FAS-PASS Maths

CSEC MATHEMATICS JANUARY 2022 PAPER 2

SECTION 1

Answer ALL questions.

All working must be clearly shown.

1. **(a)** Using a calculator, or otherwise, find:

(i) the EXACT value of $\frac{8.9+31.6}{0.75\times 5.4}$

SOLUTION:
Required to calculate: $\frac{8.9+31.6}{0.75\times 5.4}$ exactly
Calculation:

We work the numerator and the denominator separately

Numerator: $8.9+31.6=40.5$ (by the calculator)

Denominator: $0.75\times 5.4=4.05$ (By the calculator)

So,

$$\begin{aligned}\frac{8.9+31.6}{0.75\times 5.4}&=\frac{\text{Numerator}}{\text{Denominator}}\\&=\frac{40.5}{4.05}\\&=10\end{aligned}$$

(the answer is given in exact form)

(ii) the value of $3.9\tan(18^\circ)$ correct to 1 decimal place.

SOLUTION:
Required to calculate: $3.9\tan(18^\circ)$ correct to 1 decimal place
Calculation:
Using the calculator, we obtain

$$3.9\tan(18^\circ)=1.2\underline{\underline{6}}$$

$$+1\uparrow \quad \geq 5$$

$$\approx 1.3 \text{ (correct to 1 decimal place)}$$

(b) **(i)** Ria is paid at the rate of \$13.50 per hour. During a certain week she worked 40 hours. How much did she earn that week?

SOLUTION:
Data: Ria is paid at the rate of \$13.50 per hour and she worked 40 hours during a certain week.
Required to calculate: Ria's earning for that week
Calculation:

$$\begin{aligned}\text{Ria's earnings} &= \text{Hourly rate} \times \text{Number of hours worked} \\ &= \$13.50 \times 40 \\ &= \$540.00\end{aligned}$$

(ii) Ria worked 4 weeks in the month of August and her gross earnings was \$2463.75. Her regular week comprised 40 hours and overtime was paid at $1\frac{1}{2}$ times the hourly rate.

Show that Ria worked 15 hours overtime in August.

SOLUTION:
Data: Ria worked 4 weeks in August and earned \$2463.75. Her regular week comprised 40 hours and overtime was paid at time and a half.
Required to show: Ria worked 15 hours of overtime in August.
Solution:
At \$540 per regular 40-hour week for 4 weeks, Ria earnings would be

$$\begin{aligned}&= \$540 \times 4 \\ &= \$2160\end{aligned}$$

$$\begin{aligned}\text{Hence, the amount Ria earned in overtime} &= \$2463.75 - \$2160.00 \\ &= \$303.75\end{aligned}$$

$$\begin{aligned}\text{Ria's overtime rate per hour} &= 1\frac{1}{2} \times \$13.50 \\ &= \$20.25\end{aligned}$$

Hence, the number of overtime hours that Ria worked

$$\begin{aligned}&= \frac{\text{Overtime pay}}{\text{Overtime rate per hour}} \\ &= \frac{\$303.75}{\$20.25} \text{ hours} \\ &= 15 \text{ hours}\end{aligned}$$

Q.E.D.

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(iii) In August, 20% of Ria's gross earnings was deducted as tax. How much money does she have left after the deduction?

SOLUTION:
Data: 20% of Ria's gross earnings is deducted as tax.
Required to calculate: The amount of money Ria has left after the deduction.
Calculation:

$$\text{Tax} = 20\% \text{ of } \$2463.75$$
$$= \frac{20}{100} \times \$2463.75$$
$$= \$492.75$$

$\therefore$ After the deduction, Ria has $\$2463.75 - \$492.75 = \$1971$

Alternative Method:

$$\text{Amount after tax} = (100-20)\% \text{ of } \$2463.75$$
$$= \frac{80}{100} \times \$2463.75$$
$$= \$1971$$

(iv) Ria invested \$219 of her earnings for 3 years at a rate of 4.5% per annum simple interest. How much interest does she receive after 3 years?

SOLUTION:
Data: Ria invested \$219 of her earnings for 3 years at a rate of 4.5% per annum simple interest.
Required to calculate: The interest Ria earns after 3 years
Calculation:

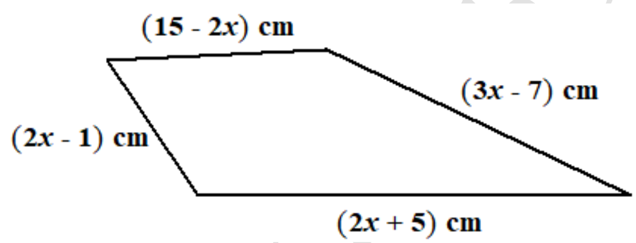
$$\text{Simple interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$
$$= \frac{\$219 \times 4.5 \times 3}{100}$$
$$= \$29.56\underline{\underline{5}}$$
$$= \$29.57 \text{ (correct to nearest cent)}$$

2. **(a)** Factorise completely

$$3n^2 + 15np$$

SOLUTION:
Required to factorise: $3n^2 + 15np$
Solution:
$3n^2 + 15np = 3.n.n + 3.5.n.p$
(3 and n are common to both terms)
So, $3n^2 + 15np = 3n(n + 5p)$

(b) The diagram below shows a quadrilateral with the length of its sides written in terms of x.



(i) Write an expression, in terms of x, for the perimeter of the quadrilateral. Express your answer in its simplest form.

SOLUTION:
Data: Diagram showing a quadrilateral with the lengths of its sides written in terms of x.
Required to write: An expression for the perimeter of the quadrilateral, in terms of x, in its simplest form.
Solution:

$$\begin{aligned}\text{Perimeter } &= (15 - 2x) + (3x - 7) + (2x + 5) + (2x - 1) \\ &= 15 - 7 + 5 - 1 - 2x + 3x + 2x + 2x \\ &= 12 + 5x\end{aligned}$$

(expressed in its simplest form)

(ii) The perimeter of the quadrilateral is 32 cm.

Find the length of the longest side of the quadrilateral.

SOLUTION:
Data: The perimeter of the quadrilateral is 32 cm.
Required to find: The length of the longest side of the quadrilateral.
Solution:

Perimeter $=32$

Hence, $12+5x=32$

$$5x=32-12$$
$$5x=20$$
$$x=\frac{20}{5}$$
$$x=4$$

Let us consider separately the length of each side of the quadrilateral when $x=4$:

$15-2x=15-2(4)=15-8=7$

$3x-7=3(4)-7=12-7=5$

$2x+5=2(4)+5=8+5=13$

$2x-1=2(4)-1=8-1=7$

Hence, the longest side is $2x+5=13$ cm.

(c) Determine ALL the integer values of *x* which satisfy the inequality

$$-1<\frac{2-4x}{3}<5.$$

SOLUTION:

Required to determine: All the integer values of *x* for which $-1<\frac{2-4x}{3}<5$

Solution:

Consider $-1<\frac{2-4x}{3}$

$\times 3$

$$-3<2-4x$$
$$-5<-4x$$

$\times -1$

$$5>4x$$
$$4x<5$$
$$x<1\frac{1}{4}$$

Consider $\frac{2-4x}{3}<5$

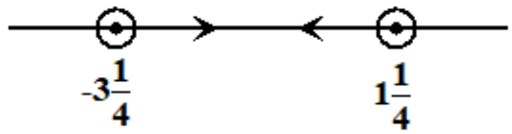
$\times 3$

$$2-4x<15$$
$$-4x<13$$

$\times -1$

$$4x>-13$$
$$x>-3\frac{1}{4}$$

Representing the solution on a number line, we have:



Hence, the integer values of x between $-3\frac{1}{4}$ and $1\frac{1}{4}$ are $\{-3, -2, -1, 0, 1\}$

3. **(a)** The box below contains the names of 5 quadrilaterals.

Trapezium	Rhombus	
Kite	Square	Rectangle

Choose the name of one quadrilateral from the box that BEST completes each statement.

(i) A …………………… has no lines of symmetry and has rotational symmetry of order one.

SOLUTION:
Data: List of five quadrilaterals
Required to complete: The statement given using one of the quadrilaterals given.
Solution:
A trapezium has no lines of 'line or reflective' symmetry and has rotational symmetry of order one.

A trapezium was considered the best fit although, strictly speaking, it has no rotational symmetry.
Rotational symmetry of order one does NOT really exist. If a shape matches itself ONLY ONCE after a full rotation about its center, then there is no rotational symmetry. In fact, every object will match itself once after a rotation of 360^0.
The order of rotational symmetry is defined as the number of times that a shape fits or matches itself in a 360^0 rotation about its center. The smallest order is two. We only count the original position if the shape matched itself at least once before returning to its original position. Hence, the order of rotational symmetry of a rectangle is two. This shape matches itself once after a rotation of 180^0 and then again when it is back to its original position.

(ii) A ……………………. has EXACTLY two lines of symmetry and 4 right angles.

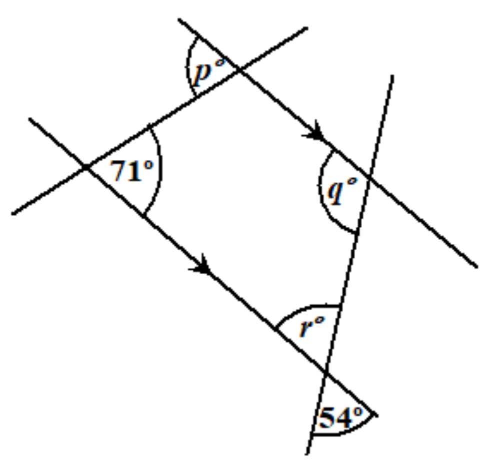
SOLUTION:
Required to complete: The statement given using one of the quadrilaterals given.
Solution:
A rectangle has EXACTLY two lines of line or reflective symmetry and 4 right angles.

(iii) A ………………………. has one line of symmetry but no rotational symmetry.

SOLUTION:
Required to complete: The statement given using one of the quadrilaterals given.
Solution:
A kite has one line of line or reflective symmetry but no rotational symmetry.

