FAS-PASS
Maths

CSEC MATHEMATICS JANUARY 2021 PAPER 2

SECTION 1

1. (a) (i) Using a calculator, or otherwise, calculate the EXACT value of

$$1\frac{4}{7}+\frac{2}{3}-1\frac{5}{6}$$

SOLUTION:

Required to find: The exact value of $1\frac{4}{7}+\frac{2}{3}-1\frac{5}{6}$

Calculation:

$$1\frac{4}{7}+\frac{2}{3}-1\frac{5}{6}$$
$$=\frac{7(1)+4}{7}+\frac{2}{3}-\frac{6(1)+5}{6}$$
$$=\frac{11}{7}+\frac{2}{3}-\frac{11}{6}$$
$$=\frac{6(11)+14(2)-7(11)}{42} \quad \text{(L.C.M. of 7, 3, and 6 is 42)}$$
$$=\frac{66+28-77}{42}$$
$$=\frac{17}{42}\ (\text{in exact form})$$

(ii) Write the value of $\frac{\sqrt[3]{27}}{9^2}$ as a fraction in its LOWEST terms.

SOLUTION:

Required to find: The value of $\frac{\sqrt[3]{27}}{9^2}$ in its lowest terms

Solution:

$$\frac{\sqrt[3]{27}}{9^2}=\frac{3}{9\times 9}$$
$$=\frac{1}{27}\ (\text{in its lowest terms})$$

(b) The thickness of one sheet of cardboard is given as 485×10^{-2}mm. A construction worker uses 75 sheets of the cardboard, stacked together, to insulate a wall.

(i) Show that the exact thickness of the insulation is 363.75 mm.

SOLUTION:

Data: Thickness of one sheet of cardboard is 485×10^{-2}mm.
The number of sheets used is 75.

Required to show: The thickness of the insulation is 363.75 mm

Proof:

The thickness of the insulation $= 485 \times 10^{-2} \times 75$ mm

$= 36375 \times 10^{-2}$ mm

We shift the decimal point two places to the left.
The thickness = 363.75 mm.

Q.E.D.

(ii) Write the thickness of the insulation

a) correct to 2 significant figures

SOLUTION:

Required to write: The thickness of the insulation correct to 2 significant figures

Solution:

$36\underset{=}{3}.75 = 360$ mm (expressed to 2 significant figures)

$\uparrow$

Deciding digit

= 360 mm (written to 2 significant figures)

b) Correct to 1 decimal place

SOLUTION:

Required to write: The thickness of the insulation correct to 1 decimal place

Solution:

$363.7\underline{\underline{5}}$ mm
$\uparrow$
Deciding digit

$$\begin{array}{rl} = 363.7 & \\ + \underline{\quad\ \ 1} & \\ \underline{363.8} & \text{mm} \end{array}$$

$= 363.8$ mm (correct to one d.p.)

c) In standard form

SOLUTION:
Required to write: The thickness of the insulation in standard form, $A \times 10^n, 1 \leq A < 10, n \in Z$
Solution:
$\underset{\curvearrowleft\curvearrowleft}{363}.75$ mm

To form a number greater than or equal to1but less than 10, we move the decimal point 2 places to the left (this is the same as dividing by 10^2). Then, we multiply by 10^2 to preserve the value of the original number.

$363.75 \text{ mm} = 3.6375\times10^2$ or 3.638×10^2 or 3.64×10^2 or 3.6×10^2
(expressed in standard form or scientific notation)

(c) Marko is on vacation in the Caribbean. He changes 4500 Mexican pesos (MXN) to Eastern Caribbean dollars (ECD). He receives 630 ECD.

Complete the statement below about the exchange rate.

1 ECD = ………………… MXN

SOLUTION:

Data: $4500 \text{ MXN} = 630 \text{ ECD}$
Required to complete: The statement 1 ECD = ………………… MXN
Solution:

$$630 \text{ ECD} = 4500 \text{ MXN}$$

$$1 \text{ ECD} = \frac{4500}{630} \text{ MXN}$$

$$= 7.1428$$

$$1 \text{ ECD} = 7.143 \text{ MXN (correct to 3 decimal places)}$$

2. (a) Factorise the following expression completely;

$$12n^2 - 4mn$$

SOLUTION:

Required to factorize: $12n^2 - 4mn$

Solution:

$$12n^2 - 4mn = \underline{\underline{4n}}(3n) - \underline{\underline{4n}}(m)$$
$$= 4n(3n - m)$$

(b) (i) Show that $\frac{x}{1-x} - 4x = \frac{x(4x-3)}{1-x}$.

SOLUTION:

Required to show: $\frac{x}{1-x} - 4x = \frac{x(4x-3)}{1-x}$

Proof:

$$\frac{x}{1-x} - 4x$$
$$\frac{x}{1-x} - \frac{4x}{1}$$
$$\frac{1(x) - 4x(1-x)}{(1-x)} = \frac{x - 4x + 4x^2}{(1-x)}$$
$$= \frac{4x^2 - 3x}{(1-x)}$$
$$= \frac{x(4x) - x(3)}{(1-x)}$$
$$= \frac{x(4x-3)}{1-x}$$

Q.E.D.

(ii) Hence, solve the equation

$$\frac{x}{1-x} - 4x = 0$$

SOLUTION:

Required to solve: $\frac{x}{1-x} - 4x = 0$

Solution:

$$\frac{x}{1-x}-4x=\frac{x(4x-3)}{1-x}$$ as shown in part (i)

So $\frac{x(4x-3)}{1-x}=0$

$$x(4x-3)=0\times(1-x)$$

$$x(4x-3)=0$$

$\therefore$ $x=0$ OR $4x-3=0$

$x=0$ OR $x=\frac{3}{4}$

(c) Make v the subject of the formula $p=\sqrt{5+vt}$

SOLUTION:

Required to make: v the subject of the formula in $p=\sqrt{5+vt}$

Solution:

$$p=\sqrt{5+vt}$$

Squaring both sides to remove the square root sign

$$p^2=5+vt$$

$$5+vt=p^2$$

$$vt=p^2-5$$

$$v=\frac{p^2-5}{t}$$

(d) The distance needed to stop a car, *d*, varies directly as the square of the speed, *s*, at which it is travelling. A car travelling at a speed of 70 km/h requires a distance of 40 m to make a stop. What distance is required to stop a car travelling at 80 km/h?

SOLUTION:

Data: *d* varies as the square of *s*. $s=70$ km/h when $d=40$ m

Required to find: *d* when $s=80$ km/h

Solution:

$$d\propto s^2$$

$\therefore d=ks^2$ (where k is the constant of proportion)

Given that $d=40$ when $s=70$

$$40=k(70)^2$$

$$k=\frac{40}{(70)^2}$$

Hence, $d=\frac{40}{(70)^2}s^2$

When $s = 80$

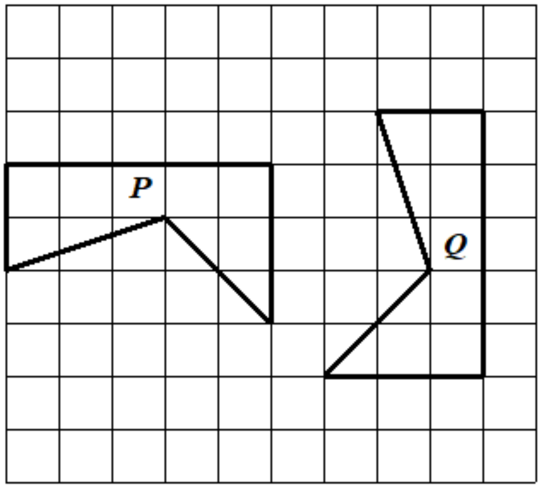
$$d = \frac{40}{(70)^2} \times (80)^2$$

$$= \frac{40 \times 80 \times 80}{70 \times 70}$$

$$= 52.24\underline{\underline{4}}$$

$$\approx 52.24 \text{ m (correct to 2 decimal places)}$$

3. **(a)** The diagram below shows two pentagons, *P* and *Q*, drawn on a grid made up of squares.



(i) Select the correct word from the following list to complete the statement below.

opposite	reflected	congruent	Translated

Pentagon *P* is …………………………………………… to Pentagon *Q*.

SOLUTION:
Data: Diagram showing two pentagons, *P* and *Q* on a grid of squares.
Require to complete: The statement given.
Solution:
Pentagon *P* is **congruent** to Pentagon *Q*.

(ii) Give the reason for your choice in (a) (i).

SOLUTION:

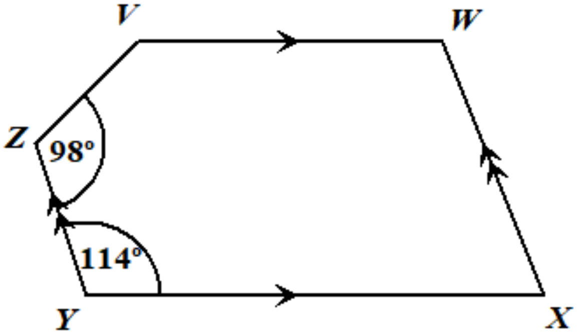
Required to give: A reason for the answer in (a) (i)

Solution:

The five sides of *P* are equal in length to the corresponding fives sides of *Q*. Hence, the figures are congruent.

OR We may say that both pentagons have the same shape and exact in size - their areas are both equal to $8\frac{1}{2}$ square units.

(b) The diagram below, **not drawn to scale**, shows the pentagon *VWXYZ*. In the pentagon, *YZ* is parallel to *XW* and *YX* is parallel to *VW*. Angle $XYZ = 114°$ while angle $VZY = 98°$.

