FAS-PASS Maths

CSEC MATHEMATICS JULY 2021 PAPER 2

SECTION 1

Answer ALL questions.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of

$$1\frac{4}{7} \div \frac{2}{3} - 1\frac{5}{7}$$

SOLUTION:

Required to calculate: $1\frac{4}{7} \div \frac{2}{3} - 1\frac{5}{7}$ exactly

Calculation:

$$1\frac{4}{7} \div \frac{2}{3} = \frac{11}{7} \div \frac{2}{3}$$
$$= \frac{11}{7} \times \frac{3}{2}$$
$$= \frac{33}{14}$$

The question reduces to $\frac{33}{14} - 1\frac{5}{7} = \frac{33}{14} - \frac{12}{7}$

$$= \frac{33-24}{14}$$
$$= \frac{9}{14} \text{ (in exact form)}$$

(b) When Meghan started working, she was paid \$85 each week. After a six-month probationary period, her pay was increased by 20%. How much was she paid each week **after** the increase?

SOLUTION:

Data: At the start of her job, Meghan was paid \$85 per week. After 6 months, her pay was increased by 20%.

Required to find: The amount of money Meghan is paid per week, after the increase.

Solution:

Initial pay per week $= \$85$

The increase $= 20\%$

$$= \frac{20}{100} \times \$85$$
$$= \$17$$

Hence, the pay after the increase $= \$(85 + 17)$
$= \$102$

Alternative Method:

Increase $= 20\%$

So, the new weekly pay is $(100 + 20)\%$ of \$85 $= \frac{120}{100} \times \$85$
$= \$102$

(c) In 1965, the population of Country *A* was 2 714 000. In 2015, the population was 3 663 900.

(i) **a)** Write the population in 2015 correct to 3 significant figures.

SOLUTION:
Data: The population of Country A in 1965 was 2 714 000 and in 2015, it was 3 663 900.
Required to write: The population in 2015 correct to 3 significant figures.
Solution:

The 4th digit is the deciding digit in writing to 3 sf.
3 663 900
↑
deciding digit < 5
So, we round down and the third digit remains unaltered.
All digits to the right of the third digit are now written as 0.

Hence, $3\ 663\ 900 \approx 3\ 660\ 000$ (correct to 3 significant figures)

b) Write the population in 1965 in standard form.

SOLUTION:
Required to write: The population is 1965 in standard form.

Solution:
The population in 1965 was 2 714 000. To write this in standard form we position the decimal point to obtain a number, A, such that $1 \le A < 10$.

2 714 000.

The decimal point is shifted 6 places to the left, so that $A = 2.714$. To restore the number to its original value we multiply A by 10^6.

Hence, $2\ 714\ 000 = 2.714 \times 10^6$ (in standard form)

(ii) Determine the percentage increase in the population from 1965 to 2015.

SOLUTION:
Required to determine: The percentage increase in the population from 1965 to 2015.
Solution:

$$\begin{aligned}\text{Increase} &= 3\ 663\ 900 - 2\ 714\ 000 \\ &= 949\ 000\end{aligned}$$

$$\begin{aligned}\text{Percentage increase} &= \frac{\text{Increase}}{\text{Original population}} \times 100\% \\ &= \frac{949\ 000}{2\ 714\ 000} \times 100\% \\ &= 35\%\end{aligned}$$

(d) The ratio of teachers to male students to female students in a school is $3:17:18$. If the TOTAL **number of students** in the school is 630, determine the **number of teachers** in the school.

SOLUTION:
Data: Ratio of teachers to male students to female students in a school is $3:17:18$. The total number of students in the school is 630.
Required to determine: The number of teachers in the school.
Solution:

Teachers: Male students: Female students
3 : 17 : 18

Number of students in the school $= 630$
Hence, 35 (17+18) parts of the ratio = 630
One part of the ration =630 ÷ 35 = 18
The number of teachers amounts to 3 parts.
So, the number of teachers in the school = 18 × 3
$= 54$ teachers

2. (a) Two quantities, *n* and *T*, are related as follows:

$$n = \sqrt{T}$$

(i) Find the value of *n* when $T = 49$.

SOLUTION:
Data: Two quantities *n* and *T* are related by the equation $n = \sqrt{T}$.
Required to find: *n* when $T = 49$
Solution:
When $T = 49$

$$n = \sqrt{49}$$
$$= \pm 7$$

Taking the positive value, $n = 7$.

(ii) Make *T* the subject of the formula.

SOLUTION:
Required to make: *T* the subject of the formula.
Solution:
$n = \sqrt{T}$
Squaring both sides to remove the root sign
$n^2 = T$
$T = n^2$

(b) Ally is *x* years. Jim is 5 years older than Ally and Chris is twice as old as Ally.

(i) Write expressions in terms of *x* for Jim's age and Chris' age.

SOLUTION:
Data: Ally is *x* years. Jim is 5 years older than Ally and Chris is twice as old as Ally.
Required to write: Expressions for Jim's age and Chris' age, in terms of *x*
Solution:

Ally's age is *x*

So, Jim's age, which is 5 more than Ally's $= x + 5$

Chris' age, which is twice as much as Ally's $= x \times 2 = 2x$

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(ii) In **two years' time**, the product of Ally's age and Chris' age will be the same as the square of Jim's **present** age.

Show that the equation $x^2 - 4x - 21 = 0$ represents the information given above.

SOLUTION:
Data: In **two years' time**, the product of Ally's age and Chris' age will be the same as the square of Jim's **present** age.
Required to show: The data given can be represented by the equation $x^2 - 4x - 21 = 0$.

Proof:
In two years' time, the ages of:

Ally will be $= x + 2$

Chris will $= 2x + 2$

Jim is at present $x + 5$

Hence,
$$(x+2)(2x+2) = (x+5)^2$$
$$2x^2 + 4x + 2x + 4 = x^2 + 5x + 5x + 25$$
$$2x^2 + 6x + 4 = x^2 + 10x + 25$$
So, $2x^2 + 6x + 4 - x^2 - 10x - 25 = 0$
and $x^2 - 4x - 21 = 0$

Q.E.D.

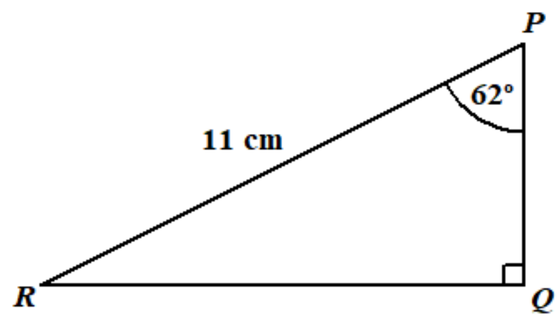
(iii) Calculate Ally's present age.

SOLUTION:
Required to calculate: Ally's present age.
Calculation:
Ally's present age $= x$

From above, $x^2 - 4x - 21 = 0$
By factorisation:
$$(x-7)(x+3) = 0$$
$x - 7 = 0$ or $x + 3 = 0$
$x = 7$ or $x = -3$
(Invalid since $x \neq 3$)

$\therefore$ Ally's present age is 7 years.

3. (a) The diagram below shows the triangle *PQR* in which angle $QPR = 62°$, angle $PQR = 90°$ and $PR = 11\text{cm}$.



Calculate:

(i) the size of angle *PRQ*

SOLUTION:
Data: Diagram showing triangle *PQR*, with $QPR = 62°$, angle $PQR = 90°$ and $PR = 11\text{cm}$.
Required to calculate: Angle *PRQ*
Calculation:

$$P\hat{R}Q = 180° - (90° + 62°)$$
$$= 28°$$

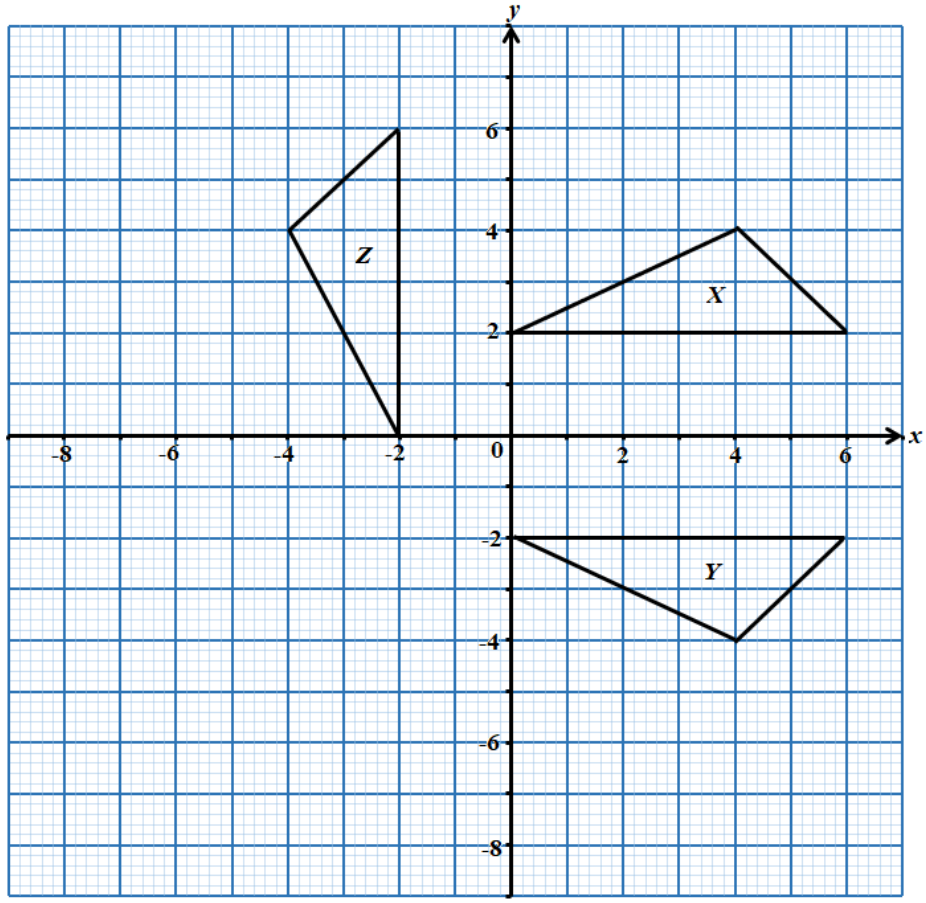
(Sum of the interior angles of a triangle is equal to 180°.)

(ii) the length of the side *RQ*.

SOLUTION:
Required to calculate: The length of side *RQ*
Calculation:

$$\sin 62° = \frac{RQ}{11}$$
$$\therefore RQ = 11 \times \sin 62°$$
$$= 9.71\underline{\underline{2}} \text{ cm}$$
$$= 9.71 \text{ cm (correct to 2 decimal places)}$$

(b) The diagram below shows three triangles, *X*, *Y* and *Z* on a square grid.



(i) Triangle *X* is mapped onto Triangle *Y* by a reflection. State the equation of the mirror line.