A kite and a trapezium, are similar in the respect that they both fit their image once in a complete rotation. Based on this explanation neither has rotational symmetry. Our answer to this part re-affirms the arguments presented above. Hence, there is a conflict in the question because part (a) suggests that a trapezium has rotational symmetry of order one and part (c) suggests that a kite has no rotational symmetry when, in fact, both shapes should be in the same category with respect to this property.

(b) The diagram below shows 4 straight lines, 2 of which are parallel.



(i) Determine the values of q and r.

SOLUTION:
Data: Diagram made up of 4 straight lines, 2 of which are parallel.
Required to determine: The value of q and of r.
Solution:

$q + r = 180°$ (Co-interior angles are supplementary.)

$\therefore q = 180° - 54°$

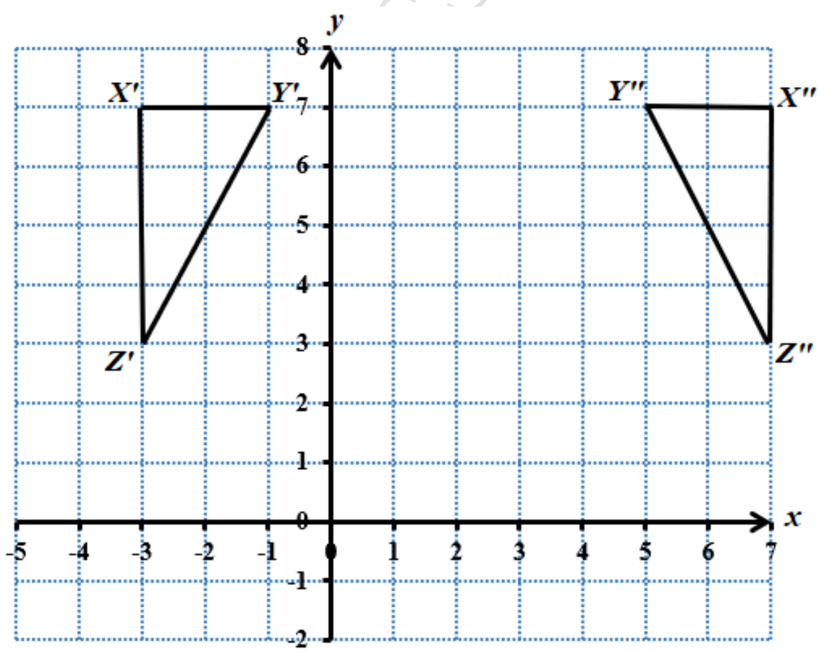
$= 126°$

OR $r = 54^0$ (Vertically opposite angle to the angle marked 54°)

(ii) Give a geometrical reason why $\angle p = 71°$.

SOLUTION:
Required to give: A reason why $\angle p = 71°$
Solution:
Angle $p = 71°$ since it is alternate to the angle marked 71° in the figure, and alternate angles are equal.

(c) The diagram below shows triangles $X'Y'Z'$ and $X''Y''Z''$ drawn on a square grid.



(i) Triangle $X'Y'Z'$ is the image of Triangle XYZ after an enlargement of scale factor 2, with center $(5, 1)$.

Draw triangle XYZ, the OBJECT for Triangle $X'Y'Z'$, on the grid above.

SOLUTION:
Data: Diagram showing triangles $X'Y'Z'$ and $X''Y''Z''$ drawn on a square grid. Triangle $X'Y'Z'$ is the image of Triangle XYZ after an enlargement of scale factor 2, with center $(5, 1)$
Required to draw: Triangle XYZ on the grid.
Solution:
Since the scale factor is 2,

$$\frac{CX'}{CX} = \frac{CY'}{CY} = \frac{CY'}{CY} = \frac{2}{1}$$

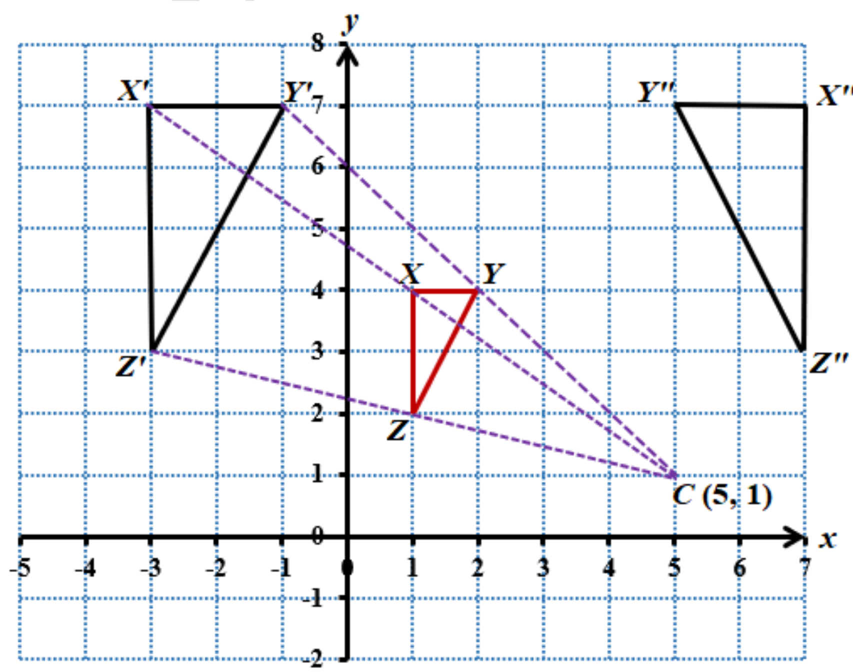
Hence, X', Y and Z are the mid-points of CX', CY' and CZ' respectively.

$C = (5, 1)$ $X' = (-3,7)$
Midpoint of $CX' = (1, 4)$
Hence, X is (1, 4)
$C = (5, 1)$ $Z' = (-3, 3)$
Midpoint of $CZ' = (1, 2)$
$\therefore Z = (1, 2)$
$C = (5, 1)$ $Y' = (-1, 7)$
Midpoint of $CY' = (2, 4)$
Hence, Y is (2, 4)

The coordinates of the mid-point of a line are $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$, so the mid-point of CX' is

$$\frac{5+(-3)}{2}, \frac{1+7}{2} = (1,4)$$

The points X, Y and Z are plotted and joined to obtained the object triangle XY as shown.



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(ii) Triangle $X'Y'Z'$ is mapped onto Triangle $X''Y''Z''$ by a reflection in the line *P*. State the equation of the mirror line, *P*.

SOLUTION:
Data: Triangle $X'Y'Z'$ is mapped onto Triangle $X''Y''Z''$ by a reflection in the line *P*.
Required to state: The equation of the mirror line, *P*.
Solution:
We choose one pair of object and image points, say Y and Y', the perpendicular bisector of $Y'Y''$ is the vertical passing through the point $x = 2$.
Hence, $x = 2$ is the equation of the mirror line, *P*.

Note: This line would also be the perpendicular bisector of $X'X''$ or $Z'Z''$.

4. Three functions *f*, *g* and *h* are defined as

$f(x) = 2x - 1$; $g(x) = 3x + 2$ and $h(x) = 5^x$.

(a) Find the value of

(i) $f\left(\frac{1}{2}\right)$

SOLUTION:
Data: $f(x) = 2x - 1$; $g(x) = 3x + 2$ and $h(x) = 5^x$
Required to find: $f\left(\frac{1}{2}\right)$
Solution:

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) - 1$$
$$= 0$$

(ii) $h(0)$

SOLUTION:
Required to find: $h(0)$
Solution:

$$h(0) = 5^0$$
$$= 1$$

(iii) $g^2(-3)$

SOLUTION:
Required to find: $g^2(-3)$
Solution:

$g(x) = 3x + 2$
$g^2(x) = 3(3x + 2) + 2$
$= 9x + 8$

$g^2(-3) = 9(-3) + 8$
$= -19$

Alternative Method:
$g^2(-3) = g[g(-3)]$
$g(-3) = 3(-3) + 2$
$= -7$
So $g^2(-3) = g(-7)$
$= 3(-7) + 2$
$= -19$

(b) Find $gf(x)$, giving your answer in its simplest form

SOLUTION:
Required to find: $gf(x)$

Solution:
$gf(x) = g[f(x)]$
$= g(2x - 1)$
$= 3(2x - 1) + 2$
$= 6x - 3 + 2$
$= 6x - 1$

(c) **(i)** Find $g^{-1}(x)$.

SOLUTION:
Required to find: $g^{-1}(x)$
Solution:
$g(x) = 3x + 2$
Let $y = 3x + 2$
Interchange x and y

$$x = 3y + 2$$
$$3y = x - 2$$

$$y = \frac{x-2}{3}$$
$$g^{-1}(x) = \frac{x-2}{3}$$

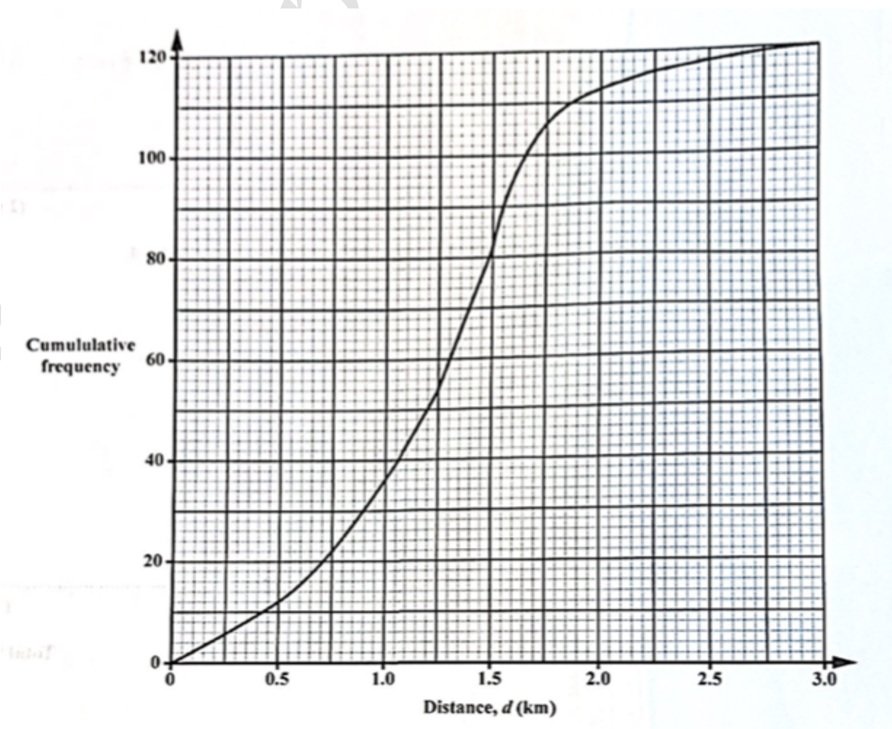
(ii) Hence, or otherwise, determine the value of x when $g^{-1}(x) = 4$.