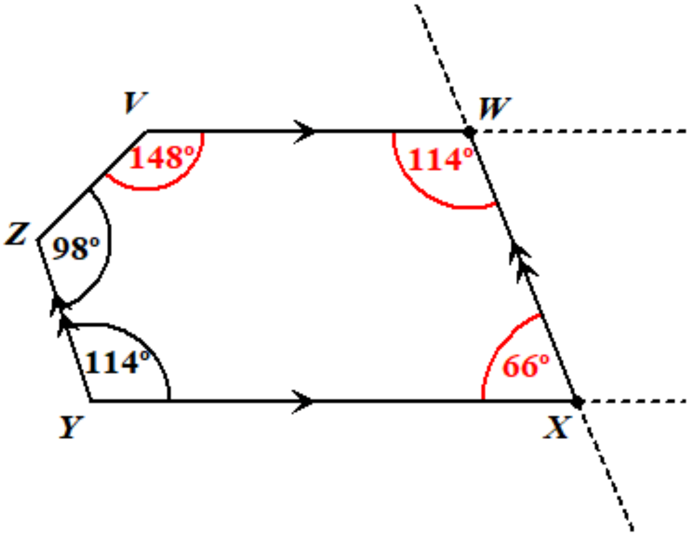


Determine the value of

(i) angle *WXY*

SOLUTION:

Data: Diagram showing pentagon *VWXYZ*.



Required to find: Angle WXY
Solution:
When parallel lines (XW and YZ) are cut by a transversal (YX) the co-interior angles are supplementary.

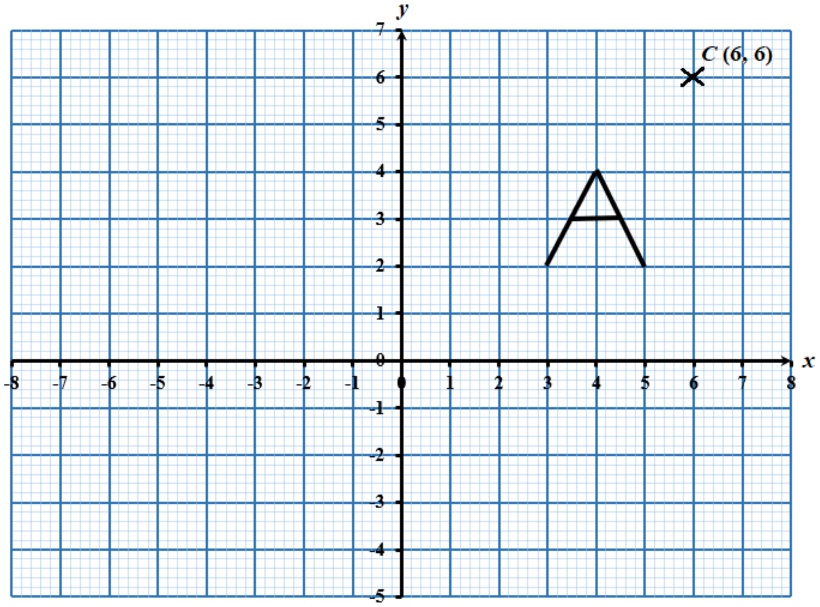
$Z\hat{Y}X + W\hat{X}Y = 180^0$ ($Z\hat{Y}X$ and $W\hat{X}Y$ are co-interior angles)
$114^0 + W\hat{X}Y = 180^0$
$W\hat{X}Y = 180° - 114°$
$= 66°$

(ii) angle ZVW

SOLUTION:
Required to find: Angle ZVW
Solution:
$X\hat{W}V + W\hat{X}Y = 180^0$ (Co-interior angles are supplementary.)
$X\hat{W}V = 180° - 66°$
$= 114°$
The sum of n interior angles of any polygon : $(2n - 4) \times 90^0$
The sum of the interior angles of a pentagon ($n = 5$)
$= [2(5) - 4] \times 90^0 = 540^0$

Hence, $Z\hat{V}W = 540° - (98° + 114° + 66° + 114°)$
$= 148°$

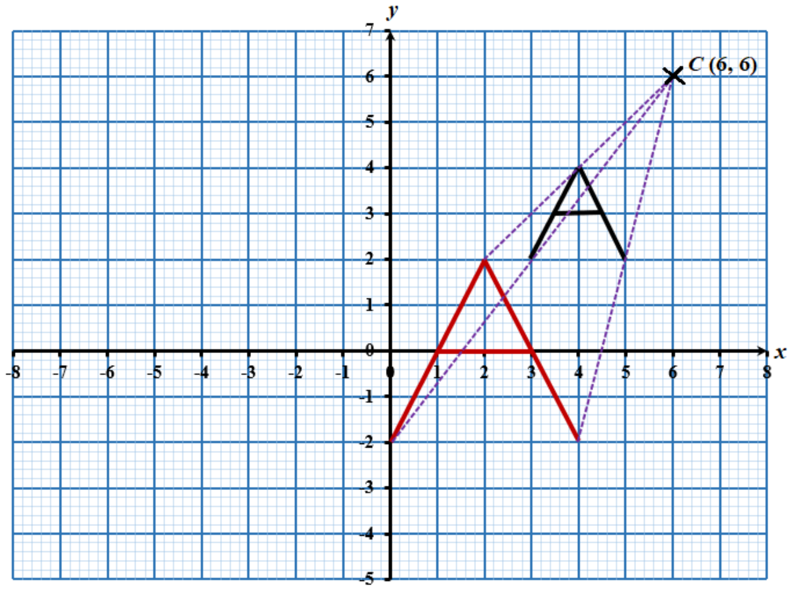
(c) The letter 'A' and a point $C(6, 6)$ are shown on the grid below.



On the diagram, draw accurately, EACH of the following transformations.

(i) The enlargement of letter 'A' by scale factor 2, about centre, $C\,(6,\,6)$.

SOLUTION:
Data: Diagram showing a letter 'A' and the point $C\,(6,\,6)$.
Required to draw: The enlargement of 'A' by scale factor 2, about centre $C\,(6,\,6)$
Solution:

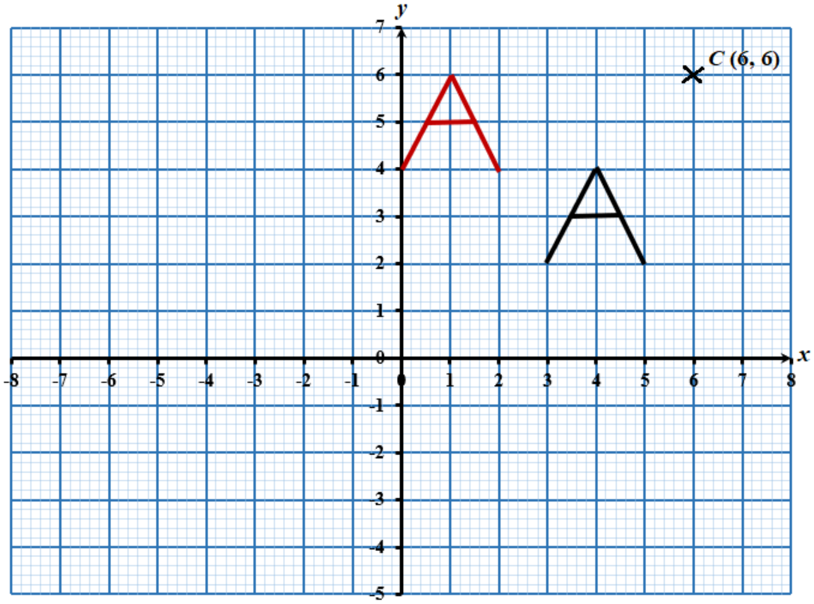


We draw straight lines from (6,6) to each of the three endpoints or vertices of the given figure A, shown in black. Then, we extend these lines (shown dotted) to two times their lengths, since the scale factor is 2. Note that all measurements are taken from the centre, C (6,6). The resulting image is shown in red.

(ii) The translation of letter 'A' using the vector $T = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$.

SOLUTION:
Required to draw: The translation of letter 'A' using the vector
$T = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$
Solution:

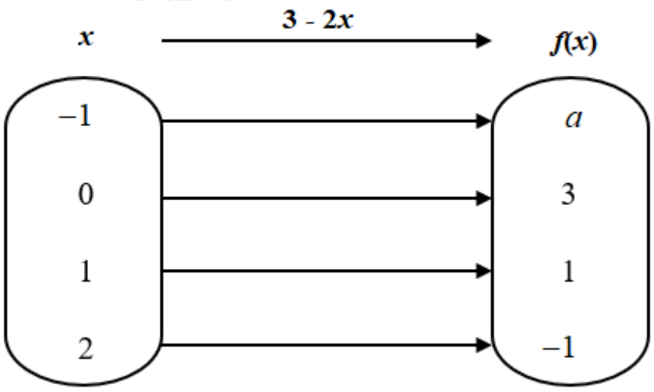
From the vector T, we deduce that each vertex of the object is to be shifted 3 units horizontally to the left (–3) and 2 units vertically upwards (+2). Either operation may be done first and the other one second. This is illustrated in the diagram.



4. (a) The function *f* is defined as

$$f : x \rightarrow 3 - 2x.$$

(i) The diagram below shows the mapping diagram of the function, *f*. Determine the value of *a*.



