CSEC MATHEMATICS JUNE 2022 PAPER 2

SECTION I

Answer ALL questions.

All working MUST be clearly shown.

1. (a) Using a calculator, or otherwise, find the

(i) **a)** EXACT value of

$$\frac{7}{8}+\frac{1}{6}\div\frac{2}{9}$$

SOLUTION:

Required to find: The exact value of $\frac{7}{8}+\frac{1}{6}\div\frac{2}{9}$

Solution:

The correct way in which such a question should be written is

$$\frac{7}{8}+\frac{1}{6}\div\frac{2}{9}=\frac{7}{8}+\left(\frac{1}{6}\div\frac{2}{9}\right)$$

First we work $\frac{1}{6}\div\frac{2}{9}=\frac{1}{6}\times\frac{9}{2}=\frac{9}{12}=\frac{3}{4}$

Hence,
$$\begin{aligned}\frac{7}{8}+\frac{1}{6}\div\frac{2}{9}&=\frac{7}{8}+\frac{3}{4}\\&=\frac{7+2(3)}{8}\\&=\frac{13}{8}\\&=1\frac{5}{8}\text{ (in exact form)}\end{aligned}$$

b) $\frac{8}{0.4^3}$

SOLUTION:

Required to find: The exact value of $\frac{8}{0.4^3}$

Solution:

$$\begin{aligned}\frac{8}{0.4^3}&=\frac{8}{0.4\times0.4\times0.4}\\&=\frac{8}{0.064}\end{aligned}$$

Using the calculator, we get $=125$ (in exact form)

(ii) value of $\sqrt{26.8} - 2.5^{\frac{3}{2}}$, correct to 2 decimal places.

SOLUTION:

Required to find: The value of $\sqrt{26.8} - 2.5^{\frac{3}{2}}$, correct to 2 decimal places.

Solution:

$$\begin{aligned}\sqrt{26.8} - 2.5^{\frac{3}{2}} &= \sqrt{26.8} - \sqrt{(2.5)^3} \\ &= 5.1768 - \sqrt{15.625} \qquad \text{(using the calculator we get)} \\ &= 5.1768 - 3.9528 \\ &= 1.22\underline{\underline{4}} \leftarrow \text{deciding digit} < 5 \\ &\approx 1.22 \text{ (correct to 2 decimal places)}\end{aligned}$$

(b) Children go to a Rodeo camp during the Easter holiday. Ms. Rekha buys bananas and oranges for the children at the camp.

(i) Bananas cost \$3.85 per kilogram. Ms. Rekha buys 25 kg of bananas and receives a discount of 12%. How much money does she spend on bananas?

SOLUTON:

Data: Bananas cost \$3.85 per kilogram. Ms. Rekha buys 25 kg of bananas. She receives a discount of 12%.

Required to find: The amount of money Ms. Rekha spends on bananas.

Solution:

25 kg of bananas at \$3.85 per kg costs $\$3.85 \times 25 = \$96.25$

Discount of 12% $= \frac{12}{100} \times \$96.25$
$= \$11.55$

Hence, the money spent on bananas $= \$96.25 - \$11.55$
$= \$84.70$

Alternative Method:

Money spent on bananas $= (100-12)\%$ of $25 \times \$3.85$

$$\begin{aligned}&= \$3.85 \times 25 \times \frac{88}{100} \\ &= \$84.70\end{aligned}$$

(ii) Ms. Rekha spends \$165.31, inclusive of a sales tax of 15%, on oranges. Calculate the original price of the oranges.

SOLUTION:

Data: Ms. Rekha spends \$165.31 inclusive of a 15% sales tax on oranges.

Required to calculate: The original price of the oranges.

Calculation:

Cost of oranges including 15% tax $= \$165.31$

Hence, $(100+15)\% = 115\%$ of the price of the oranges before tax is added $= \$165.31$

$$\text{Cost of oranges before tax is added} = \frac{\$165.31}{115} \times 100$$
$$= \$143.74\underline{\underline{7}}$$
$$\approx \$143.75 \text{ (correct to the nearest cent)}$$

(iii) The ratio of the number of bananas to the number of oranges is $2:3$. Furthermore, there are 24 more oranges than bananas.

Calculate the number of bananas Ms. Rekha bought.

SOLUTION:

Data: The ratio of the number of bananas to the number of oranges is $2:3$. There are 24 more oranges than bananas.

Required to calculate: The number of bananas Ms. Rekha bought.

Calculation:
Ratio of the number of bananas to the number of oranges is $2:3$.

The number of bananas $= 2x$ and the number of oranges $= 3x,\ x \in Z^+$

The number of oranges is 24 more than the number of bananas.
Hence, $3x = 2x + 24$

$$\therefore 3x - 2x = 24$$
$$x = 24$$

Hence, the number of bananas bought $= 2x$
$= 2\ (24)$
$= 48$

2. (a) (i) Factorise completely the following quadratic expression.

$$5x^2 - 9x + 4$$

SOLUTION:

Required to factorise: $5x^2 - 9x + 4$ completely

Solution:

$$5x^2 - 9x + 4 = (5x - 4)(x - 1)$$

Check

$$= 5x^2 - 5x$$
$$-4x + 4$$
$$= 5x^2 - 9x + 4$$

Hence, $5x^2 - 9x + 4$ = (5*x* - 4) (*x* – 1)

(ii) Hence, solve the following equation.

$$5x^2 - 9x + 4 = 0$$

SOLUTION:

Required to solve: $5x^2 - 9x + 4 = 0$

Solution:

$$5x^2 - 9x + 4 = 0$$

Hence, $(5x - 4)(x - 1) = 0$

If $5x - 4 = 0$ $\quad x = \frac{4}{5}$

If $x - 1 = 0$ $\quad x = 1$

Hence, $x = \frac{4}{5}$ OR 1.

(b) Make *v* the subject of the formula.

$$w = \frac{5 + v}{v - 3}$$

SOLUTION:

Required to make: *v* the subject of the formula

Solution:

$$w = \frac{5+v}{v-3}$$

$$\frac{w}{1} = \frac{5+v}{v-3}$$

$$\therefore w(v-3) = 1(5+v)$$

$$wv - 3w = 5 + v$$

$$wv - v = 5 + 3w$$

$$v(w-1) = 5 + 3w$$

$$\therefore v = \frac{5+3w}{w-1}$$

(c) The height, *h*, of an object is directly proportional to the square root of its perimeter, *p*.

(i) Write an equation showing the relationship between *h* and *p*.

SOLUTION:

Data: The height, *h*, of an object is directly proportional to the square root of its perimeter, *p*.

Required to write: An equation relating *h* and *p*.

Solution:

$$h \propto \sqrt{p}$$

$$\therefore h = k\sqrt{p} \quad (k \text{ is the constant of proportionality})$$

(ii) Given that $h = 5.4$ when $p = 1.44$, determine the value of *h* when $p = 2.89$.

SOLUTION:

Data: $h = 5.4$ when $p = 1.44$

Required to determine: The value of *h*

Solution:

$$5.4 = k\sqrt{1.44}$$

$$\therefore 5.4 = k(1.2)$$

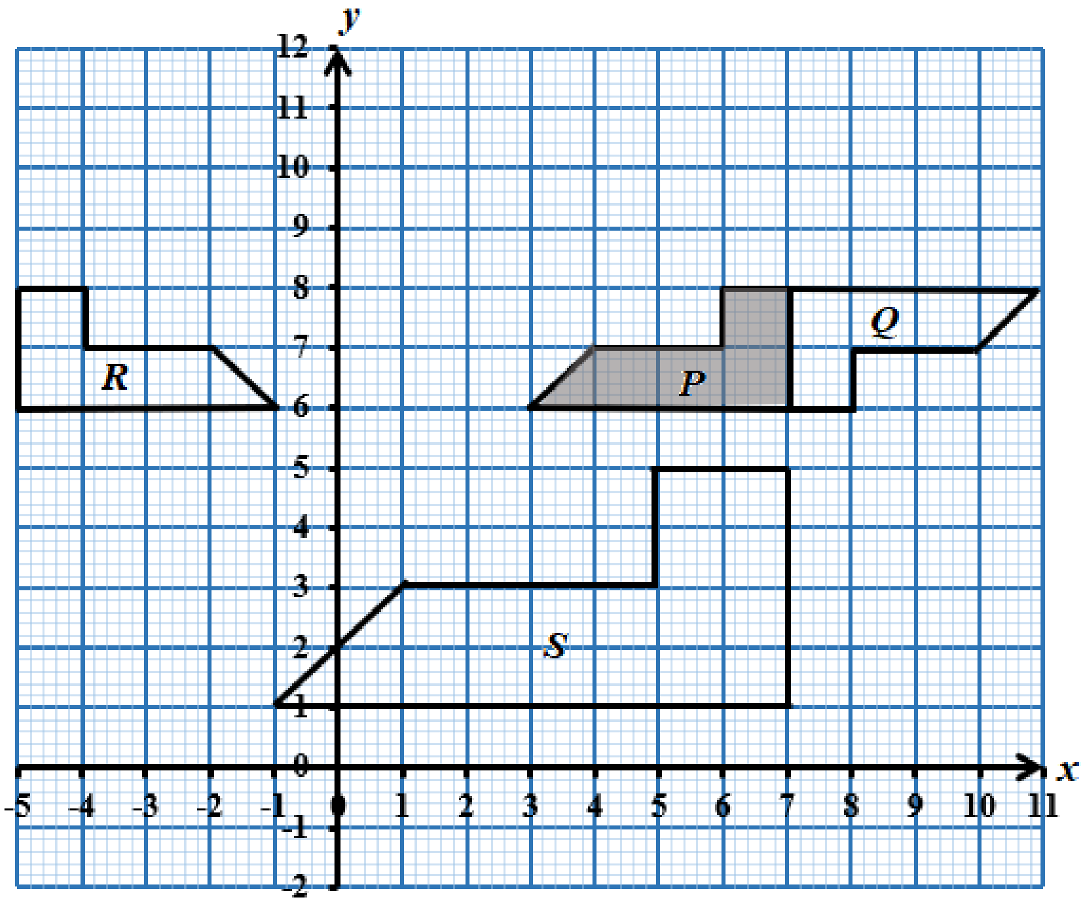
$$k = \frac{5.4}{1.2}$$

(taking the positive value of *p*)

When $p = 2.89$

$$\begin{aligned} h &= \frac{5.4}{1.2}\sqrt{2.89} \\ &= \frac{5.4 \times 1.7}{1.2} \\ &= 7.65 \end{aligned}$$

3. The diagram below shows four shapes, *P*, *Q*, *R* and *S*, on a square grid.



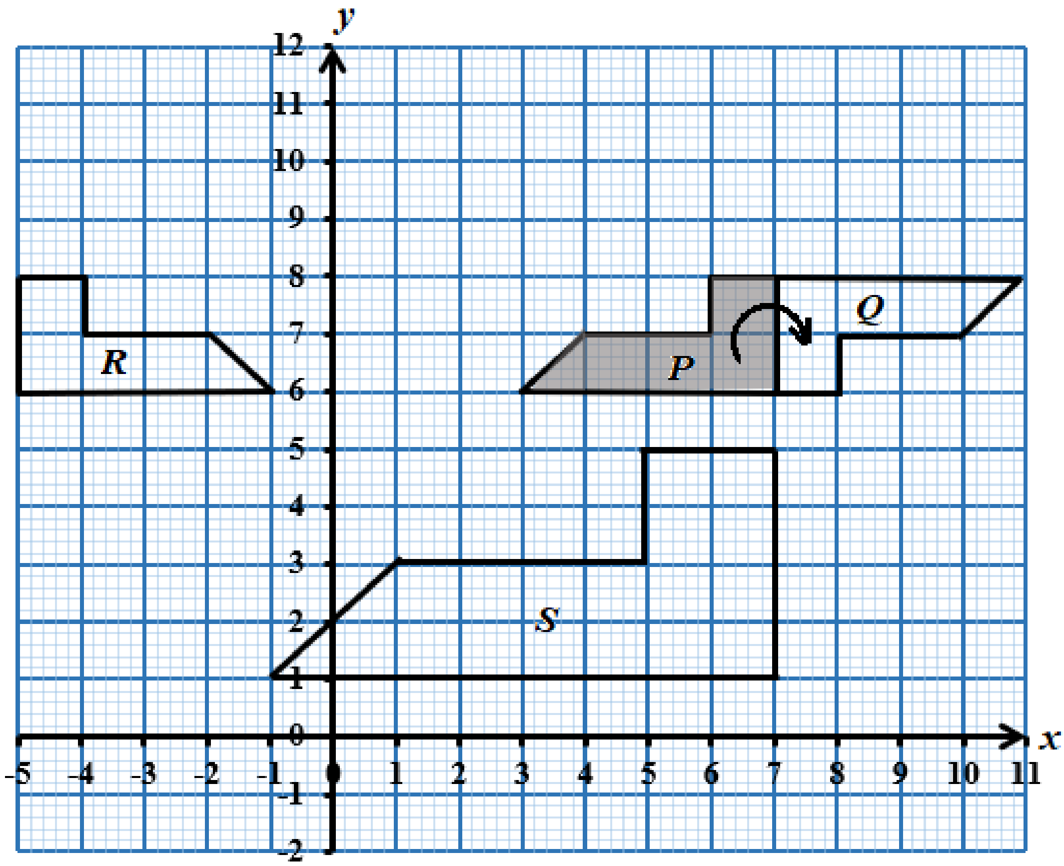
(a) Describe fully the **single** transformation that maps shape *P* onto shape:

(i) *Q*

SOLUTION:

Data: Diagram showing four shapes, *P*, *Q*, *R* and *S* on a square grid.