SOLUTION:
Data: Diagram showing three triangles, *X*, *Y* and *Z* on a grid. Triangle *X* is mapped onto Triangle *Y* by a reflection.
Required to state: The equation of the mirror line.
Solution:

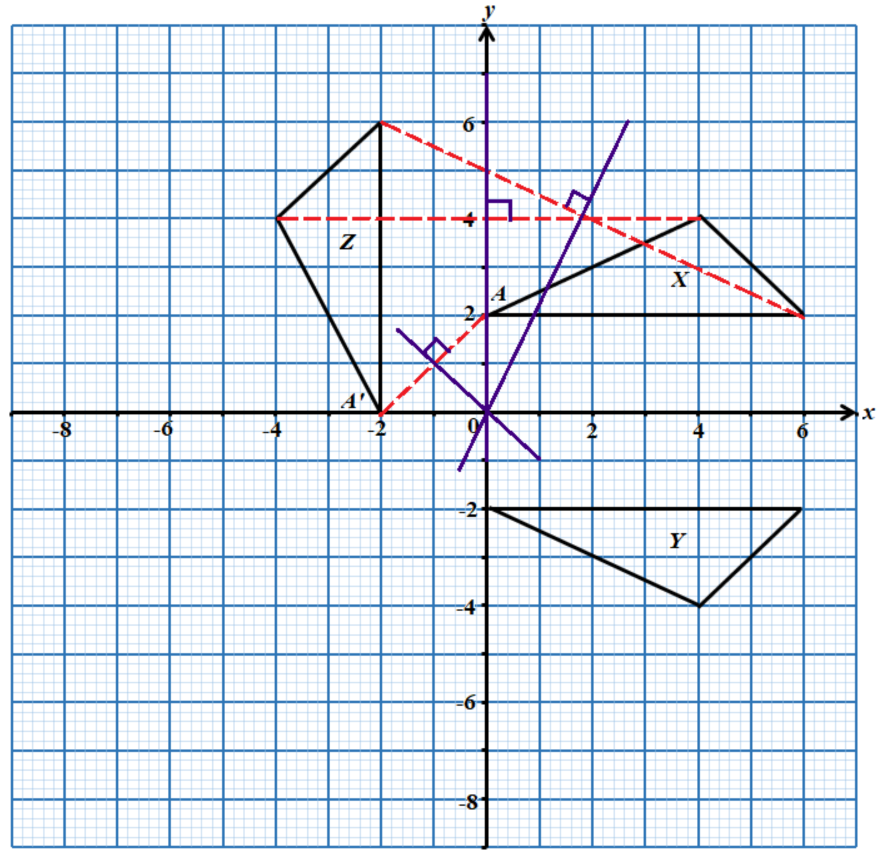
$X \xrightarrow{\text{Reflection in the } x\text{-axis}} Y$

The mirror line is the perpendicular to the line joining any object point to its corresponding image point. This is the x – axis with equation $y = 0$.

(ii) Describe fully the transformation which maps Triangle *X* onto Triangle *Z*.

SOLUTION:
Required to describe: The transformation that maps Triangle *X* onto Triangle *Z*.

Solution:



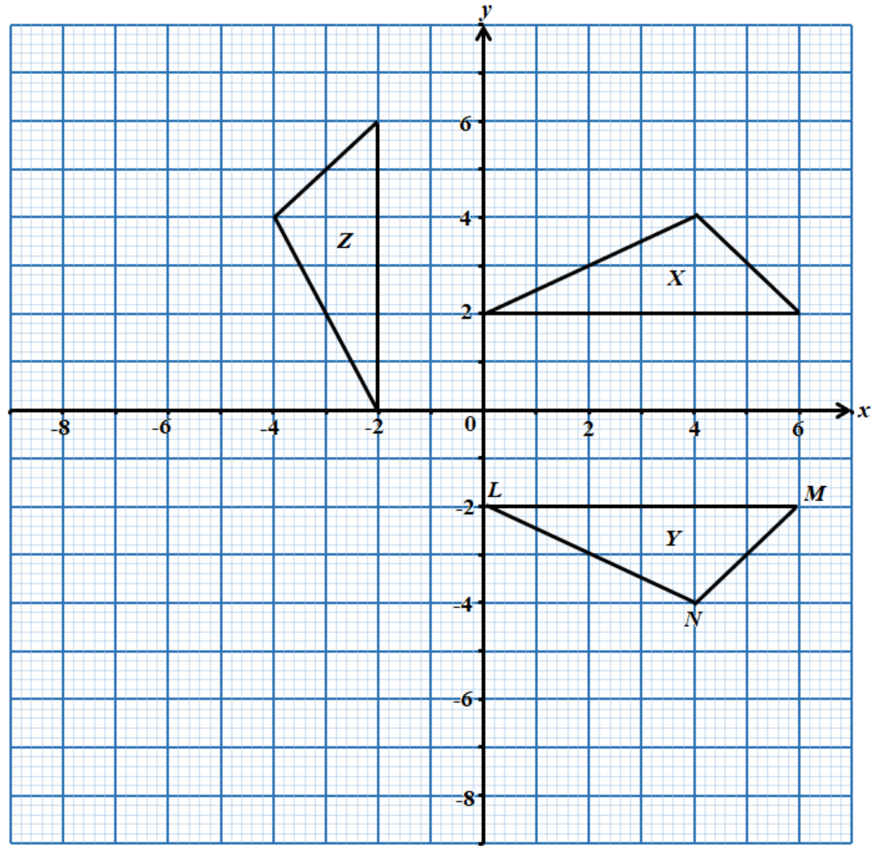
The triangles X and Z are congruent. The image, Z, is re-oriented with respect to the object, X. Hence, the transformation is a rotation. The perpendicular bisectors of the lines joining each object point to its corresponding image point intersect at O and which is the center of rotation.

A' is the image of A and O is the center of rotation. The angle of rotation is the angle between AO and OA', $A\hat{O}A' = 90°$
Hence, the rotation is 90°, anti-clockwise about O.

(iii) On the diagram, translate Triangle Y using vector $\begin{pmatrix} -7 \\ 1 \end{pmatrix}$. Label this image V.

SOLUTION:
Data: Triangle V is formed by translating Triangle Y using the vector $\begin{pmatrix} -7 \\ 1 \end{pmatrix}$.
Required to draw: Triangle V
Solution:

Let the vertices of Triangle *Y* be *LMN* as shown below.



We can determine the co-ordinates of L', M' and $N'$by calculation using column matrices as shown below.

$$L(0,-2) \xrightarrow{Translation\ of \begin{pmatrix}-7\\1\end{pmatrix}} L'(?,?)$$

$$\begin{pmatrix}0\\-2\end{pmatrix} + \begin{pmatrix}-7\\1\end{pmatrix} = \begin{pmatrix}-7\\-1\end{pmatrix}$$

Hence, $L' = (-7,-1)$

$$M(6,-2) \xrightarrow{Translation\ of \begin{pmatrix}-7\\1\end{pmatrix}} M'(?,?)$$

$$\begin{pmatrix}6\\-2\end{pmatrix} + \begin{pmatrix}-7\\1\end{pmatrix} = \begin{pmatrix}-1\\-1\end{pmatrix}$$

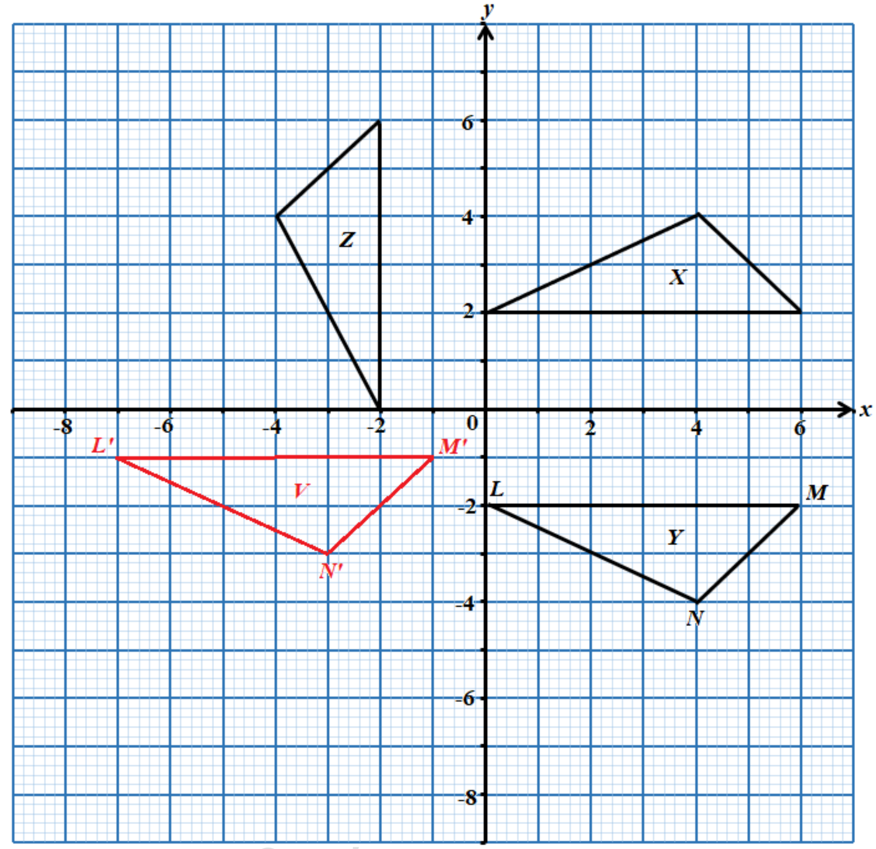
Hence, $M' = (-1,-1)$

$$N(0,-2) \xrightarrow{Translation\ of \begin{pmatrix}-7\\1\end{pmatrix}} N'(?,?)$$

$$\begin{pmatrix}4\\-4\end{pmatrix} + \begin{pmatrix}-7\\1\end{pmatrix} = \begin{pmatrix}-3\\-3\end{pmatrix}$$

Hence, $N' = (-3,-3)$

The image, V, is shown on the diagram (in red)



(iv) On the diagram, enlarge triangle X about the center, $C\ (0,\ 0)$, and scale factor $\frac{1}{2}$. Label this image W.

SOLUTION:
Data: Triangle X is mapped into Triangle W by an enlargement about the center $C\ (0,\ 0)$ and scale factor $\frac{1}{2}$.
Required to draw: Triangle W
Solution:

Let the vertices of Triangle X be P, Q and R. An enlargement of scale factor, $\frac{1}{2}$ and center O is represented by the 2×2 matrix $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$.
We obtain the coordinates of the images of P, Q and R using matrix multiplication as shown below.

$$\begin{matrix} & P & Q & R & & P' & Q' & R' \end{matrix}$$

$$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 0 & 6 & 4 \\ 2 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 3 & 2 \\ 1 & 1 & 2 \end{pmatrix}$$