SOLUTION:
Data: Mapping diagram of the function $f : x \rightarrow 3 - 2x$
Required to find: The value of a
Solution:
$f : x \rightarrow 3 - 2x$
$a = f(-1)$
$a = 3 - 2(-1)$
$a = 3 + 2$
$a = 5$

(ii) Determine, in their simplest form, expressions for

a) the inverse of the function, *f*, $f^{-1}(x)$

SOLUTION:
Required to find: $f^{-1}(x)$
Solution:
Let $y = 3 - 2x$
To obtain the inverse function, we interchange x and y.

$$x = 3 - 2y$$

Now, make y the subject

$$2y = 3 - x$$
$$y = \frac{3 - x}{2}$$

Using the inverse notation,

$$f^{-1}(x) = \frac{3 - x}{2}$$

b) the composite function $f^2(x)$

SOLUTION:
Required to find: $f^2(x)$
Solution:

$$f^2(x) = ff(x) = f[f(x)]$$

$$f[f(x)] = f(3 - 2x)$$

$$= 3 - 2(3 - 2x)$$
$$= 3 - 6 + 4x$$
$$= 4x - 3$$

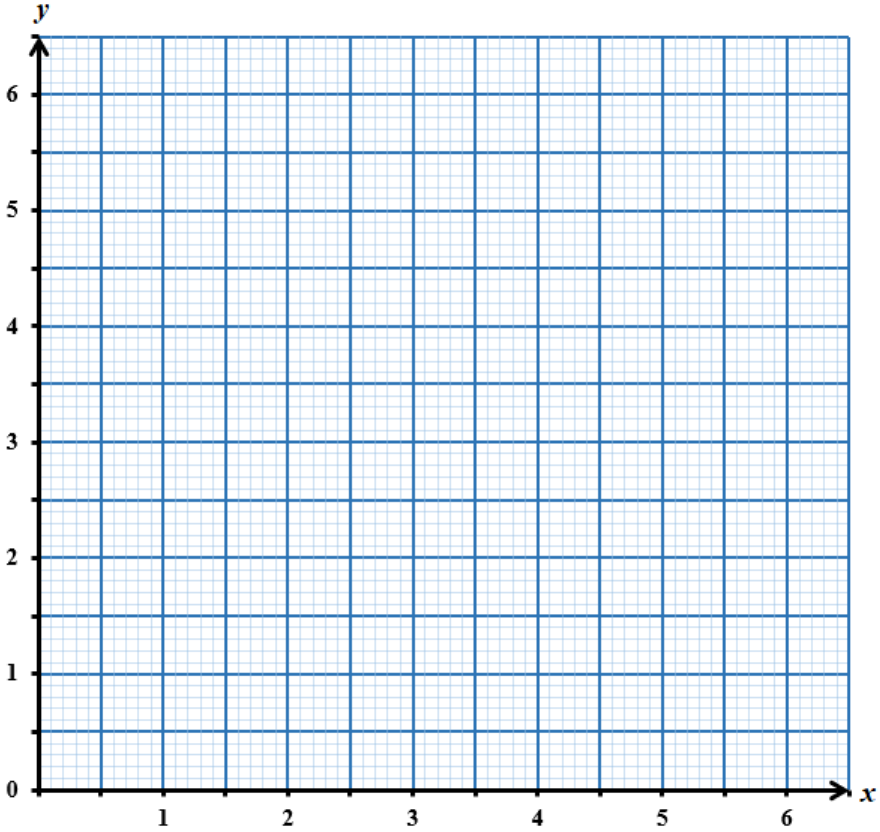
(iii) State the value of $f\,f^{-1}(-2)$.

SOLUTION:
Required to state: The value of $f\,f^{-1}(-2)$
Solution:

$$f^{-1}(x) = \frac{3 - x}{2}$$
$$f^{-1}(-2) = \frac{3 - (-2)}{2}$$
$$= \frac{5}{2}$$

$$f\,f^{-1}(-2)=f\left(\frac{5}{2}\right)$$
$$=3-2\left(\frac{5}{2}\right)$$
$$=3-5$$
$$=-2$$

(b) **(i)** Using a ruler, draw the lines $x=\frac{1}{2}$, $y=x$ and $x+y=5$, on the grid below.



SOLUTION:

Required to draw: The lines $x=\frac{1}{2}$, $y=x$ and $x+y=5$ on the grid given

Solution:

$x=\frac{1}{2}$ is the vertical line that cuts the x-axis at $\frac{1}{2}$.

We create a table of values to draw $y = x$

x	y
0	0
6	6

We plot and join the points $(0,\ 0)$ and $(6,\ 6)$ to obtain $y = x$ and which may be extended in either direction as required.

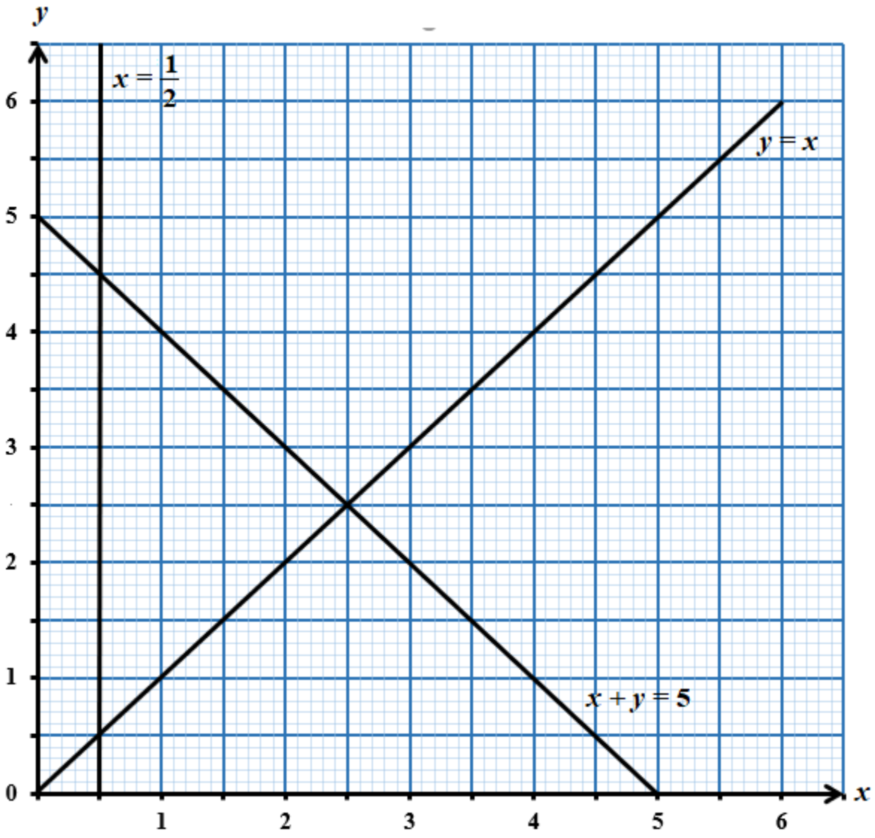
We create a table of values to draw $x + y = 5$

x	y
0	5
5	0

We plot and join the points $(0,\ 5)$ and $(5,\ 0)$to obtain $x + y = 5$.and

which

may be extended in either direction as required.



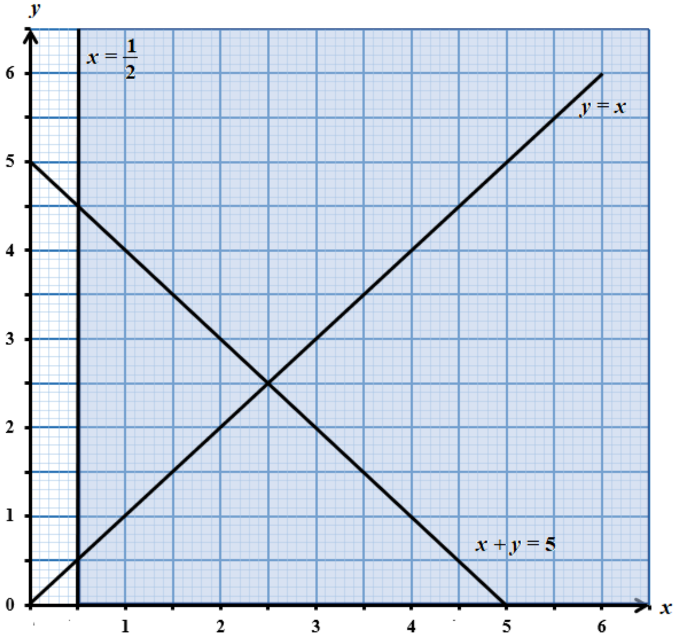
(ii) On the grid, label as R, the region where $x \geq \frac{1}{2}$, $y \geq x$ and $x+y \leq 5$.

SOLUTION:

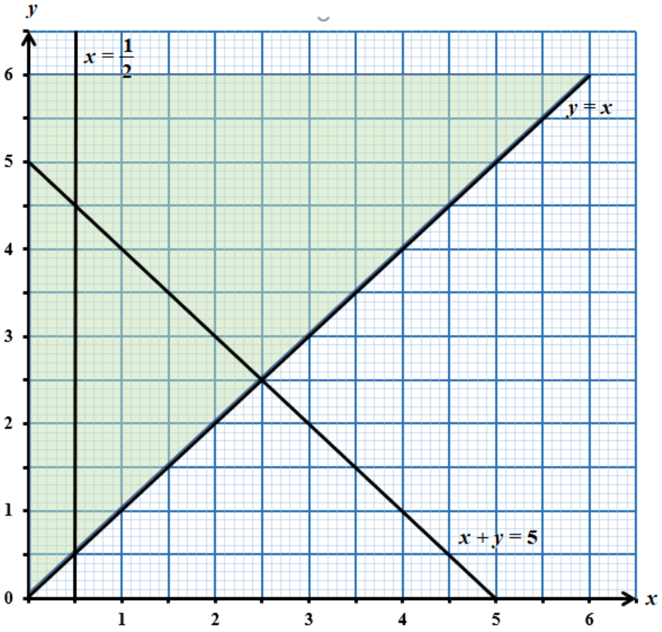
Required to label: The region, R, on the grid where $x \geq \frac{1}{2}$, $y \geq x$ and, $x+y \leq 5$

Solution:

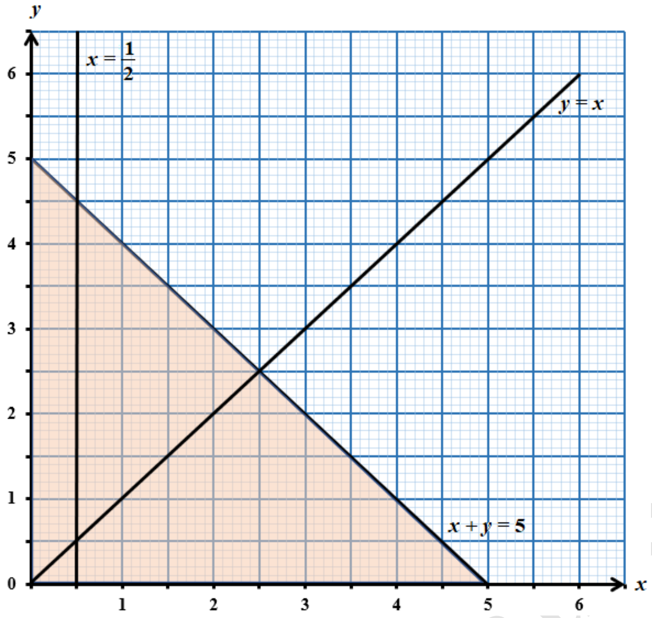
$x \geq \frac{1}{2}$ includes the line and is shown coloured in blue.