SOLUTION:
Data: $g^{-1}(x) = 4$
Required to determine: The value of x
Solution:

$$g^{-1}(x) = 4$$
$$\therefore \frac{x-2}{3} = 4$$
$$x - 2 = 12$$
$$x = 14$$

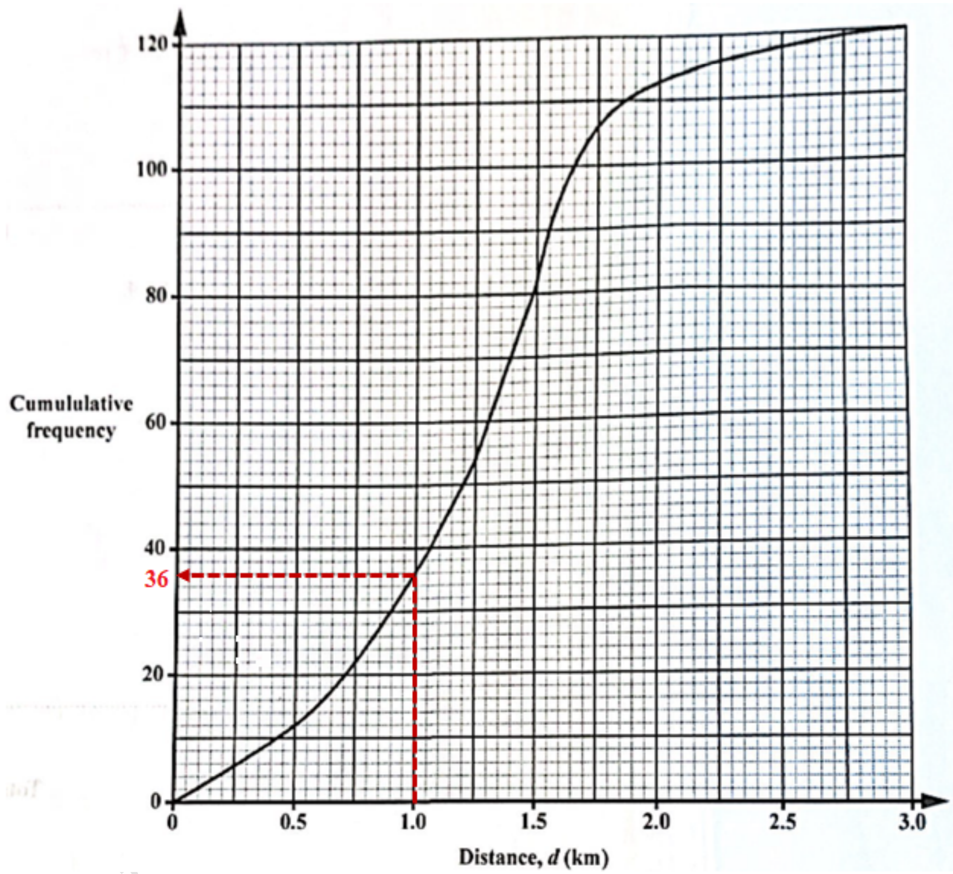
OR $g^{-1}(x) = 4$ implies $x = g(4)$
$g(4) = 3(4) + 2 = 14$

5. The cumulative frequency diagram below shows information about the distance, d km, that each of 120 students walks to school on a particular day.



(a) How many students walked AT MOST 1 km to school on that day?

SOLUTION:
Data: Cumulative frequency curve showing information about the distance, d km, that each of 120 students walks to school on a particular day.
Required to find: The number of students who walked at most 1 km on that day.
Solution:



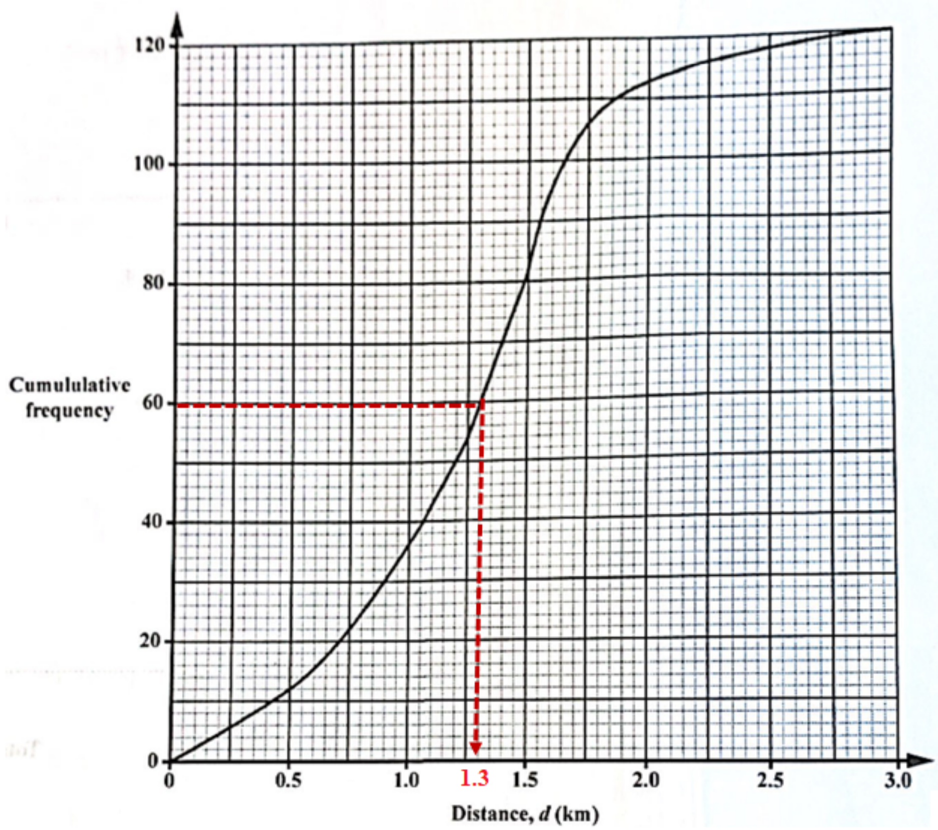
We draw a vertical from 1 km on the horizontal axis to meet the curve. At the point where the vertical meets the curve, we draw a horizontal to meet the vertical axis. We read off 36.
∴36 students walked at most 1 km on that day.

(b) Using the cumulative frequency diagram, determine an estimate of

(i) the median

SOLUTION:
Required to find: The median
Solution:

Position of median $=\frac{1}{2}(120)=60$



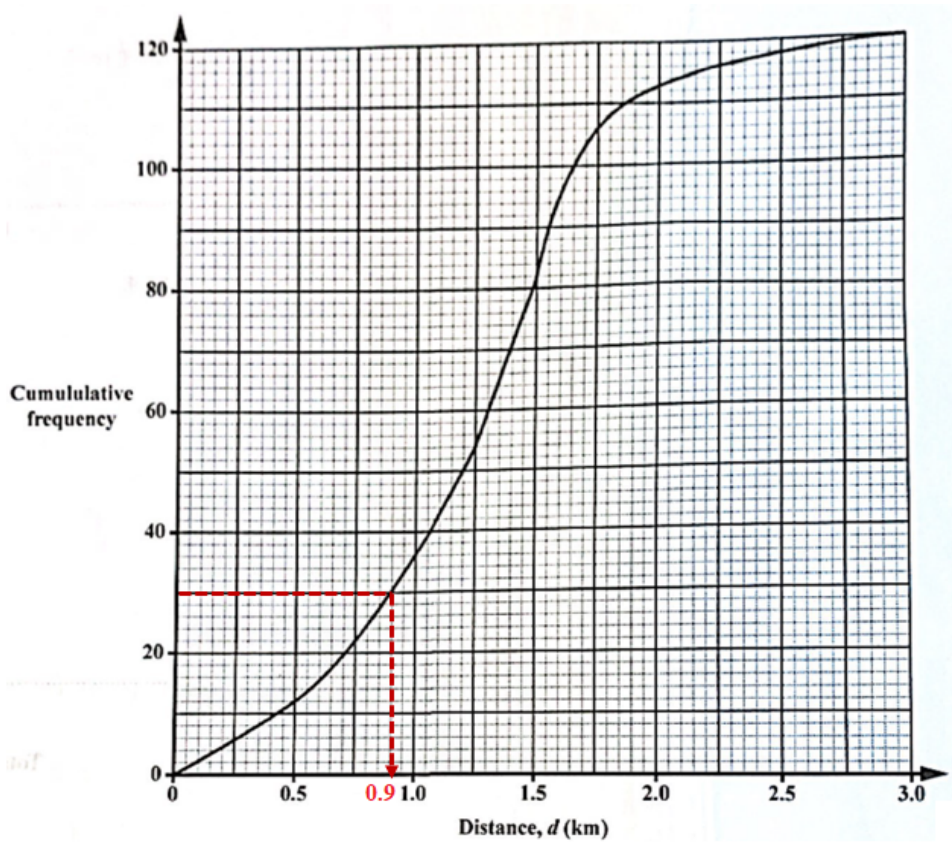
Half of the cumulative frequency is ½ (120) = 60. We draw a horizontal from 60 to meet the curve. At the point where the horizontal meets the curve, we draw a vertical to meet the horizontal axis. We read off 1.3.