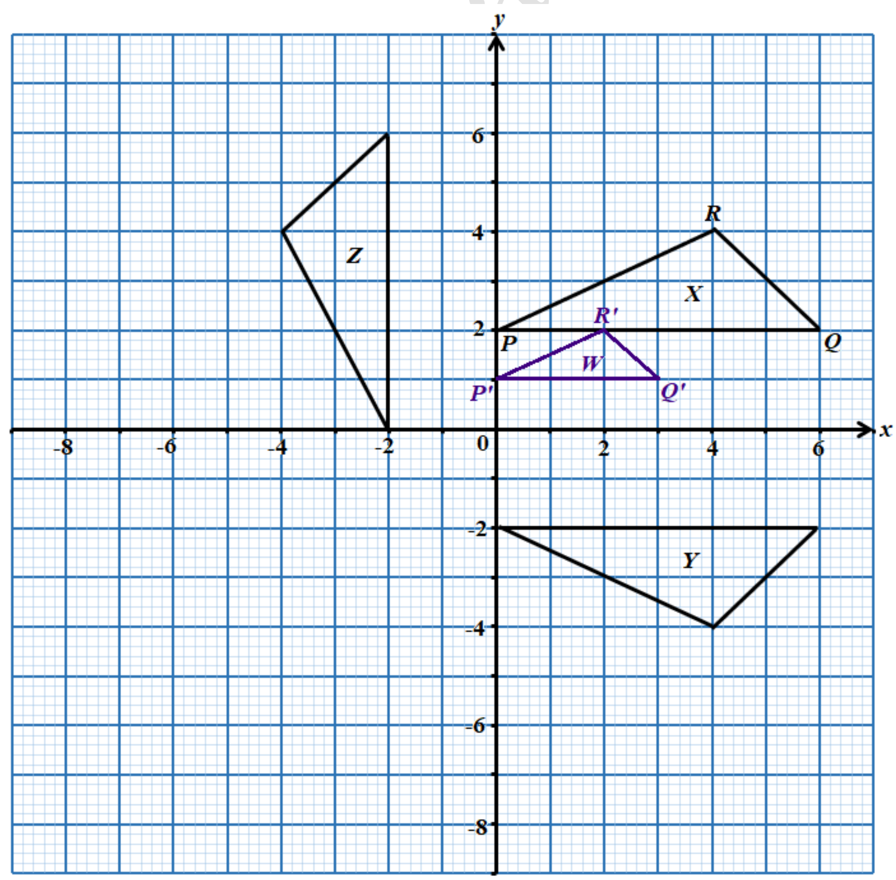
Object Point $\longrightarrow$ Image Point

$$(0,\ 2) \xrightarrow{\text{Enlargement of centre } (0,\ 0) \text{, scale factor} = \frac{1}{2}} (0,\ 1)$$

$$(6,\ 2) \xrightarrow{\text{Enlargement of centre } (0,\ 0) \text{, scale factor} = \frac{1}{2}} (3,\ 1)$$

$$(4,\ 4) \xrightarrow{\text{Enlargement of centre } (0,\ 0) \text{, scale factor} = \frac{1}{2}} (2,\ 2)$$

The triangle W is shown on the grid.



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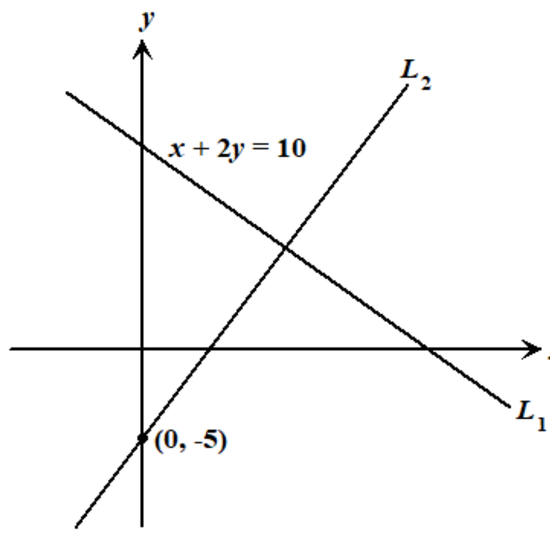
4. (a) The diagram below shows two lines, L_1 and L_2. The equation of the line L_1 is $x+2y=10$. The line L_2 passes through the point $(0,-5)$ and is **perpendicular** to L_1.



(i) Express the equation of the line L_1 in the form $y=mx+c$.

SOLUTION:
Data: Diagram showing two lines L_1 and L_2, where L_1 has equation $x+2y=10$, L_2 passes through the point $(0,-5)$ and L_1 and L_2 are perpendicular.
Required to express: The equation of L_1 in the form $y=mx+c$.
Solution:
The equation of L_1 is $x+2y=10$

$$2y=-x+10$$
$$y=-\frac{1}{2}x+5$$

This is of the form $y=mx+c$, where $m=-\frac{1}{2}$ and $c=5$.

(ii) State the gradient of the line L_1.

SOLUTION:
Required to state: The gradient of the line L_1.
Solution:
The line, L_1 when expressed in the form $y = mx + c$ was found to be

$$y=-\frac{1}{2}x+5$$

where m is the gradient of the line. Therefore, the gradient of L_1 is $-\frac{1}{2}$.

(iii) Hence, determine the equation of the line L_2.

SOLUTION:
Required to determine: The equation of the line L_2.
Solution:
The product of the gradients of perpendicular lines is -1.

Gradient of $L_1 \times$ Gradient of $L_2 = -1$

Gradient of $L_2 = \frac{-1}{Gradient\ of\ L_1}$

$$\text{Gradient of } L_2 = \frac{-1}{-\frac{1}{2}}$$
$$= 2$$

The equation of any line is y = $mx + c$ where m is the gradient and c is the intercept on the y-axis. L_2 cuts the vertical axis at -5 and has a gradient of 2.

Hence, the equation of L_2 is $y = 2x - 5$.

(b) Given that $f(x) = \frac{1}{3}x + 4$ and $g(x) = \frac{3x}{x+1}$,

(i) determine the value of $f(9)$

SOLUTION:
Data: $f(x) = \frac{1}{3}x + 4$ and $g(x) = \frac{3x}{x+1}$
Required to determine: $f(9)$
Solution:

$$f(x) = \frac{1}{3}x + 4$$
$$f(9) = \frac{1}{3}(9) + 4$$
$$= 3 + 4$$
$$= 7$$

(ii) calculate the value of $fg(-3)$.

SOLUTION:

Required to calculate: $fg(-3)$

Calculation:

$$g(x)=\frac{3x}{x+1}$$

$$g(-3)=\frac{3(-3)}{-3+1}$$

$$=\frac{-9}{-2}$$

$$=\frac{9}{2}$$

Hence, $fg(-3)=f\left(\frac{9}{2}\right)$

$$=\frac{1}{3}\left(\frac{9}{2}\right)+4$$

$$=\frac{3}{2}+4$$

$$=5\frac{1}{2}$$

Alternative Method:

$$fg(x)=\frac{1}{3}\left(\frac{3x}{x+1}\right)+4$$

$$fg(-3)=\frac{1}{3}\left(\frac{3(-3)}{-3+1}\right)+4$$

$$=\frac{-3}{-2}+4$$

$$=5\frac{1}{2}$$

(iii) determine the value of x, for which $g(x)=\frac{5}{2}$.

SOLUTION:

Required to determine: The value of x for which $g(x)=\frac{5}{2}$.

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Solution:

$$g(x)=\frac{5}{2}$$

$$\frac{3x}{x+1}=\frac{5}{2}$$

$$2(3x)=5(x+1)$$

$$6x=5x+5$$

$$x=5$$

5. (a) One hundred students were surveyed on the amount of money they spent on data for their cellphones during a week. The table below shows the results as well as the midpoint for each class interval.

Amount Spent (\$)	**Number of Students (f)**	**Midpoint (\$) (x)**
$50 < x \le 60$	7	55
$60 < x \le 70$	11	65
$70 < x \le 80$	31	75
$80 < x \le 90$	29	85
$90 < x \le 100$	22	95

Using the table,

(i) **a)** determine the modal class of the amount of money spent

SOLUTION:
Data: Grouped frequency distribution showing the amount of money that one hundred students spent on data for their cellphones during a week.
Required to determine: The modal class of the amount of money spent.
Solution:
The modal class is $70 < x \le 80$, since this class occurs most often.

b) calculate an estimate of the mean amount of money spent, giving your answer correct to 2 decimal places.

SOLUTION:
Required to calculate: An estimate of the mean amount of money, correct to 2 decimal places.
Calculation:

$\bar{x}=\frac{\sum fx}{\sum f}$, where x =midpoint, f =frequency and $\bar{x}$ =mean

$$= \frac{(7\times 55)+(11\times 65)+(31\times 75)+(29\times 85)+(22\times 95)}{100}$$

$= 79.8$

$= \$79.80$

(ii) Damion reports that the median amount spent is \$84. Briefly explain why Damion's report could be correct.

SOLUTION:
Data: Damion reports that the median amount of money spent is \$84.
Required to explain: Why this report is correct.
Solution:
In 100 scores, the median will be the score that is in the class interval with the 50th score. The first 49 (7+11+31) scores are less than 80. Therefore, the 50th score lies in the class $80 < x \leq 90$ and in the absence of raw scores, the midpoint of 85 is considered as the median. However, the actual score could be any value that is greater than 80 and less than or equal to 90. Hence, Damion could be correct.

(b) The two-way/contingency table below gives information on the mode of transportation to school for 100 students.

	Walk	**Cycle**	**Drive**	**Total**
Boy	15		14	48
Girl		18	26	
Total	23		40	100

(i) Complete the table by inserting the missing values.

SOLUTION:
Data: Incomplete two-way/contingency table showing information on the mode of transportation to school for 100 students.
Required to complete: The table given.
Solution:

	Walk	**Cycle**	**Drive**	**Total**
Boy	15	$48-(15+14)$ $=19$	14	48
Girl	$52-(18+26)$ $=8$	18	26	$100-48$ $=52$
Total	23	$19+18$ $=37$	40	100

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(ii) A student is selected at random. What is the probability that he/she was being driven to school on that day?

SOLUTION:
Required to find: The probability that a randomly selected student is being driven to school on a particular day.
Solution:

$$\text{P(student is being driven to school)} = \frac{\text{Number of students being driven to school}}{\text{Total number of students}}$$
$$= \frac{40}{100}$$
$$= \frac{2}{5}$$

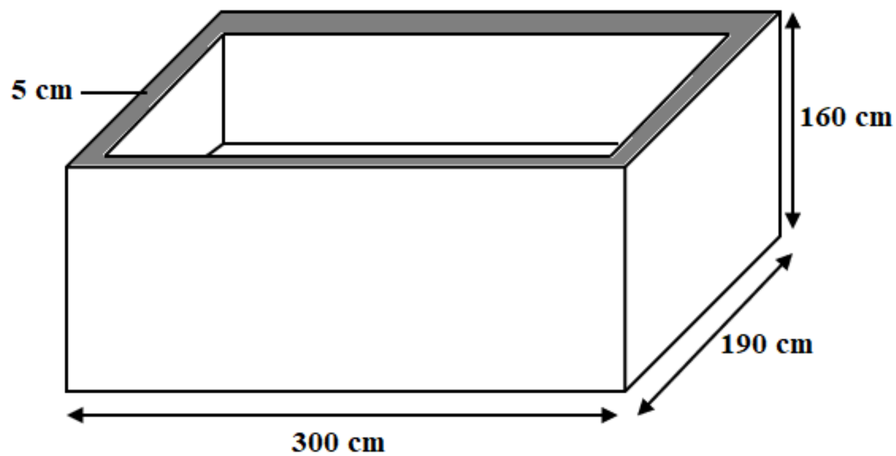
(iii) One of the girls is selected at random. What is the probability that she did NOT cycle to school?

SOLUTION:
Required to find: The probability that a randomly selected girl did not cycle to school.
Solution:

$$\text{P(girl did not cycle to school)} = \frac{\text{Number of girls who did not cycle to school}}{\text{Total number of girls}}$$
$$= \frac{8+26}{52}$$
$$= \frac{34}{52}$$
$$= \frac{17}{26}$$

We could also have found this by 1 – P (girl who cycles to school)
This would be $1-\frac{18}{52}=1-\frac{9}{26}$
$$= \frac{17}{26}$$

6. Farmer Brown makes troughs to feed his farm animals, using wood that is 5 cm thick. As shown in the diagram below, the troughs are rectangular-based, open at the top and have external dimensions of 300 cm by 190 cm by 160 cm.