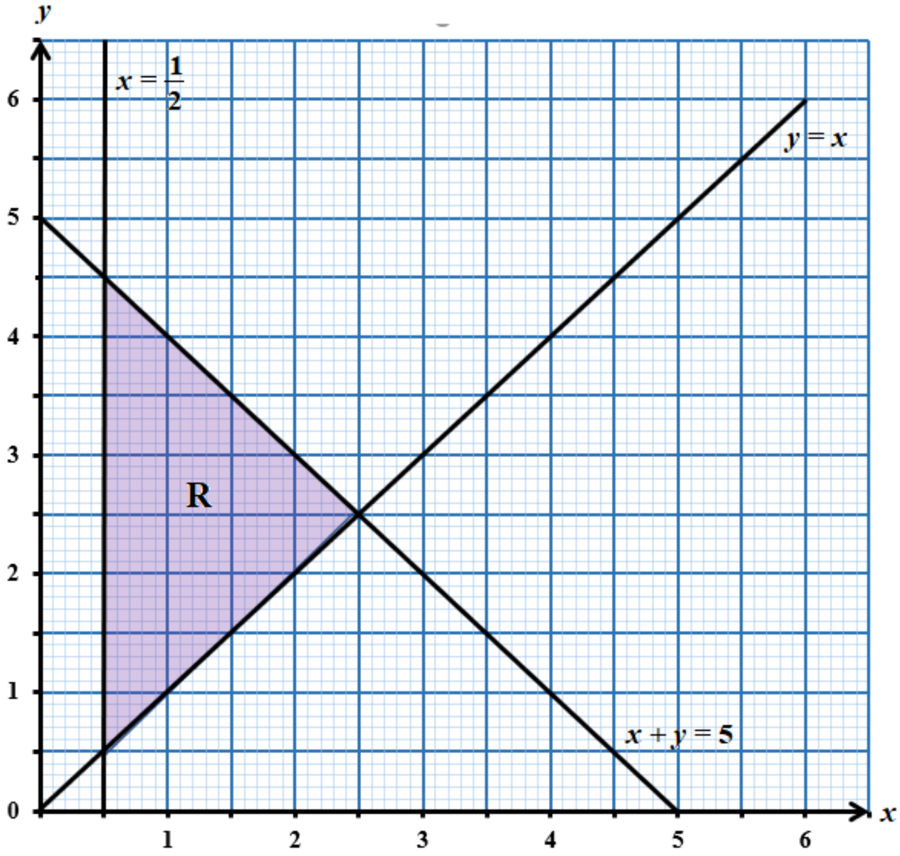
$y \geq x$ includes the line and is shown coloured in green.



$x + y \leq 5$ includes the line and is shown coloured



Hence, R is the region that is common to all the shaded regions. Points in this region that satisfies all three inequalities.



5. (a) Sixty students took an algebra test, which comprised 15 multiple-choice questions. The number of correct answers that each student obtained is recorded in the table below.

Number of Correct Answers	**Number of Students**
8	6
9	14
10	2
11	6
12	2
13	11
14	9
15	10

Using the table, determine

(i) the number of students who had exactly 13 correct answers.

SOLUTION:
Data: Table showing the number of questions student get correct in a multiple-choice test comprising of 15 questions.
Required to find: The number of students who has exactly 13 correct answers.

Solution:
From the table, the number of students who had exactly 13 correct answers is 11.

Number of Correct Answers	**Number of Students**
8	6
9	14
10	2
11	6
12	2
13	11
14	9
15	10

(ii) the modal number of correct answers

SOLUTION:

Required to find: The modal number of correct answers
Solution:
The modal number of correct answers is 9 since it is the score that occurred the most in the distribution.

Number of Correct Answers	Number of Students
8	6
9	14
10	2
11	6
12	2
13	11
14	9
15	10

(iii) the median number of correct answers.

SOLUTION:
Required to find: The median number of correct answers.
Solution:

$$\sum f = 6+14+2+6+2+11+9+10$$
$$= 60$$

For a set of 60 scores, the middle scores are the 30th and 31st score.

From the table, the 30th score is 12

Scores 1-29, $\underset{30^{th}}{12}$, $\underset{31^{st}}{13}$,…, Scores 32-60

The median is the middle value when arranged in either ascending or descending order or magnitude.

The median number of correct answers is $\frac{12+13}{2} = 12.5$.

(iv) the probability that a student chosen at random had at least 12 correct answers.

SOLUTION:
Required to find: The probability that a student chosen at random had at least 12 correct answers.
Solution:
P(student chosen at random has at least 12 correct answers)

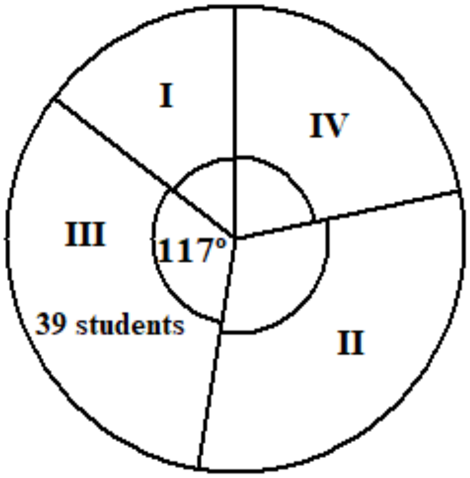
$$=\frac{\text{Number of students with at least 12 correct answers}}{\text{Total number of students}}$$

$$=\frac{2+11+9+10}{60}$$

$$=\frac{32}{60}$$

$$=\frac{8}{15}$$

(b) A group of students wrote a Physics examination. Each of the students achieved a Grade I, II, III or IV. The pie chart below shows the results.



Thirty-nine students achieved a Grade III.

(i) Determine the TOTAL number of students who wrote the examination.

SOLUTION:
Data: Pie chart showing the grades received by a group of students in a Physics examination. Thirty-nine students achieved a Grade III.
Required to find: The total number of students who wrote the examination.

Solution:
117° represents 39 students.

1° will represent $\frac{39}{117}$

The number of students who wrote the examination is represented by the whole pie chart or an angle of 360°

Number of students who wrote the examination $= \frac{39}{117^\circ} \times 360^\circ$
$= 120$ students

(ii) The ratio of the number of students who achieved a Grade I, II or IV is $2:4:3$. A student passed the examination if he/she achieved a Grade I, II or III.

How many students passed the examination?

SOLUTION:
Data: The ratio of the number of students who achieved a Grade I, II or IV is $2:4:3$. The passing grades are I, II or III. The number of students who received Grade III is 39, as seen on the Pie Chart.
Required to find: The number of students who passed the examination.

Solution:
The ratio of the number of students who achieved Grade I, II and IV is $2:4:3$
The number of students who received Grade III = 39
The number of students achieving Grade I, II or IV $= 120 - 39$
$= 81$ students
The number of students who passed the examination
= The total number of students receiving Grade I, II or III.

The ratio of students who received these Grades are shown below:

Grade I		Grade II		Grade IV
2	:	4	:	3

The number of students who attained Grade I $= \dfrac{2}{2+3+4} \times 81$
$= 18$ students

The number of students who attained Grade II $= \dfrac{4}{2+3+4} \times 81$
$= 36$ students

The number of students who attained Grade IV $= \dfrac{3}{2+3+4} \times 81$
$= 27$ students

The number who passed, that is, the number who achieved a Grade I, II or III $= 18 + 36 + 39$
$= 93$

(iii) Determine the value of the angle for the sector representing Grade I in the pie chart.

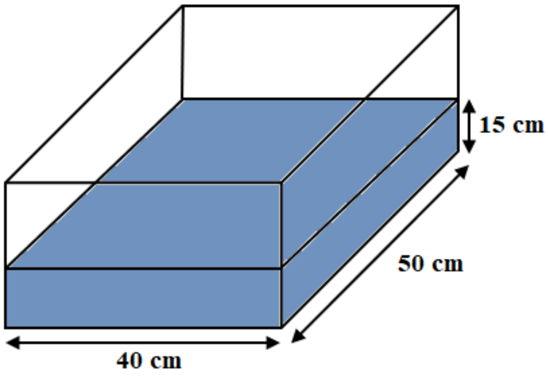
SOLUTION:
Required to determine: The value of the angle for the sector representing Grade I

Solution:
The number of students who attained Grade I = 18

The angle of the sector representing Grade I $= \dfrac{18}{120} \times 360°$
$= 54°$

6. In this question, take π to be $\dfrac{22}{7}$.

The diagram below shows a rectangular tank, with base 50 cm by 40 cm, that is used to store water. The tank is filled with water to a depth of 15 cm.

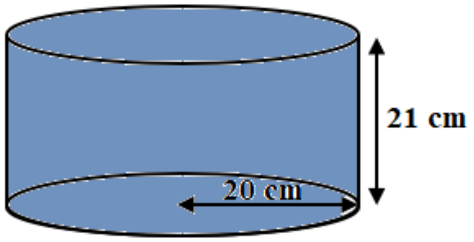


(a) Calculate the volume of water in the tank.

