FAS-PASS
Maths

JUNE 2018 PAPER 2

SECTION I

1. (a) Using a calculator, or otherwise, evaluate EACH of the following, giving your answers to two decimal places.

(i) $73.18-5.23\times 9.34$

SOLUTION:
Required to calculate: $73.18-5.23\times 9.34$
Calculation:

$$
\begin{aligned}
73.18-(5.23\times 9.34) &= 73.18-48.8482 \text{ (by the calculator)}\\
&= 24.33\underline{\underline{1}}8\\
&= 24.33 \text{ (correct to 2 decimal places)}
\end{aligned}
$$

(ii) $\dfrac{3.1^2}{6.17}+1.12$

SOLUTION:
Required to calculate: $\dfrac{3.1^2}{6.17}+1.12$
Calculation:

$$
\begin{aligned}
\frac{3.1^2}{6.17}+1.12 &= \frac{3.1\times 3.1}{6.17}+1.12\\
&= \frac{9.61}{6.17}+1.12 \text{ (by the calculator)}\\
&= 1.5575+1.12\\
&= 2.67\underline{\underline{7}}5\\
&= 2.68 \text{ (correct to 2 decimal places)}
\end{aligned}
$$

(b) Jenny works at Sammy's Restaurant and is paid according to the rates in the following table.

Jenny's weekly wage agreement
Basic wage \$600.00 PLUS \$0.90 for each customer served

In a week when Jenny serves n customers, her weekly wage, W_J, in dollars, is given by the formula

$W_J = 600 + 0.90n$

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(i) Determine Jenny's weekly wage if she serves 230 customers.

SOLUTION:
Data: Formula showing how Jenny is paid, $W_J = 600 + 0.90n$
Required to find: Jenny's wage when $n = 230$
Solution:

$$\begin{aligned} W_J &= 600 + (0.9 \times 230) \\ &= 600 + 207 \\ &= \$807 \end{aligned}$$

(ii) In a **good** week, Jenny's wage is $\$1000.00$ or more. What is the LEAST number of customers that Jenny must serve in order to have a **good** week?

SOLUTION:
Data: Jenny's wage is $\$1000.00$ or more in a good week.
Required to find: The least number of customers that Jenny served
Solution:

$$W_J = 600 + 0.9n$$
$$600 + 0.9n = W_J$$
$$W_J \geq 1000$$

Hence, $600 + 0.9n \geq 1000$

$$0.9n \geq 1000 - 600$$
$$0.9n \geq 400$$
$$n \geq \frac{400}{0.9}$$
$$n \geq 444.4$$

However, n is a positive integer and 444 customers will NOT earn Jenny \$1 000. So we must find the next positive integer that is greater than 444.4 and which is 445. So, 445 customers will have Jenny cross the \$1 000 mark.

$\therefore$ The least number of customers $= 445$

(iii) At the same restaurant, Shawna is paid a weekly wage of \$270.00 plus \$1.50 for each customer she serves.

If W_S is Shawna's weekly wage, in dollars, write a formula for calculating Shawna's weekly wage when she serves m customers.

SOLUTION:
Data: Shawna is paid a basic salary of \$270.00 plus \$1.50 per customer.
Required to write: A formula for Shawna's salary

Solution:

$W_S =$ Shawna's salary

$\therefore W_S = 270 + 1.50 \times$ Number of customers

When the number of customers $= m$

Then $W_S = 270 + 1.5m$

(iv) In a certain week, Jenny and Shawna received the same wage for serving the same number of customers.

How many customers did EACH serve?

SOLUTION:
Data: Jenny and Shawna received the same salary.
Required to calculate: The number of customers each served
Calculation:

$W_J = 600 + 0.9n$

$W_S = 270 + 1.5m$

If the number of customers served by both Jenny and Shawna is the same, then $n = m$

Let C be the equal number of customers that Jenny and Shawna served.

So
$$600 + 0.9C = 270 + 1.5C$$
$$600 - 270 = 1.5C - 0.9C$$
$$330 = 0.6C$$
$$C = \frac{330}{0.6}$$
$$= 550$$

$\therefore$ Both Jenny and Shawna served 550 customers.

2. (a) Factorise completely, EACH of the following expressions.