(a) Show, by calculation, that the internal capacity (volume) of the trough is 8 091 000 cm^3.

SOLUTION:
Data: Diagram showing an open rectangular-based trough with dimensions 300 cm by 190 cm by 160 cm made with wood that is 5 cm thick.
Required to show: The internal capacity (volume) of the trough is 8 091 000 cm^3.
Proof:
The internal width $=190-(5+5)=180$ cm
The internal length $=300-(5+5)=290$ cm
The internal height $=160-5=155$ cm

The internal capacity $=(180\times290\times155)\text{ cm}^2$
$=8091000\text{ cm}^3$

Q.E.D.

(b) Calculate the volume of wood needed to make a trough.

SOLUTION:
Required to calculate: The volume of wood needed to make a trough.
Calculation:
The external volume of the trough $=(300\times190\times160)\text{ cm}^3$
$=9120000\text{ cm}^3$

Hence, the volume of wood used to make the trough

$= \text{External volume} - \text{Internal volume}$

$= (9120000 - 8091000)\ \text{cm}^3$

$= 1029000\ \text{cm}^3$

(c) Farmer Brown must paint the INTERNAL surface of the trough. Given that one gallon of paint covers approximately 280 000 cm^2 of surface, determine the TOTAL amount of paint, **in litres**, that is needed to paint the internal surface of the trough.

(1 gallon ≈ 3.79 litres)

SOLUTION:
Data: 1 gallon of paint covers approximately 280 000 cm^2 of surface and 1 gallon ≈ 3.79 litres
Required to determine: The total amount of paint needed to paint the internal surface of the trough, in litres.
Solution:
The internal surface to be painted in cm^2 is the sum of the four vertical sides and the base of the trough

$= (155 \times 180) \times 2 + (155 \times 290) \times 2 + (290 \times 180)$

$= 55800 + 89900 + 52200$

$= 197900\ \text{cm}^2$

Hence, the number of litres of paint required

$= \dfrac{197900}{280000} \times 3.79\ \text{litres}$

$= 2.68\ \text{litres (correct to 2 decimal places)}$

$\therefore$ The farmer will likely have to buy 3 litres of paint.

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7. The first 3 figures in a sequence of shapes, formed by connecting lines of unit length, are shown below.

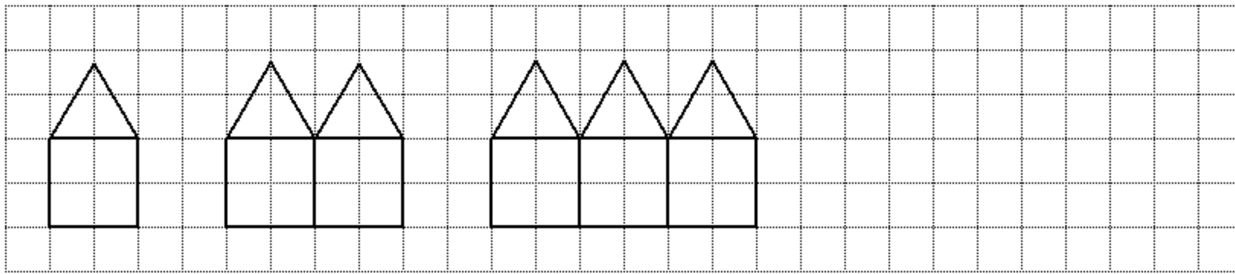

Figure 1 **Figure 2** **Figure 3** **Figure 4**

(a) Draw Figure 4 of the pattern in the space provided below.

SOLUTION:
Data: Diagram showing the first 3 figures of a sequence of shapes formed by connecting lines of unit length.
Required to draw: The 4th figure in the sequence.
Solution:

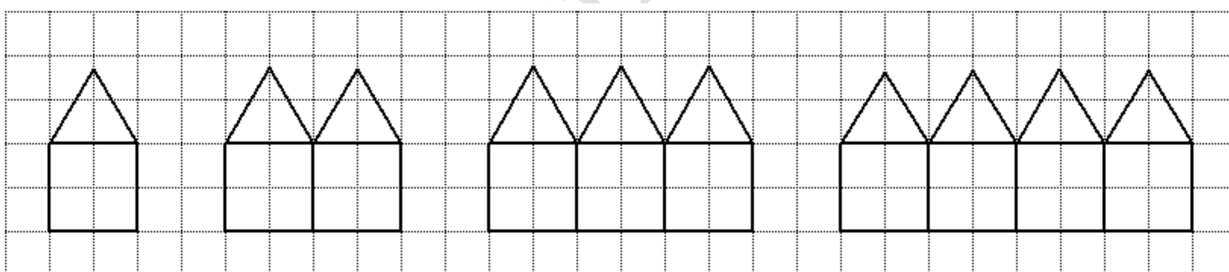

Figure 1 **Figure 2** **Figure 3** **Figure 4**

(b) The number of lines, L, in each shape and the perimeter, P, of the shape follow a pattern. Study the pattern of numbers in each row of the table below and answer the questions that follow.

Complete the table below showing the number of lines and the perimeter of each figure.

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	Figure	Number of Lines (L)	Perimeter (P)
	1	6	5
	2	11	8
	3	16	11
	⋮	⋮	⋮
(i)	5	________	________
	⋮	⋮	⋮
(ii)	________	66	________
	⋮	⋮	⋮
(iii)	n	________	________

SOLUTION:
Data: Incomplete table showing the relationship between the number of lines, L, and perimeter, P, of the shapes in the sequence.
Required to complete: The table given.
Solution:

The number of lines, L, increases by 5 and which are 6, 11, 16…
Consider, $L = 5n \pm C$, where C is a number.

When $n = 1$, $L = 6$ $6 = 5(1) + 1$ $\therefore C = 1$ Hence, $L = 5n + 1$	Test: When $n = 2$ $L = 5(2) + 1$ $= 10 + 1$ $= 11$	Test: When $n = 3$ $L = 5(3) + 1$ $= 15 + 1$ $= 16$

The perimeter of the figure, P increases by 3 and which are 5, 8, 11, …
Consider, $P = 3n \pm K$, where K is a number.

When $n = 1$, $P = 5$ $5 = 3(1) + 2$ $\therefore K = 2$ Hence, $P = 3n + 2$	Test: When $n = 2$ $P = 3(2) + 2$ $= 8$	Test: When $n = 3$ $P = 3(3) + 2$ $= 11$

Alternative Solution:

Figure	Number of Lines (L)	Perimeter (P)
1	$1 + 1(5) = 6$	$2 + 1(3) = 5$
2	$1 + 2(5) = 11$	$2 + 2(3) = 8$
3	$1 + 3(5) = 16$	$2 + 3(3) = 11$
⋮	⋮	⋮
n	$1 + n(5) = 5n + 1$	$2 + n(3) = 3n + 2$

Hence, $L = 5n + 1$ **and** $P = 3n + 2$

The completed table looks like:

	Figure	Number of Lines (L)	Perimeter (P)
	1	6	5
	2	11	8
	3	16	11
	⋮	⋮	⋮
(i)	5	$5(5)+1=26$	$3(5)+2=17$
	⋮	⋮	⋮
(ii)	13	66 $5n+1=66$ $5n=65$ $n=13$	$3(13)+2=41$
	⋮	⋮	⋮
(iii)	n	$L=5n+1$	$P=3n+2$

(c) Write a simplified expression, in terms of *n*, for the difference, *d*, between the number of lines and the perimeter of any figure, *n*.

SOLUTION:
Required to write: A simplified expression for the difference between the number of lines, *L*, and the perimeter, *P*, in terms of *n*.
Solution:
The number of lines, $L = 5n + 1$
The perimeter, $P = 3n + 2$

$$d = L - P$$
$$= (5n+1) - (3n+2)$$
$$d = 2n - 1$$

SECTION II

Answer ALL questions.

ALL working MUST be clearly shown.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. Marla buys 2 types of mobile phones, B-Flo and C-Flex, from a company to retail. One B-Flo mobile phone costs \$60 while one C-Flex costs \$80. She buys x number of B-Flo phones and y number of C-Flex phones.

(a) **(i)** Marla must not spend more than \$1 200. Write an inequality to represent this information.

SOLUTION:
Data: One B-Flo mobile phone costs \$60 and one B-Flex mobile phone costs \$80. Marla buys x B-Flo phones and y B-Flex phones, but must not spend more than \$1 200.
Required to write: An inequality to represent the data given.
Solution:
x phones are \$60 each and y phones at \$80 each will cost

$\$(60\times x)+\$(80\times y)=6x+80y$

The maximum amount of money available to spend is \$1 200.
Hence, $60x+80y\le 1200$

$(\div 20)$

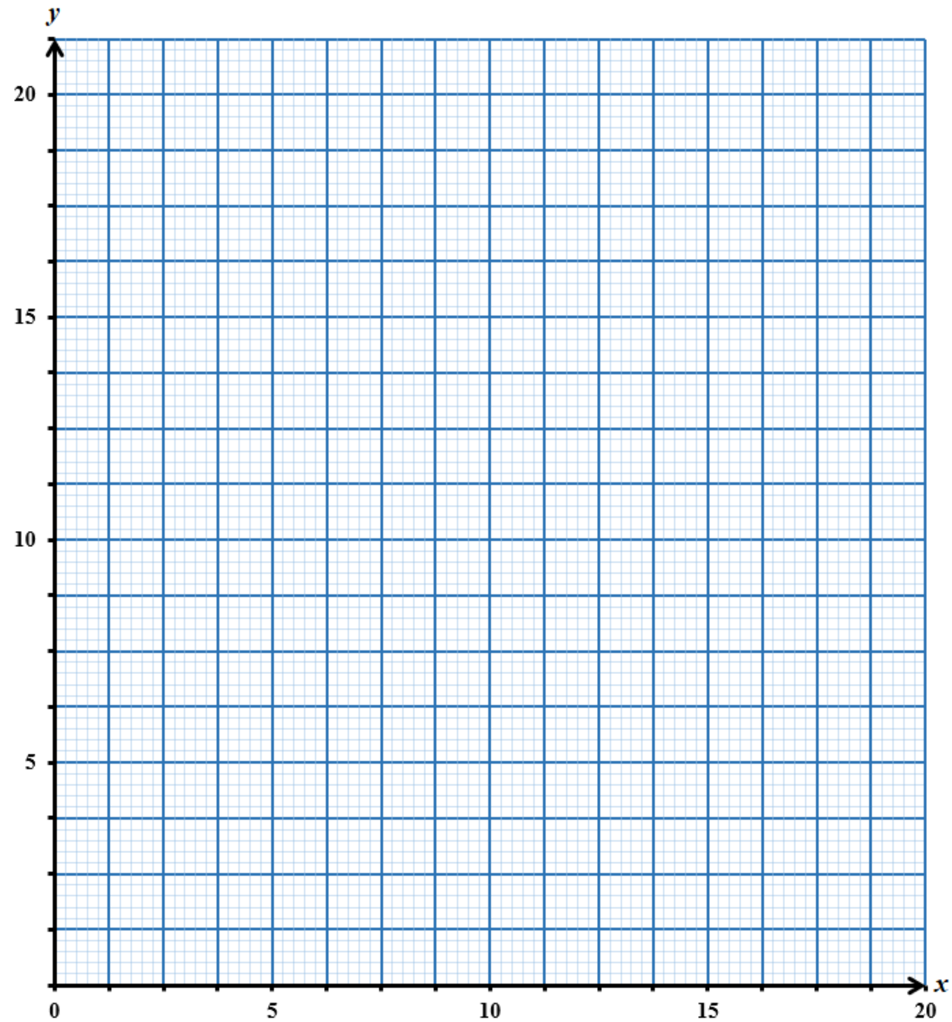
$3x+4y\le 60$

(ii) The number of B-Flo phones must be greater than or equal to the number of C-Flex phones. Write down an inequality in x and y to show this information.