Required to describe: The single transformation that maps shape *P* onto shape *Q*.
Solution:



The image, Q, is congruent to the object, P, and it is re-oriented with respect to P. The image is not flipped, nor is it translated. Hence, it is a rotation.

In general, to find the centre of rotation we are expected to join the image points to their respective object points and construct their perpendicular bisectors. The perpendicular bisectors are all concurrent and pass through the centre of rotation.

The object and the image share a common side. This is the side that joins (7, 6) to (7, 8). The midpoint of this line is (7, 7).

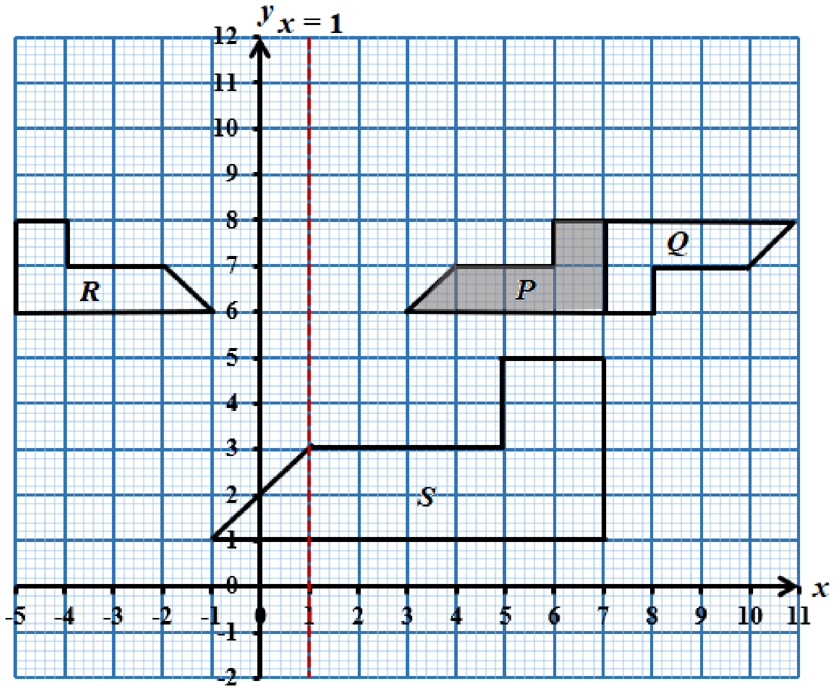
Hence, P is mapped onto Q by a rotation of 180° (clockwise or anti-clockwise) about the centre $(7, 7)$.

(ii) R

SOLUTION:

Required to describe: The single transformation that maps shape P onto shape R.

Solution:

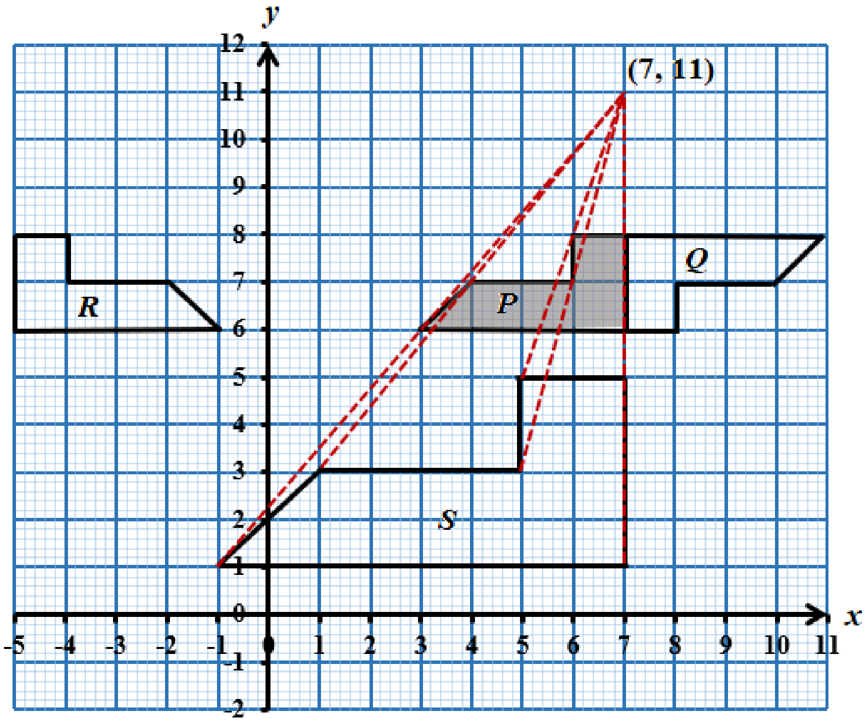


The image *R* is congruent to the object *P* and is flipped or laterally inverted. The perpendicular bisector of the line joining *P* to *R* is $x = 1$.
Hence, *P* is mapped onto *R* under a reflection in the line $x = 1$.

(iii) *S*

SOLUTION:

Required to describe: The single transformation that maps shape *P* onto shape *S*.
Solution:



The object and the image are similar, that is, they are of the same shape but differ in size. Hence the transformation is an enlargement. We choose one side of the image and its corresponding object side

$$\frac{\text{Image length } S}{\text{Object length } P} = \frac{8}{4}$$
$$= 2$$

We join the image points to their corresponding object points and produce the lines backwards to meet at the centre of enlargement. This is found to be at the point (7, 11).

Hence, *P* is mapped onto *S* by an enlargement, centre $(7, 11)$ and scale factor 2.

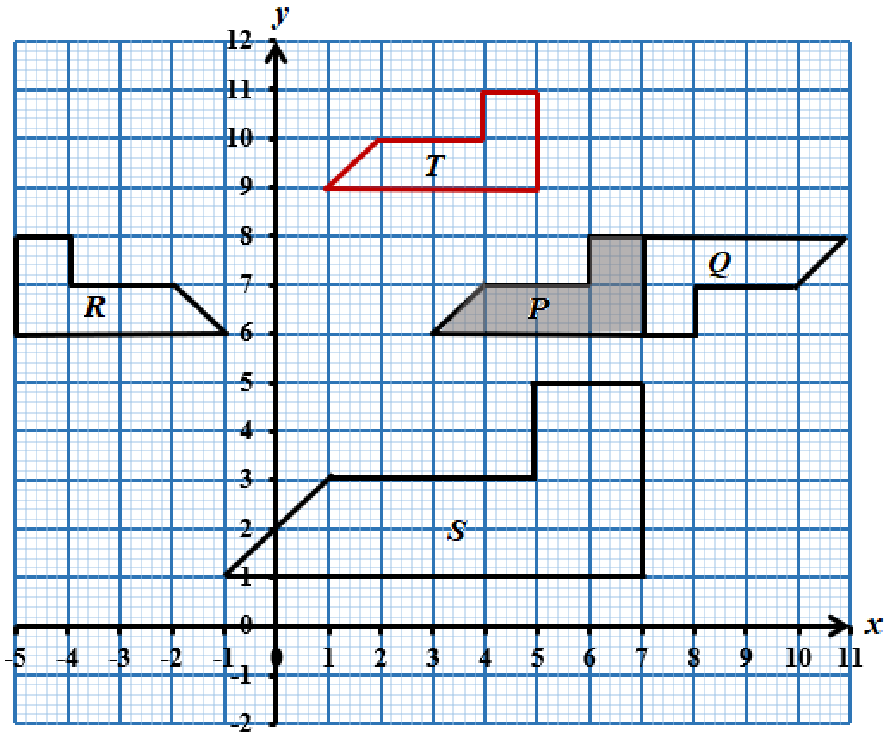
(b) On the grid provided, draw the image of shape *P* after a translation by the vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$. Label this image *T*.

SOLUTION:

Data: Shape *P* undergoes a translation by vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ to form shape *T*.

Required to draw: Shape *T*, on the grid provided.

Solution:



Each of the vertices of *P* is shifted two units horizontally to the left (-2) and three units vertically upwards (+3) to obtain the image *T*, as shown on the diagram.

4. (a) The functions *f* and *g* are defined as follows:

$$f(x)=5x+7 \text{ and } g(x)=3x-1.$$

For the functions given above, determine

(i) $g\left(\frac{1}{3}\right)$

SOLUTION:

Data: $f(x)=5x+7$ and $g(x)=3x-1$

Required to determine: $g\left(\frac{1}{3}\right)$

Solution:

$$g\left(\frac{1}{3}\right)=3\left(\frac{1}{3}\right)-1$$
$$=1-1$$
$$=0$$

(ii) $f^{-1}(-3)$

SOLUTION:

Required to determine: $f^{-1}(-3)$

Solution:

Let $y=5x+7$

$$\therefore y-7=5x$$
$$\frac{y-7}{5}=x$$

Replace *y* by *x* to get $f^{-1}(x)=\frac{x-7}{5}$

Hence, $f^{-1}(-3)=\frac{(-3)-7}{5} \quad =\frac{-10}{5}=-2$

$$f^{-1}(-3)=-2$$

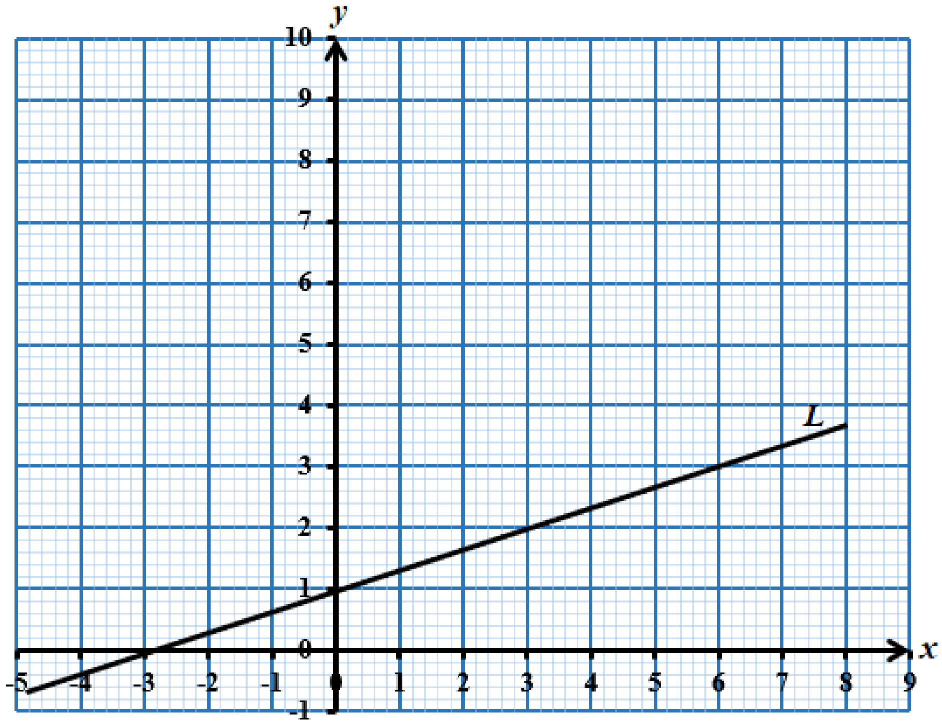
Alternative Method

To find $f^{-1}(-3)$, we must find the pre-image of -3 or what value of x is mapped onto -3.

$$f(x) = 5x + 7$$
$$-3 = 5x + 7$$
$$5x = -10$$
$$x = -2$$

So, $f^{-1}(-3)=-2$

(b) The line L is shown on the grid below.