$\therefore$Median 1.3 km

(ii) the lower quartile

SOLUTION:
Required to find: The lower quartile
Solution:

Position of lower quartile $= \frac{1}{4}(120)$
$= 30$



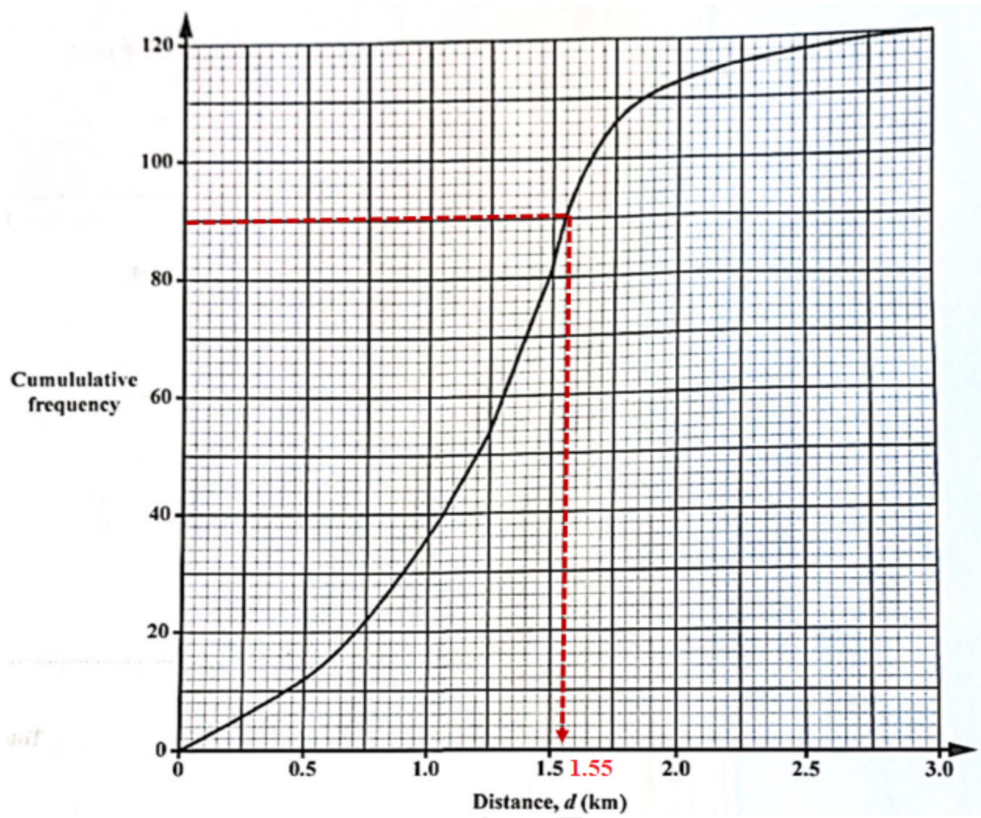
One quarter of the cumulative frequency is ¼ (120) = 30. We draw a horizontal from 30 to meet the curve. At the point where the horizontal meets the curve, we draw a vertical to meet the horizontal axis. We read off 0.9.

$\therefore$ Lower quartile $= 0.9$ km

(iii) the interquartile range.

SOLUTION:
Required to find: The interquartile range
Solution:

$$\text{Position of upper quartile } = \frac{3}{4}(120)$$
$$= 90$$



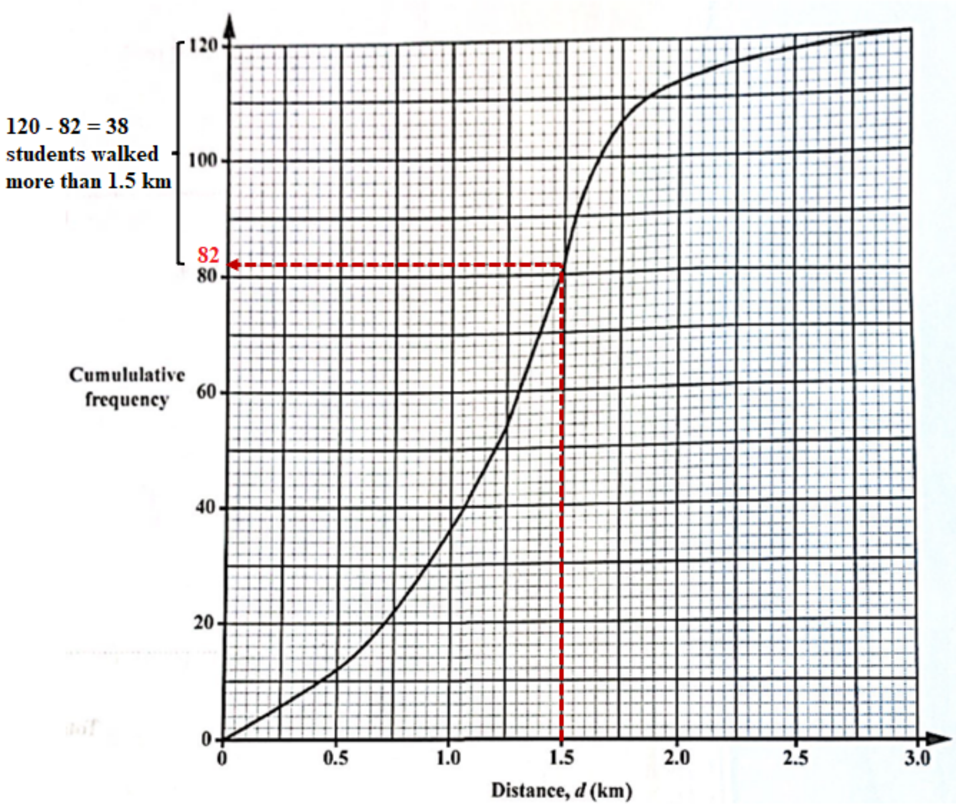
Three quarters of the cumulative frequency is ¾ (120) = 90. We draw a horizontal from 90 to meet the curve. At the point where the horizontal meets the curve, we draw a vertical to meet the horizontal axis. We read off 1.6.

$\therefore$ Upper quartile 1.55 km

$$\text{Interquartile range} = \text{Upper quartile} - \text{Lower quartile}$$
$$= 1.55 - 0.9 = 0.65$$

(c) A student is chosen at random. What is the probability that the student walked for **more than** 1.5 km to school that day?

SOLUTION:
Required to find: The probability that a randomly chosen student walked for more than 1.5 km that day.
Solution:



We draw a vertical from 1.5 km on the horizontal axis to meet the curve. At the point where the vertical meets the curve, we draw a horizontal to meet the vertical axis. From the diagram, 82 students walked for 1.5 km or less.
Hence, $(120-82)=38$ students walked for more than 1.5 km.

P(Student walked for more than 1.5 km)

$$= \frac{\text{No. of students who walked for more than 1.5 km}}{\text{Total no. of students}}$$

$$= \frac{38}{120}$$

$$= \frac{19}{60}$$

(d) Complete the table below and use the information to calculate an estimate of the mean distance walked by the students on that day.

Distance, *d* (km)	**Midpoint (*x*)**	**Number of Students (*f*)**	$f \times x$
$0 < d \leq 0.5$	0.25	12	3.0
$0.5 < d \leq 1.0$	0.75	24	18
$1.0 < d \leq 1.5$	1.25	46	57.5
$1.5 < d \leq 2.0$			
$2.0 < d \leq 2.5$	2.25		
$2.5 < d \leq 3.0$	2.75	2	5.5

SOLUTION:
Data: Incomplete grouped frequency table for the cumulative frequency curve given.
Required to complete: The table and find an estimate of the mean.
Solution:
The mid-point for the interval $1.5 < d \leq 2.0$ is calculated as follows:

$$\frac{1.5+2}{2} = 1.75$$

To calculate the number of students in the interval $1.5 < d \leq 2.0$, we read the the cumulative frequencies (CF) as follows:

CF for 2.0 = 112 (the number of students who walked less than 2.0 km)
CF for 1.5 = 82 (the number of students who walked less than 1.5 km)
Difference = 30

Similarly, the number of students in the interval $2.0 < d \leq 2.5$
$= 118 - 112 = 6$

Distance, *d* (km)	**Midpoint (*x*)**	**Number of Students (*f*)**	$f \times x$
$0 < d \leq 0.5$	0.25	12	3.0
$0.5 < d \leq 1.0$	0.75	24	18.0
$1.0 < d \leq 1.5$	1.25	46	57.5
$1.5 < d \leq 2.0$	1.75	30	52.5
$2.0 < d \leq 2.5$	2.25	6	13.5
$2.5 < d \leq 3.0$	2.75	2	5.5

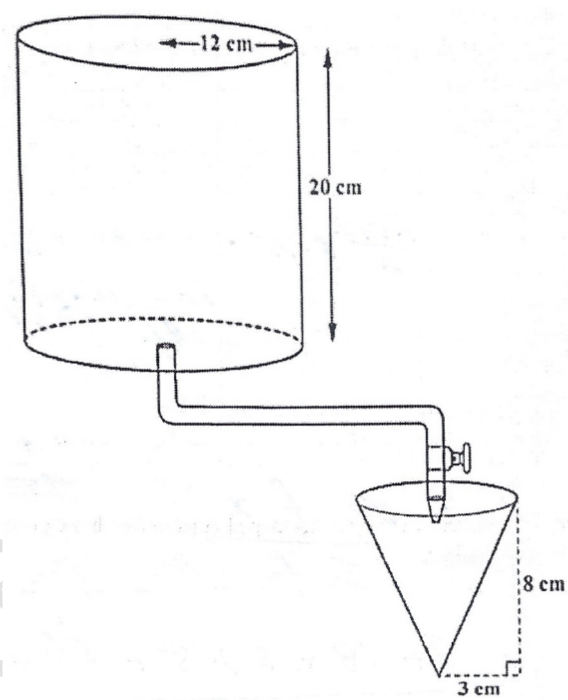
The mean, $\bar{x} = \frac{\sum fx}{\sum f}$

$$= \frac{3 + 18 + 57.5 + 52.5 + 13.5 + 5.5}{120}$$

$$= \frac{150}{120}$$

$$= 1.25\ km$$

6. At a track meet, a cylindrical container, fitted with a pipe as shown in the diagram below, is used to serve water to athletes. The cylindrical container of radius 12 cm and height 20 cm is completely filled with water and the pipe fitted at the bottom dispenses water into cone-shaped cups. The cone-shaped cups have a radius of 3 cm and a height of 8 cm.