SOLUTION:
Data: Diagram showing a rectangular tank with base 40 cm by 50 cm. Water is filled in the tank to a depth of 15 cm.

Required to calculate: The volume of water in the tank.
Calculation:
The water in the tank has the shape of a cuboid.
The volume of water in the tank = $L \times B \times H$
The volume of water in the tank $= (40 \times 50 \times 15)\ \text{cm}^3$
$= 30\,000\ \text{cm}^3$

(b) The cylindrical container shown in the diagram below is used to fetch more water to fill the rectangular tank. The container, which is completely filled with water has a radius of 20 cm and a height of 21 cm.



All the water in this container is added to the water in the rectangular tank. Calculate the TOTAL volume of water that is now in the rectangular tank.

SOLUTION:
Data: Diagram showing a cylindrical container of radius 20 cm and height 21 cm which is filled with water and is added to the water in the tank.

Required to calculate: The total volume of water in the rectangular tank.
Calculation:
The volume of the cylindrical container $= \pi r^2 h$

$$= \left(\frac{22}{7} \times (20)^2 \times 21\right)\ \text{cm}^3$$

$$= 26\,400\ \text{cm}^3$$

Hence, the volume of water in the rectangular tank after the water from the cylindrical container is added $= (30\,000 + 26\,400)\ \text{cm}^3$
$= 56\ 400\ cm^3$

(c) Show that the new depth of water in the rectangular tank is 28.2 cm.

SOLUTION:
Required to show: The new depth of water in the rectangular tank is 28.2 cm.
Proof:
The volume of water in the rectangular tank $= 56\,400$
Let h represent the new depth (or height) of water,

$$50 \times 40 \times h = 56\,400$$

$$h = \frac{56\,400}{40 \times 50}\ cm$$

$$h = 28.2\ cm$$

(d) The vertical height of the rectangular tank is 48 cm. Determine how many more cylindrical containers of water must be poured into the rectangular tank for it to be completely filled.

SOLUTION:
Data: The vertical height of the rectangular tank is 48 cm.
Required to determine: The number of cylindrical containers of water must be poured into the rectangular tank to completely fill it.
Solution:
The volume of the rectangular tank $= (40 \times 50 \times 48)\ \text{cm}^3$
$= 96\,000\ \text{cm}^3$
The volume of water already in the tank = 56 400 cm^3
The additional volume of water needed to fill the rectangular tank
$96\,000 - 56\,400\ \text{cm}^3$
$= 39\,600\ cm^3$
One cylindrical container has a volume of 26 400 cm^3

The number of cylindrical containers of water needed to completely fill the tank
$= \frac{39\,600}{26\,400}$
$= 1.5$ containers
(Two filled cylindrical containers will be more than sufficient to fill the tank)

7. The diagrams below show a sequence of figures made up of circles with dots. Each figure has one dot at the centre and four dots on the circumference of each circle. The radius of the first circle is one unit. The radius of each new circle is one unit greater than the radius of the previous circle. Except for the first figure, a portion of each of the other figures is shaded.

Figure 1

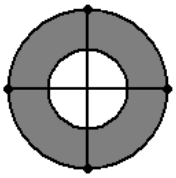


Figure 2

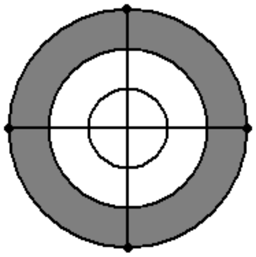


Figure 3

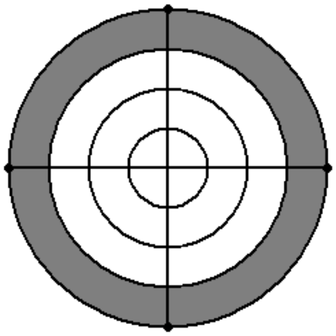


Figure 4

(a) Complete the rows in the table below for Figure 5 and Figure *n*.

	Figure Number	**Number of Dots**	**Area of Outer (Largest) Circle**	**Area of Shaded Region**	**Total Length of Circumference of all Circles**
	1	5	π	π	2π
	2	9	4π	3π	6π
	3	13	9π	5π	12π
	4	17	16π	7π	20π
(i)	5		25π		
	⋮	⋮	⋮	⋮	⋮
(ii)	*n*				

SOLUTION:
Data: Diagram showing a sequence of figures made up of circles with dots.
Required to complete: Rows in the table given.
Solution:

Column 2

The number of dots (D) in each subsequent figure increase by 4.
5, 9, 13, 17,…

The number of dots $D = 4n \pm C$, where n = number of the figure and C is a constant to be determined.

When $n = 1,\ d = 5$

i.e. $4(1)+1$ $\quad\therefore C = 1$

So we realise that the number of dots in the n th pattern is $4n + 1$

If we test when n =1, 2, 3 and 4, we would see that the formula holds true.

When $n = 5$ $\quad D = 4(5)+1$

$= 21$

The pattern can also be explained as follows:

n		
1	$5 + 0 \times 4$	5
2	$5 + 1 \times 4$	9
3	$5 + 2 \times 4$	13
n	$5 + (n-1)(4)$	

The number of dots in the nth pattern is

$$5 + (n-1)(4)$$
$$= 5 + 4n - 4$$
$$= 4n + 1$$

Column 3

Area of largest circle: π, 4π, 9π, 16π, 25π,…

The numbers in each term are all square numbers: $1^2, 2^2, 3^2, 4^2, 5^2, \ldots n^2$

The area of the outer circle is $n^2 \times \pi = n^2\pi$.

If we test when n =1, 2, 3, 4 and 5 we would see that the formula holds true.

Column 4

The area of the shaded region: π, 3π, 5π, 7π, …

This can be explained as:

{(two times the figure number, n) -1} $\times \pi$

$$(2n - 1) \times \pi$$

If we test when n =1, 2, 3, and 4 we would see that the formula holds true.

Hence, when $n = 5$, the area of the shaded region in square units $= \left(2(5) - 1\right) \times \pi$

$= 9\pi$

The pattern can also be explained as follows:

n		
1	$\pi(1 + 0 \times 2)$	π
2	$\pi(1 + 1 \times 2)$	3π
3	$\pi(1 + 2 \times 2)$	5π
4	$\pi(1 + 3 \times 2)$	7π
n	$\pi[1 + (n-1) \times 2]$	

The area of the shaded region is

$$\pi[1 + (n-1) \times 2]$$
$$= \pi[1 + 2n - 2]$$
$$= \pi(2n - 1)$$

Column 5

The total length of circumference of all circles: $2\pi, 6\pi, 12\pi, 20\pi, \ldots$

The total length of all the circumferences of all the circles is
(1 more than the number of the figure, $n + 1$) × (the number of figure, n) $\times\pi$

Figure 1 $(1+1)\times(1)\times\pi = 2\pi$

Figure 2 $(2+1)\times(2)\times\pi = 6\pi$

Figure 3 $(3+1)\times(3)\times\pi = 12\pi$

Figure 4 $(4+1)\times(4)\times\pi = 20\pi$

When $n=5$ $(5+1)\times(5)\times\pi = 30\pi$

For Figure n, this will be $(n+1)n\times\pi = n(n+1)\pi$.

OR

We can tabulate the sequence $2\pi, 6\pi, 12\pi, 20\pi, \ldots$ as follows:

n	Length of all the circumferences	Length of all the circumferences
1	$\pi(2)$	$\pi(1+1)$
2	$\pi(6)$	$\pi(2+4)$
3	$\pi(12)$	$\pi(3+9)$
4	$\pi(20)$	$\pi(4+16)$
n		$\pi(n+n^2)$

The completed table looks like:

	Figure Number	**Number of Dots**	**Area of Outer (Largest) Circle**	**Area of Shaded Region**	**Total Length of Circumference of all Circles**
	1	5	π	π	2π
	2	9	4π	3π	6π
	3	13	9π	5π	12π
	4	17	16π	7π	20π
(i)	5	21	25π	9π	30π
	⋮	⋮	⋮	⋮	⋮
(ii)	n	$4n+1$	$n^2\pi$	$(2n-1)\pi$	$n(n+1)\pi$

(b) Determine the value of *n* when the number of dots in Figure *n* is 541.

SOLUTION:
Data: The number of dots in Figure *n* is 541.
Required to find: The value of *n*
Solution:
Number of dots $= 4n + 1$

$$541 = 4n + 1$$
$$541 - 1 = 4n$$
$$540 = 4n$$
$$n = \frac{540}{4}$$
$$n = 135$$

(c) Write down, in terms of p and π, the area of the LARGEST circle in Figure $3p$.

SOLUTION:
Required to write: The area of the largest circle in Figure $3p$, in terms of p and π.
Solution:

In Figure $3p$, $n = 3p$

Area of the largest circle $= n^2\pi$
$$= (3p)^2 \pi$$
$$= 9p^2\pi$$

SECTION II

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) The straight line graph of $x = 5 - 3y$ intersects the x – axis at P and the y – axis at Q.

(i) Determine the coordinates of P and Q.

SOLUTION:
Data: The graph of the straight line $x = 5 - 3y$ intersects the x – axis at P and the y – axis at Q.
Required To determine: The coordinates of P and Q
Solution:
$x = 5 - 3y$

A line cuts x – axis at $y = 0$

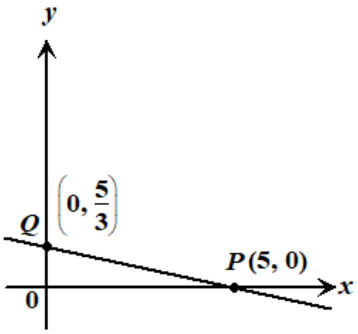
$$x = 5 - 3(0)$$
$$= 5$$
$$\therefore P = (5,\ 0)$$

A line cuts the y – axis at $x = 0$

$$0 = 5 - 3y$$
$$3y = 5$$
$$y = \frac{5}{3}$$
$$\therefore Q\left(0,\ \frac{5}{3}\right)$$

(ii) Calculate the length of PQ, giving your answer to 2 decimal places.