(i) $1 - 4h^2$

SOLUTION:
Required to factorise: $1 - 4h^2$
Solution:

$1 - 4h^2 = (1)^2 - (2h)^2$

This is now in the form of the difference of two squares

So $1 - 4h^2 = (1 - 2h)(1 + 2h)$

(ii) $pq - q^2 - 3p + 3q$

SOLUTION:
Required to factorise:
Solution:

$$\underbrace{pq-q^2}\underbrace{-3p+3q}=q(p-q)-3(p-q)$$
$$=(p-q)(q-3)$$

(b) Solve each of the following equations.

(i) $\frac{3}{2}y=12$

SOLUTION:
Required to solve: $\frac{3}{2}y=12$
Solution:

$$\frac{3}{2}y=12$$
$$\times\frac{2}{3}$$
$$\frac{2}{3}\times\frac{3y}{2}=\frac{2}{3}\times 12$$
$$y=8$$

Alternative Method:

$$\frac{3}{2}y=12$$
$$\frac{3y}{2}=\frac{12}{1}$$
$$3y\times 1=12\times 2$$
$$3y=24$$
$$y=\frac{24}{3}$$
$$y=8$$

(ii) $2x^2+5x-3=0$

SOLUTION:
Required to solve: $2x^2+5x-3=0$
Solution:

$$2x^2+5x-3=0$$
$$(2x-1)(x+3)=0$$
$2x-1=0$ **OR** $x+3=0$

and $x=\frac{1}{2}$ and $x=-3$

(c) The quantities *F*, *m*, *u*, *v* and *t* are related according to the formula

$$F=\frac{m(v-u)}{t}$$

(i) Find the value of *F* when $m=3,\ u=-1,\ v=2$ and $t=1$.

SOLUTION:
Data: $F=\frac{m(v-u)}{t}$
Required to find: *F* when $m=3,\ u=-1,\ v=2$ and $t=1$
Solution:

$$F=\frac{(3)\{(2)-(-1)\}}{1}$$
$$=\frac{3\{2+1\}}{1}$$
$$=\frac{3\times 3}{1}$$
$$=9$$

(ii) Make *v* the subject of the formula.

SOLUTION:
Required to make: *v* the subject of the formula
Solution:

$$F=\frac{m(v-u)}{t}$$
$$\frac{F}{1}=\frac{m(v-u)}{t}$$
$$F\times t=m(v-u)$$
$$m(v-u)=F\times t$$
$$mv-mu=Ft$$

$$mv = Ft + mu$$

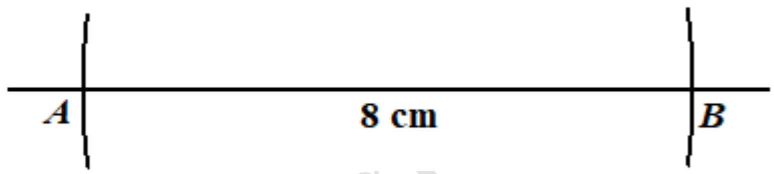
$$v = \frac{Ft + mu}{m}$$

3. (a) Using a ruler, a pencil and a pair of compasses, construct the triangle ABC, such that $AB = 8$cm, $\angle BAC = 30°$ and $AC = 10$cm.

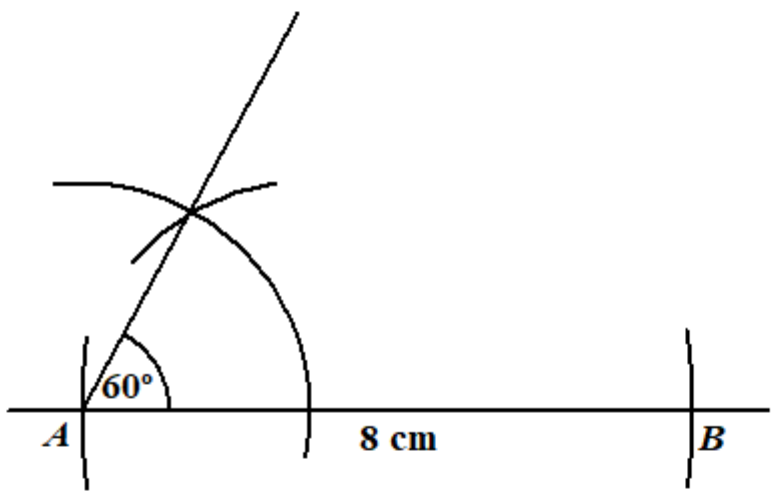
SOLUTION:

Required to construct: ΔABC with $AB = 8$cm, $\angle BAC = 30°$ and $AC = 10$cm.
Construction:

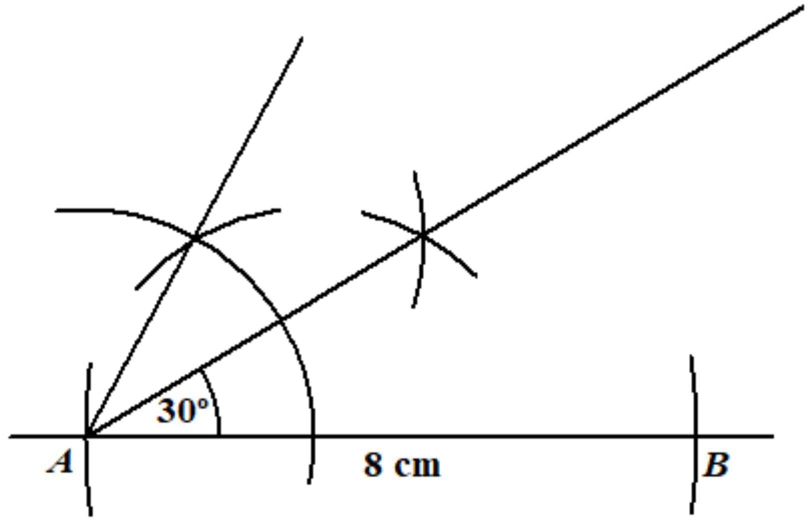
Although the steps are all done on the same diagram, we perform them in separate steps so that the construction may be more easily understood.
Step 1:



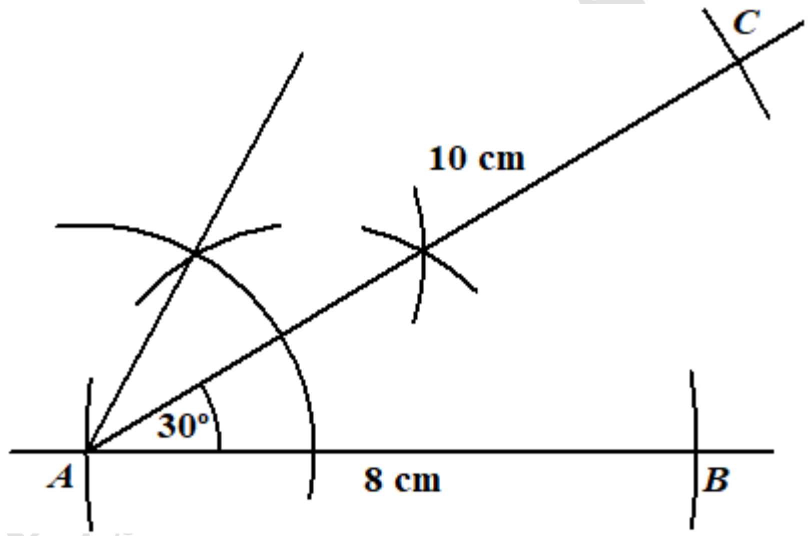
Step 2:



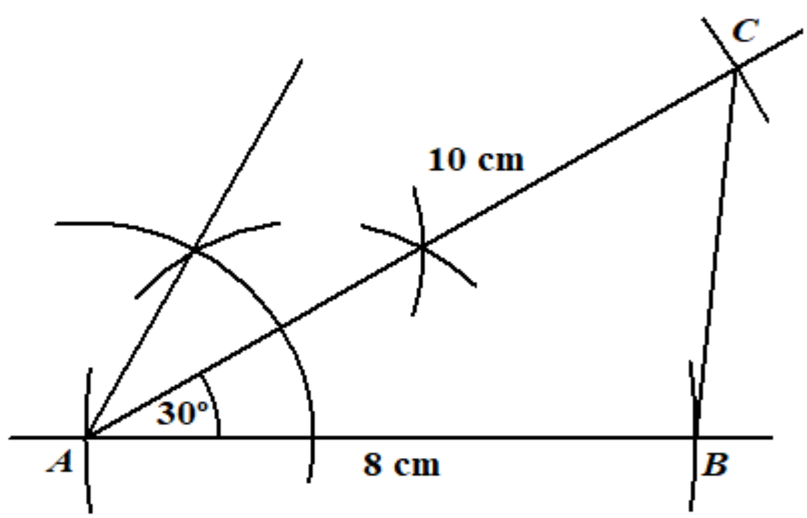
Step 3:



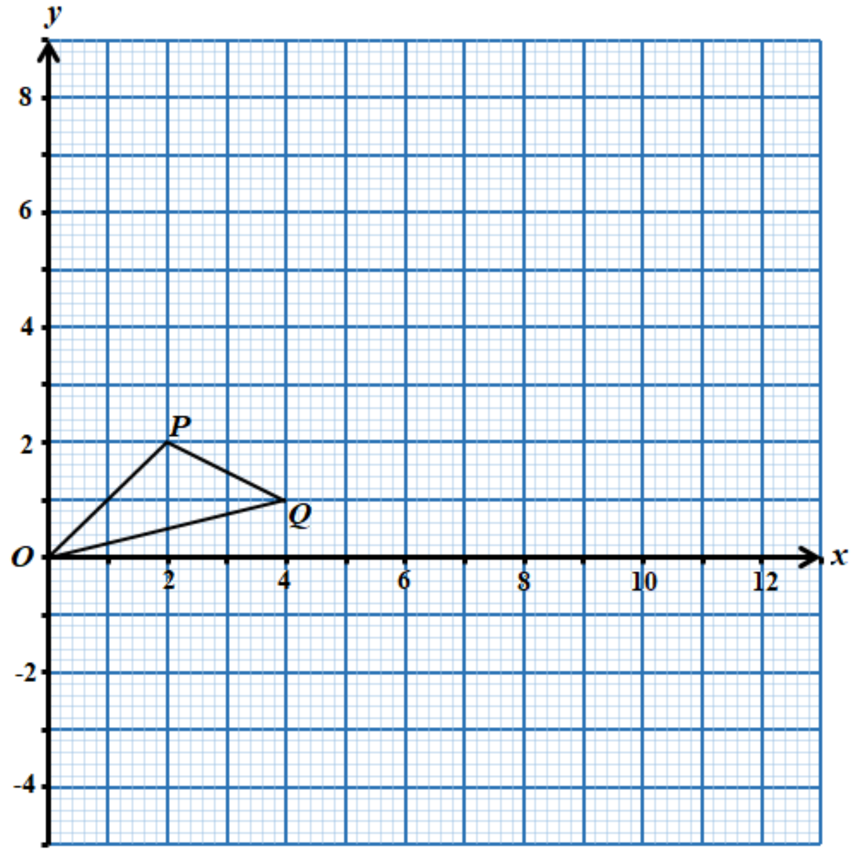
Step 4:



Step 5:

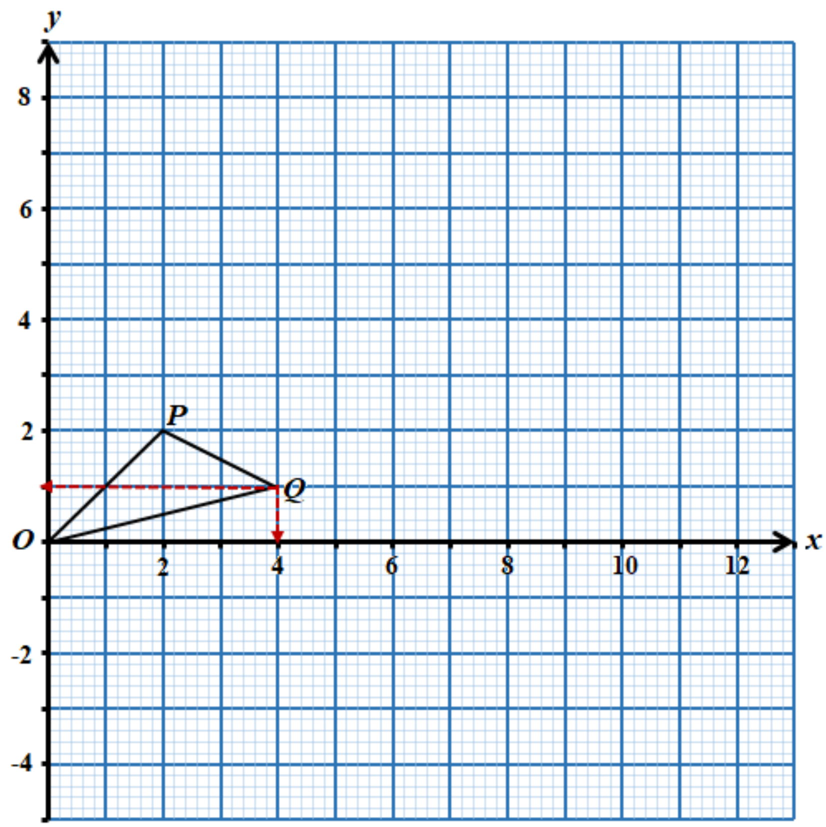


(b) The diagram below shows the triangle *OPQ*.