(a) Calculate the volume of water in the cylindrical container, in litres. Write your answer correct to 2 decimal places. [1000 cm^3 = 1 litre]

SOLUTION:
Data: Diagram showing a cylindrical container of radius 12 cm and height 20 cm that is completely filled with water and fitted with a pipe at the bottom that is used to dispense water into cone-shaped cups of radius 3 cm and height 8 cm.

Required to calculate: The volume of water in the cylindrical container, in litres, correct to 2 decimal places.

Calculation:

Volume of cylinder $= \pi r^2 h$ [where $\pi = 3.142$]

$= \pi (12)^2 \times 20$

$= 9048.96 \text{ cm}^3$

$= \dfrac{9048.96}{1000}$ litres

$= 9.048$ litres

≈ 9.05 litres (correct to 2 decimal places)

(b) Water flows from the cylindrical container along the pipe into the cone-shaped cups at a rate of 7.8 ml per second.

Calculate the time taken to fill ONE of the empty cone-shaped cups. Give your answer correct to nearest second.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

SOLUTION:

Data: Water flows from the cylindrical container along the pipe into the cone-shaped cups at a rate of 7.8 ml per second.

Required to calculate: The time taken to fill ONE empty cone-shaped cup, correct to the nearest second.

Calculation:

Volume of the cone $= \dfrac{1}{3} \times \pi \times (3)^2 \times 8$ ml

$= 75.408$ ml

Rate of dispensing $= 7.8 \text{ ml s}^{-1}$

$\therefore$ Time taken to fill one cone-shaped cup $= \dfrac{75.408}{7.8}$ s

$= 9.667$ s

≈ 10 s (correct to the nearest second)

(c) Determine the number of empty cone-shaped cups in (b) that can be **completely** filled from the cylindrical container.

SOLUTION:
Required to determine: The number of cone-shaped cups that can be completely filled from the cylindrical container.
Solution:
The number of cone-shaped cups that can be filled from the cylindrical container

$$= \frac{\text{Volume of the cylindrical container}}{\text{Volume of the cone-shaped cup}}$$

$$= \frac{9048.96}{75.408}$$

$$= 120$$

7. A sequence of figures is made from lines of unit length and dots. The lines form a series of octagons and squares. The dots are placed at each vertex.

The first 3 figures in the sequence are shown below.

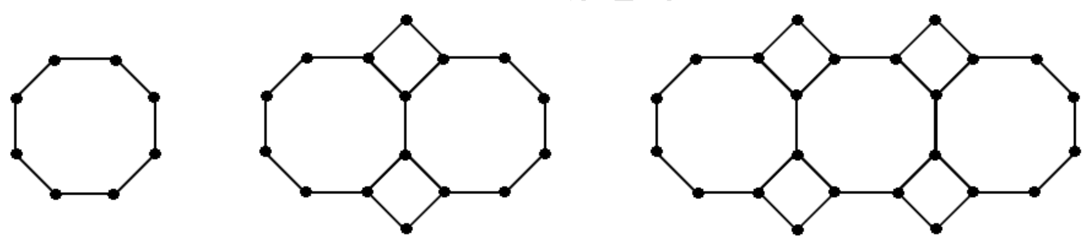
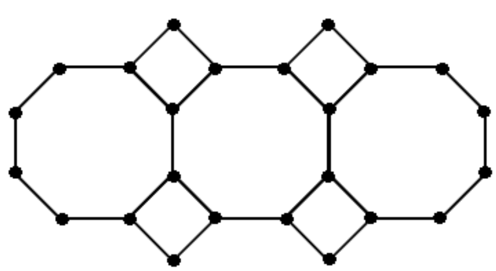
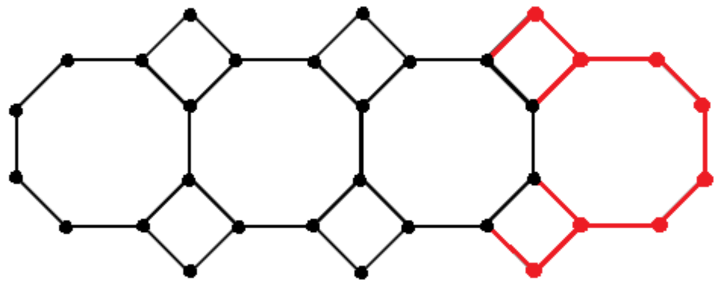


Figure 1 **Figure 2** **Figure 3**

(a) Figure 3 of the sequence shown below. Add more lines of unit length and dots to Figure 3 to correctly represent Figure 4.



SOLUTION:
Data: Diagrams showing the first three figures in a sequence of figures made from lines of unit length and dots.
Required to add: Lines to Figure 3 in order to create Figure 4.
Solution:



(b) The number of dots, *D*, and the number of unit lines that form the perimeter of the shape, *P*, form a pattern. The values for *D* and *P* for the first 3 figures are written in the table below. Study the pattern of numbers in each row of the table.

Complete the rows numbered (i), (ii) and (iii).

	Figure Number	Number of Dots (*D*)	Perimeter of Figure (*P*)
	1	8	8
	2	16	14
	3	24	20
(i)	4		
	⋮	⋮	⋮
(ii)			86
	⋮	⋮	⋮
(iii)	*n*		

SOLUTION:
Data: Incomplete table showing the pattern formed by the number of dots, *D*, and the number of unit lines that form the perimeter of the shape, *P*.
(i) **Required to calculate**: the value of *D* and *P* when $n = 4$.

By Fayad W. Ali and Shereen A. Khan.

Solution:

(i)

Number of Dots (D)	Perimeter of figure (P)
The number sequence for D is 8, 16, 24, … The number of dots, D is a multiple of 8. $8 \times 1, 8 \times 2, 8 \times 3, \ldots..$ So, the fourth term in the sequence is $8 \times 4 = 32$	The number sequence for P is 8, 14, 20, … The perimeter, P increases by 6. 8, $8+6$, $8+6+6$, So, the fourth term in the sequence is $8 + 6 + 6 + 6 = 26$

(ii)**Required to calculate**: the value of D and the Figure number when $P = 86$.
Solution:
Based on the pattern for P above, we have thus far the table values for $n = 1$ to $n = 4$, as shown below.

In the last row, we replace the Figure by n and obtain general expressions for D and P.

Figure number	Number of Dots (D)	Perimeter of figure (P)
1	8×1	$8 + 0(6)$
2	8×2	$8 + 1(6)$
3	8×3	$8 + 2(6)$
4	8×4	$8 + 3(6)$
n	$8 \times n$	$8 + (n-1)(6)$

When $P = 86$,

$$8 + (n-1)(6) = 86$$
$$8 + 6n - 6 = 86$$
$$6n + 2 = 86$$
$$6n = 84$$
$$n = 14$$

So, $D = 8n = 8 \times 14 = 112$

Alternatively,

$P = 8, 14, 20, ...$ P increases by 6 Let $P = 6n \pm c$, where c is a number. When $n = 1$, $P = 8$: $8 = 6(1) + c$ $c = 2$ $\therefore P = 6n + 2$	Test: When $n = 2$ Test: When $n = 3$ $P = 6(2) + 2$ $P = 6(3) + 2$ $= 14$ $= 20$

To calculate D and n when $P = 86$,

$6n + 2 = 86$ $D = 8n$

$6n = 84$ $D = 8(14)$

$n = 14$ $= 112$

(iii) $D = 8n$ and $P = 6n + 2$, where n is the figure number.

The completed table is shown below.

	Figure Number	Number of Dots (*D*)	Perimeter of Figure (*P*)
	1	8	8
	2	16	14
	3	24	20
(i)	4	$8(4) = 32$	$6(4) + 2 = 26$
	$\vdots$	$\vdots$	$\vdots$
(ii)	14	112	86
	$\vdots$	$\vdots$	$\vdots$
(iii)	n	$8n$	$6n + 2$

(c) For any figure, $n > 1$, the number of dots, *D*, is greater than its perimeter, *P*. Determine the value of *n* for a figure in which the difference between *D* and *P* is 36.

SOLUTION:
Data: For any figure, $n > 1$, the number of dots, *D*, is greater than its perimeter, *P*.
Required to determine: The value of n for a figure in which the difference between *D* and *P* is 36.
Solution:

$$D > P$$
$$D - P = 36$$
$$8n - (6n + 2) = 36$$
$$2n - 2 = 36$$
$$2n = 38$$
$$n = 19$$

SECTION II

Answer ALL questions.

ALL working MUST be clearly shown.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. The function $f : x \to 3+5x-x^2$.