(i) Write the equation of the line L in the form $y = mx + c$.

SOLUTION:

Data: Graph showing a straight line L.

Required to write: The equation of the line L in the form $y = mx + c$.
Solution:
Choosing two points on the line, $(0, 1)$ and $(-3, 0)$.

$$\text{Gradient } = \frac{1-0}{0-(-3)}$$
$$= \frac{1}{3}$$

The equation of a line is $y = mx + c$

$m =$ gradient $\quad$ $c =$ intercept on the y – axis

$\therefore$ Equation of the line is $y = \frac{1}{3}x + 1$, where $m = \frac{1}{3}$ and $c = 1$.

(ii) The equation of a different line, *Q*, is $y = -2x + 8$.

a) Write down the coordinates of the point where the line *Q* crosses the *x* – axis.

SOLUTION:

Data: The equation of the line *Q* is $y = -2x + 8$.

Required to write: The coordinates of the point where *Q* crosses the *x* – axis.

Solution:

$y = -2x + 8$

A line cuts the *x* – axis at $y = 0$.

Let $y = 0$

$\therefore 0 = -2x + 8$

$2x = 8$

$(\div 2)$

$x = 4$

Hence, the line, Q, cuts the *x* – axis at $(4,\ 0)$.

b) Write down the coordinates of the point where *Q* crosses the *y* – axis.

SOLUTION:

Required to write: The coordinates of the point where *Q* crosses the *y* – axis.

Solution:

A line crosses the *y* – axis at $x = 0$.

Let $x = 0$

$y = -2(0) + 8$

$= 8$

$\therefore$ The line crosses the *y* – axis at $(0,\ 8)$.

Alternative Method:

$y = -2x + 8$ is of the form $y = mx + c$, where $c = 8$ is the intercept on the *y* – axis.

Therefore, the line crosses the *y* – axis at $x = 0$ *and* $y = 8$, that is at the point (0, 8).

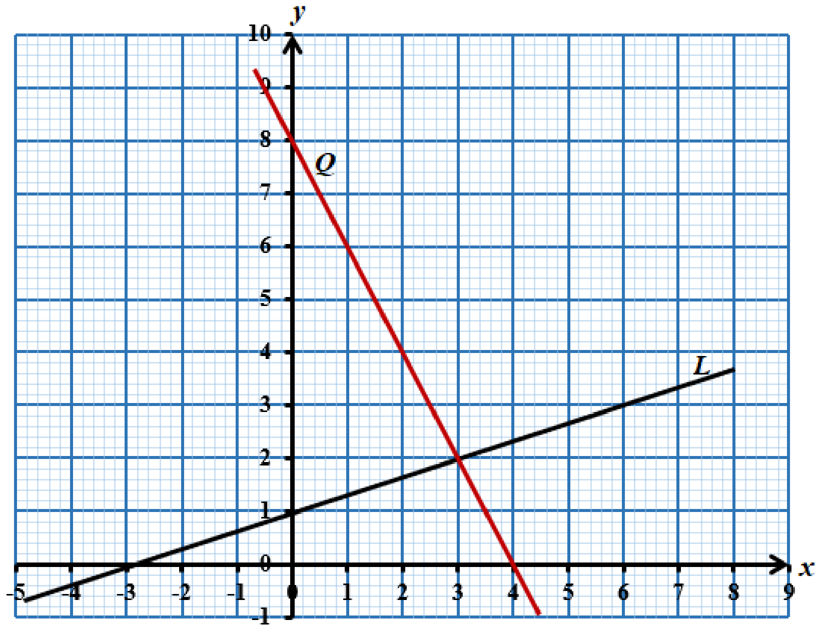
c) On the grid, draw the graph of the line *Q*.

SOLUTION:

Required to draw: The graph of the line Q on the grid provided.

Solution:

We know two points on the line Q. We plot these points $(0, 8)$ and $(4, 0)$ to draw line Q, and which can be extended to any length.



(iii) Complete the statement below.

According to the graph, the solution of the system of equations consisting of L and Q is ....

SOLUTION:

Required to complete: The statement given.

Solution:
The solution of the system of equations is actually the value of x and of y that satisfy both equations. This is determined by the point of intersection of both lines on the $x-y$ or Cartesian plane. The point of intersection is read-off as $(3, 2)$. Hence, $x=3$ and $y=2$, according to the diagram. The completed statement is as follows:

According to the graph, the solution of the system of equations consisting of L and Q is $x=3$ and $y=2$.

5. A school nurse records the heights, h cm, of each of the 150 students in Class A who was vaccinated. The table below shows the information.

Height, h (cm)	**Number of Students (f)**
$60 < h \leq 80$	4
$80 < h \leq 100$	20
$100 < h \leq 120$	35
$120 < h \leq 140$	67
$140 < h \leq 160$	20
$160 < h \leq 180$	4

(a) Complete the table below and use the information to calculate an estimate of the mean height of the students. Give your answer correct to 1 decimal place.

Height, h (cm)	**Number of Students (f)**	**Midpoint (x)**	$f \times x$
$60 < h \leq 80$	4	70	280
$80 < h \leq 100$	20	90	1 800
$100 < h \leq 120$	35	110	3 850
$120 < h \leq 140$	67	___________	___________
$140 < h \leq 160$	20	150	3 000
$160 < h \leq 180$	4	170	680

SOLUTION:

Data: Table showing the heights, h cm, of each of the 150 students in Class A who was vaccinated and an incomplete table showing midpoint, x, and $f \times x$.

Required to complete: The table given and calculate an estimate of the mean height of the students, correct to 1 decimal place.

Solution:
The completed table is as follows:

Height, *h* (cm)	Number of Students (*f*)	Midpoint (*x*)	$f \times x$
$60 < h \le 80$	4	70	280
$80 < h \le 100$	20	90	1 800
$100 < h \le 120$	35	110	3 850
$120 < h \le 140$	67	$\frac{120+140}{2}=130$	$67\times130=8710$
$140 < h \le 160$	20	150	3 000
$160 < h \le 180$	4	170	680

Mean, $\bar{x}=\frac{\sum fx}{\sum f}$, where $\bar{x}=$ mean, $f=$ frequency,
$x=$ midpoint of the class
and $\sum=$ the sum of

$$\bar{x}=\frac{280+1800+3850+8710+3000+680}{150}$$
$$=\frac{18320}{150}$$
$$=122.1\underline{\underline{3}}$$
$$\approx 122.1 \text{ (correct to 1 decimal place)}$$

(b) In Class B, the mean height of the students is 123.5 cm, and the standard deviation is 29.87. For Class A, the standard deviation is 21.38.

Using the information provided, and your response in (a), comment on the distribution of the heights of the students in both Class A and Class B.

SOLUTION:

Data: In Class B, the mean height of the students is 123.5 cm, and the standard deviation is 29.87. For Class A, the standard deviation is 21.38.

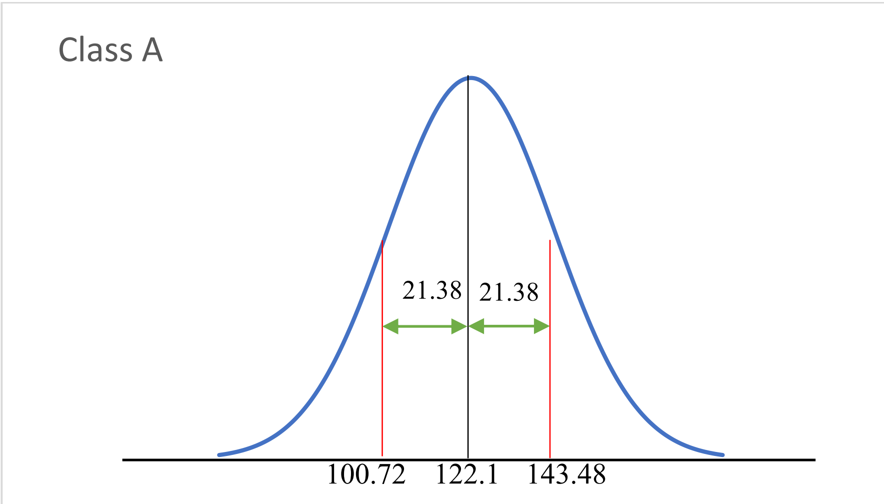
Required to comment: On the distribution of the heights of the students in both Class A and Class B.

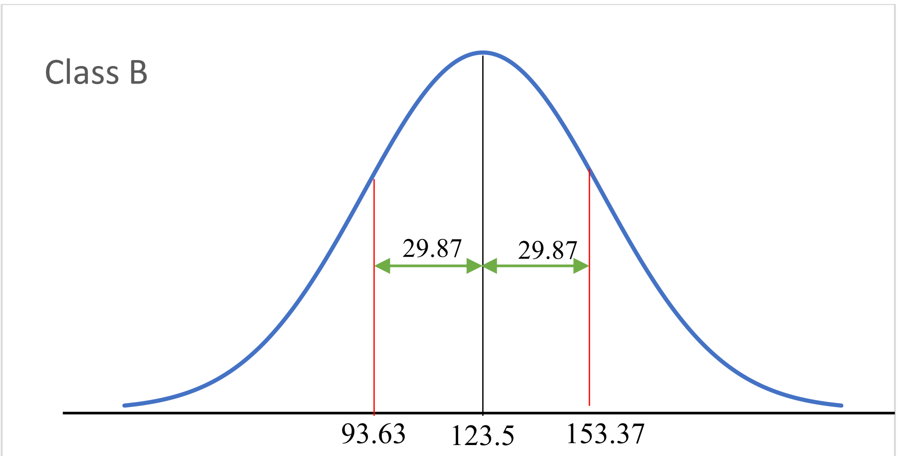
Solution:

The mean height of students in Class B (123.5) is approximately the same as that of the students in Class A (122.2). However, the standard deviation of the scores in Class B (29.87) is significantly higher than that in Class A (21.38). Standard deviation is a measure of the spread of data. This suggests that the heights of students in Class B are more 'spread out', while those in Class A are more clustered around the mean.

We can also illustrate this information on a diagram. If we assume that the distributions are normal, then 68% of the scores will fall within one standard deviation of the mean.
Hence, in Class A, 68% of the scores will lie between 100.72 and 143.48.
In Class B, 68% of the scores will lie between 93.63 and 153.37.