(i) State the coordinates of the point *Q*.

SOLUTION:
Data: Diagram showing triangle *OPQ*
Required to state: The coordinates of point *Q*
Solution:



The coordinates of Q are $(4, 1)$.

(ii) The line PQ is mapped onto $P'Q'$ by an enlargement, center O and scale factor 3. On the diagram above, draw the line $P'Q'$.

SOLUTION:
Data: PQ is mapped onto $P'Q'$ by an enlargement, center O and scale factor 3
Required to draw: $P'Q'$
Solution:

$$P = (2, 2)$$

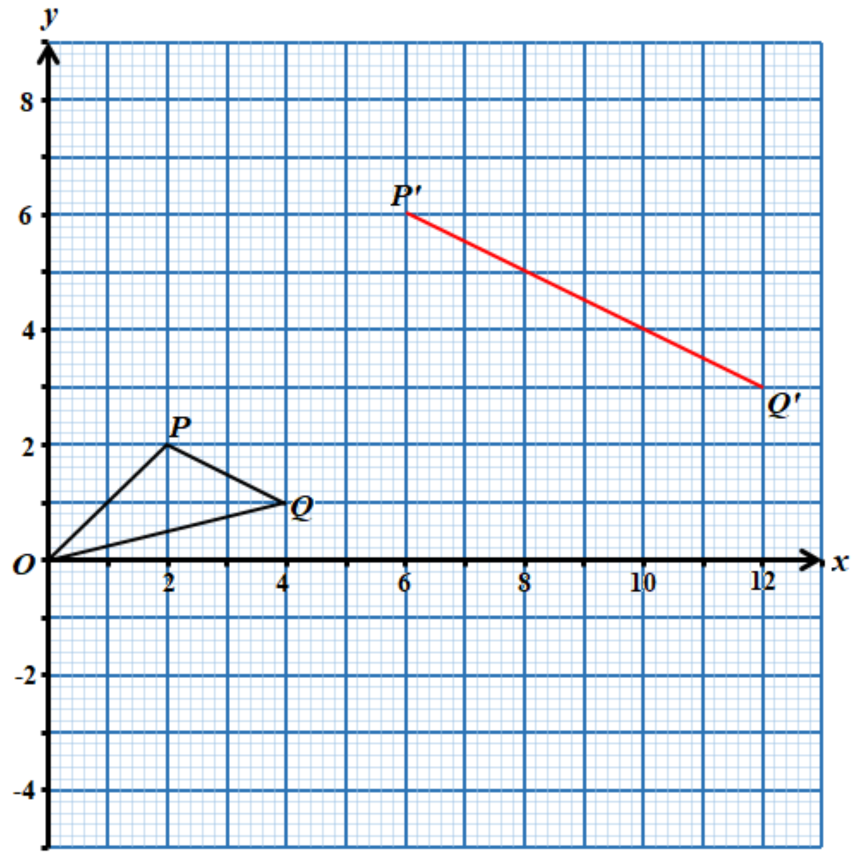
$$\therefore \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$$