SOLUTION:
Data: The number of B-Flo phones must be greater than or equal to the number of C-Flex phones.
Required to write: An inequality to represent the data given.
Solution:
x is greater than or equal to y

$x\ge y$ or $y\le x$

(iii) Represent the two inequalities on the grid shown below. Label as R the region that satisfies both inequalities.



The grid presented in the examination booklet showed 4 cm = 5 units, from 0 to 15 on the y-axis but 5 cm = 5 units from 15 to 20. This was indeed an error, and as such the scale on the y – axis was modified to reflect 4 cm ≡ 5 units throughout. The above diagram shows the grid with the modified scale.

SOLUTION:
Data: Grid given on which to represent both inequalities.
Required to represent: The two inequalities on the grid given.
Solution:

Drawing $3x+4y=60$
Choosing two values of x and finding the corresponding two values of y.

x	y
0	15
20	0

We plot $(0, 15)$ and $(20, 0)$ to draw the graph of $3x+4y=60$.

Drawing $y=x$

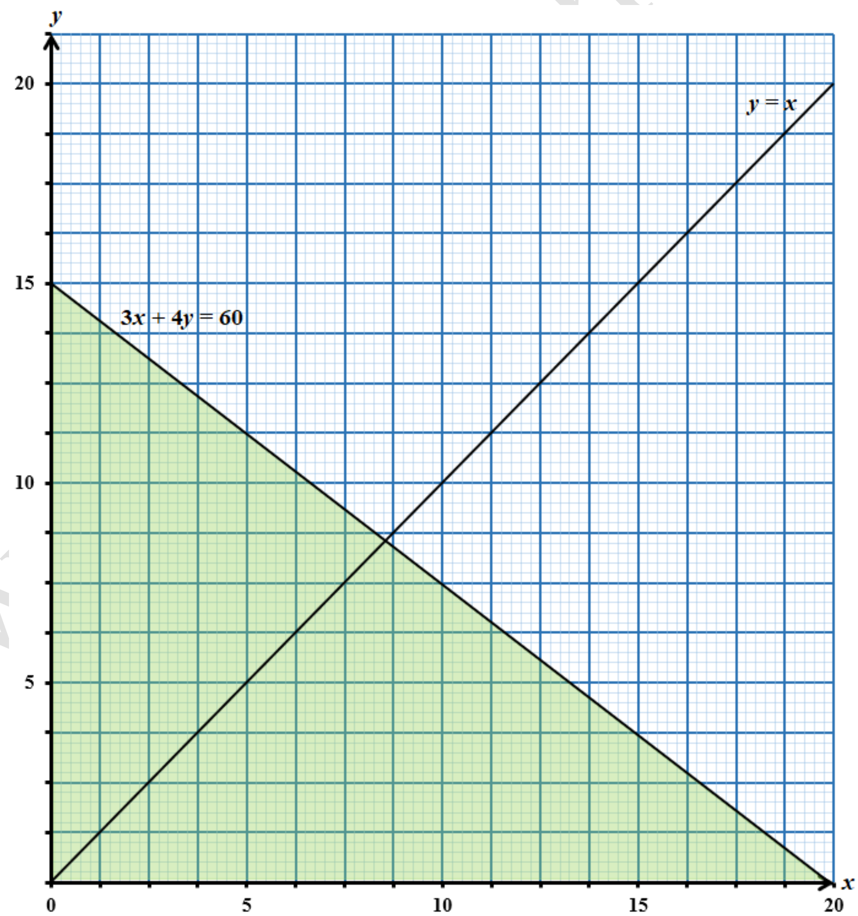
Choosing two values of x and finding the corresponding two values of y.

x	y
0	0
20	20

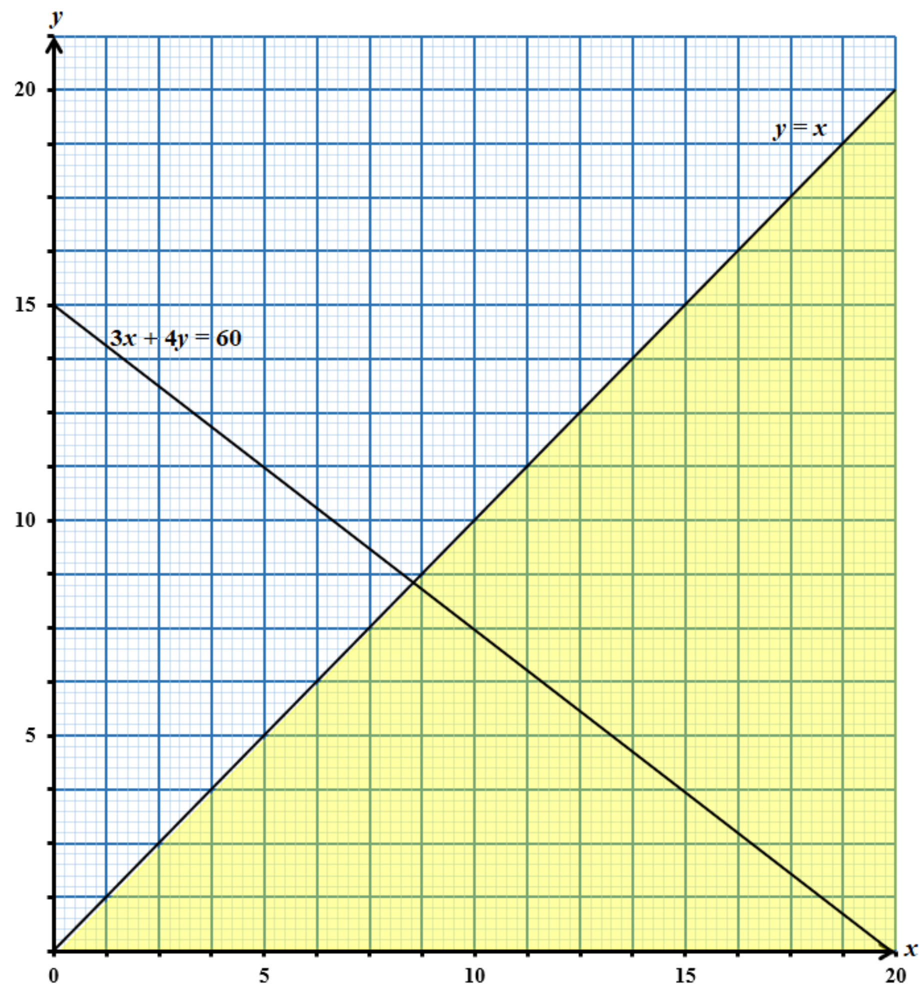
Plot $(0, 0)$ and $(20, 20)$ to draw the graph of $y=x$

(As indicated above, if the point (20, 20) was plotted this would have automatically led to an incorrect line on the grid because of incorrect labelling and which would hardly be detected by a candidate)

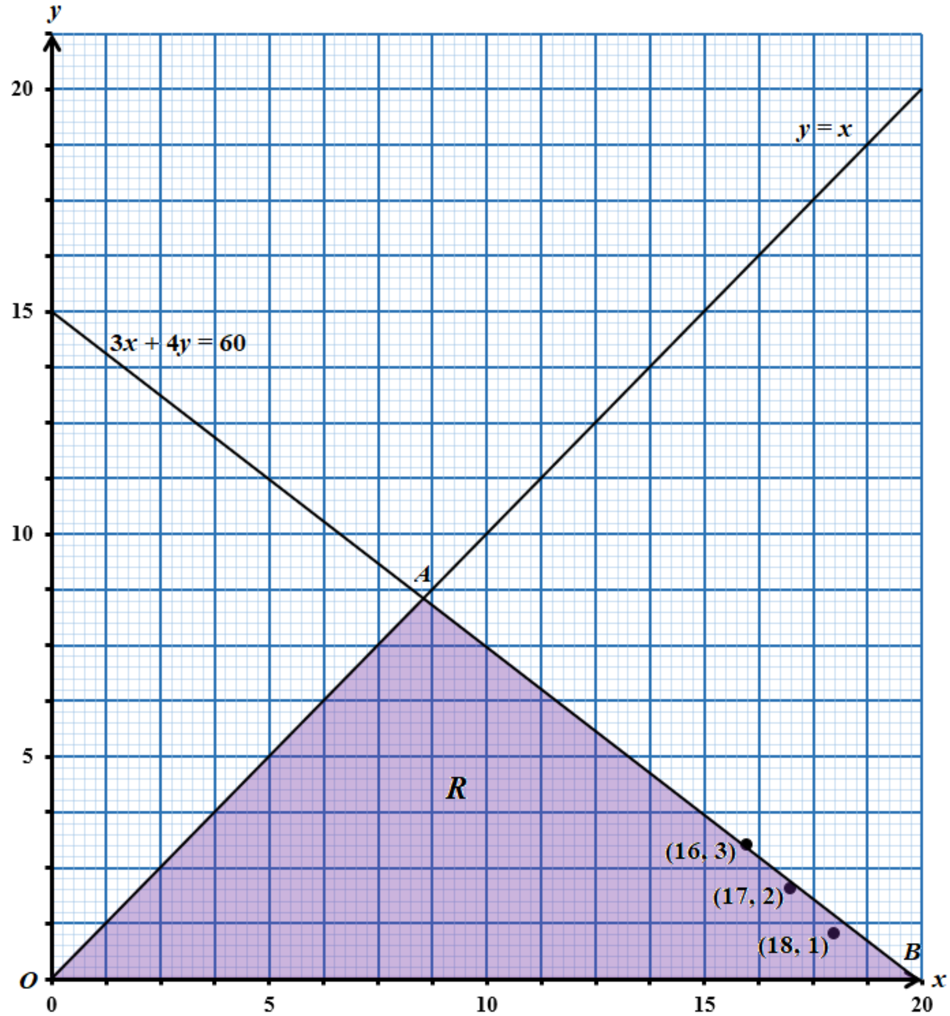
$3x+4y \le 60$ includes the line and is shown coloured in green.



$y \le x$ includes the line and is shown coloured in yellow.



Hence, R is the region that is common to both shaded regions. This is coloured purple below.



(iv) The total number of mobile phones is represented by $x+y$. According to the graph, what is the largest possible value of $x+y$?

SOLUTION:
Data: The total number of mobile phones is represented by $x+y$.
Required to find: The maximum possible value of $x+y$.
Solution:
The vertices of the region, R, are $O\ (0,\ 0)$, A (8.6, 8.6) and $B\ (20,\ 0)$
Considering the vertex $B\ (20,\ 0)$ $x=20,\ y=0$

$$x+y=20+0$$
$$=20$$

$\therefore$ The maximum value of $x+y$ appears to be 20.