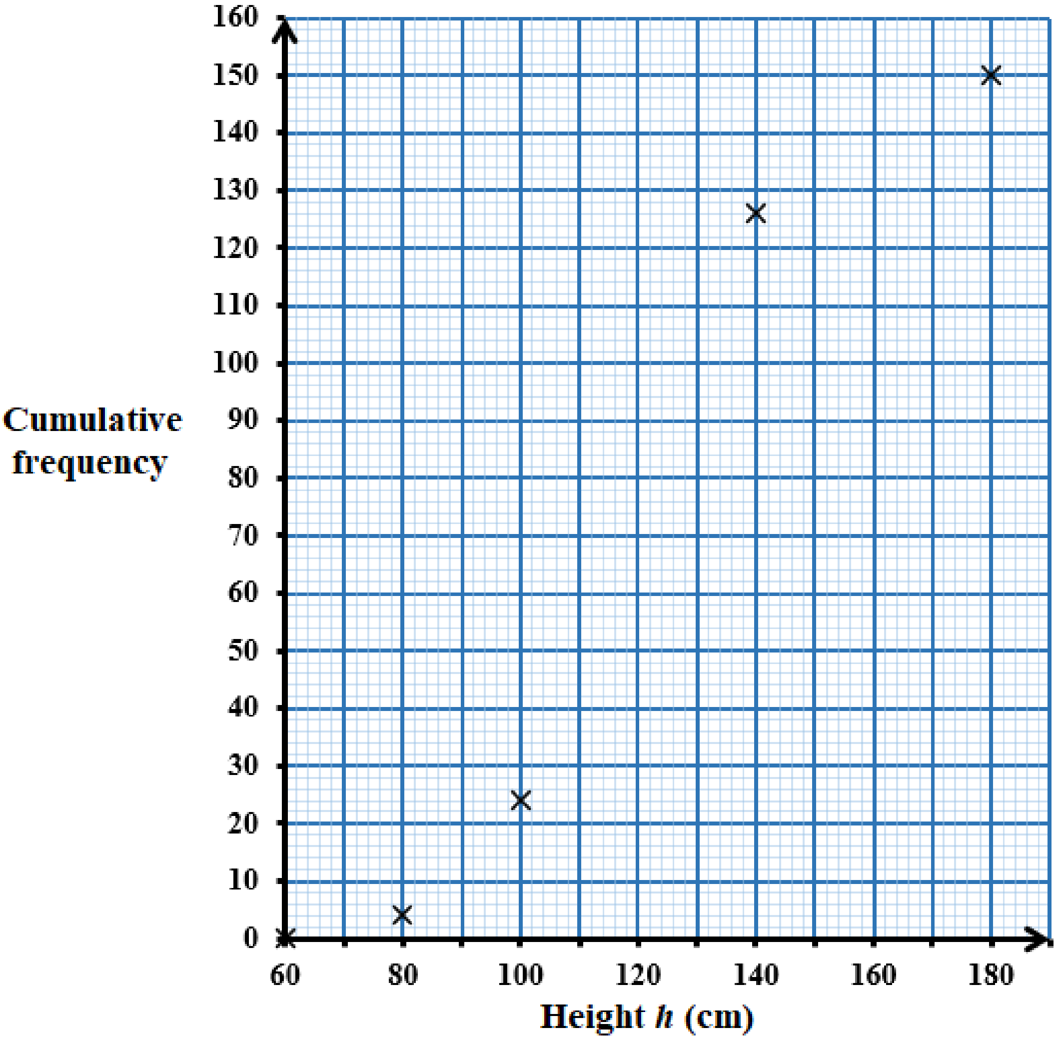
Class	**Mean**	**Standard Deviation**	$\bar{X} \pm 1$ S.D.
A	122.1	21.38	$[100.72, 143.48]$
B	123.5	29.87	$[93.63, 153.37]$





(c) **(i)** Complete the cumulative frequency table below and use the information to construct the cumulative frequency curve on the grid provided.

Height, *h* (cm)	Number of Students (*f*)	Cumulative Frequency
$60 < h \leq 80$	4	4
$80 < h \leq 100$	20	24
$100 < h \leq 120$	35	________
$120 < h \leq 140$	67	126
$140 < h \leq 160$	20	________
$160 < h \leq 180$	4	150



SOLUTION:

Data: Incomplete cumulative frequency table showing the heights of vaccinated students and an incomplete cumulative frequency curve illustrating the data.

Required to complete: The cumulative frequency table and the cumulative frequency curve for the data.

Solution:

Note: Height is a continuous variable, so

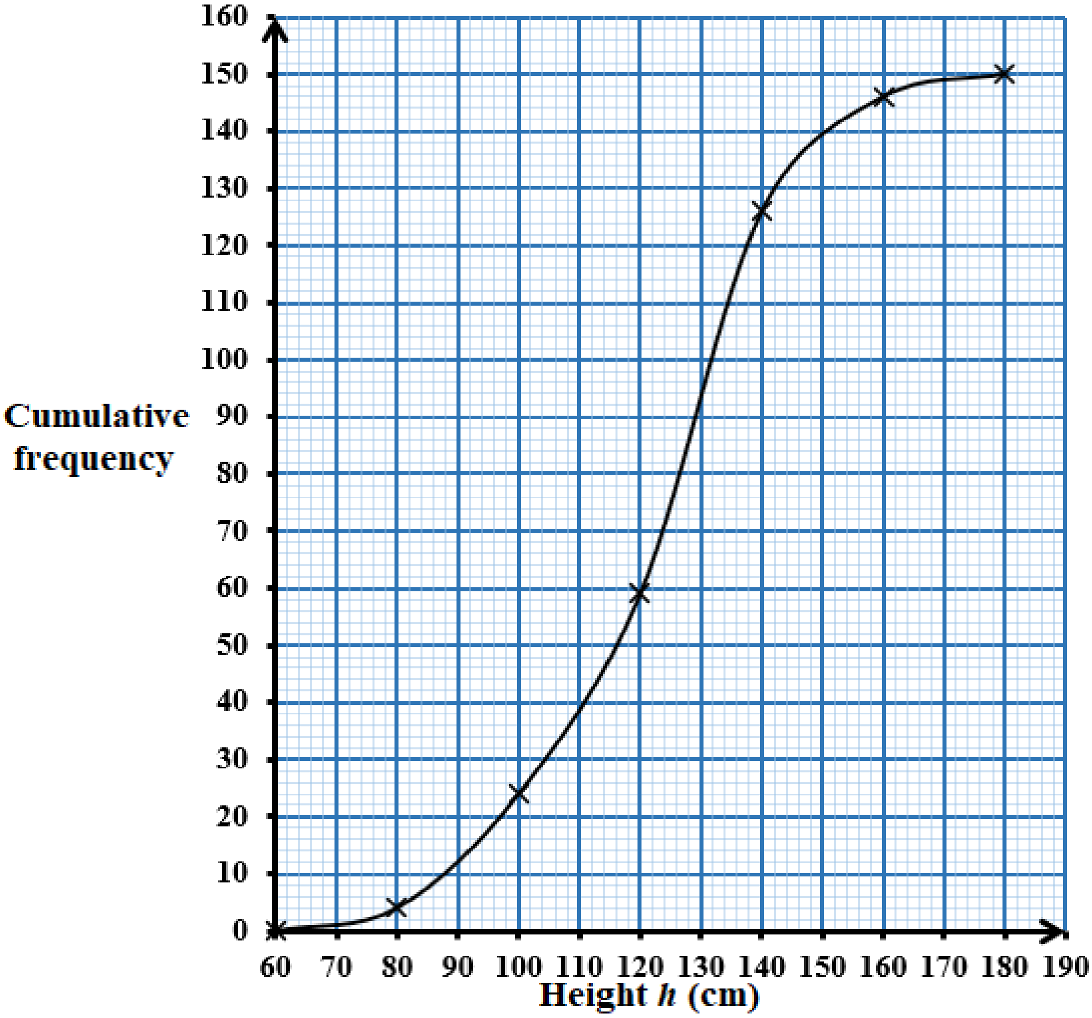
Lower class limit				Upper class limit
60	$<$	h	$\leq$	80

The modified and completed cumulative frequency table is shown below:

Height, *h* (cm)	**Class Boundaries** **L.C.B** (lower class boundary **U.C.B.** (Upper class boundary)	**Number of Students (*f*)** (frequency)	**Points to Plot** (Upper class boundary, cumulative frequency)	**Cumulative Frequency**
			$(60.5,\ 0)$	
$60 < h \leq 80$	$59.5 \leq h < 80.5$	4	$(80.5,\ 4)$	4
$80 < h \leq 100$	$80.5 \leq h < 100.5$	20	$(100.5,\ 24)$	24
$100 < h \leq 120$	$100.5 \leq h < 120.5$	35	$(120.5,\ 59)$	$35 + 24 = 59$
$120 < h \leq 140$	$120.5 \leq h < 140.5$	67	$(140.5,\ 126)$	126
$140 < h \leq 160$	$140.5 \leq h < 160.5$	20	$(160.5,\ 146)$	$20 + 126 = 146$
$160 < h \leq 180$	$160.5 \leq h < 180.5$	4	$(180.5,\ 150)$	150

The point $(60.5,\ 0)$ is obtained by checking backwards so that the cumulative frequency curve starts from the horizontal axis.

The completed graph looks like:



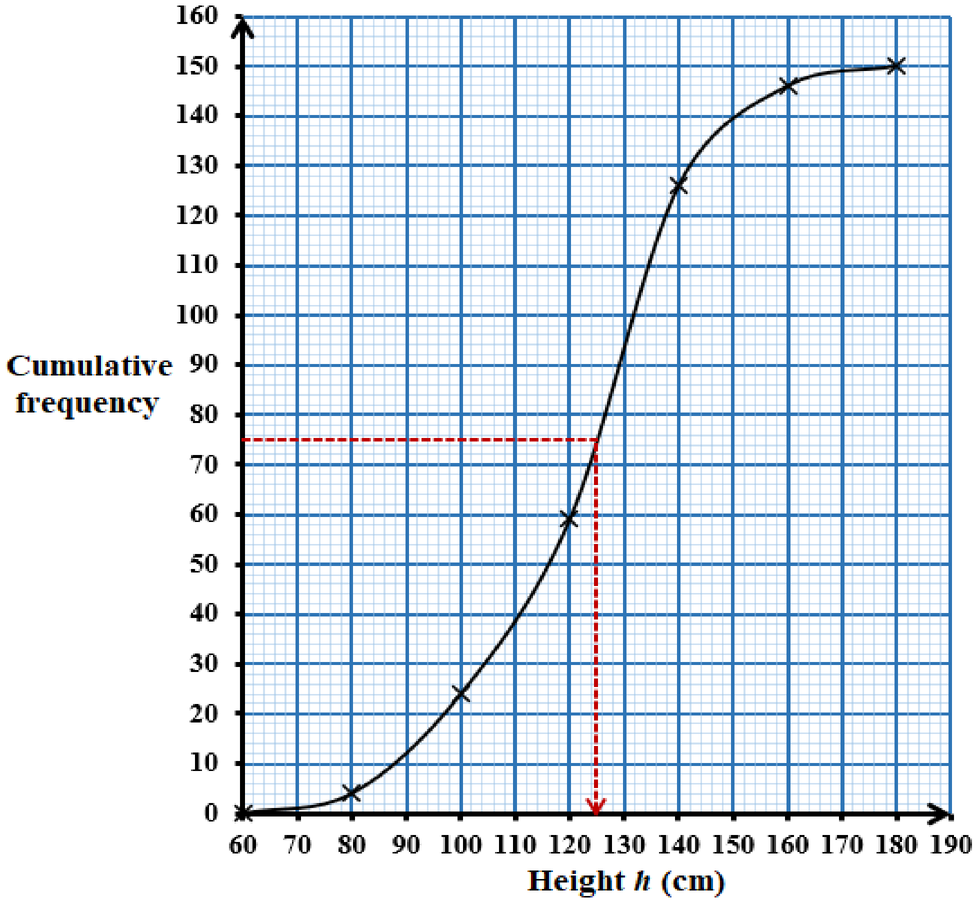
(ii) Use your cumulative frequency curve to find:

a) an estimate of the median height of the group of students

SOLUTION:

Required to find: An estimate of the median height of the group of students, using the cumulative frequency curve drawn.

Solution:
We draw a horizontal line at one-half of the cumulative frequency which is $\frac{1}{2}(150)=75$.
We then drop a vertical line where this horizontal line meets the curve and read off the value for the median.



The estimated median $= 125$ cm

b) the probability that a student chosen at random would be taller than 130 cm.