$$P' = (6, 6)$$

$$Q = (4, 1)$$

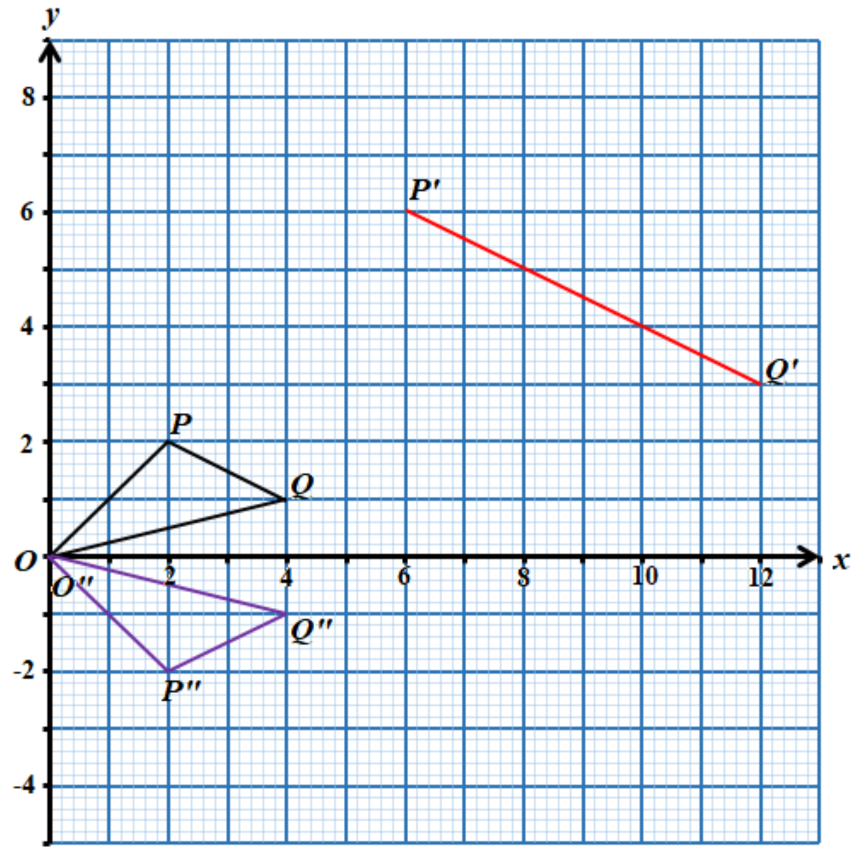
$$\therefore \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 12 \\ 3 \end{pmatrix}$$

$$Q' = (12, 3)$$



(iii) The ΔOPQ undergoes a reflection in the line $y = 0$ to produce the image $O''P''Q''$. On the diagram above, show the $\Delta O''P''Q''$.

SOLUTION:
Data: ΔOPQ is reflected in the line $y = 0$ to give $\Delta O''P''Q''$
Required to draw: $\Delta O''P''Q''$
$y = 0$ is the equation of the $x-$axis.
$OPQ \xrightarrow[\text{in } y=0]{\text{Reflection}} O''P''Q''$



4. (a) The function f with domain, $A = \{1, 2, 3\}$ is given by

$$f(x) = \frac{1}{2}x - 3$$

(i) What is the value of $f(1)$?

SOLUTION:

Data: $f(x) = \frac{1}{2}x - 3$ and the domain of f is $A = \{1, 2, 3\}$

Required to find: $f(1)$

Solution:

$$f(1) = \frac{1}{2}(1) - 3$$
$$= \frac{1}{2} - 3$$
$$= -2\frac{1}{2}$$

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(ii) Find the value of x for which $f(x) = -2$.

SOLUTION:
Data: $f(x) = -2$
Required to find: x
Solution:

$$\frac{1}{2}x - 3 = -2$$
$$\frac{1}{2}x = -2 + 3$$
$$\frac{1}{2}x = 1$$
$$x = 1 \times 2$$
$$x = 2$$

(iii) An ordered pair for the function is expressed in the form (a, b). Using your answers to (a) (i) and (a) (ii), or otherwise, list the ordered pairs for the function, f.

SOLUTION:
Data: (a, b) is an ordered pair of $f(x)$
Required to list: The ordered pairs for f
Solution:

From (a) (i), $f(1) = -2\frac{1}{2}$

From (a) (ii), $f(2) = -2$

$$f(3) = \frac{1}{2}(3) - 3$$
$$= 1\frac{1}{2} - 3$$
$$= -1\frac{1}{2}$$

So the list of ordered pairs is $\left\{\left(1, -2\frac{1}{2}\right), (2, -2), \left(3, -1\frac{1}{2}\right)\right\}$.

(iv) Explain why $f(x) \neq 5$ for the function specified.

SOLUTION:

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Required to explain: Why $f(x) \neq 5$

Proof:

If $f(x) = 5$, then

$$\frac{1}{2}x - 3 = 5$$

$$\therefore \frac{1}{2}x = 8$$

$$x = 16$$

But $16 \not\subset A$ according to the data and so $f(x) \neq 5$.

(b) **(i)** Solve the inequalities

a) $3x - 1 < 11$

SOLUTION:
Data: $3x - 1 < 11$
Required to find: x
Solution:

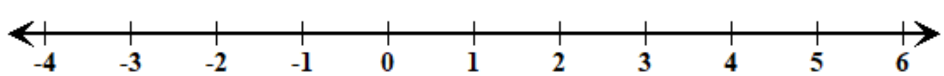
$$3x - 1 < 11$$
$$3x < 11 + 1$$
$$3x < 12$$
$$(\div 3)\quad x < 4$$

b) $2 \leq 3x - 1$

SOLUTION:
Data: $2 \leq 3x - 1$
Required to find: x
Solution:

$$2 \leq 3x - 1$$
$$2 + 1 \leq 3x$$
$$3 \leq 3x$$
$$(\div 3)\quad 1 \leq x$$
$$\therefore x \geq 1$$

(ii) Represent the solution to $2 \leq 3x - 1 < 11$ on the number line shown below.