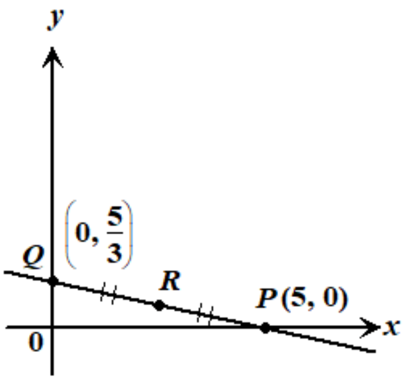
SOLUTION:
Required to calculate: The length of PQ, correct to 2 decimal places.
Calculation:



$$\text{Length of } PQ = \sqrt{(0-5)^2 + \left(\frac{5}{3}-0\right)^2}$$
$$= \sqrt{25+\frac{25}{9}}$$
$$= \sqrt{25+2\frac{7}{9}}$$
$$= \sqrt{27\frac{7}{9}}$$
$$= \sqrt{\frac{250}{9}}$$
$$= \frac{\sqrt{250}}{3}$$
$$= 5.270$$
$$\approx 5.27 \text{ units (correct to 2 decimal places)}$$

(iii) *R* is the midpoint of *PQ*. Determine the coordinates of *R*.

SOLUTION:
Data: *R* is the midpoint of *PQ*
Required to determine: The coordinates of *R*
Solution:



$$\text{Coordinates of } R = \left(\frac{0+5}{2}, \frac{\frac{5}{3}+0}{2} \right)$$

$$= \left(2\frac{1}{2}, \frac{5}{6} \right)$$

(b) The funtions *f* and *g* are defined as follows

$$f : x \rightarrow 5 - x \text{ and } g : x \rightarrow x^2 - 2x - 1$$

The graphs of $f(x)$ and $g(x)$ meet at points *M* and *N*. Determine the coordinates of the points *M* and *N*.

SOLUTION:

Data: $f : x \rightarrow 5 - x$ and $g : x \rightarrow x^2 - 2x - 1$. The graphs of $f(x)$ and $g(x)$ meet at points *M* and *N*

Required to determine: The coordinates of *M* and *N*

Solution:

$f : x \rightarrow 5 - x$

$g : x \rightarrow x^2 - 2x - 1$

Let $y = 5 - x$ …❶

$y = x^2 - 2x - 1$ …❷

Solving simultaneously to find *M* and *N*

Substitute ❶ into ❷

$$5 - x = x^2 - 2x - 1$$

$$x^2 - 2x - 1 + x - 5 = 0$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3 \text{ OR } x = -2$$

When $x = 3$ $\quad y = 5 - 3$

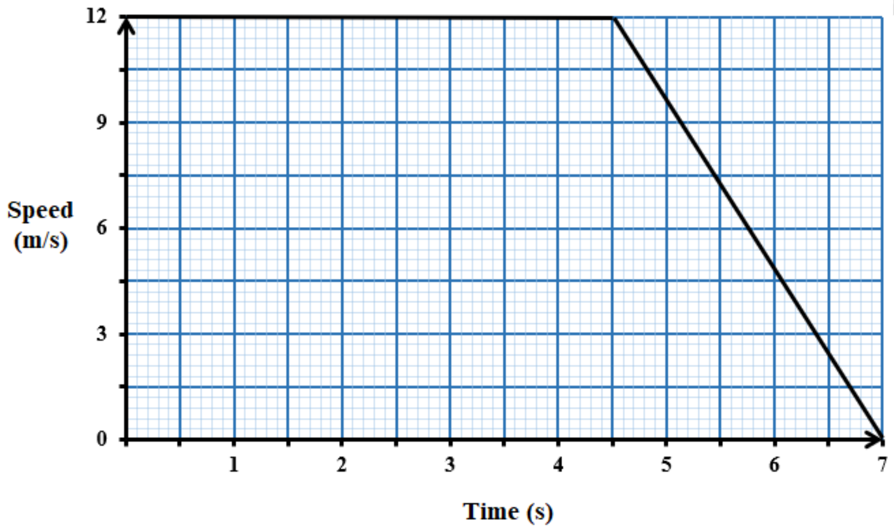
$= 2$

When $x = -2$ $\quad y = 5 - (-2)$

$= 7$

We may say $M = (3, 2)$ and $N = (-2, 7)$

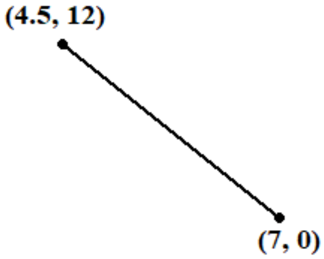
(c) Monty is cycling at 12 metres per second (m/s). After 4.5 second he starts to decelerate and after a further 2.5 seconds he stops. The speed-time graph is shown below.



Calculate

(i) the constant deceleration

SOLUTION:
Data: Speed-time graph showing Monty's cycling journey
Required to calculate: The constant deceleration
Calculation:
Looking at the branch when Monty is decelerating



The gradient of the speed-time line will give the acceleration

$$\text{Gradient } = \frac{12-0}{4.5-7}$$
$$= \frac{12}{-2.5}$$
$$= -4.8$$

Acceleration $= -4.8 \text{ ms}^{-2}$

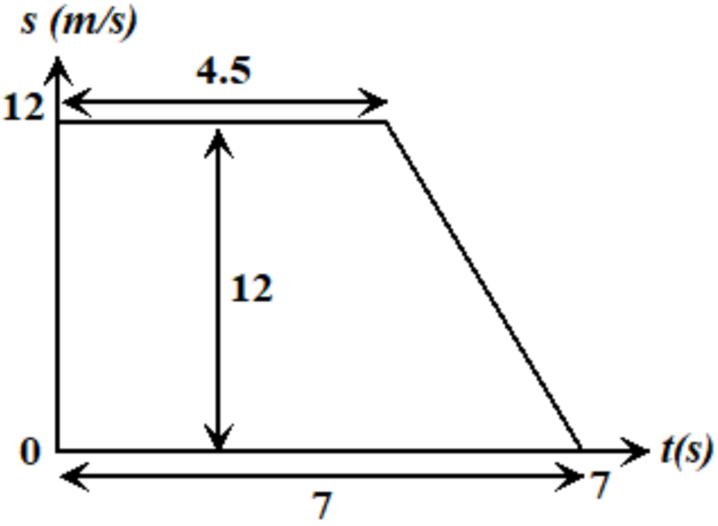
Hence, the deceleration is 4.8 ms^{-2}.

(ii) Monty's average speed over the 7 seconds.

SOLUTION:

Required to calculate: Monty's average speed over the 7 seconds

Calculation:



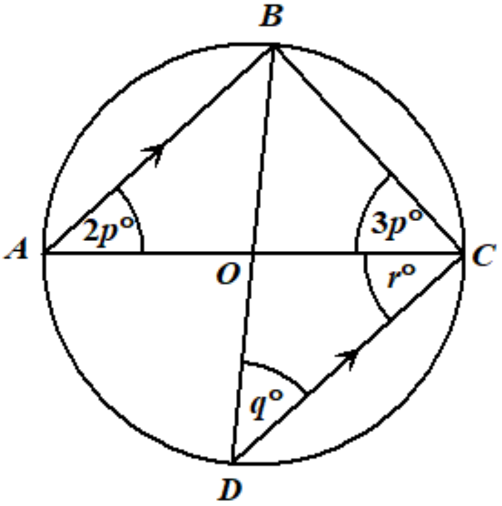
Area under the distance-timegraph is the distance covered

$$= \frac{1}{2}(4.5+7)\times 12$$
$$= 69 \text{ m}$$

$$\text{Average speed } = \frac{\text{Total distance covered}}{\text{Total time taken}}$$
$$= \frac{69 \text{ m}}{7 \text{ s}}$$
$$= 9.857142 \text{ m/s}$$
$\approx 9.86\ m/s$ (correct to 3 sf or 1 dp)

GEOMETRY AND TRIGONOMETRY

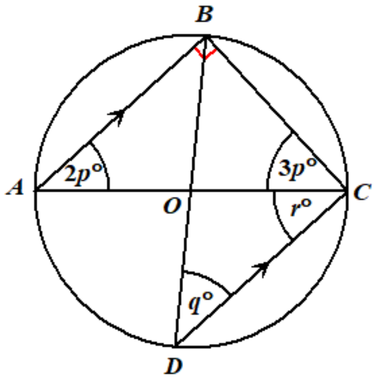
9. (a) In the diagram below, *A*, *B*, *C* and *D* are points on the circumference of a circle, with centre *O*. *AOC* and *BOD* are diameters of the circle. *AB* and *DC* are parallel.



(i) State the reason why angle *ABC* is 90°.

Data: Diagram showing a circle with centre *O* and the points *A*, *B*, *C* and *D* along the circumference. *AOC* and *BOD* are diameters of the circle. *AB* and *DC* are parallel.
Required to state: The reason why angle *ABC* is 90°
Solution:

O is the centre of the circle, hence, the line AOC is a diameter. A diameter subtends a right angle on the circumference of a circle (angle in a semi-circle is 90°). Since B is on the circumference of the circle, subtended by the diameter, AC, $A\hat{B}C = 90°$.