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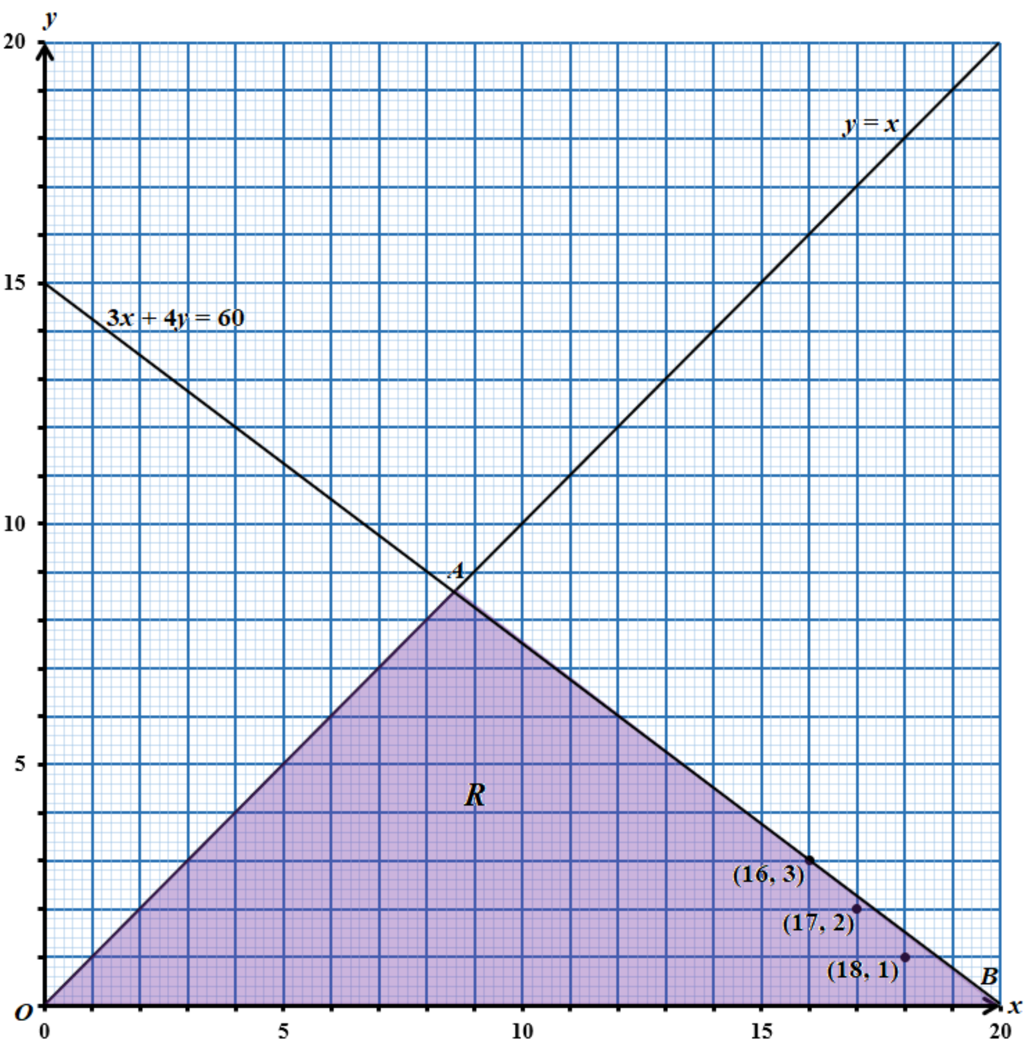
However, from the wording of the question, Maria bought **both** types of phones so the value of y cannot be 0 as she must buy at least one of each type.

We need to search within the feasible region for the maximum value of $x + y$. The maximum can occur at any of the three points (18, 1), (17, 2) and (16, 3).

Hence, the maximum value of $x + y$ is 19, which is obtained by using the coordinates of any of these three points,

$x = 18$ and $y = 1$
$x = 17$ and $y = 2$
$x = 16$ and $y = 3$.

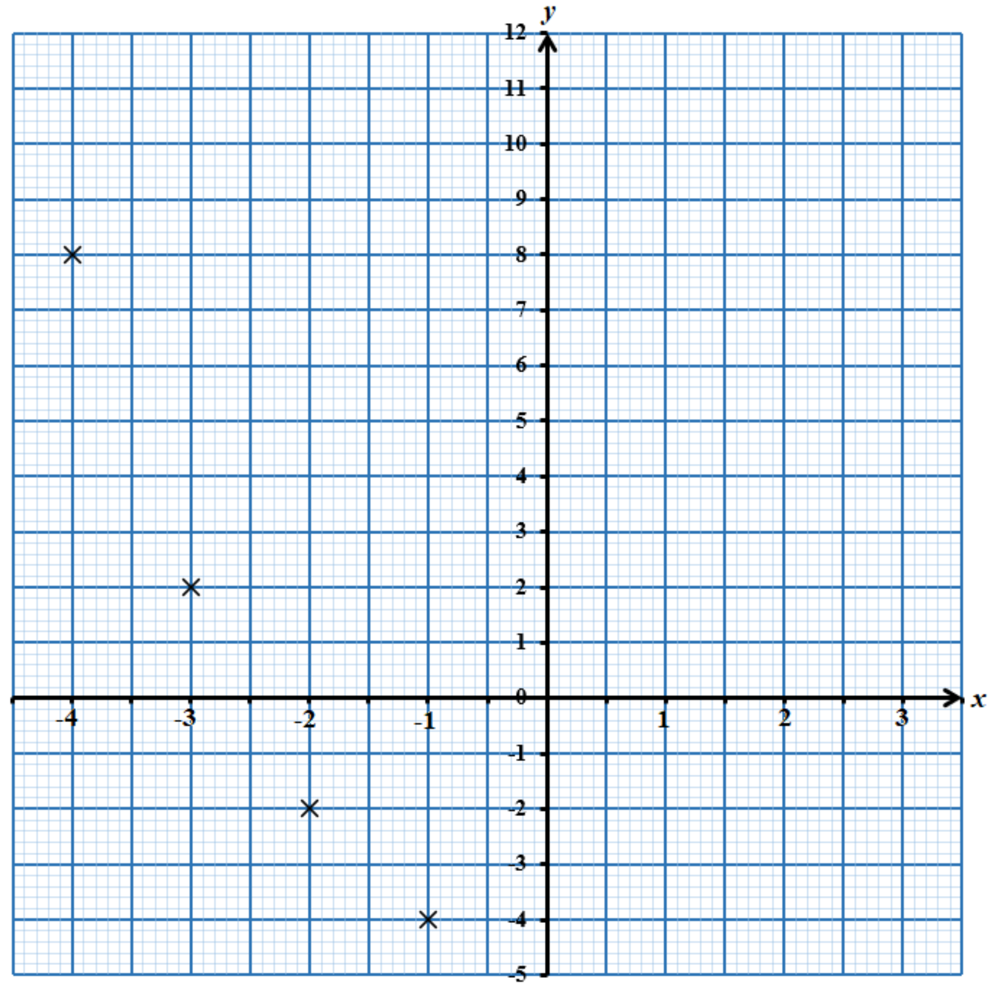
We now present the solution on a new graph using a scale of 5cm equal to 5 units so that the three points can be easily read off from the graph.



(b) The table below shows pairs of values for the function $y = x^2 + x - 4$.

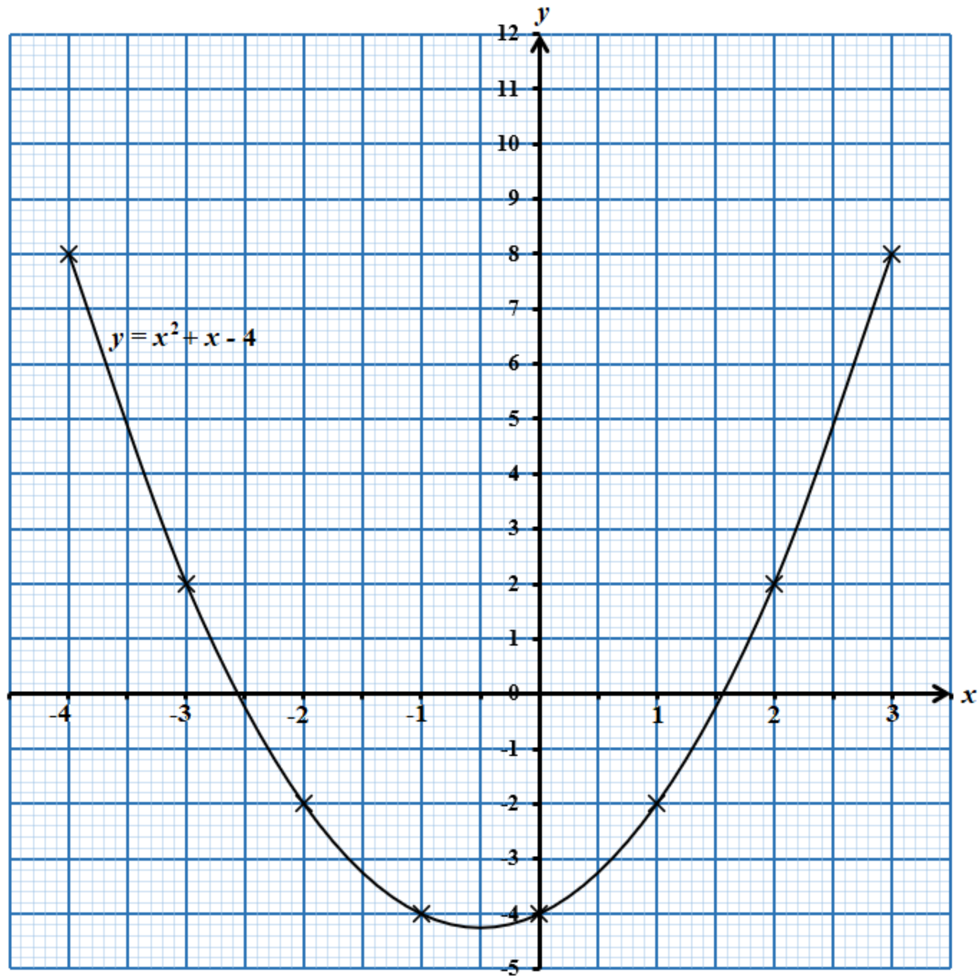
x	-4	-3	-2	-1	0	1	2	3
y	8	2	-2	-4	-4	-2	2	8

(i) On the grid provided, plot the remaining 4 points and draw the graph of the function $y = x^2 + x - 4$ for $-4 \le x \le 3$.



SOLUTION:
Data: Table of values for the function $y = x^2 + x - 4$ for $-4 \le x \le 3$.
Required to plot: The last 4 points and draw the graph of $y = x^2 + x - 4$ for $-4 \le x \le 3$.

Solution:



(ii) Write down the maximum or minimum value of the function.

SOLUTION:
Required to write: The maximum or minimum value of the function $y = x^2 + x - 4$.
Solution:

The minimum value of the function occurs at $x = -\frac{1}{2}$
From the graph, the y- co-ordinate appears to be -4.25.

We can calculate the y-co-ordinate by substitution as follows:
When $x = -\frac{1}{2}$, $y = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 4 = \frac{1}{4} - \frac{1}{2} - 4 = -4\frac{1}{4}$
The minimum value of the function is $-4\frac{1}{4}$.