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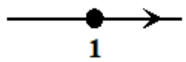
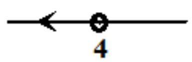
SOLUTION:
Required to represent: $2 \le 3x - 1 < 11$ on the number line
Solution:
First we consider $\quad 2 \le 3x - 1$

$$2 + 1 \le 3x$$
$$3 \le 3x$$
$$1 \le x$$
$$x \ge 1$$

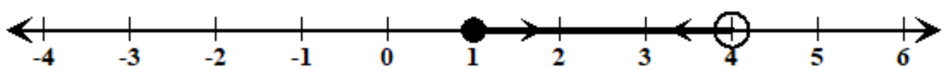


Now, we consider $3x - 1 < 11$

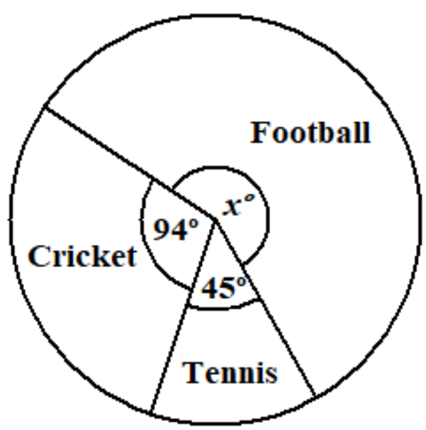
$$3x < 11 + 1$$
$$3x < 12$$
$$x < 4$$



Expressing both inequalities on the same number line we get,



5. (a) Students in a group were asked to name their favourite sport. Their responses are shown on the pie chart below.



(i) Calculate the value of x.

SOLUTION:
Data: Pie chart showing students' favourite sport.

Required to calculate: x
Calculation:

$x = 360° - (94° + 45°)$ (Sum of the angles in a circle $= 360°$)

$= 221°$

(ii) What percentage of the students chose cricket?

SOLUTION:
Required to calculate: The percentage of students who chose cricket.
Calculation:
The percentage of students who chose cricket

$$= \frac{\text{Angle corresponding to cricket}}{360°} \times 100\%$$

$$= \frac{94°}{360°} \times 100\%$$

$= 26.11\%$ (correct to 2 decimal places)

(iii) Given that 40 students chose tennis, calculate the TOTAL number of students in the group.

SOLUTION:
Data: 40 students chose tennis
Required to calculate: The total number of students in the group
Calculation:
$45°$ represent 40 students

$\therefore 1°$ represents $\frac{40}{45}$ students

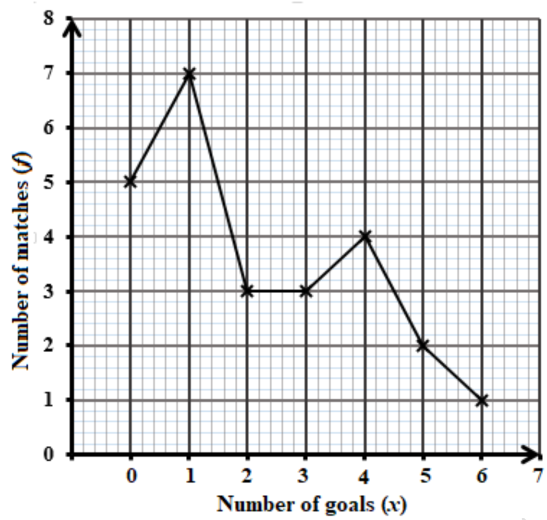
The entire group $= \frac{40}{45} \times 360°$

$= 320$ students

It appears that there is a misprint in this question and the angle of the sector representing cricket should have been 90^0. If not, the number of students who chose cricket will be 83.6 and who chose football will be 196.4

(b) The diagram below shows a frequency polygon of the number of goals scored by a football team in 25 matches.

As a point of interest, the given diagram is NOT a frequency polygon. It is NOT enclosed and such data (discrete) is best illustrated on a bar chart.



(i) Complete the following table using the information in the diagram.

