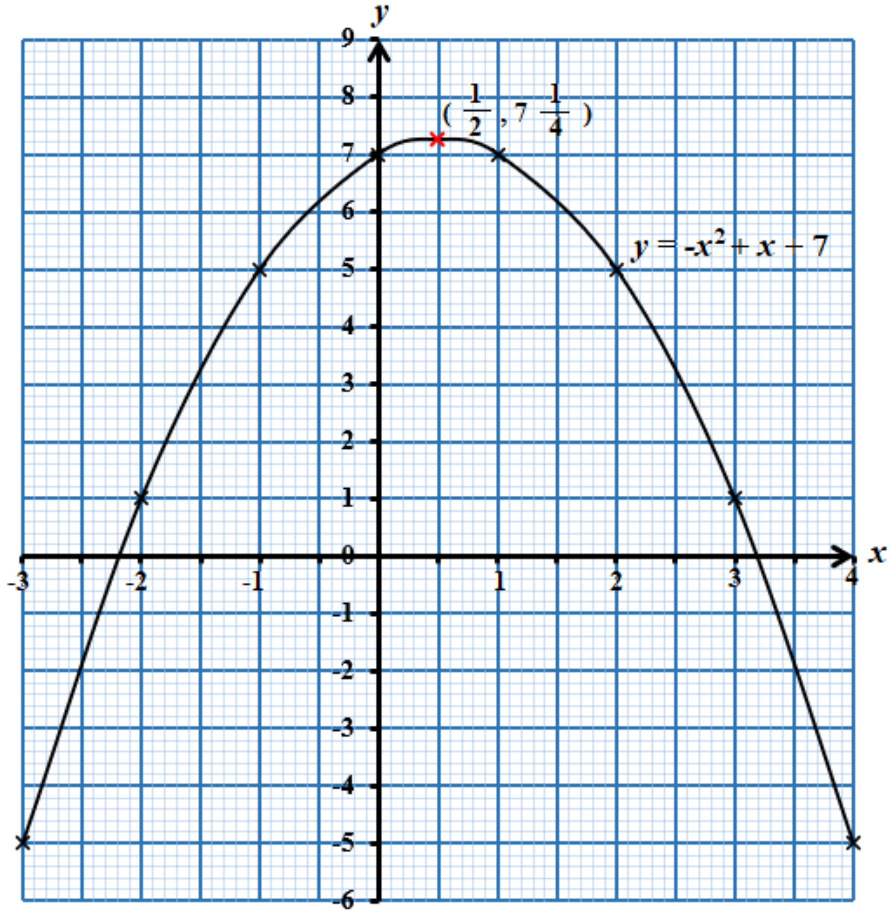


By Fayad W. Ali and Shereen A. Khan.

(c) Write down the coordinates of the maximum/minimum point of the graph.

(.........., ...)

SOLUTION:
Required to write: The coordinates of the maximum/minimum point of the graph.
Solution:

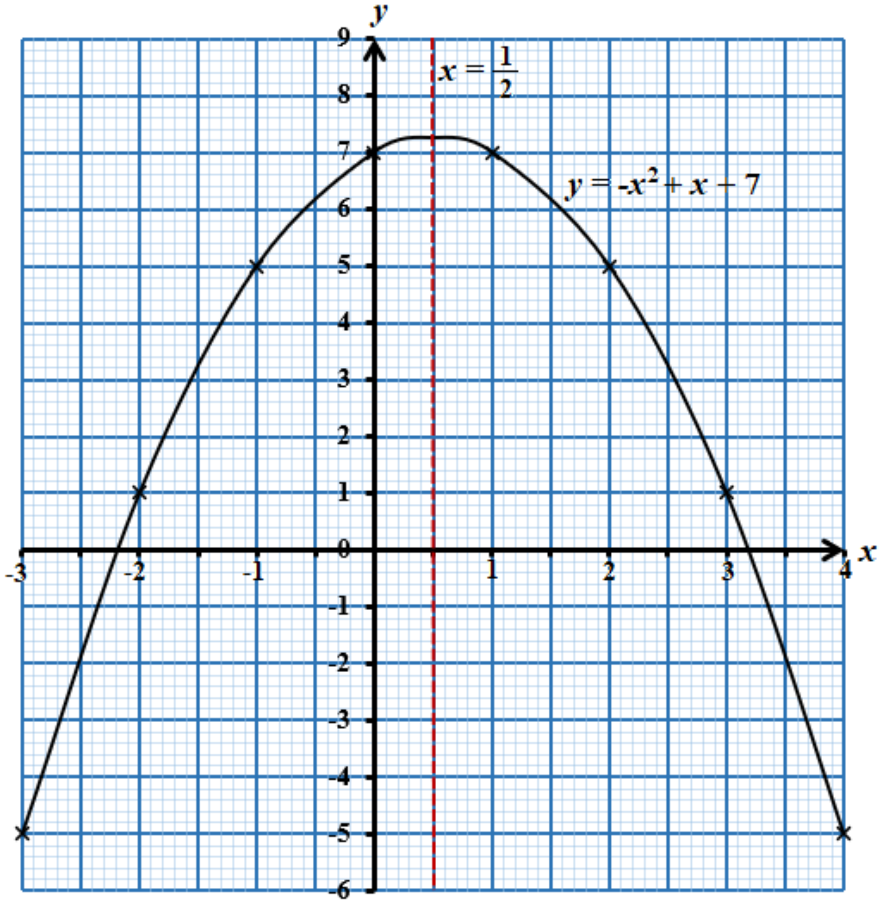


The maximum point is estimated to be $\left(\frac{1}{2}, 7\frac{1}{4}\right)$ obtained by a read-off.

By Fayad W. Ali and Shereen A. Khan.

(d) Write down the equation of the axis of symmetry of the graph.

SOLUTION:
Required to write: The equation of the axis of symmetry of the graph
Solution:



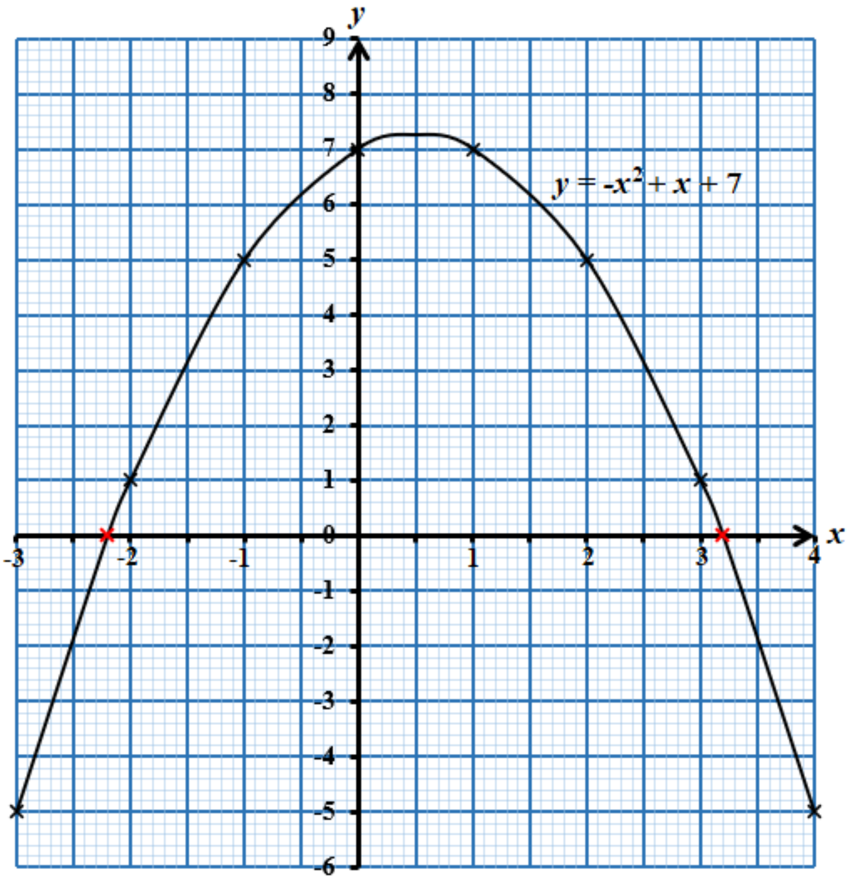
$y = -x^2 + x + 7$ has the axis of symmetry $x = \frac{-(1)}{2(-1)}$ (using $x = -\frac{b}{2a}$)

$x = \frac{1}{2}$ (OR by Read-off)

(e) Use your graph to find the solutions of the equation $-x^2+x+7=0$.