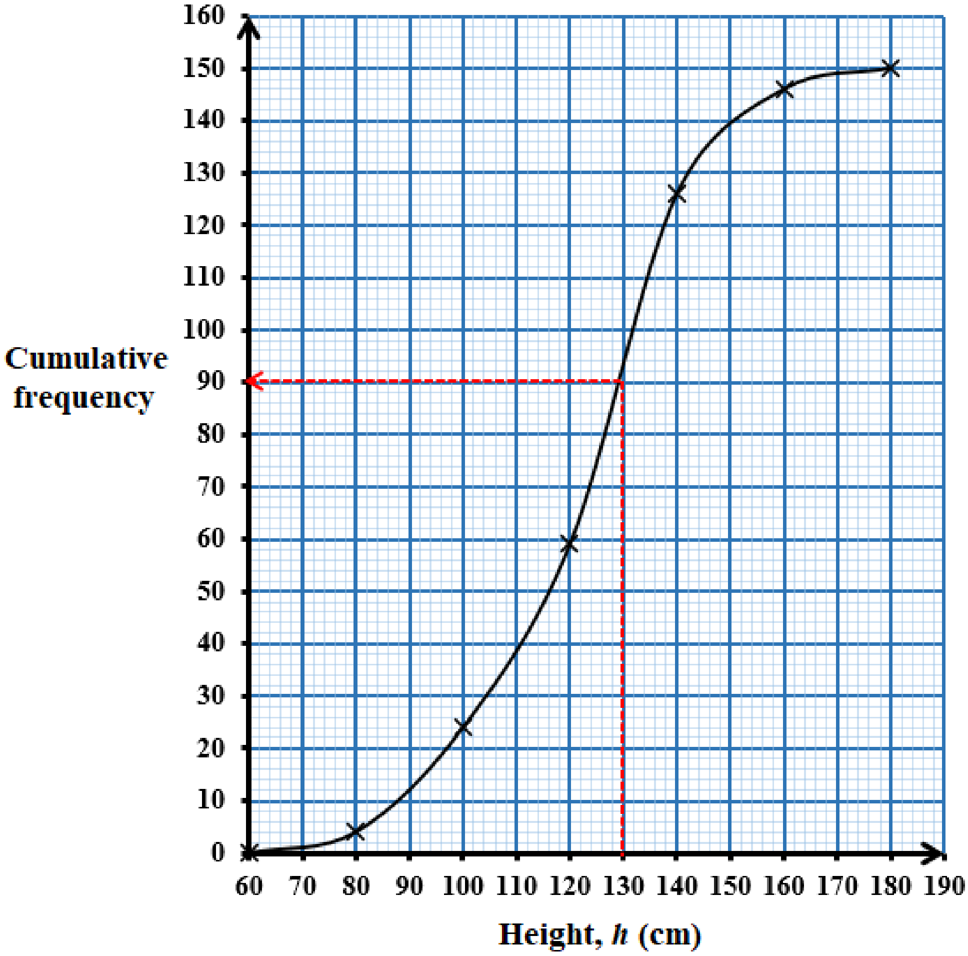
SOLUTION:

Required to calculate: The probability that a student chosen at random is taller than 130 cm.

Calculation:

We draw a vertical line at 130 cm to meet the curve, then draw a horizontal from the point to meet the vertical axis.

This meets at 90.



From the read-off, we deduce that 150 – 90 = 60 students are taller than 130 cm.

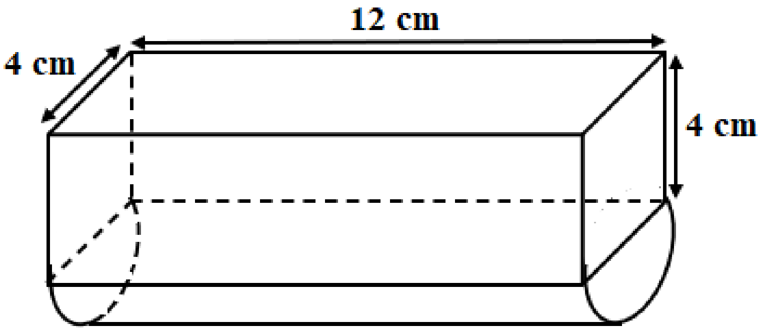
P (student is taller than 130 cm)

$$= \frac{\text{Number of students taller than 130 cm}}{\text{Total number of students}}$$

$$= \frac{60}{150}$$

$$= \frac{2}{5}$$

6. The diagram below shows a solid made from a semi-circular cylindrical base, with a rectangular prism above it. The diameter of the cylindrical base and the width of the rectangular prism are 4 cm each.



(a) Calculate the TOTAL surface area of the solid.
[The surface area, A, of a cylinder with radius r is $A = 2\pi r^2 + 2\pi rh$.]

SOLUTION:

Data: Diagram showing a solid made from a semi-circular cylindrical base, with a rectangular prism above it. The diameter of the cylindrical base and the width of the rectangular prism are 4 cm each. The surface area, A, of a cylinder with radius r is $A = 2\pi r^2 + 2\pi rh$.

Required to calculate: The total surface area of the solid.

Calculation:

Surface area of the semi-cylindrical base $= \frac{1}{2}(2\pi rh) + \frac{1}{2}(2\pi r^2)$

$$= \pi rh + \pi r^2$$
$$= (\pi \times 2 \times 12) + (\pi \times 4)\ \text{cm}^2$$
$$= 75.398\ \text{cm}^2 + 12.566\ \text{cm}^2$$
$$= 87.964\ \text{cm}^2$$

Surface area of the top rectangular part of the prism

$$= 2(12 \times 4) + 2(4 \times 4) + (12 \times 4)\ \text{cm}^2$$
$$= (96 + 32 + 48)\ \text{cm}^2$$
$$= 176\ \text{cm}^2$$

Hence the total surface area of the solid is (87.96 + 176) cm^2

= 263.96 cm^2 (correct to 2 d.p)

(b) Calculate the volume of the solid.

SOLUTION:

Required to calculate: The volume of the solid.

Calculation:

Volume of the semi-cylindrical base $=\dfrac{1}{2}\pi r^2 h$

$$=\frac{1}{2}\pi \times 2^2 \times 12 = 24\pi = 75.398$$

Volume of rectangular top of the prism $=12\times 4\times 4 \text{ cm}^3 = 192 \text{ cm}^3$

Hence, the volume of solid $=75.398+192 \text{ cm}^3$

$$= 267.40 \text{ cm}^3 \ (\text{correct to 2 decimal places})$$

(c) The solid is made from gold. One cubic centimetre of gold has a mass of 19.3 grams. The cost of 1 gram of gold is \$42.62.

Calculate the cost of the gold used to make the solid.

SOLUTION:

Data: The solid is made from gold. One **cubic centimetre** of gold has a mass of 19.3 grams. The cost of 1 gram of gold is \$42.62.

Required to calculate: The cost of the gold use to make the solid.

Calculation:

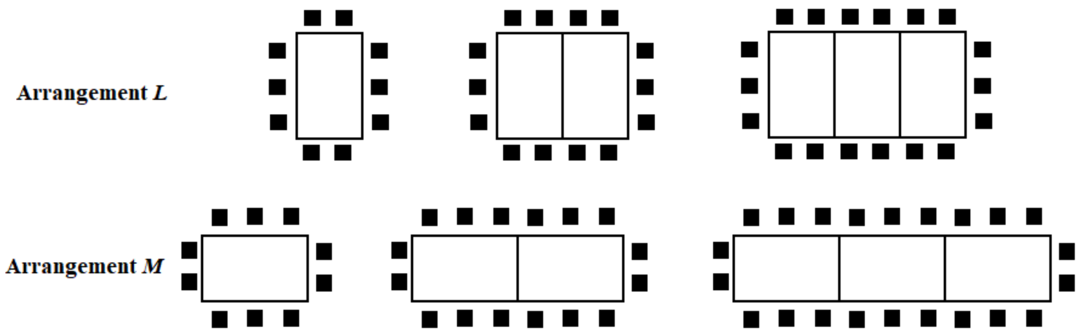
Mass of solid $=276.40\times 19.3$ grams

$= 5\ 160.82$ grams

Cost of the solid $=5160.82\times \$42.62$

$= \$219\ 954.15$

7. At an entertainment hall, tables and chairs can be arranged in two different ways as shown the diagrams below.



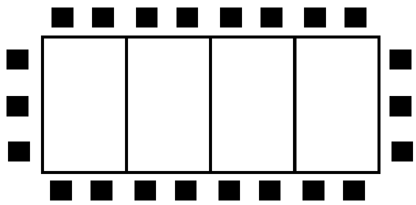
(a) Draw the diagram for **4 tables** using Arrangement *L*.

SOLUTION:

Data: Diagrams showing two different arrangements of tables and chairs at an entertainment hall.

Required to draw: The arrangement of 4 tables using arrangement *L*.

Solution:



(b) The number of chairs, ***C***, that can be placed around a given number of tables, ***T***, for either arrangement, ***L*** or ***M***, forms a pattern. The values for ***C*** for the first 3 diagrams for both arrangements are shown in the table below. Study the pattern of numbers in each row of the table.

Complete the rows numbered (i), (ii) and (iii).

	Number of Tables (*T*)	**Arrangement *L*** Number of Chairs (*C*)	**Arrangement *M*** Number of Chairs (*C*)
	1	10	10
	2	14	16
	3	18	22
(i)	4	________	________
	⋮	⋮	⋮
(ii)	________	________	130
	⋮	⋮	⋮
(iii)	*n*	________	________

SOLUTION:

Data: Incomplete table showing the pattern of the number of chairs placed around the tables for both arrangements ***L*** and ***M***.

Required to complete: The table given.

Solution:

(i) For Arrangement ***L:***
Values of ***C*** are 10, 14, 18, …
There is an increase by 4.

When $T = 4$, $C = 18 + 4 = 22$

For Arrangement M:
The values of C are 10, 16, 22, …

There is an increase by 6.

When $T = 4$, $C = 22 + 4 = 26$

Parts (ii) and (iii)

For Arrangement ***L:***
Values of ***C*** are 10, 14, 18, …
Since there is an increase by 4.

$\therefore C = 4T + \alpha$, α is a constant
When $T = 1$, $C = 10$
$10 = 4(1) + 6$
$\therefore \alpha = 6$
$\therefore C = 4T + 6$
Test:

When $T = 2$	When $T = 3$
$C = 4(2) + 6$	$C = 4(3) + 6$
$= 14$	$= 18$

Hence, the formula $C = 4\,T + 6$ holds true for arrangement L.

For Arrangement M:
The values of C are 10, 16, 22, …
Since there is an increase by 6.

$C = 6T + \beta$, β is a constant
When $T = 1$, $C = 10$
$10 = 6(1) + \beta$
$\therefore \beta = 4$
$\therefore C = 6T + 4$

Test:

When $T = 2$	When $T = 3$
$C = 6(2) + 4$	$C = 6(3) + 4$
$= 16$	$= 22$

Hence, the formula $C = 6\,T + 6$ holds true for arrangement M

(ii) When $C = 130$

$$130 = 6T + 4$$

$$T = \frac{130 - 4}{6}$$

$$T = 21$$

When $T = 21$

$$4T + 6 = 4(21) + 6$$

$$= 90$$

Alternative Method for part (iii):

Arrangement L					
No. of tables (T)	1	2	3	4	T
No. of Chairs (C)	10	10+4(1) =14	10+4(2) =18	10+4(3) =22	10+ 4(T–1) =10+4T–4 =4T+6

Arrangement M					
No. of tables (T)	1	2	3	4	T
No. of Chairs (C)	10	10+6(1) =14	10+6(2) =18	10+6(3) =22	10+ 6(T–1) =10+6T–6 =6T+4

The completed table looks like:

	Number of Tables (T)	**Arrangement L** Number of Chairs (C)	**Arrangement M** Number of Chairs (C)
	1	10	10
	2	14	16
	3	18	22
(i)	4	22	28
	⋮	⋮	⋮
(ii)	21	90	130
	⋮	⋮	⋮
(iii)	n	$4n + 6$	$6n + 4$

(c) Leon needs to arrange the tables to seat 70 people for a birthday party. Which of the arrangements, *L* or *M*, will allow him to rent the LEAST number of tables?

Use calculations to justify your answer.

SOLUTION:

Data: Leon needs to arrange the tables to seat 70 people for a birthday party.

Required to determine: The arrangement, *L* or *M*, that will allow Leon to rent the least number of tables.

Solution:
For arrangement *L*:
$4T + 6 = 70$
$4T = 70 - 6$
$4T = 64$
$T = 16$
Arrangement *L* will use 16 tables.

For arrangement *M*:
$6T + 4 = 70$
$6T = 70 - 4$
$6T = 66$
$T = 11$
Arrangement *M* will use 11 tables.

Therefore, arrangement *M* allows a smaller number of tables to be rented.

SECTION II

Answer ALL questions.

ALL working must be CLEARLY shown.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. A rental company has x cars and y minivans. The company has at least 8 vehicles altogether. The number of minivans is less than or equal to the number of cars. The number of cars is no more than 9.

(a) Write down THREE inequalities, in terms of x and/or y, other than $x \geq 0$ and $y \geq 0$, to represent this information.

SOLUTION:

Data: A rental company has x cars and y minivans. The company has at least 8 vehicles altogether. The number of minivans is less than or equal to the number of cars. The number of cars is no more than 9.

Require to write: Three inequalities in terms of x and/or y, other than $x \geq 0$ and $y \geq 0$, to represent the information given.