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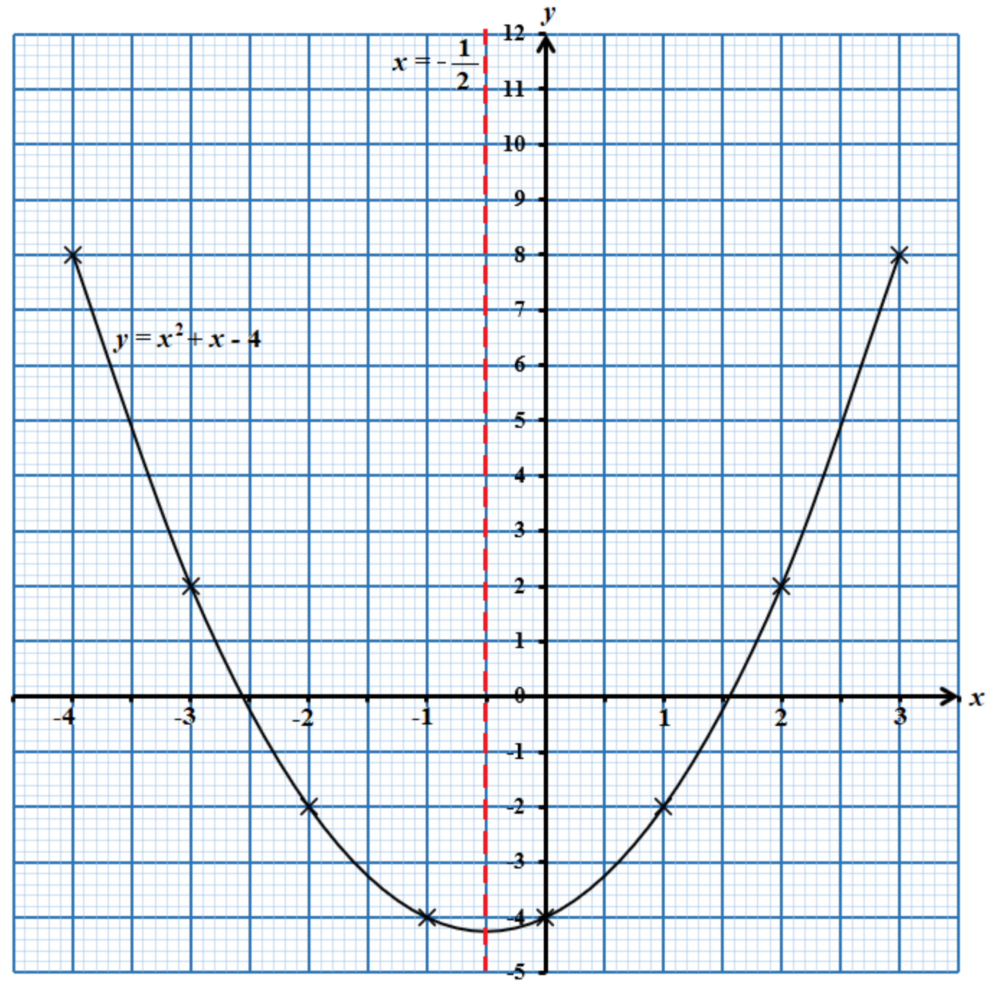
(iii) Using a ruler, draw the axis of symmetry on the graph.

SOLUTION:
Required to draw: The axis of symmetry on the graph.
Solution:

The axis of symmetry is at $x = -\frac{1}{2}$.



The axis of symmetry $x = -\frac{1}{2}$ is shown in the red dotted line.

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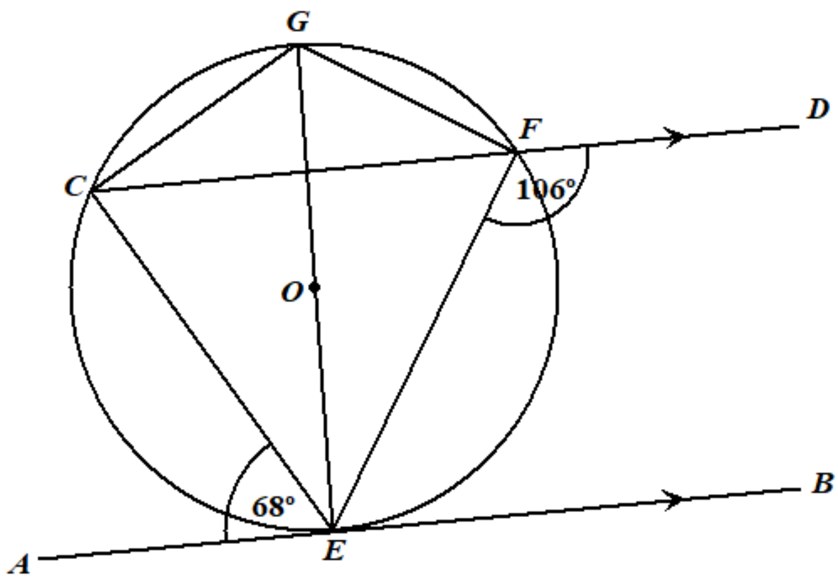
9. (a) In the diagram below, *E*, *C*, *G* and *F* are points on the circumference of a circle. *EG* is a diameter of the circle. The tangent *AEB* is parallel to *CD*. Angle $AEC = 68°$ and angle $EFD = 106°$.



Determine the value of EACH of the following angles. Show detailed working where necessary and give a reason to support your answer.

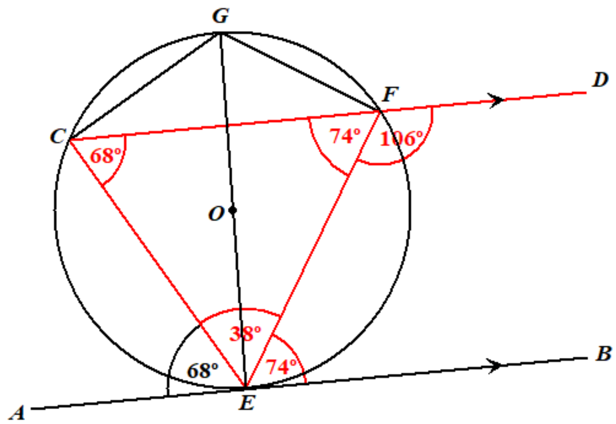
The angles given on the diagram are geometrically incorrect because the angle *CFE* should be 68° since the angle between a tangent (*AEB*) to a circle and a chord (*CE*) at the point of contact (*E*) is equal to the angle in the alternate segment and which is angle *CFE.* Based on the information given, this angle will be calculated to be 74°and so create an ambiguity.

The question would have had no ambiguities if the angle *EFD*, given as 106° was not given. A candidate who uses angle CFE as 68^0 will have a different answer for part (iii) of the question.

(i) *ECD*

SOLUTION:
Data: Diagram showing a circle with the points *E*, *C*, *G* and *F* on its circumference. *EG* is a diameter of the circle. The tangent *AEB* is parallel to *CD*. Angle $AEC = 68°$ and angle $EFD = 106°$.
Required To Determine: The size of angle *ECD*.

Solution:



$F\hat{E}B = 180° - 106°$

$= 74°$

(Co-interior angles are supplementary.)

$C\hat{E}F = 180° - (68° + 74°)$

$= 38°$

(Angles in a straight line total 180°.)

$C\hat{F}E = 180° - 106°$

$= 74°$

(Angles in a straight line total 180°.)

Consider ΔECF:

$E\hat{C}F = E\hat{C}D$

$= 180° - (74° + 38°)$

$= 68°$

(Sum of the interior angles in a triangle is equal to 180°.)

Alternatively,

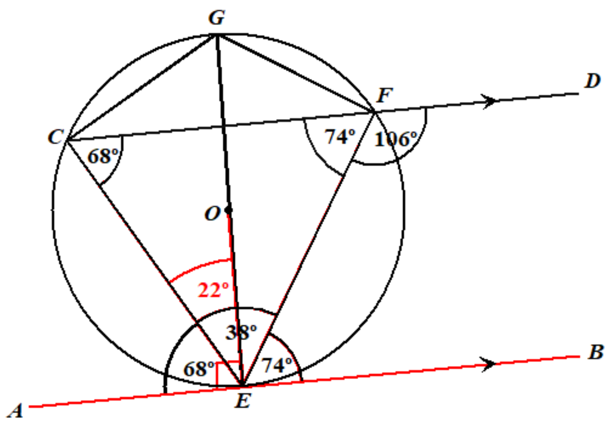
$E\hat{C}F = 68^0$ (CD is parallel to AB and $E\hat{C}F$ is alternate to angle $C\hat{E}A = 68^0$

(ii) *CEG*

SOLUTION:

Required to determine: The size of angle *CEG*.

Solution:

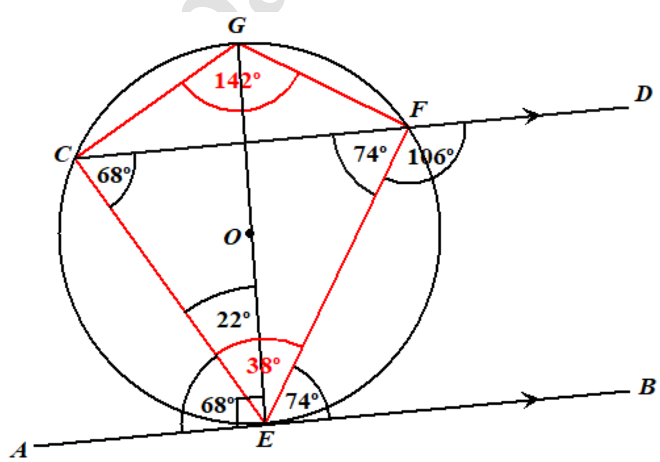


$O\hat{A}E = 90°$
(The angle made by the tangent to a circle (*EA*) and a radius (*OE*) at the point of contact (*E*) is 90°.)

Hence, $C\hat{E}G = 90° - 68°$
$= 22°$

(iii) *CGF*

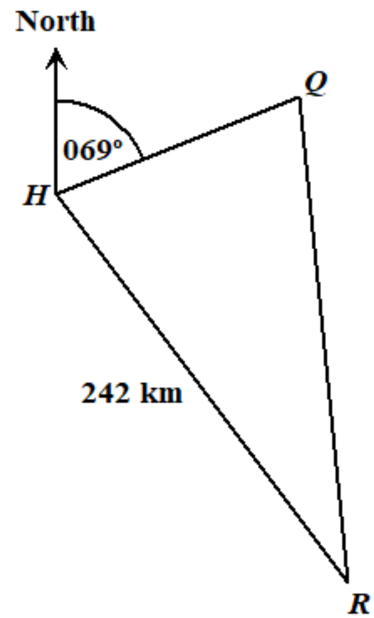
SOLUTION:
Required to determine: The size of the angle *CGF*.
Solution:



Consider the quadrilateral *CEFG*.
$C\hat{G}F = 180° - 38°$
$= 142°$
(Opposite angles in a cyclic quadrilateral are supplementary.)

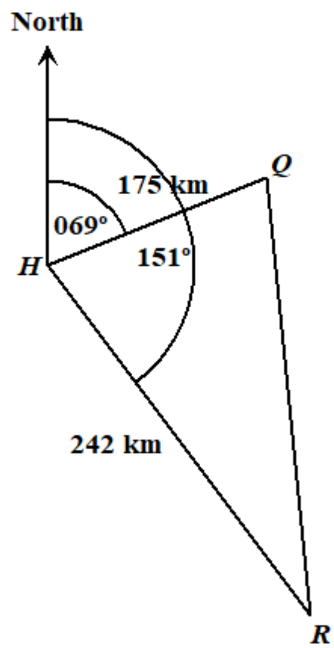
Note: if $C\hat{F}E$ was used as 68^0, then $C\hat{G}F = 180^0 - 2(22^0) = 136^0$

(b) From a harbour, *H*, the bearing of two ships, *Q* and *R*, are 069° and 151° respectively. *Q* is 175 km from *H* while *R* is 242 km from *H*.