$x=$……………………………. or $x=$……………………………….

SOLUTION:
Required to find: The solutions of $-x^2+x+7=0$, using the graph.
Solution:
The solutions will be where the graph cuts the horizontal or x-axis
We obtain this by a read-off.

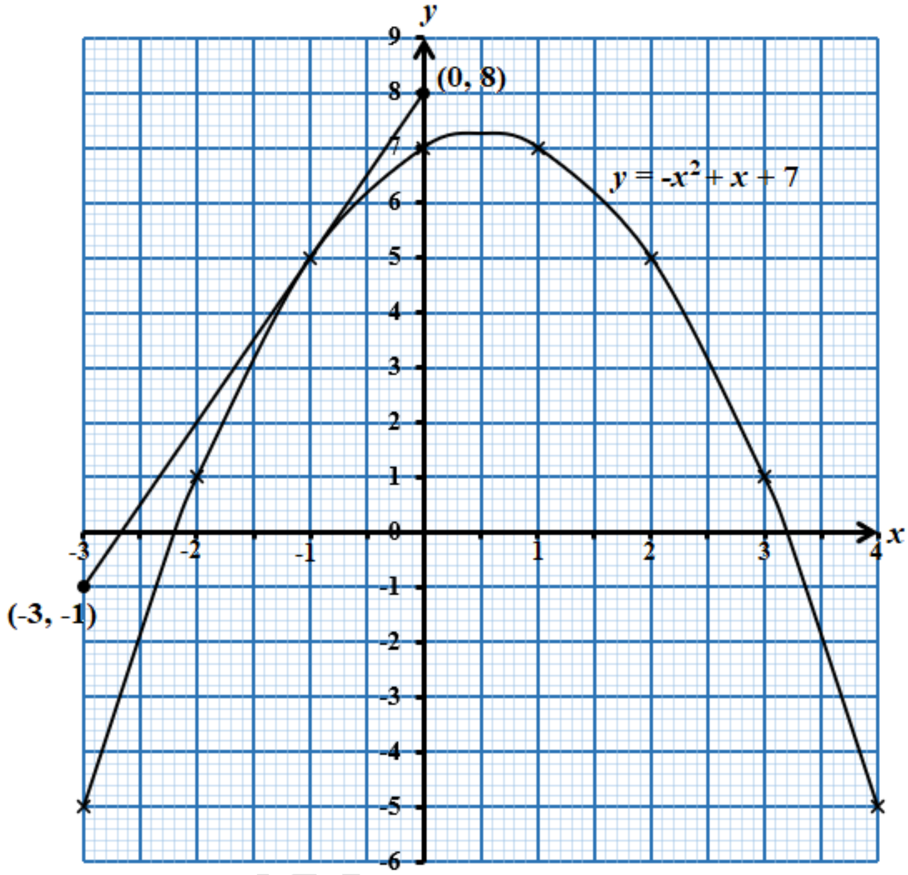


$x=-2.2$ or $x=3.2$

By Fayad W. Ali and Shereen A. Khan.

(f) **(i)** On the grid, draw a line through the points $(-3, -1)$ and $(0, 8)$.

SOLUTION:
Required to draw: A line through the points $(-3, -1)$ and $(0, 8)$.
Solution:



(ii) Determine the equation of this line in the form $y = mx + c$.