Solution:
Number of vehicles is at least 8.
Hence, $x + y \geq 8$ …❶

Number of minivans is less than or equal to the number of cars.
Hence, $y \leq x$ …❷

Number of cars is not more than 9.
Hence, $x \ngtr 9$ or $x \leq 9$ …❸

(b) A car can carry 4 people and a minivan can carry 6 people. There are at most 60 persons to be taken on a tour.

Show that $2x + 3y \leq 30$.

SOLUTION:

Data: A car can carry 4 people and a minivan can carry 6 people. There are at most 60 persons to be taken on a tour.

Required to show: $2x + 3y \leq 30$

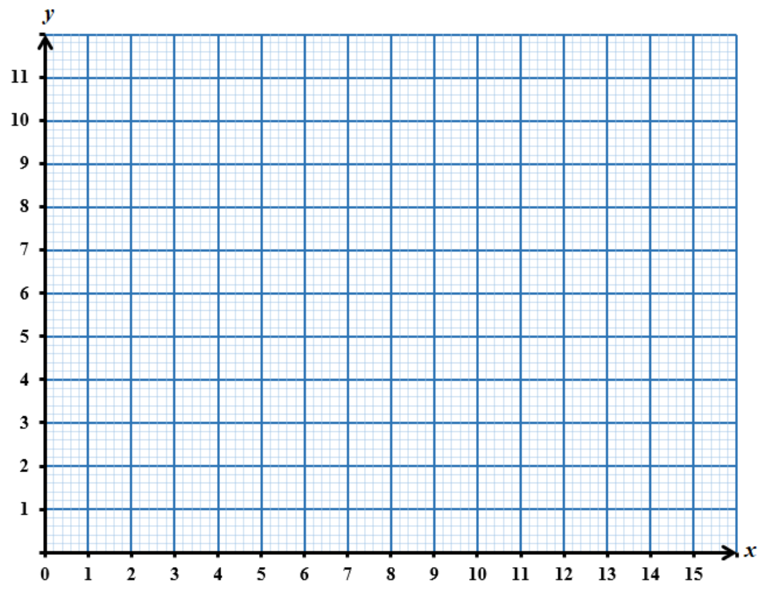
Proof:
Number of persons in cars + Number of persons in minivans is not more than 60.
Hence, $4(x) + 6(y) \leq 60$

$(\div 2)$

$2x + 3y \leq 30$

Q.E.D.

(c) On the grid below, plot the four lines associated with the inequalities in (a) and (b). Shade and label the region that satisfies all four inequalities.



SOLUTION:

Required to plot: The four lines associated with the inequalities in (a) and (b) and to shade the region that satisfies them.

Solution:

Consider $x + y \geq 8$:
To draw the line $x + y = 8$ we obtain two points on the line.

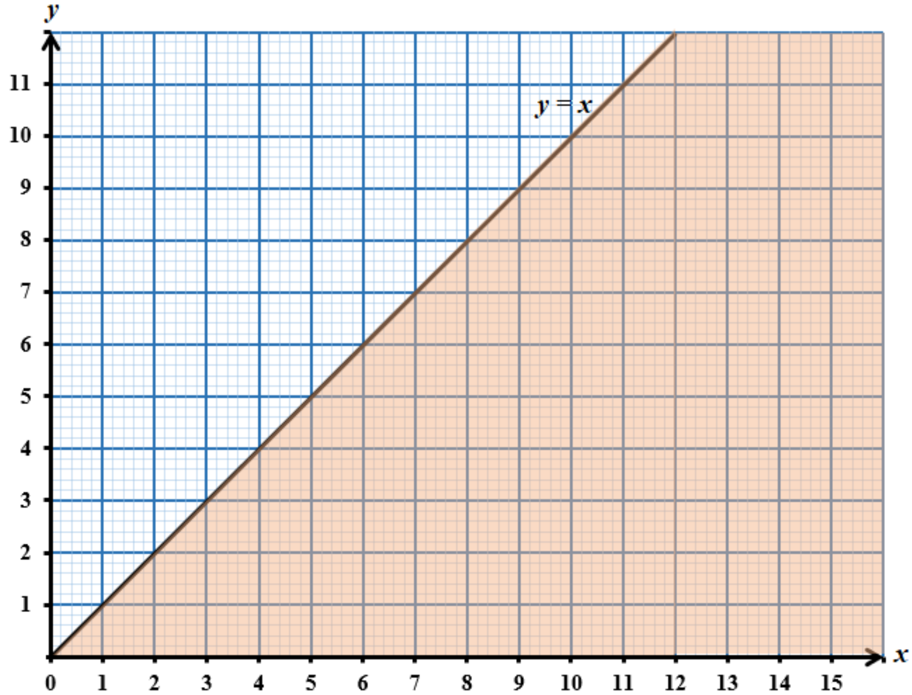
x	*y*
0	8
8	0

The shaded region shows $x + y \geq 8$

Consider $y \leq x$:

To draw the line $y = x$ we obtain two points on the line

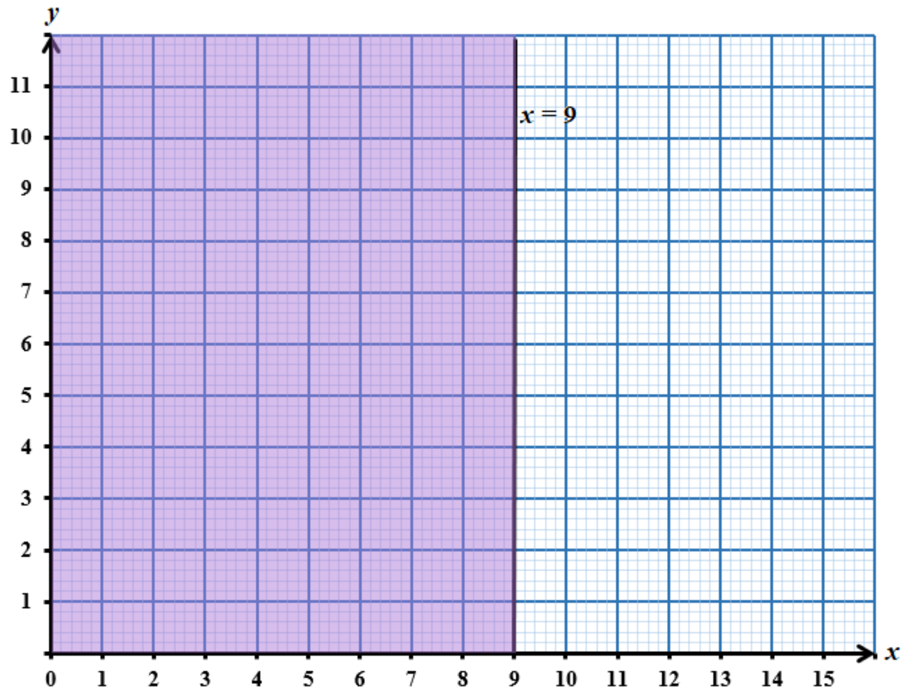
x	y
0	0
12	12



The shaded region shows $y \leq x$

Consider $x \leq 9$:

The line $x = 9$ is a vertical line passing through 9 on the x – axis.

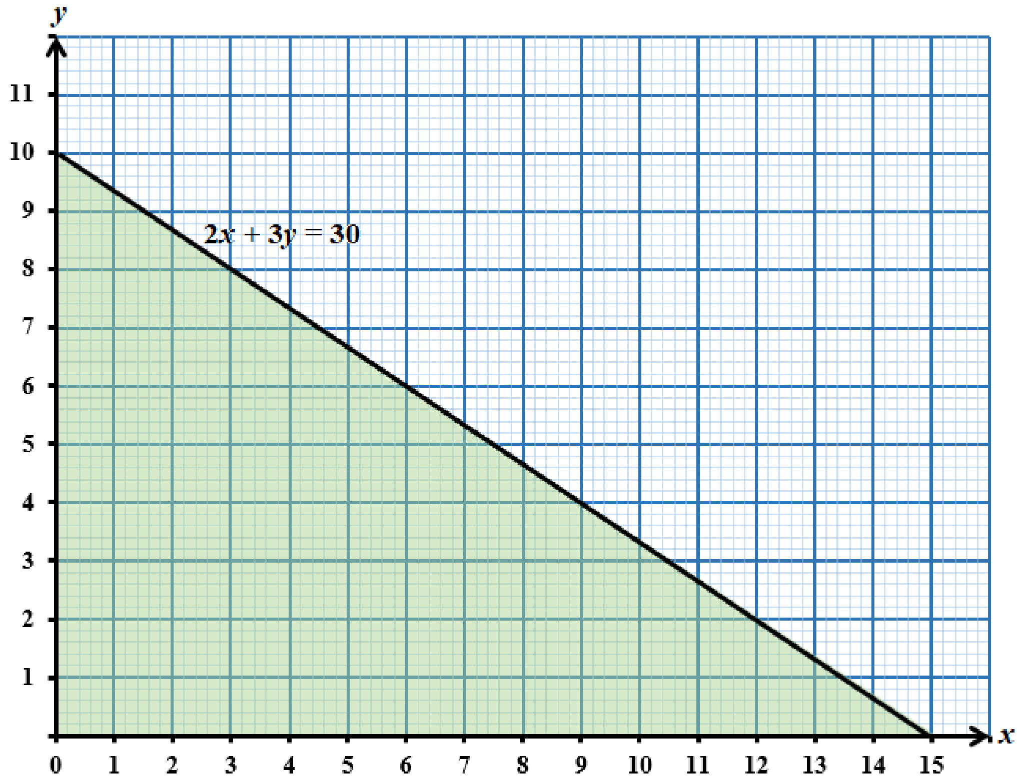


The shaded region shows $x \leq 9$

Consider $2x+3y \leq 30$:

To draw the line $2x+3y=30$ we obtain two points on the line

x	y
0	10
15	0

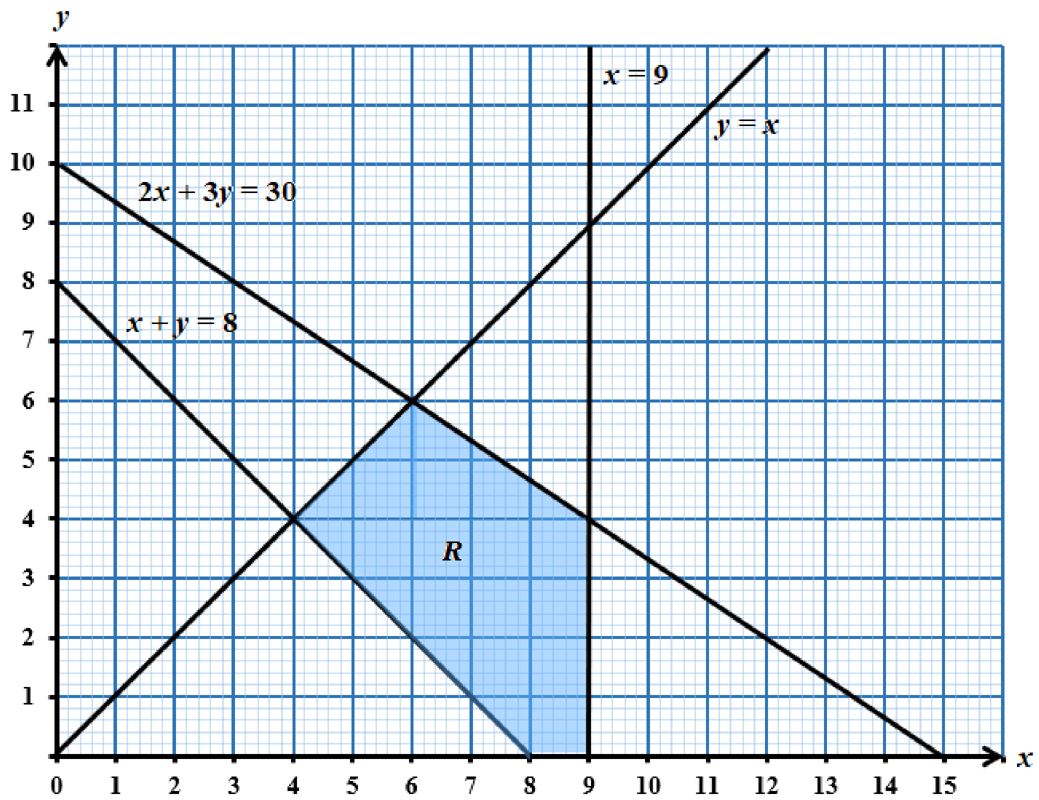


The shaded region shows $2x+3y \leq 30$

We now draw all four inequalities on the same diagram and identify the region that is common to all four inequalities.