(ii) Determine the value of EACH of the following angles. Show detailed working where necessary and give a reason to support your answer.

a) Angle *BAC*

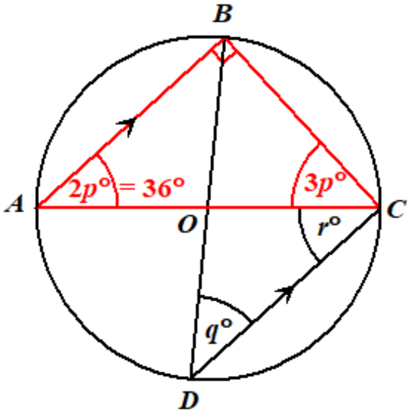
SOLUTION:
Required to determine: Angle *BAC*, giving a reason for your answer
Solution:

Consider ΔABC

The sum of the angles in a triangle is 180^0

$$90° + 2p° + 3p° = 180°$$
$$5p° = 180° - 90°$$
$$5p° = 90°$$
$$p = 18$$

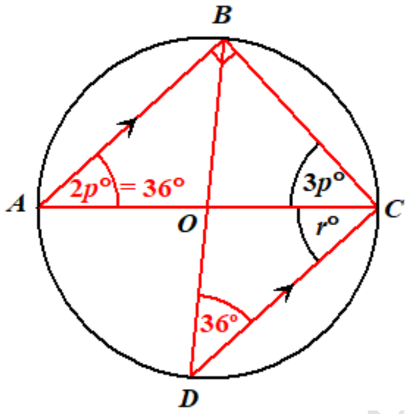
Hence, $B\hat{A}C = 2p$
$$= 2(18°)$$
$$= 36°$$



b) Angle *q*

SOLUTION:
Required to determine: Angle q
Solution:

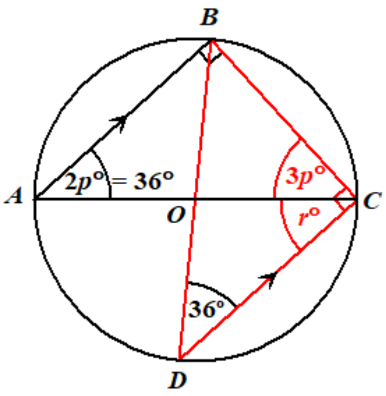
$B\hat{D}C = B\hat{A}C = 36°$
(The angles subtended by a chord (BC) at the circumference of a circle ($B\hat{A}C$ and $B\hat{D}C$) and standing on the same arc are equal.)



(iii) Calculate the value of r.

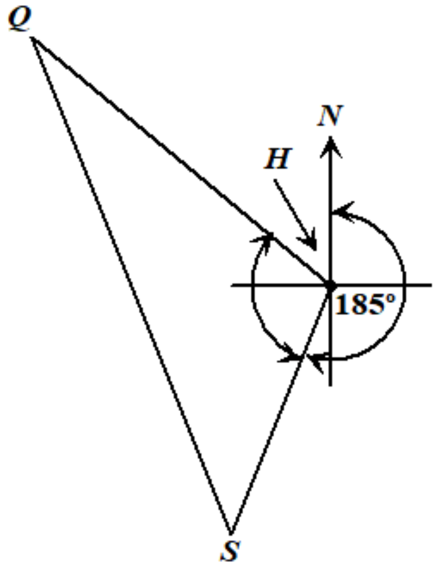
SOLUTION:
Required to calculate: The value of r
Calculation:

Consider ΔBCD $B\hat{C}D = 90°$ (angle in a semi-circle $= 90°$) $r + 3p = 90^0$ $r° + 3(18°) = 90°$ $r° = 90° - 54°$ $= 36°$ $\therefore r = 36$	AB is parallel to DC, and AC is a transversal, hence, $B\hat{A}C = A\hat{C}D$ (alternate angles) From part (ii)a $B\hat{A}C = 36^0$ Therefore $A\hat{C}D = 36^0$ But, $A\hat{C}D = r,$ $So, \ r = 36^0$



(b) From a harbour, *H*, the bearing of two buoys, *S* and *Q*, are 185° and 311° respectively. *Q* is 5.4 km from *H* while *S* is 3.5 km from *H*.

(i) On the diagram below, which shows the sketch of this information, insert the value of the marked angle, *QHS*.



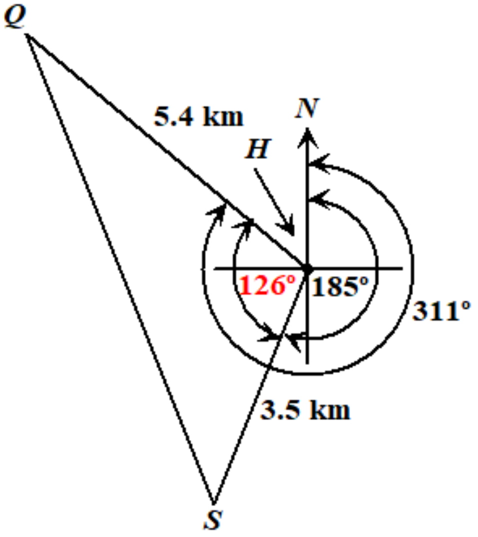
SOLUTION:

Data: The bearings of two buoys *S* and *Q* from a harbour *H* are 185° and 311° respectively. $HQ = 5.4$km and $HS = 3.5$km.

Required to insert: The value of the angle *QHS* on the diagram given.

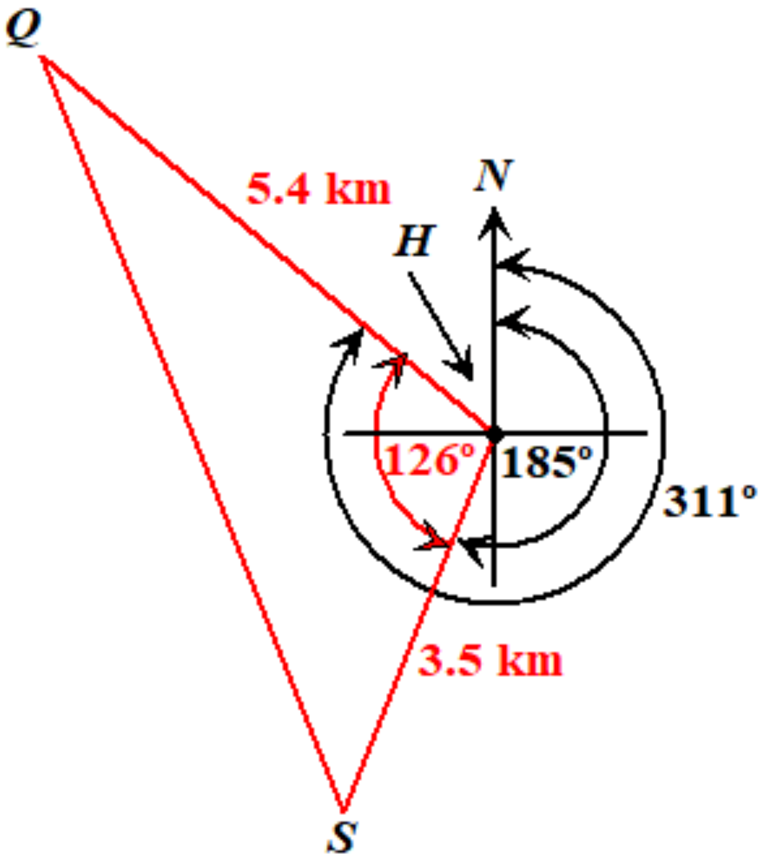
Solution:

$$Q\hat{H}S = 311° - 185°$$
$$= 126°$$



(ii) Calculate *QS*, the distance between the two buoys.

SOLUTION:
Required to calculate: *QS*
Calculation:
Consider ΔQHS



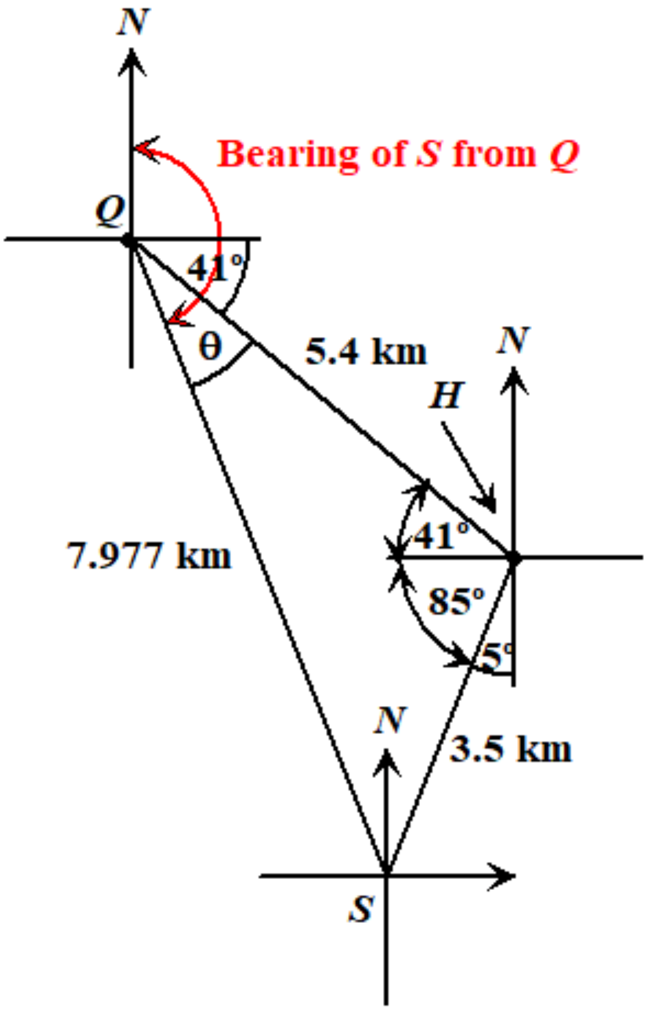
Applying the cosine law

$$QS^2 = (5.4)^2 + (3.5)^2 - 2(5.4)(3.5)\cos 126°$$
$$= 29.16 + 12.25 - (-22.218)$$
$$= 63.628$$
$$QS = 7.977 \text{ km}$$
$$\approx 7.98 \text{ km (correct to 2 decimal places)}$$

(iii) Calculate the bearing of *S* from *Q*.