SOLUTION:
Required to determine: The equation of the line through $(0, 8)$ and $(-3, -1)$, in the form $y = mx + c$.
Solution:

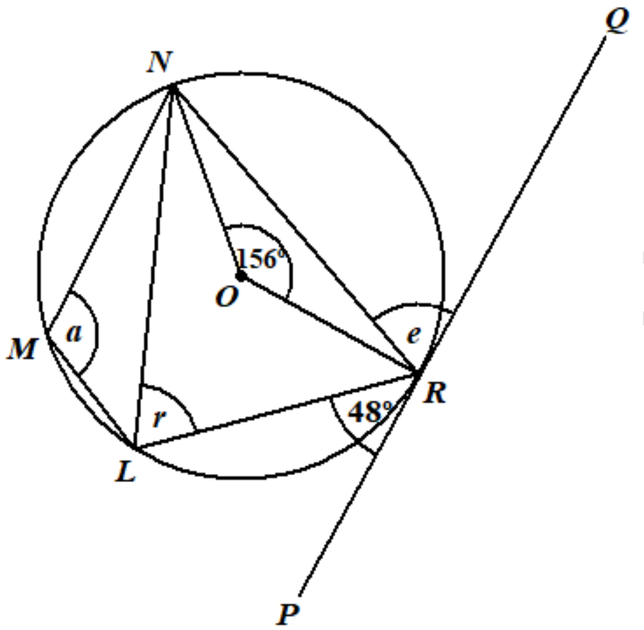
Gradient of the line joining $(-3, -1)$ and $(0, 8)$ is $\dfrac{8-(-1)}{0-(-3)} = \dfrac{9}{3} = 3$

The equation of a straight is $y = mx + c$, where m is the gradient and c is the intercept on the $y-$ axis.

Using $m = 3$ and $c = 8$, the equation of the line through $(0, 8)$ and $(-3, -1)$, in the form $y = mx + c$ is $y = 3x + 8$.

GEOMETRY AND TRIGONOMETRY

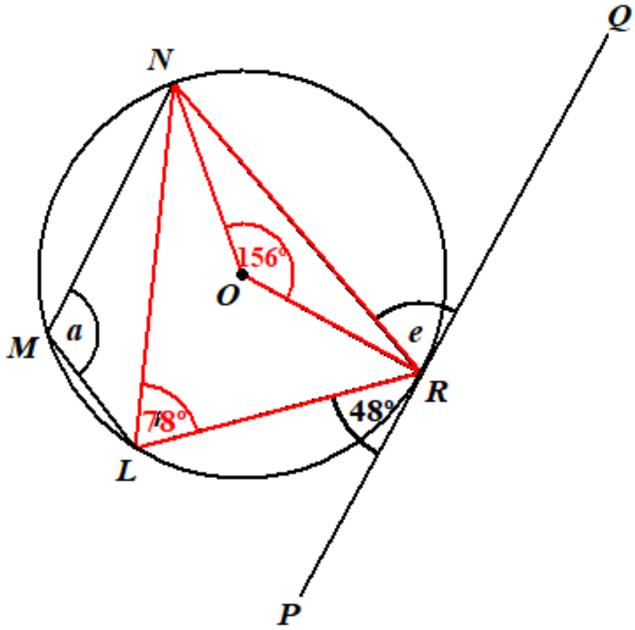
9. (a) L, M, N and R are points on the circumference of a circle, with centre O. PQ is a tangent to the circle at R. Angle $PRL = 48°$ and angle $RON = 156°$.



Find the value of EACH of the following angles, giving reasons for EACH of your answers. Show ALL working where appropriate.

(i) Angle r

SOLUTION:
Data: Diagram showing a circle with centre and the points L, M, N and R along the circumference. PQ is a tangent to the circle at R. Angle $PRL = 48°$ and angle $RON = 156°$.
Required to find: Angle r
Solution:



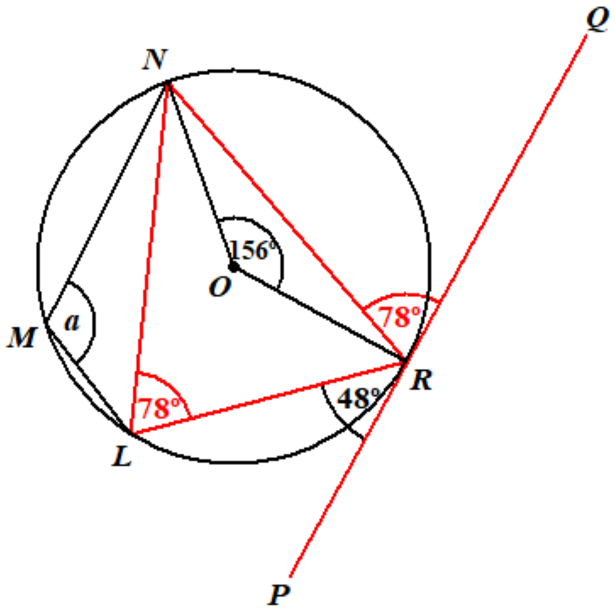
$N\hat{O}R = 2 \times N\hat{L}R$

[The angle subtended by a chord, (*NR*,) at the centre of a circle, ($N\hat{O}R$) , is twice the angle that the chord subtends at the circumference, ($N\hat{L}R$), standing on the same arc.]

$$\therefore r = \frac{1}{2}(156°)$$
$$= 78°$$

(ii) Angle *e*

SOLUTION:
Required to find: Angle *e*
Solution:

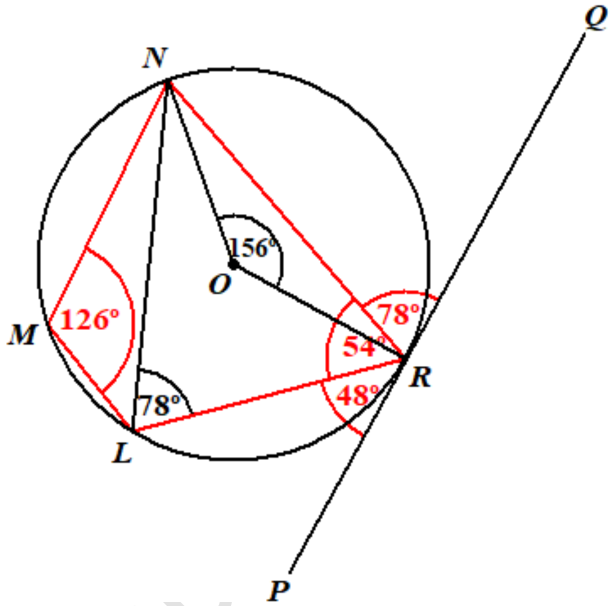


By Fayad W. Ali and Shereen A. Khan.

$N\hat{R}Q = e$ is equal to $N\hat{L}R = r = 78°$
(the angle made by a tangent to a circle (*RQ*) and a chord (*RN*) at the point of contact (*R*) is equal to the angle in the alternate segment (angle *NLR*))
$\therefore e = 78°$

(iii) Angle *a*

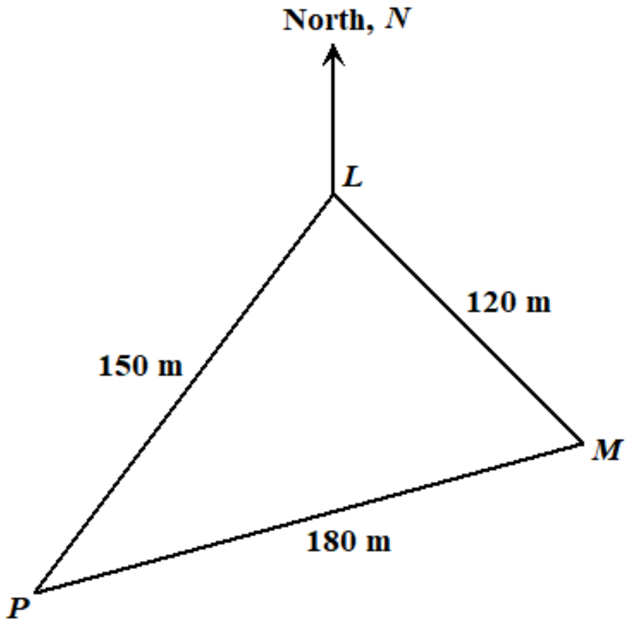
SOLUTION:
Required to find: Angle *a*
Solution:



$N\hat{R}L = 180° - (78° + 48°)$
$= 54°$
(Angles in a straight line total 180°.)