(a) **(i)** Complete the table of values for $f(x)=3+5x-x^2$.

x	–1	0	1	2	3	4	5	6
f(x)	–3			9	9		3	

SOLUTION:
Data: $f : x \to 3+5x-x^2$
Required to complete: The incomplete table of values given.
Solution:

When $x=0: f(0)=3+5(0)-(0)^2=3$

When $x=1: f(1)=3+5(1)-(1)^2=7$

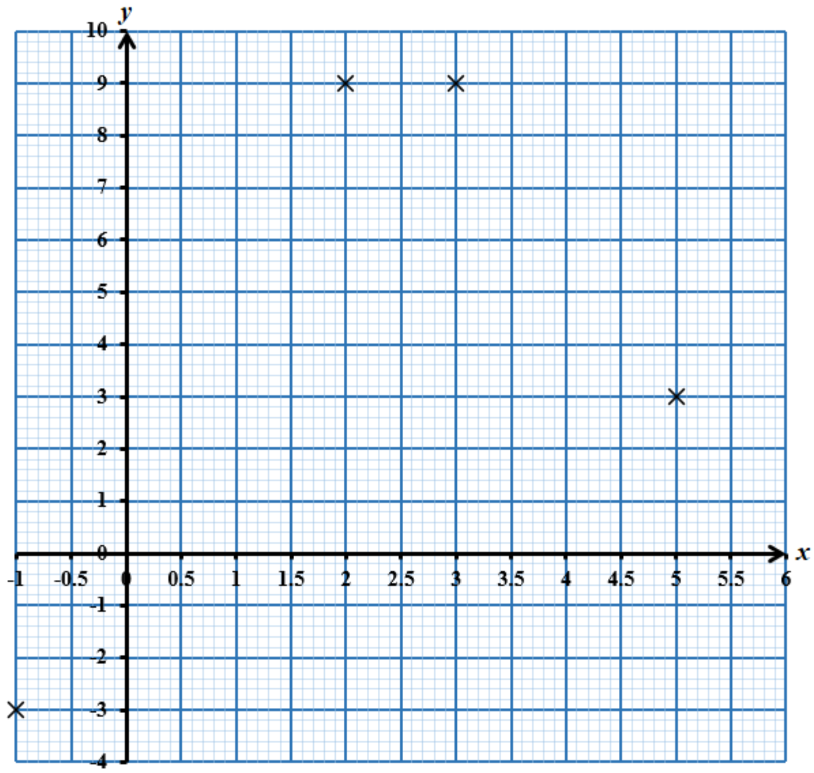
When $x=4: f(4)=3+5(4)-(4)^2=7$

When $x=6: f(6)=3+5(6)-(6)^2=-3$

The completed table of values is shown below.

x	–1	0	1	2	3	4	5	6
f(x)	–3	3	7	9	9	7	3	–3

(ii) On the grid below, complete the graph of $f(x)=3+5x-x^2$ for $-1 \le x \le 6$.

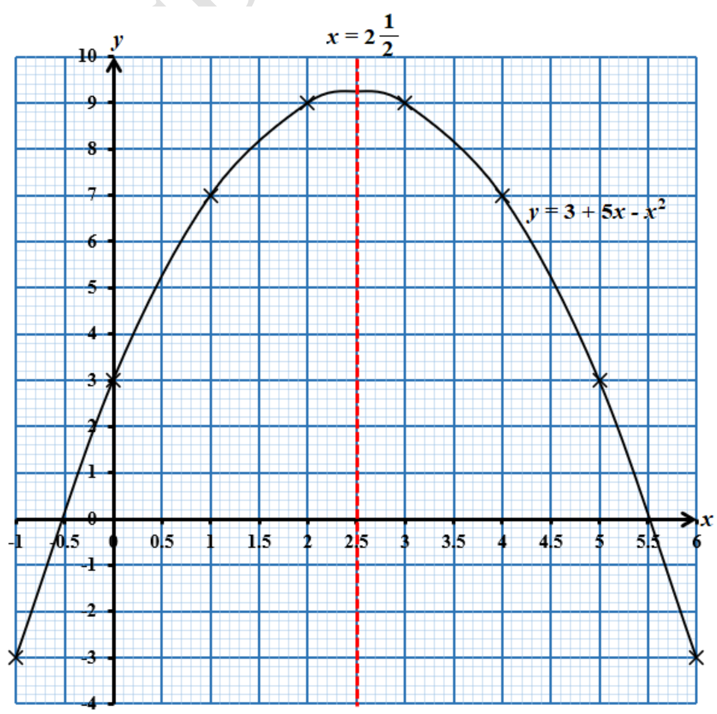


SOLUTION:

Data: Incomplete graph of $f(x) = 3 + 5x - x^2$.

Required to complete: The graph of $f(x) = 3 + 5x - x^2$.

Solution:



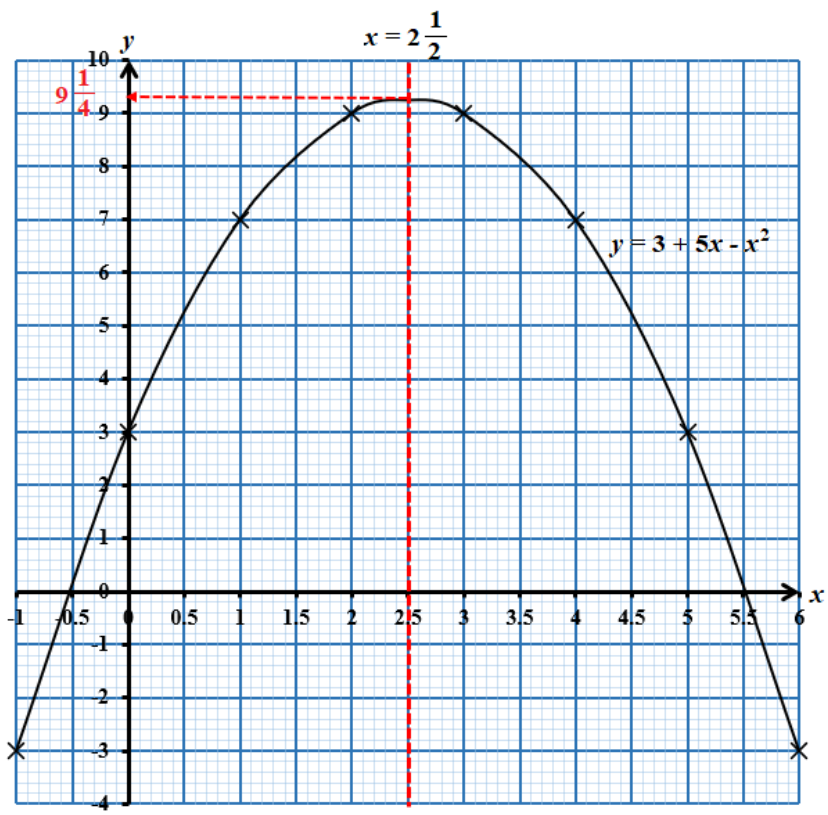
(b) **(i)** Write down the equation of the axis of symmetry of the graph of $f(x) = 3 + 5x - x^2$.

SOLUTION:
Required to write: The equation of the axis of symmetry of the graph of $f(x) = 3 + 5x - x^2$.
Solution:
$f(x) = -x^2 + 5x + 3$

The axis of symmetry is $x = 2\frac{1}{2}$ (Read off)

(ii) State the maximum value of the function.

SOLUTION:
Required to state: The maximum value of the function.
Solution:



The maximum value of $f(x)$ is $f\left(2\frac{1}{2}\right) = 9\frac{1}{4}$.

(c) Write down the co-ordinates of the point where the line $y = 3 - \frac{1}{2}x$

(i) crosses the x – axis

SOLUTION:
Required to write: The coordinates of the point where the line $y = 3 - \frac{1}{2}x$ crosses the x – axis.
Solution:
Let $y = 0$

$$3 - \frac{1}{2}x = 0$$
$$\frac{1}{2}x = 3$$
$$x = 6$$

$\therefore$ The line crosses the x – axis at the point $(6, 0)$.

(ii) crosses the y – axis.

SOLUTION:
Required to write: The coordinates of the point where the line $y = 3 - \frac{1}{2}x$ crosses the y – axis.
Solution:
Let $x = 0$

$$y = 3 - \frac{1}{2}(0)$$
$$y = 3$$

$\therefore$ The line crosses the y – axis at the point $(0, 3)$.

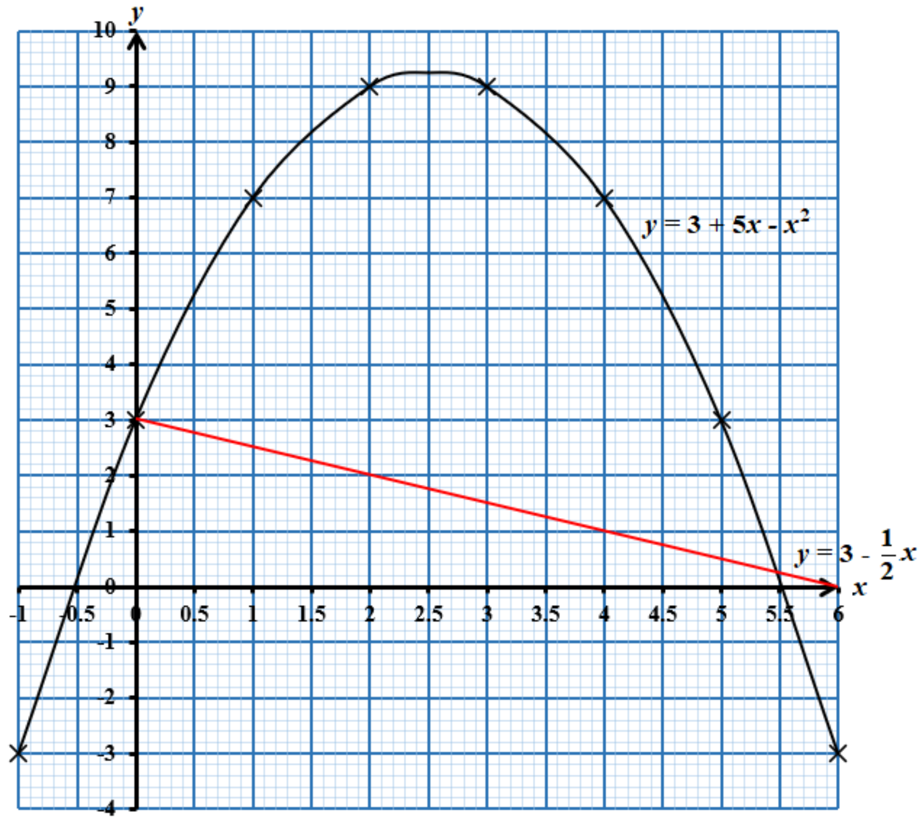
(d) On the grid, draw the line $y = 3 - \frac{1}{2}x$.