SOLUTION:
Required to calculate: The bearing of *S* from *Q*

Calculation:



Let $H\hat{Q}S = \theta^0$

$$\frac{3.5}{\sin\theta}=\frac{7.977}{\sin 126^\circ} \qquad \text{(Sine rule)}$$

$$\sin\theta=\frac{3.5\times\sin 126^\circ}{7.977}$$

$$=0.3550$$

$$\theta=\sin^{-1}(0.3550)$$

$$=20.79^\circ$$

$\therefore$The bearing of S from Q $=90^\circ+41^\circ+20.79^\circ$
$=151.79^\circ$
$=151.8^\circ$ (correct to the nearest 0.1°)

VECTORS AND MATRICES

10. (a) Given the matrix $W=\begin{pmatrix}3 & 6\\ -2 & 5\end{pmatrix}$, determine

(i) the 2×2 matrix, L, such that $W+L=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$.

SOLUTION:

Data: $W=\begin{pmatrix}3 & 6\\ -2 & 5\end{pmatrix}$ and $W+L=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$

Required to determine: the 2×2 matrix, L

Solution:

$$W=\begin{pmatrix}3 & 6\\ -2 & 5\end{pmatrix}$$

$$W+L=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

Let $$L=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$$

$$\therefore\begin{pmatrix}3 & 6\\ -2 & 5\end{pmatrix}+\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

$$\begin{pmatrix}3+a & 6+b\\ -2+c & 5+d\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

Equating corresponding entries:

$3+a=0 \qquad \therefore a=-3$

$6+b=0 \qquad \therefore b=-6$

$-2+c=0 \qquad \therefore c=2$

$5+d=0 \qquad \therefore d=-5$

$$\therefore L=\begin{pmatrix}-3 & -6\\ 2 & -5\end{pmatrix}$$

OR we could have said that L is $-W$

$$-W = -\begin{pmatrix}3 & 6\\ -2 & 5\end{pmatrix} = \begin{pmatrix}-3 & -6\\ 2 & 5\end{pmatrix} = L$$

(ii) the 2×2 matrix, P, such that $WP=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$.

SOLUTION:

Data: $WP=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ $W=\begin{pmatrix}3 & 6\\ -2 & 5\end{pmatrix}$

Required to determine: The 2×2 matrix, P

Solution:

$WP=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$,

Hence, $P=W^{-1}$

Since $WW^{-1}=I$

Finding W^{-1}

$$\begin{aligned}\det W &= (3\times 5)-(6\times -2)\\ &=15+12\\ &=27\end{aligned}$$

$$W^{-1}=\frac{1}{27}\begin{pmatrix}5 & -6\\ 2 & 3\end{pmatrix}$$

$$W^{-1}=\begin{pmatrix}\frac{5}{27} & \frac{-6}{27}\\ \frac{2}{27} & \frac{3}{27}\end{pmatrix}$$

$$W^{-1}=\begin{pmatrix}\frac{5}{27} & \frac{-2}{9}\\ \frac{2}{27} & \frac{1}{9}\end{pmatrix}$$

(b) A right-angled triangle, *M*, has vertices $X\left(1, 1\right)$, $Y\left(3, 1\right)$ and $Z\left(3, 4\right)$. When *M* is transformed by the matrix $N=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, the image is M'.

Find the coordinates of the vertices of M'.

SOLUTION:

Data: A right-angled triangle, *M*, has vertices $X\left(1, 1\right)$, $Y\left(3, 1\right)$ and $Z\left(3, 4\right)$.

M is transformed by the matrix $N=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ to produce M'.

Required to find: The coordinates of the vertices of M'.

Solution:

$$\Delta XYZ \xrightarrow{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}} \Delta X'Y'Z'$$

We represent the coordinates of triangle XYZ using a 2×3 matrix.

$$\text{Let } T = \begin{array}{c} \begin{matrix} X & Y & Z \end{matrix} \\ \begin{pmatrix} 1 & 3 & 3 \\ 1 & 1 & 4 \end{pmatrix} \end{array}$$

$$N(T) = M'$$

$$\underset{2\times 2}{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}} \underset{\times}{} \underset{2\times 3}{\overset{X\ \ Y\ \ Z}{\begin{pmatrix}1 & 3 & 3\\ 1 & 1 & 4\end{pmatrix}}} \underset{=}{=} \underset{2\times 3}{\overset{X'\ \ Y'\ \ Z'}{\begin{pmatrix}e_{11} & e_{12} & e_{13}\\ e_{21} & e_{22} & e_{23}\end{pmatrix}}}$$

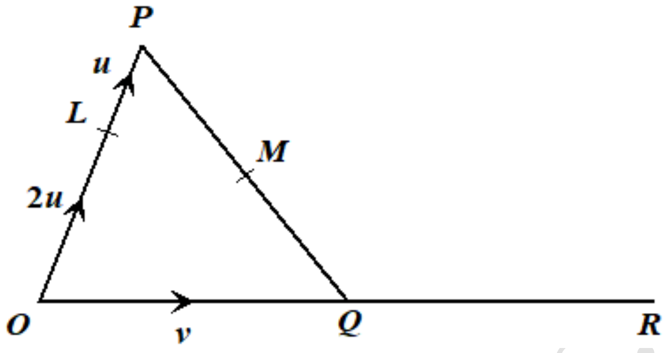
$$=\begin{pmatrix}1 & 1 & 4\\ 1 & 3 & 3\end{pmatrix}$$

$$e_{11}=\left(0\times 1\right)+\left(1\times 1\right)=1$$
$$e_{12}=\left(0\times 3\right)+\left(1\times 1\right)=1$$
$$e_{13}=\left(0\times 3\right)+\left(1\times 4\right)=4$$
$$e_{21}=\left(1\times 1\right)+\left(0\times 1\right)=1$$
$$e_{22}=\left(1\times 3\right)+\left(0\times 1\right)=3$$
$$e_{23}=\left(1\times 3\right)+\left(0\times 4\right)=3$$

$\therefore$ The image of *X* is $\left(1, 1\right)$, the image of *Y* is $\left(1, 3\right)$ and the image of *Z* is $\left(4, 3\right)$.

The coordinates of the vertices of triangle XYZ are:
$X' = (1,1), Y' = (1,3), Z' = (4,3)$

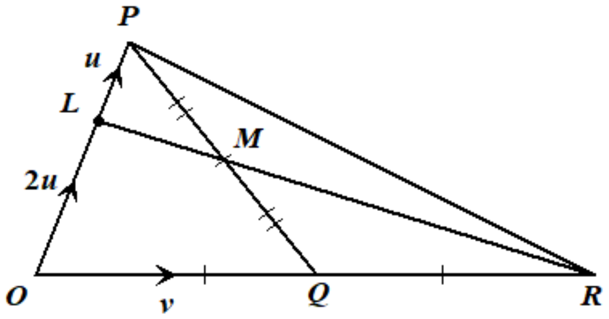
(c) The diagram below shows a triangle OPQ in which $\overrightarrow{OP} = 3u$ and $\overrightarrow{OQ} = v$. Q is the midpoint of OR and M is the midpoint of PQ. L is a point on OP such that $OL = \frac{2}{3}OP$.



(i) Write in terms of u and v, an expression for

a) $\overrightarrow{LM}$

SOLUTION:
Data: Diagram showing triangle OPQ in which $\overrightarrow{OP} = 3u$ and $\overrightarrow{OQ} = v$. Q is the midpoint of OR and M is the midpoint of PQ. L is a point on OP such that $OL = \frac{2}{3}OP$.
Required To Express: $\overrightarrow{LM}$ in terms of u and v
Solution:



$\overrightarrow{LP} = u$

$$\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ}$$
$$= \frac{1}{2}\left(\overrightarrow{PO} + \overrightarrow{OQ}\right)$$
$$= \frac{1}{2}(-3u + v)$$

$$\overrightarrow{LM} = \overrightarrow{LP} + \overrightarrow{PM}$$
$$= u + \frac{1}{2}(-3u + v)$$
$$= u - 1\frac{1}{2}u + \frac{1}{2}v$$
$$= -\frac{1}{2}u + \frac{1}{2}v$$
$$= \frac{1}{2}(-u + v) \text{ OR } \frac{1}{2}(v - u)$$

b) $\overrightarrow{PR}$

SOLUTION:

Required to express: $\overrightarrow{PR}$ in terms of *u* and *v*

Solution:

$$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR}$$
$$= -(3u) + 2v$$
$$= -3u + 2v$$

(ii) Prove that the points *L*, *M* and *R* are collinear.

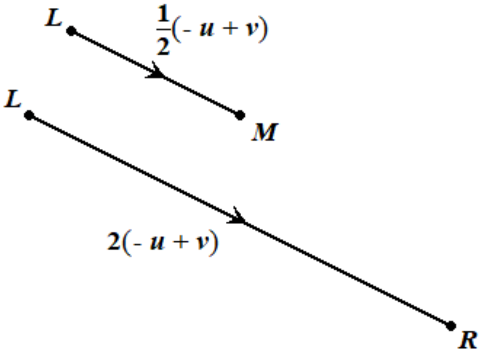
SOLUTION:

Required to prove: The points *L*, *M* and *R* are collinear.

Proof:

$$\overrightarrow{LM} = \frac{1}{2}(-u + v)$$
$$= -\frac{1}{2}u + \frac{1}{2}v$$
$$= \frac{1}{2}(-u + v)$$
$$2\overrightarrow{LM} = (-u + v)$$

$$\overrightarrow{LR} = \overrightarrow{LO} + \overrightarrow{OR}$$
$$= -2u + 2v$$
$$= 2(-u + v)$$
$$= 2(2\overrightarrow{LM})$$
$$= 4\overrightarrow{LM}$$



$\overrightarrow{LR}$ is scalar multiple of $\overrightarrow{LM}$.
This proves that *LR* is parallel to *LM*.
Since *L* is a common point to both lines then *L*, *M* and *R* are collinear.

Q.E.D