This region is to be named as R and is called the 'Feasible Region'.

The completed diagram looks like:



(d) **(i)** Determine the two combinations for the MINIMUM number of cars and minivans that can be used to carry EXACTLY 60 people on the tour.

SOLUTION:

Required to determine: Two combinations for the MINIMUM number of cars and minivans that can be used to carry EXACTLY 60 people on the tour.

Solution:

We list the coordinates of the five vertices of the feasible region. The number of passengers is calculated for each combination of (x, y).

(x, y)	$4x + 6y$	**Number of Passengers**
$(4, 4)$	$4(4)+6(4)$	30
$(6, 6)$	$4(6)+6(6)$	60
$(9, 4)$	$4(9)+6(4)$	60
$(8, 0)$	$4(8)+6(0)$	32
$(9, 0)$	$4(9)+6(0)$	36

The two combinations are 6 cars and 6 minivans OR 9 cars and 4 minivans as shown in the table.

(ii) The company charges \$75 to rent a car and \$90 to rent a minivan. Show that the MINIMUM rental cost for this tour is \$990.

SOLUTION:

Data: The company charges \$75 to rent a car and \$90 to rent a minivan.

Required to show: The minimum cost for this tour is \$990.

Proof:

Cost of renting 6 cars and 6 minivans $=(6\times\$75)+(6\times\$90)$
$=\$990$

Cost of renting 9 cars and 4 minivans $=(9\times\$75)+(4\times\$90)$
$=\$1035$

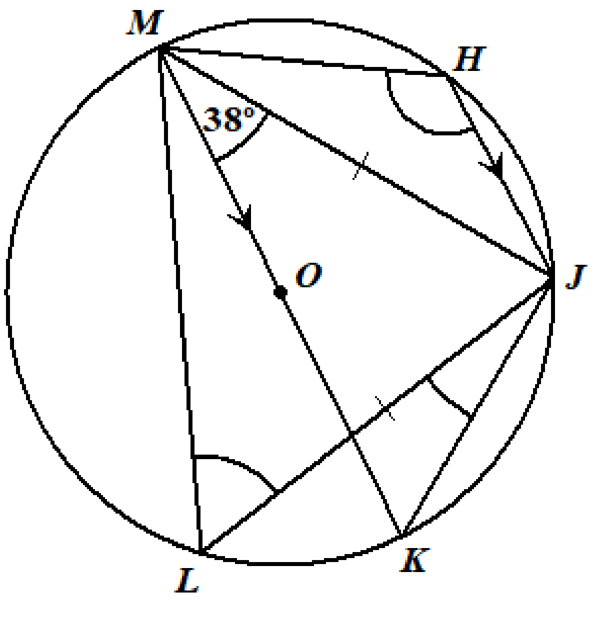
$\$990<\$1035$

$\therefore$ Minimal rental cost is \$990.

Q.E.D.

GEOMETRY AND TRIGONOMETRY

9. (a) H, J, K, L and M are points on the circumference of a circle with centre O. MK is a diameter of the circle and it is parallel to HJ. $MJ=JL$ and angle $JMK=38°$.



(i) Explain, giving a reason, why angle:

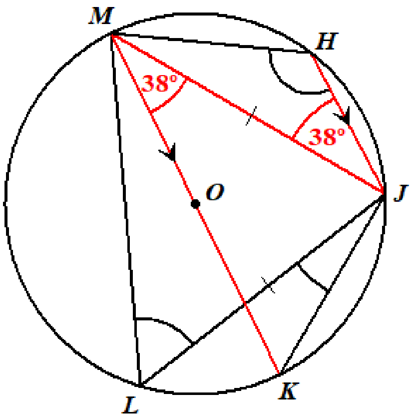
a) $HJM = 38°$

SOLUTION:

Data: Diagram showing a circle with centre *O* with the points *H*, *J*, *K*, *L* and *M* along its circumference. *MK* is a diameter of the circle and it is parallel to *HJ*. $MJ = JL$ and angle $JMK = 38°$.

Required to explain: Why angle $HJM = 38°$

Solution:



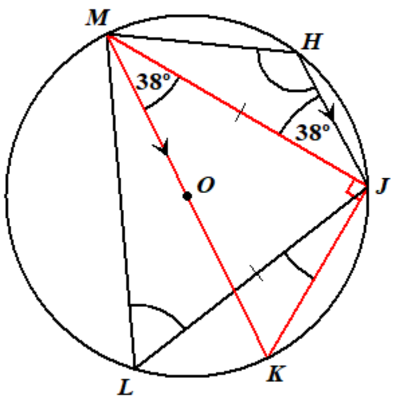
$H\hat{J}M$ is alternate to $J\hat{M}K$. Hence, $H\hat{J}M = 38°$.
(When the parallel lines, *HJ* and *MK*, are cut by the transversal, *JM*, alternate angles are equal).

b) $MJK = 90°$

SOLUTION:

Required to explain: Why angle $MJK = 90°$

Solution:



$M\hat{J}K = 90°$
(Angles in a semi-circle $= 90°$)
MK is a diameter and *J* is the point on the circumference.

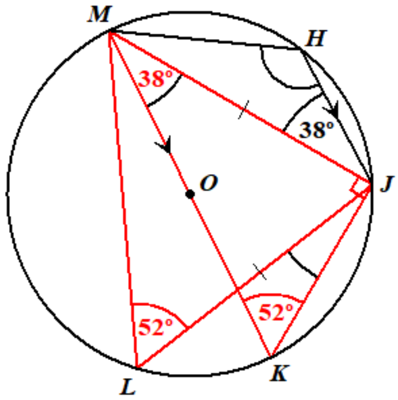
(ii) Determine the value of EACH of the following angles. Show detailed working where appropriate.

a) Angle MLJ

SOLUTION:

Required to determine: The value of angle *MLJ*

Solution:



Consider ΔMKJ :
$M\hat{K}J = 180° - (90° + 38°)$
$= 52°$
(Sum of the interior angles in a triangle is equal to 180°.)

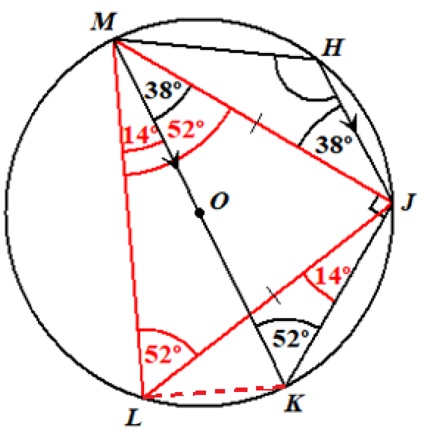
Hence, $M\hat{L}J = 52°$
(Angles $M\hat{K}J$ and $M\hat{L}J$ are subtended by the chord, *MJ*, at the circumference of a circle, and are standing on the same arc, hence, they are equal.)

b) Angle *LJK*

SOLUTION:

Required to determine: The value of angle *LJK*

Solution:



$JM = JL$ (Data)
Therefore, triangle JLM is isosceles
$\therefore J\hat{M}L = 52°$ (Base angles in an isosceles triangle are equal.)

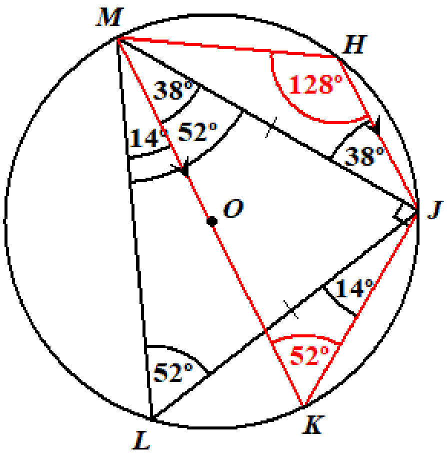
$\therefore L\hat{M}K = 52° - 38°$
$= 14°$
$\therefore L\hat{J}K = 14°$
(Angles subtended by a chord, (LK) at the circumference of a circle, $L\hat{M}K$ and $L\hat{J}K$, and standing on the same arc are equal).
The chord LK may be drawn to illustrate this.

c) Angle JHM

SOLUTION:

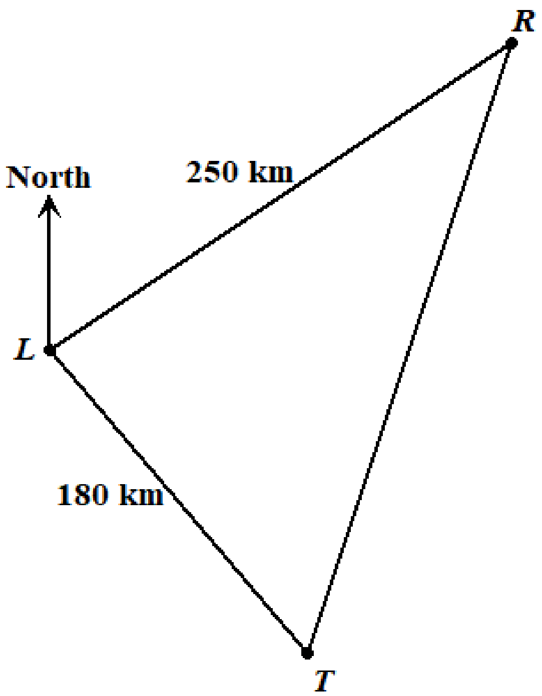
Required to determine: The value of angle JHM

Solution:



Consider the cyclic quadrilateral $JHML$:
$J\hat{H}M = 180° - 52°$
$= 128°$
(The opposite angles in a cyclic quadrilateral are supplementary.)

(b) From a port, *L*, ship *R* is 250 kilometres on a bearing of 065°. Ship *T* is 180 kilometres from *L* on a bearing of 148°. This information is illustrated in the diagram below.



(i) Complete the diagram above by inserting the value of angle *RLT*.

SOLUTION:

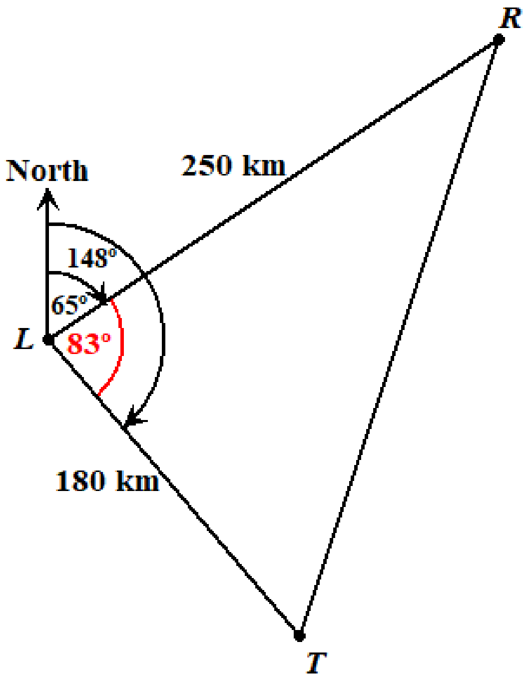
Data: Diagram showing ship *R* 250 kilometres away from port *L* on a bearing of 065°. Ship *T* is 180 kilometres from *L* on a bearing of 148°.

Required to complete: The diagram by inserting the value of angle *RLT*.

Solution:

The angle $RLT = 148^0 - 65^0 = 83^0$

This is illustrated on the diagram shown below



(ii) Calculate *RT*, the distance between the two ships.