SOLUTION:

Required to draw: The line $y = 3 - \frac{1}{2}x$ on the grid.

Solution:

We draw the line by plotting the points (6, 0) and (0, 3) and joining these points. This is shown in red



(e) Using your graph, determine the solutions to the equations

$$y = 3 + 5x - x^2$$

$$y = 3 - \frac{1}{2}x.$$

SOLUTION:

Required to determine: The solutions of the equations $y = 3 + 5x - x^2$ and $y = 3 - \frac{1}{2}x$, using the graph.

Solution:

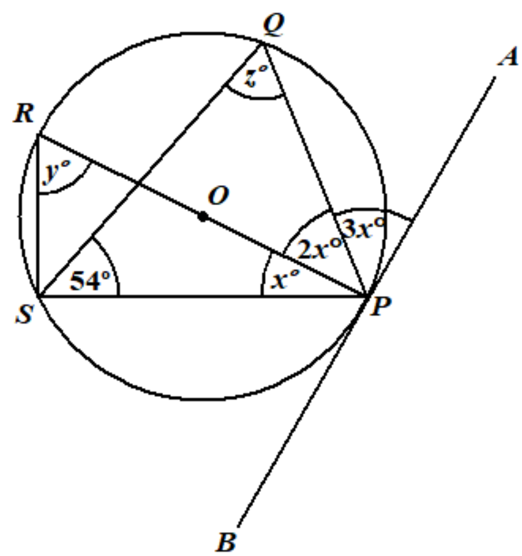
The solution of the equations will be at the points of intersections of both graphs.

Points of intersection are $(0, 3)$ and $\left(5\frac{1}{2}, \frac{1}{4}\right)$.

Hence, $x = 0$ and $y = 3$ or $x = 5\frac{1}{2}$ and $y = \frac{1}{4}$.

GEOMETRY AND TRIGONOMETRY

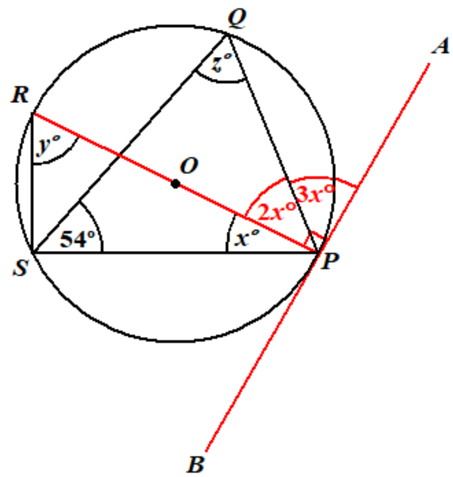
9. **(a)** The diagram below shows a circle, with the points *P*, *Q*, *R* and *S* lying on its circumference and its center marked *O*. *RP* is a diameter of the circle and *AB* is a tangent to the circle at *P*. Angle $APQ = 3x°$, angle $QPR = 2x°$, $RPS = x°$ and angle $QSP = 54°$.



Determine the value of EACH of the following angles. Show detailed working where possible and give a reason for your answer.

(i) x

SOLUTION:
Data: Diagram showing a circle, with the points *P*, *Q*, *R* and *S* lying on its circumference and its center marked *O*. *RP* is a diameter of the circle and *AB* is a tangent to the circle at *P*. Angle $APQ = 3x°$, angle $QPR = 2x°$, $RPS = x°$ and angle $QSP = 54°$.
Required to determine: The value of x
Solution:



$O\hat{P}A = 2x° + 3x°$ (Angle made by a tangent (PA) and a radius (OP) at the point of contact (P) is 90°.)

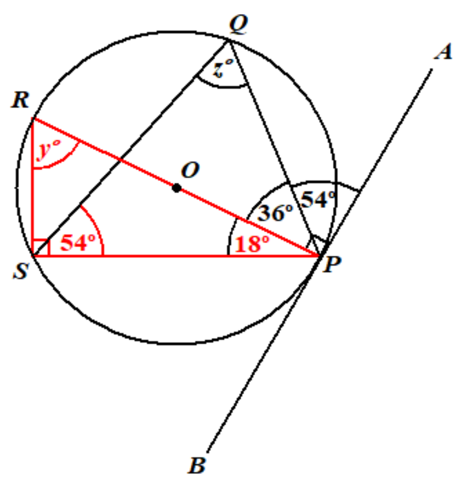
$= 90°$

$5x° = 90°$

$x = 18°$

(ii) y

SOLUTION:
Required to find: The value of y
Solution:



$R\hat{S}P = 90°$ (Angle in a semi-circle is a right angle.)

Consider the ΔRSP:

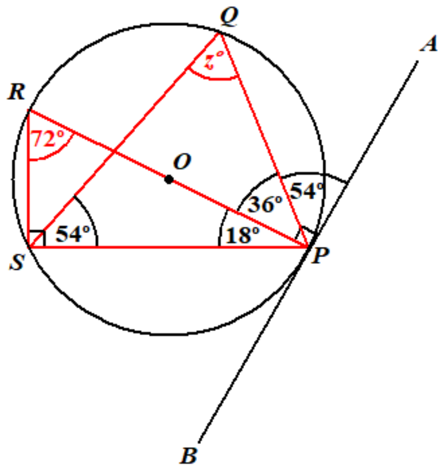
$y° + 90° + 18° = 180°$ (Sum of the interior angles in a triangle $= 180°$)

$y° = 180° - (90° + 18°)$

$= 72°$

(iii) z

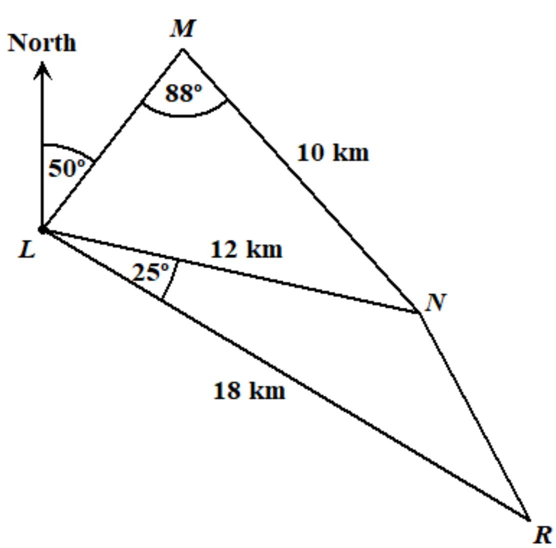
SOLUTION:
Required to determine: The value of z
Solution:



$z = 72°$

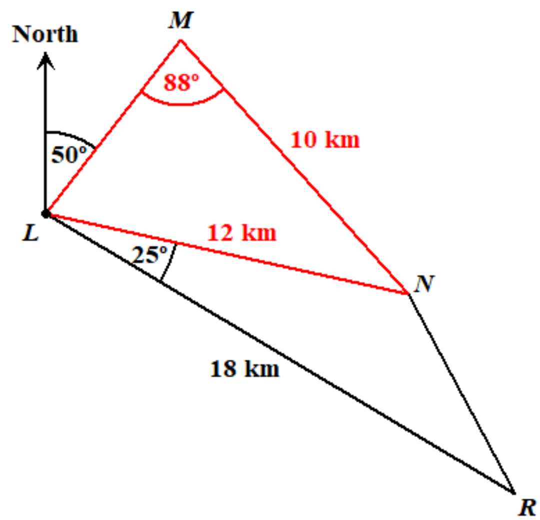
(Angles subtended by a chord (SP) at the circumference of a circle, and standing on the same arc ($S\hat{R}P$ and $S\hat{Q}P$) are equal.)

(b) The diagram below shows straight roads connecting the towns L, M, N and R. $LR = 18$ km, $LN = 12$ km and $MN = 10$ km. Angle $RLN = 25°$ and angle $LMN = 88°$.



(i) Calculate the angle *MLN*.

SOLUTION:
Data: Diagram showing straight roads connecting the towns *L*, *M*, *N* and *R*. $LR = 18$ km, $LN = 12$ km and $MN = 10$ km. Angle $RLN = 25°$ and angle $LMN = 88°$.
Required to calculate: Angle *MLN*
Calculation:



Consider the ΔMLN:

$$\frac{10}{\sin M\hat{L}N} = \frac{12}{\sin 88°} \text{ (Sine Rule)}$$

$$\sin M\hat{L}N = \frac{10 \times \sin 88°}{12}$$

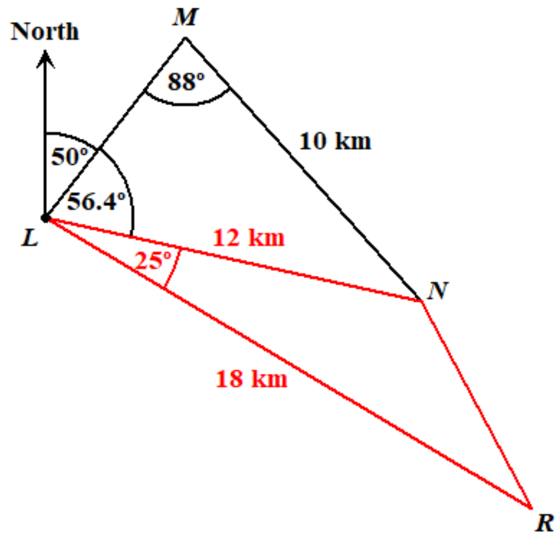
$$\sin M\hat{L}N = 0.8328$$

$$M\hat{L}N = \sin^{-1}(0.8328)$$

$$M\hat{L}N = 56.4° \text{ (correct to the nearest } 0.1°\text{)}$$

(ii) Calculate the distance *NR*.