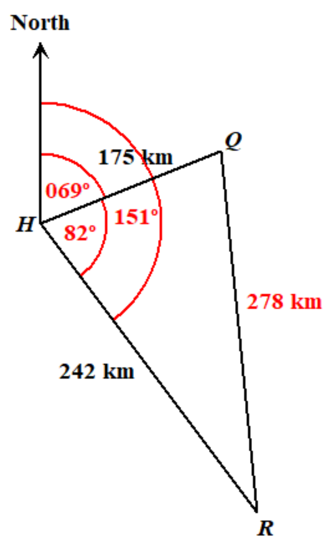
(i) Complete the diagram above to show the information given.

SOLUTION:
Data: Incomplete diagram showing the position of two ships, *R* and *Q*, from a harbour, *H*. The bearing of two ships, *Q* and *R*, from *H* are 069° and 151° respectively. *Q* is 175 km from *H* while *R* is 242 km from *H*.
Required to complete: The diagram given using the information given.
Solution:



(ii) Calculate QR, the distance between the two ships, to the nearest km.

SOLUTION:
Required to calculate: The distance QR, correct to nearest km.
Calculation:



Using the Cosine Rule:

$$QR^2 = (175)^2 + (242)^2 - 2(175)(242)\cos 82°$$

$$QR^2 = 30\,625 + 58\,564 - 11\,787.96$$

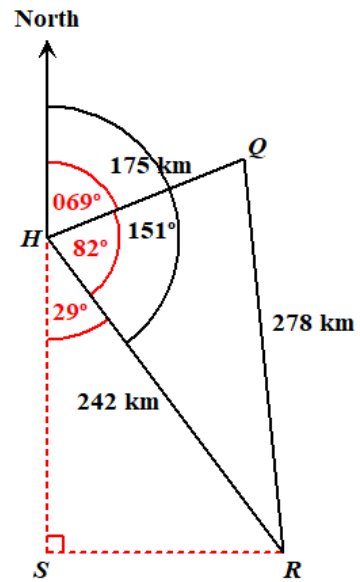
$$QR^2 = 77\,401.04$$

$$QR = 278.\underline{\underline{2}}10$$

$$QR \approx 278 \text{ km (correct to the nearest km)}$$

(iii) Calculate how far due south is Ship R of the harbour, H.

SOLUTION:
Required to calculate: The distance due south that Ship R is from the harbour, H.
Calculation:



$$S\hat{H}R = 180° - (69° + 82°)$$
$$= 29°$$

(Angles along a straight line total 180°.)

Consider ΔHSR

$$\cos 29° = \frac{HS}{242}$$
$$HS = 242 \cos 29°$$
$$= 211.65\underline{\underline{8}}$$
$$\approx 211.66 \text{ km (correct to 2 decimal places)}$$

Hence, Ship R is 211.66 km due south of the harbour, H.

10. (a) **(i)** Calculate the matrix product $\begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 & -4 \\ 0 & 3 & 6 \end{pmatrix}$.

SOLUTION:

Required to calculate: The matrix product $\begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 & -4 \\ 0 & 3 & 6 \end{pmatrix}$.

Calculation:

$$\begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 & -4 \\ 0 & 3 & 6 \end{pmatrix}$$

$$2\times 2 \quad \times \quad 2\times 3 \quad = 2\times 3$$

$$\begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 & -4 \\ 0 & 3 & 6 \end{pmatrix} = \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \end{pmatrix}$$

$$e_{11} = (5\times 2)+(4\times 0) = 10$$
$$e_{12} = (5\times 1)+(4\times 3) = 17$$
$$e_{13} = (5\times -4)+(4\times 6) = 4$$
$$e_{21} = (-3\times 2)+(-2\times 0) = -6$$
$$e_{22} = (-3\times 1)+(-2\times 3) = -9$$
$$e_{23} = (-3\times -4)+(-2\times 6) = 0$$

$$\therefore \begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 & -4 \\ 0 & 3 & 6 \end{pmatrix} = \begin{pmatrix} 10 & 17 & 4 \\ -6 & -9 & 0 \end{pmatrix}$$

(ii) State why the two matrices in (a) (i) are conformable for multiplication.

SOLUTION:
Required to state: The reason why the two matrices in (a) (i) are conformable for multiplication.
Solution:
$A = \begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 & -4 \\ 0 & 3 & 6 \end{pmatrix}$. In the product $A\times B$ the number of columns of A is equal to the number of rows of B and which is the condition necessary for a matrix multiplication.

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(b) Determine the inverse of $\begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$.

SOLUTION:

Required to determine: The inverse of $\begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$.

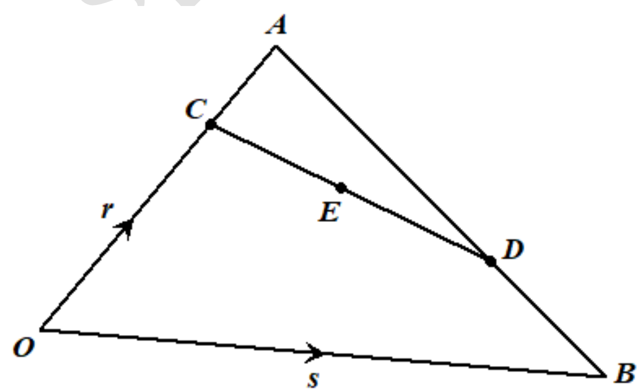
Solution:

Let $A = \begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$

$$|A| = (5\times -2) - (4\times -3)$$
$$= -10 - (-12)$$
$$= 2$$

$$A^{-1} = \frac{1}{2}\begin{pmatrix} -2 & -(4) \\ -(-3) & 5 \end{pmatrix}$$
$$= \begin{pmatrix} -1 & -2 \\ 1\frac{1}{2} & 2\frac{1}{2} \end{pmatrix}$$

(c) The diagram below shows triangle OAB in which $\overrightarrow{OA} = r$ and $\overrightarrow{OB} = s$. In addition, E is the midpoint of CD, $OC = \frac{3}{4}OA$ and $AD = \frac{2}{3}AB$.



Write, in terms of r and s, in the simplest form, an expression for:

(i) $\overrightarrow{CD}$

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SOLUTION:

Data: Diagram showing triangle OAB in which $\overrightarrow{OA} = r$ and $\overrightarrow{OB} = s$, E is the midpoint of CD, $OC = \frac{3}{4}OA$ and $AD = \frac{2}{3}AB$.

Required to write: $\overrightarrow{CD}$, in terms of r and s.

Solution:

If $\overrightarrow{OC} = \frac{3}{4}r$

then $\overrightarrow{CA} = \frac{1}{4}r$

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= -(r) + s$$
$$\therefore \overrightarrow{AD} = \frac{2}{3}(-r + s)$$

$$\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{AD}$$
$$= \frac{1}{4}r + \frac{2}{3}(-r + s)$$
$$= \frac{1}{4}r - \frac{2}{3}r + \frac{2}{3}s$$
$$= -\frac{5}{12}r + \frac{2}{3}s$$

(ii) $\overrightarrow{OE}$

SOLUTION:

Required to write: $\overrightarrow{OE}$, in terms of r and s.

Solution:

$$\overrightarrow{OE} = \overrightarrow{OC} + \overrightarrow{CE}$$
$$= \frac{3}{4}r + \frac{1}{2}\left(-\frac{5}{12}r + \frac{2}{3}s\right)$$
$$= \frac{3}{4}r - \frac{5}{24}r + \frac{1}{3}s$$
$$= \frac{13}{24}r + \frac{1}{3}s$$

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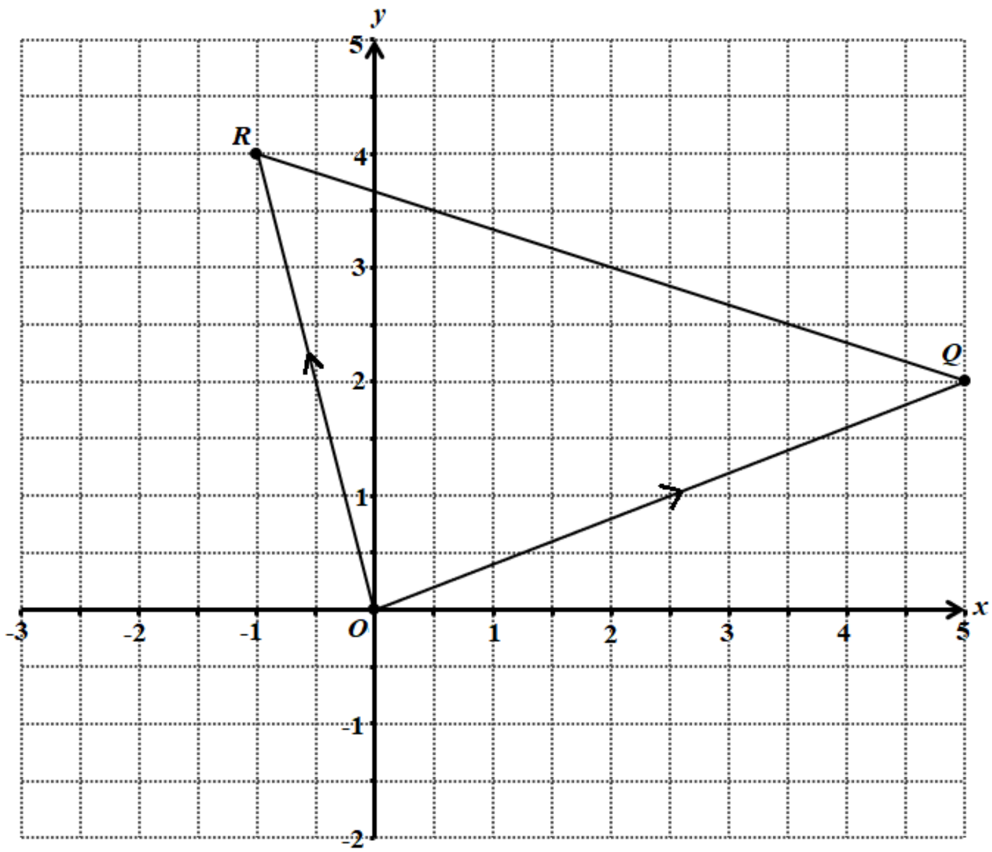
(d) The points O, Q and R have coordinates $(0,\ 0)$, $(5,\ 2)$ and $(-1,\ 4)$ respectively.



(i) Write $\overrightarrow{OR}$ as a column vector.

SOLUTION:
Data: Grid showing the points O, Q and R with coordinates $(0,\ 0)$, $(5,\ 2)$ and $(-1,\ 4)$ respectively.
Required to write: $\overrightarrow{OR}$ as a column vector.
Solution:

$$R = (-1,\ 4)$$

$$\therefore \overrightarrow{OR} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

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(ii) Determine $\left|\overrightarrow{QR}\right|$.

SOLUTION:
Required to determine: $\left|\overrightarrow{QR}\right|$
Solution:

$$Q = (5,\ 2)$$

$$\therefore \overrightarrow{OQ} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$\overrightarrow{QR} = \overrightarrow{QO} + \overrightarrow{OR}$$

$$= -\begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

$$\left|\overrightarrow{QR}\right| = \sqrt{(-6)^2 + (2)^2}$$

$$= \sqrt{40} \text{ units}$$

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.