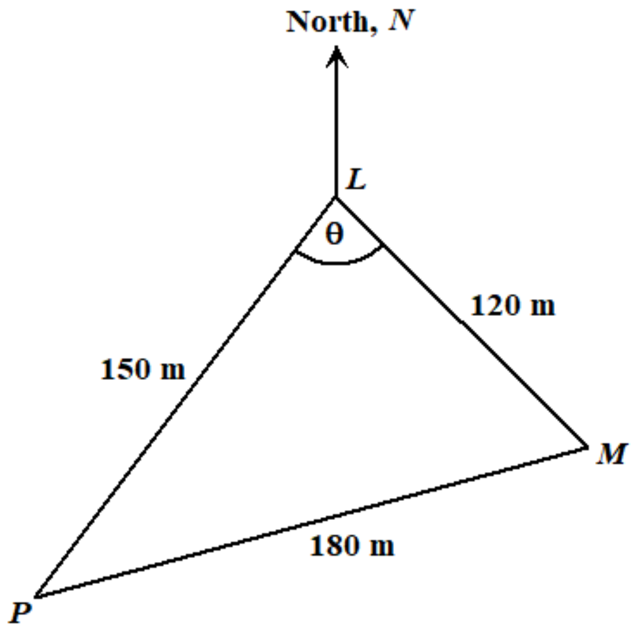
$N\hat{M}L = a$
$= 180° - 54°$
$= 126°$
(Opposite angles in a cyclic quadrilateral (*LMNR*) are supplementary.)

(b) The diagram below shows a triangular field, *LMP*, on horizontal ground.



(i) Calculate the value of Angle *MLP*.

SOLUTION:
Data: Diagram showing a triangular field, *LMP*.
Required to calculate: Angle *MLP*
Calculation:



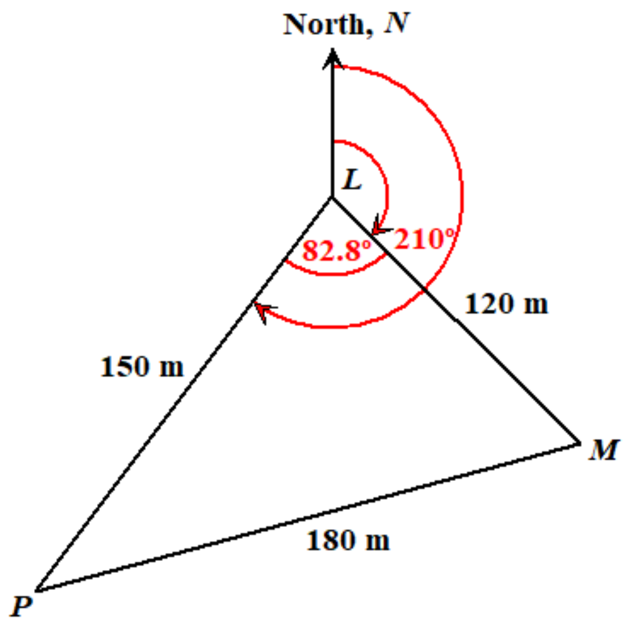
Let $M\hat{L}P = \theta$
Using the Cosine Rule:

$$MP^2 = LP^2 + ML^2 - 2LP.ML\cos M\hat{L}P$$
$$(180)^2 = (150)^2 + (120)^2 - 2(150)(120)\cos\theta$$
$$32400 = 22500 + 14400 - 36000\cos\theta$$
$$36000\cos\theta = 4500$$
$$\cos\theta = \frac{4500}{36000}$$
$$\cos\theta = 0.125$$
$$\theta = \cos^{-1}(0.125)$$
$$= 82.8\underline{1}9^\circ$$
$$\approx 82.8^\circ \text{ (correct to the nearest } 0.1^\circ)$$

(ii) The bearing of P from L is 210°.

a) Find the bearing of M from L.