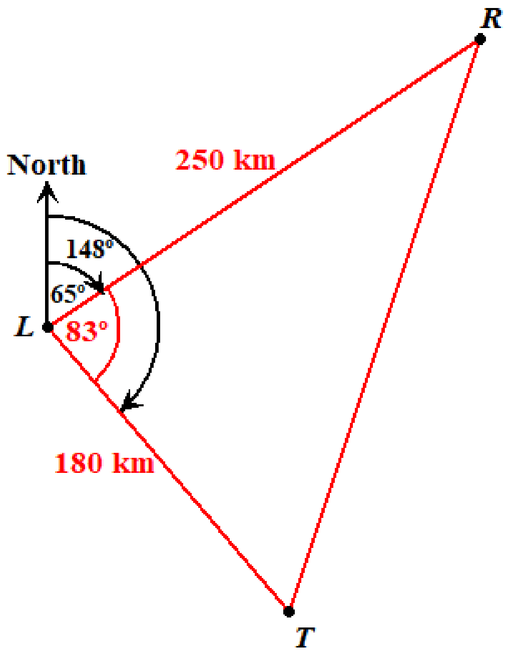
SOLUTION:

Required to calculate: The distance *RT*

Calculation:

$$R\hat{L}T = 148° - 65°$$
$$= 83°$$

Consider ΔRLT



Using the cosine law we get:

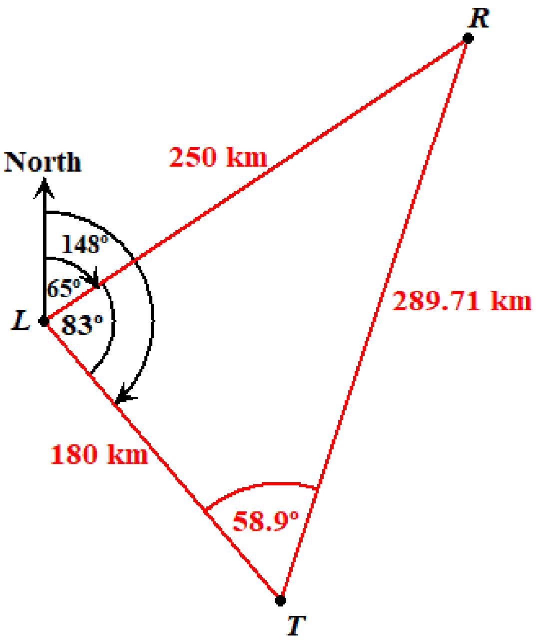
$$RT^2 = (250)^2 + (180)^2 - 2(250)(180)\cos 83°$$
$$= 62\,500 + 32\,400 - 90\,000(0.1219)$$
$$= 83931.75909$$
$$RT = 289.71 \text{ km (correct to 2 decimal places)}$$

(iii) Determine the bearing of T from R.

SOLUTION:

Required to determine: The bearing of T from R

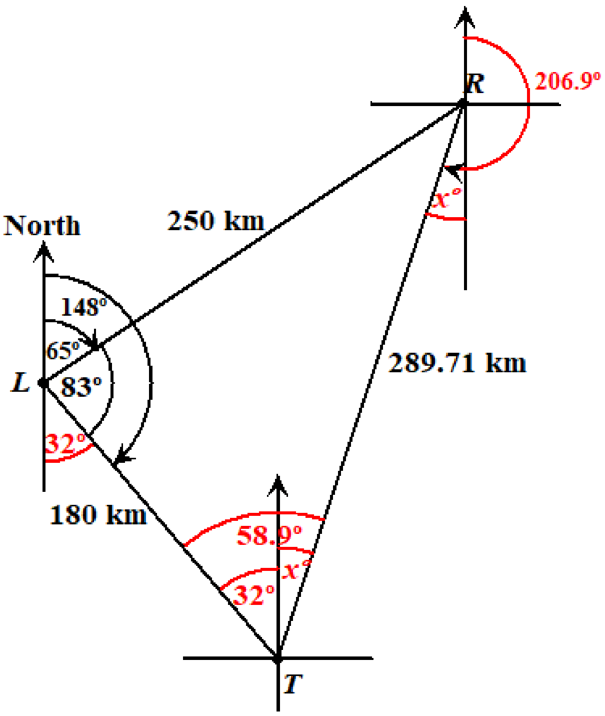
Solution:



Using the sine rule in ΔRLT:

$$\frac{RT}{\sin 83°} = \frac{LR}{\sin L\hat{T}R}$$
$$\frac{289.71}{\sin 83°} = \frac{250}{\sin L\hat{T}R}$$
$$\sin L\hat{T}R = \frac{250 \times \sin 83°}{289.71}$$
$$= 0.8565$$
$$L\hat{T}R = \sin^{-1}(0.8565)$$
$$= 58.9°$$

The bearing of T from R is $(180 + x)^0$, where x is the angle formed by TR and the South line at R (see diagram below).



The angle between the North at L and LT is 148^0. So, the angle between LT and the North line at T will be $180^0 – 148^0 = 32^0$ (co-interior angles are supplementary)

$$x = 58.9° - 32°$$
$$= 26.9°$$

The angle between TR and the South line is x^0 (alternate angles)

Hence, the bearing of T from R $= 180° + x°$

$\therefore$ Bearing of T from R $= 180° + 26.9°$
$= 206.9°$

VECTORS AND MATRICES

10. (a) The transformation matrix $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ represents a rotation of 90° anticlockwise about the origin O.

The transformation matrix $B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ represents a reflection in the straight line with equation $y = -x$.

(i) Write the coordinates of P', the image of the point $P\,(7, 11)$ after it undergoes a rotation of 90° anticlockwise about the origin, *O*.

SOLUTION:

Data: $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ represents a rotation of 90° anticlockwise about the origin *O* and $B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ represents a reflection in the straight line with equation $y = -x$.

Required to write: the coordinates of P', the image of the point $P\,(7, 11)$
after it undergoes a rotation of 90° anticlockwise about the origin, *O*.

Solution:

$$P \xrightarrow{A} P'$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 7 \\ 11 \end{pmatrix} = \begin{pmatrix} e_{11} \\ e_{21} \end{pmatrix}$$

$$2\times 2 \;\times 2\times 1 = 2\times 1$$

$$\begin{aligned} e_{11} &= (0\times 7) + (-1\times 11) \\ &= -11 \\ e_{21} &= (1\times 7) + (0\times 11) \\ &= 7 \end{aligned}$$

Hence, the image $P' = (-11, 7)$.

(ii) *T* is the combined transformation of *A* followed by *B*. Determine the elements of the matrix representing the transformation *T*.

SOLUTION:

Data: *T* is the combined transformation of *A* followed by *B*.

Required to determine: The elements of the matrix representing the transformation *T*.

Solution:
$T \to$ A followed by B

$\therefore T = BA$

$$T = \underset{2\times 2}{\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}} \underset{\times}{} \underset{2\times 2}{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}$$

$$T = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$$

$$\begin{aligned} e_{11} &= (0\times 0)+(-1\times 1) \\ &= -1 \\ e_{12} &= (0\times -1)+(-1\times 0) \\ &= 0 \\ e_{21} &= (-1\times 0)+(0\times 1) \\ &= 0 \\ e_{22} &= (-1\times -1)+(0\times 0) \\ &= 1 \end{aligned}$$

$$\therefore T = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(iii) Describe geometrically, the transformation represented by *T*.

SOLUTION:

Required to describe: The transformation represented by *T*

Solution:
T is a reflection in the x – axis.

(b) The 2×2 matrix *C* is defined, in terms of a scalar constant *k*, by $C = \begin{pmatrix} 3 & k \\ 6 & 4 \end{pmatrix}$.

Determine the value of *k*, given that the matrix *C* is singular.

SOLUTION:

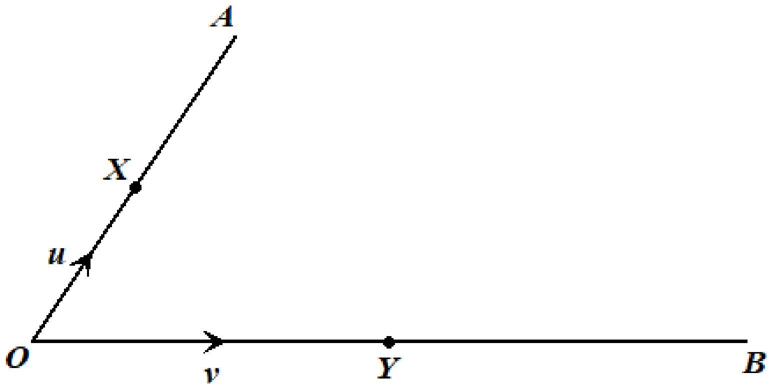
Data: The singular matrix *C* is such that $C = \begin{pmatrix} 3 & k \\ 6 & 4 \end{pmatrix}$.

Required to determine: The value of *k*

Solution:
When C is singular, then the determinant, $|C| = 0$

$$(3 \times 4) - (k \times 6) = 0$$
$$12 - 6k = 0$$
Hence, $6k = 12$
$(\div 6)$
$$k = 2$$

(c) In the diagram below, O is the origin, $\overrightarrow{OX} = u$ and $\overrightarrow{OY} = v$. OX and OY are extended so that X and Y are the midpoints of OA and OB respectively.



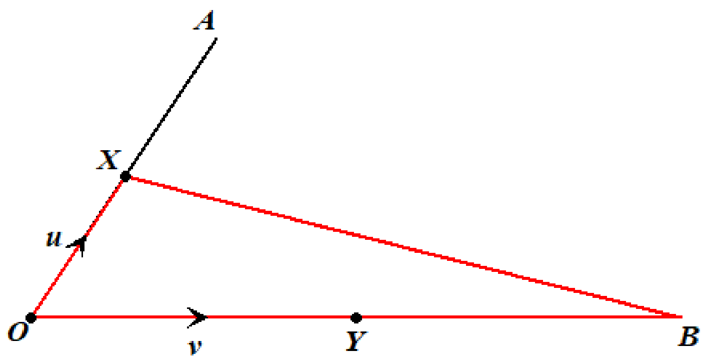
(i) Express $\overrightarrow{BX}$ in terms of u an v.

SOLUTION:

Data: Diagram showing $\overrightarrow{OX} = u$ and $\overrightarrow{OY} = v$, where O is the origin. OX and OY are extended so that X and Y are the midpoints of OA and OB respectively.

Required to express: $\overrightarrow{BX}$ in terms of u an v

Solution:



$\overrightarrow{XA} = u$ and $\overrightarrow{OA} = 2u$
$\overrightarrow{YB} = v$ and $\overrightarrow{OB} = 2v$

$$\overrightarrow{BX} = \overrightarrow{BO} + \overrightarrow{OX}$$
$$= -(2v) + u$$
$$= u - 2v$$

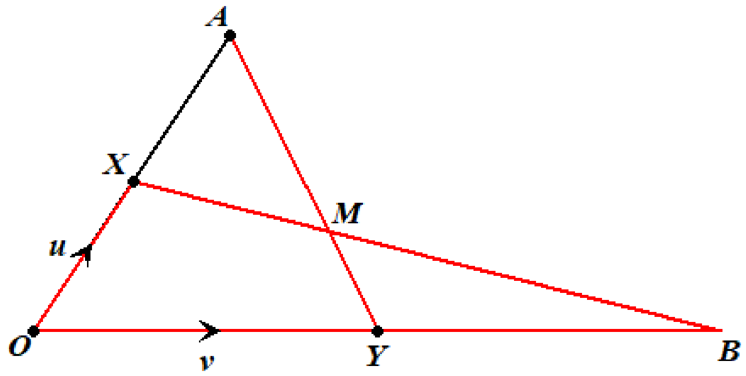
(ii) Given that YA and BX intersect at M and $BM = 2MX$.

a) express $\overrightarrow{BM}$ in terms of u and v

SOLUTION:

Data: YA and BX intersect at M and $BM = 2MX$.
Required to express: $\overrightarrow{BM}$ in terms of u and v

Solution:



$$BM = 2MX$$
$$\therefore BM = \frac{2}{3}BX$$
$$\overrightarrow{BM} = \frac{2}{3}(u - 2v)$$
$$= \frac{2}{3}u - \frac{4}{3}v$$

b) Using a vector method, show that the ratio $YM : YA$ is $1:3$. Show ALL working.

SOLUTION:

Required to show: The ratio $YM : YA$ is $1:3$.

Proof:

Using the triangle law of vector addition

$$\overrightarrow{YM} = \overrightarrow{YB} + \overrightarrow{BM}$$

$$= v + \left(\frac{2}{3}u - \frac{4}{3}v\right)$$

$$= \frac{2}{3}u - \frac{1}{3}v$$

$$= \frac{1}{3}(2u - v)$$

$$\overrightarrow{YA} = \overrightarrow{YO} + \overrightarrow{OA}$$

$$= -(v) + 2u$$

$$= 2u - v$$

$$YM : YA$$

$$= \frac{1}{3}(2u - v) : (2u - v)$$

$$= \frac{1}{3} : 1$$

$$= 1 : 3$$

Q.E.D.