SOLUTION:
Required to calculate: The distance *NR*.
Calculation:



Consider the ΔNLR:

$NR^2 = (12)^2 + (18)^2 - 2(12)(18)\cos 25°$ (Cosine law)

$NR^2 = 144 + 324 - 432(0.906)$

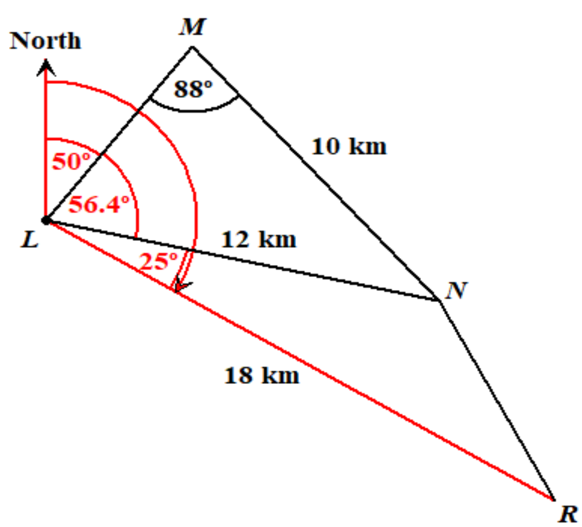
$= 76.61$

$NR = \sqrt{76.61}$

$= 8.75$ km (correct to 2 decimal places)

(iii) Determine the bearing of Town *R* from Town *L*.

SOLUTION:
Required to determine: The bearing of Town *R* from Town *L*.
Solution:

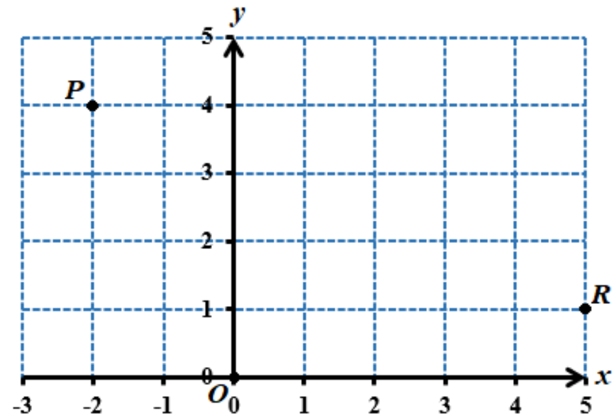


The bearing of Town *R* from Town *L* $= 50° + 56.4° + 25°$

$= 131.4°$

VECTORS AND MATRICES

10. (a) Three points O, P and R are shown in the grid below. O is the origin.



(i) Write the position vector of R, $\overrightarrow{OR}$, in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

SOLUTION:
Data: Grid showing three points O, P and R.

Required to write: The position vector of R, $\overrightarrow{OR}$, in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

Solution:

$R = (5, 1)$

$\therefore \overrightarrow{OR} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ is of the form $\begin{pmatrix} a \\ b \end{pmatrix}$, where $a = 5$ and $b = 1$.

(ii) Another point, Q, is located in such a way that $\overrightarrow{QR} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$.
Using this information, plot the point Q on the graph.

SOLUTION:

Data: Another point, Q, is located in such a way that $\overrightarrow{QR} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

Required to plot: The point Q on the graph.
Solution:
Let $Q = (x, y)$

$\therefore \overrightarrow{OQ} = \begin{pmatrix} x \\ y \end{pmatrix}$

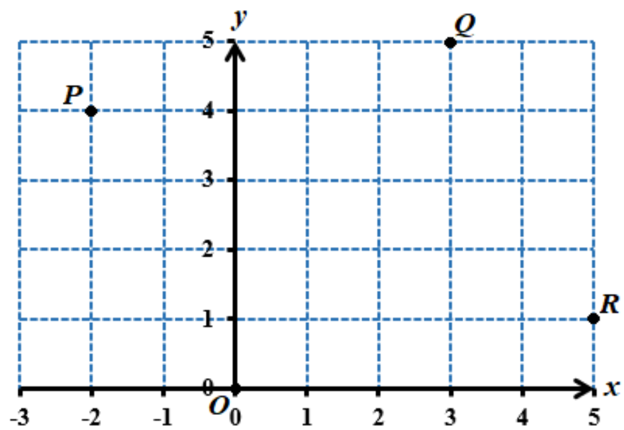
$$\overrightarrow{QR} = \overrightarrow{QO} + \overrightarrow{OR}$$

$$\begin{pmatrix} 2 \\ -4 \end{pmatrix} = -\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$\therefore 2 = -x + 5$ and $-4 + y = 1$

$x = 3$ $y = 5$

$$\therefore \overrightarrow{OQ} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \text{ and } Q = (3,\ 5)$$



(iii) Determine $\left|\overrightarrow{QR}\right|$, the magnitude of $\overrightarrow{QR}$.

SOLUTION:
Required to determine: $\left|\overrightarrow{QR}\right|$
Solution:

$$\overrightarrow{QR} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

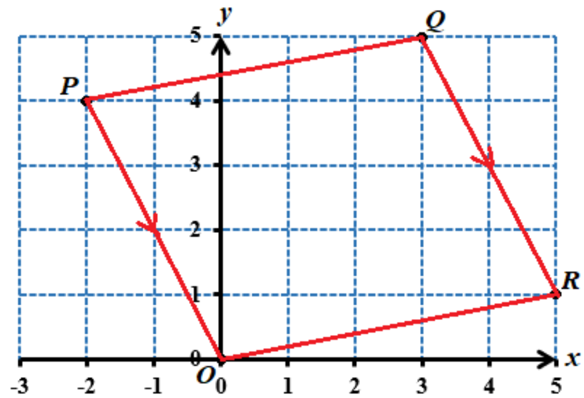
$$\left|\overrightarrow{QR}\right| = \sqrt{(2)^2 + (-4)^2}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5} \text{ units}$$

(iv) Show, by calculation, that *OPQR* is a parallelogram.

SOLUTION:
Required to show: *OPQR* is a parallelogram.
Solution:



$$P = (-2, 4)$$

$$\therefore \overrightarrow{OP} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$\overrightarrow{PO} = -\begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$\overrightarrow{QR} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \text{ (given)}$$

Hence, $\left|\overrightarrow{QR}\right| = \left|\overrightarrow{PO}\right|$ AND $\overrightarrow{QR}$ is parallel to $\overrightarrow{PO}$.

If one pair of opposite sides of a quadrilateral is both parallel and equal, then the quadrilateral is a parallelogram.

Q.E.D.

Note, there are other conditions that are sufficient to prove that the quadrilateral *OPQR* is a parallelogram

We could prove also that in the quadrilateral *OPQR*

i. both pairs of opposite sides are equal to each other
ii. both pairs of opposite sides are parallel to each other
iii. both pairs of opposite angles are equal to each other
iv. any two adjacent angles are supplementary
v. the diagonals bisect each other

(b) Calculate the value of x and the value of y in the matrix equation below.

$$\begin{pmatrix} 1 & 5 \\ 2 & y \end{pmatrix}\begin{pmatrix} -4 & 1 \\ 2 & 9 \end{pmatrix} = \begin{pmatrix} x & 46 \\ 6 & 65 \end{pmatrix}$$

SOLUTION:

Data: $\begin{pmatrix} 1 & 5 \\ 2 & y \end{pmatrix}\begin{pmatrix} -4 & 1 \\ 2 & 9 \end{pmatrix} = \begin{pmatrix} x & 46 \\ 6 & 65 \end{pmatrix}$

Required to calculate: x and y

Calculation:

Consider the left hand side:

$$\begin{pmatrix} 1 & 5 \\ 2 & y \end{pmatrix}\begin{pmatrix} -4 & 1 \\ 2 & 9 \end{pmatrix} = \begin{pmatrix} x & 46 \\ 6 & 65 \end{pmatrix}$$

$$2\times 2 \times \quad 2\times 2 \quad = \quad 2\times 2$$

$$\begin{pmatrix} 1 & 5 \\ 2 & y \end{pmatrix}\begin{pmatrix} -4 & 1 \\ 2 & 9 \end{pmatrix} = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$$

$$e_{11} = (1\times -4) + (5\times 2) = 6$$
$$e_{21} = (2\times -4) + (y\times 2) = -8 + 2y$$

$$\Rightarrow \begin{pmatrix} 6 & e_{12} \\ -8+2y & e_{22} \end{pmatrix} = \begin{pmatrix} x & 46 \\ 6 & 65 \end{pmatrix}$$

Equate corresponding entries:

$$6 = x$$

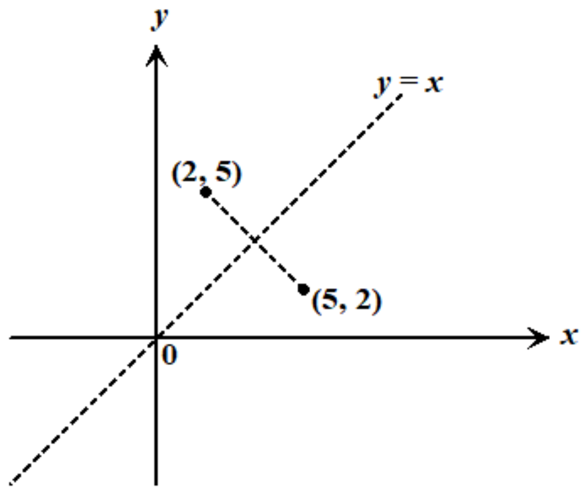
$$-8 + 2y = 6$$
$$2y = 14$$
$$y = 7$$

Hence, $x = 6$ and $y = 7$.

(c) A transformation, T, represented by the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, maps $S\,(2, 5)$ onto $S'\,(5, 2)$. Describe fully the transformation T.

SOLUTION:

Data: A transformation, T, represented by the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, maps $S\,(2,\,5)$ onto $S'\,(5,\,2)$.
Required to describe: The transformation, T.
Solution:
T represents a reflection in the line $y = x$.



END OF TEST