SOLUTION:
Data: The bearing of P from L is 210°.
Required to find: The bearing of M from L.
Solution:

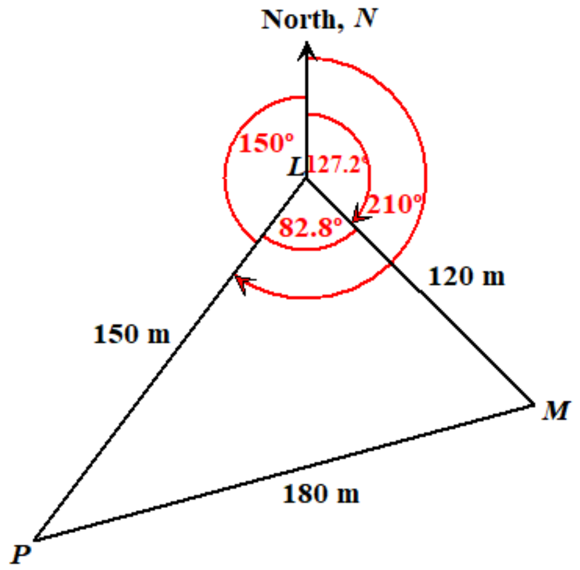


The bearing of M from L $= 210^\circ - 82.8^\circ$
$= 127.2^\circ$

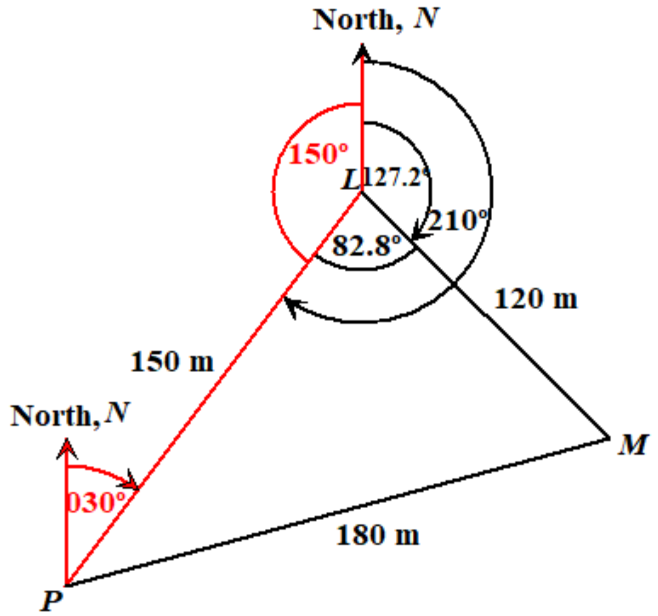
b) Calculate the value of angle NLP and hence, find the bearing of L from P.

SOLUTION:
Required To Calculate: Angle NLP and the bearing of L from P.

Calculation:



$N\hat{L}P = 360° - 210°$
$= 150°$



Bearing of L from P $= 180° - 150°$
$= 030°$
(When parallel lines are cut by a transversal, co-interior angles are supplementary)

By Fayad W. Ali and Shereen A. Khan.

VECTORS AND MATRICES

10. (a) The matrices A and B represent the transformations given below.

$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ represents an anticlockwise rotation of 90° about the origin, O.

$B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ represents a reflection in the straight line $y = -x$.

(i) Determine the elements of the matrix C which represents an anticlockwise rotation of 90° about the origin, O, followed by a reflection in the straight line $y = -x$.

SOLUTION:

Data: $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ represents an anticlockwise rotation of 90° about the origin, O. and $B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ represents a reflection in the straight line $y = -x$.

Required to determine: The matrix, C, that represents an anticlockwise rotation of 90° about the origin, O, followed by a reflection in the straight line $y = -x$.

Solution:

The transformation using A followed by the transformation using B is

$$\begin{tikzcd} \begin{pmatrix} x \\ y \end{pmatrix} \arrow[r, "A"] \arrow[rr, bend right, "BA = C"'] & \begin{pmatrix} x' \\ y' \end{pmatrix} \arrow[r, "B"] & \begin{pmatrix} x'' \\ y'' \end{pmatrix} \end{tikzcd}$$

$C = B \times A$

$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$= \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$

$e_{11} = (0 \times 0) + (-1 \times 1) = -1$

$e_{12} = (0 \times -1) + (-1 \times 0) = 0$

$e_{21} = (-1 \times 0) + (0 \times -1) = 0$

$e_{12} = (-1 \times -1) + (0 \times 0) = 1$

$$\therefore C = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(ii) Describe, geometrically, the single transformation represented by C.

SOLUTION:
Required to describe: The single transformation represented by C.
Solution:
Let C transform (a, b) to (a', b')

Then $\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -a \\ b \end{pmatrix}$

Since $(a, b) \to (-a, b)$ under C

C represents the transformation matrix for a reflection in the $y-$axis.

(b) A transformation, T, is defined by the following 2×2 matrix.

$$T = \begin{pmatrix} 1 & 2 \\ k & -1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

T maps the point $(2, 3)$ onto the point $(8,15)$.

Determine the value of k.

SOLUTION:
Data: $T = \begin{pmatrix} 1 & 2 \\ k & -1 \end{pmatrix}$, where k is a constant and T maps $(2, 3)$ onto $(8,15)$.
Required to determine: k
Solution:

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \xrightarrow{T = \begin{pmatrix} 1 & 2 \\ k & -1 \end{pmatrix}} \begin{pmatrix} 8 \\ 15 \end{pmatrix}$$

$$\underset{2 \times 2}{\begin{pmatrix} 1 & 2 \\ k & -1 \end{pmatrix}}\underset{2 \times 1}{\begin{pmatrix} 2 \\ 3 \end{pmatrix}} = \underset{2 \times 1}{\begin{pmatrix} e_{11} \\ e_{21} \end{pmatrix}}$$

$$= \begin{pmatrix} 8 \\ 2k-3 \end{pmatrix}$$

$e_{11} = (1 \times 2) + (2 \times 3) = 2 + 6 = 8$
$e_{21} = (k \times 2) + (-1 \times 3) = 2k - 3$
Equating corresponding entries
$\therefore 2k - 3 = 15$
$2k = 18$
$k = 9$

(c) The following vectors are defined as shown below.

$$\overrightarrow{WX} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} \qquad \overrightarrow{XY} = \begin{pmatrix} -3 \\ 7 \end{pmatrix} \qquad \overrightarrow{ZY} = \begin{pmatrix} 8 \\ -7 \end{pmatrix}$$

Determine EACH of the following.

(i) A vector, other than $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$, that is parallel to $\overrightarrow{WX}$.

SOLUTION:
Data: $\overrightarrow{WX} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$, $\overrightarrow{XY} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$ and $\overrightarrow{ZY} = \begin{pmatrix} 8 \\ -7 \end{pmatrix}$

Required to determine: A vector parallel to $\overrightarrow{WX}$, other than $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

Solution:
$$\overrightarrow{WX} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

Any vector parallel to $\overrightarrow{WX}$ is a scalar multiple of $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

For example, $\alpha\begin{pmatrix} 5 \\ -1 \end{pmatrix}$, where α is a scalar.

For example, if $\alpha = 2$, $2\begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 10 \\ -2 \end{pmatrix}$ is parallel to $\overrightarrow{WX}$.

(α is any real value and can take an infinite number of values)

(ii) $\overrightarrow{WY}$

SOLUTION:
Required to determine: $\overrightarrow{WY}$
Solution:
$$\overrightarrow{WY} = \overrightarrow{WX} + \overrightarrow{XY}$$
$$= \begin{pmatrix} 5 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ 7 \end{pmatrix}$$
$$= \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

(iii) $\overrightarrow{XZ}$

SOLUTION:
Required to determine: $\overrightarrow{XZ}$

Solution:

$$\overrightarrow{YZ} = -\overrightarrow{ZY}$$
$$= -\begin{pmatrix} 8 \\ -7 \end{pmatrix}$$
$$= \begin{pmatrix} -8 \\ 7 \end{pmatrix}$$
$$\overrightarrow{XZ} = \overrightarrow{XY} + \overrightarrow{YZ}$$
$$= \begin{pmatrix} -3 \\ 7 \end{pmatrix} + \begin{pmatrix} -8 \\ 7 \end{pmatrix}$$
$$= \begin{pmatrix} -11 \\ 14 \end{pmatrix}$$

(iv) $\left|\overrightarrow{XY}\right|$

SOLUTION:

Required to determine: $\left|\overrightarrow{XY}\right|$

Solution:

$$\overrightarrow{XY} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$$
$$\therefore \left|\overrightarrow{XY}\right| = \sqrt{(-3)^2 + (7)^2}$$
$$= 9 + 49$$
$$= \sqrt{58} \text{ units}$$
(in exact form)