Number of matches (f)	5	7		3	4		1
Number of goals scored (x)	0	1	2	3	4	5	6

SOLUTION:
Data: Diagram showing the number of goals scored in 25 matches by a football team
Required to complete: The table given
Solution:
From the diagram, 2 goals were scored in 3 matches and 5 goals were scored in 2 matches. This was obtained by a read-off.
So the complete table looks like:

Number of matches (f)	5	7	3	3	4	2	1
Number of goals scored (x)	0	1	2	3	4	5	6

(ii) What is the modal number of goals scored by the team?

SOLUTION:
Required to find: Modal number of goals scored
Solution:
The modal number of goals scored by the team is 1 since this score occurred the most number of times, that is, it had the greatest frequency.

(iii) Determine the median number of goals scored by the team.

SOLUTION:
Required to find: Median number of goals scored
Solution:
There were 25 matches played. Hence, the median number of goals scored will be the number of goals scored at the 13^{th} game when the scores are

arranged in numerically ascending or descending order of magnitude. The 13^{th} score is 2 when arranged in order of magnitude.
$\therefore$ The median number of goals scored is 2.

(iv) Calculate the mean number of goals scored by the team.

SOLUTION:
Required to find: Mean number of goals scored
Solution:
Mean number of goals scored,

$$\bar{x} = \frac{\sum fx}{\sum f}, \quad \bar{x} = \text{mean}, \quad x = \text{score}, \quad f = \text{frequency}$$

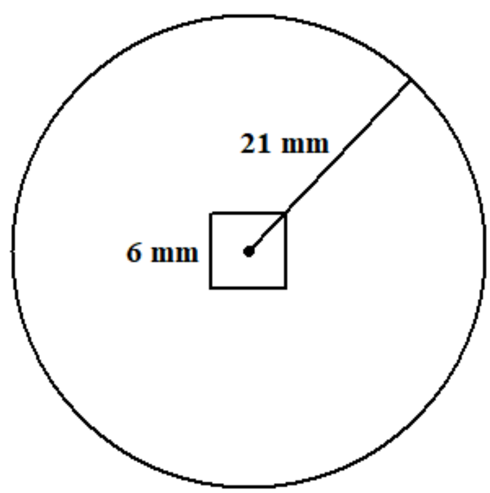
$$= \frac{\{(0\times5)+(1\times7)+(2\times3)+(3\times3)+(4\times4)+(5\times2)+(1\times6)\}}{25}$$

$$= \frac{54}{25}$$

$$= 2.16 \text{ goals}$$

6. (a) In this question, take the value of π to be $\frac{22}{7}$.

The diagram below, **not drawn to scale**, shows the cross-section of a circular metal disc of radius 21 mm. A square hole with sides 6 mm is located at the center of the disc.



Calculate:

(i) the circumference of the disc

SOLUTION:
Data: Disc of radius 21 mm with a square hole of 6 mm located at the center
Required to calculate: The circumference of the disc
Calculation:
Circumference $= 2 \times \pi \times \text{radius}$

$$= 2 \times \frac{22}{7} \times 21$$

$$= 132 \text{ mm}$$

(ii) the area, in mm^2, of the cross-section of the disc.

SOLUTION:
Required to calculate: The area of the cross-section of the disc
Calculation:
Area of the cross-section of the disc

$$= \text{Area of the circle} - \text{Area of the square hole}$$

$$= \pi(21)^2 - (6)^2$$

$$= \left(\frac{22}{7} \times 21 \times 21\right) - (6)^2 \text{ mm}^2$$

$$= 1386 - 36 \text{ mm}^2$$

$$= 1350 \text{ mm}^2$$

(iii) Given that the thickness of the disc is 2 mm, calculate the **maximum** number of discs that can be constructed from 1000 cm^3 of metal.

$$\left(1 \text{ cm}^3 = 1000 \text{ mm}^3\right)$$

SOLUTION:
Data: The thickness of the disc is 2 mm
Required to calculate: The maximum number of discs that can be constructed from 1000 cm^3 of metal
Calculation:
Volume of 1 disc $= \text{Cross-sectional area} \times \text{thickness}$

$$= 1350 \times 2 \text{ mm}^3$$

$$= 2700 \text{ mm}^3$$

Volume of available metal $= 1000 \times 1000 \text{ mm}^3$

$\therefore$ Number of discs that can be made $= \dfrac{\text{Volume of metal}}{\text{Volume of 1 disc}}$

$$= \frac{1000 \times 1000}{2700}$$
$$= 370.4 \text{ discs}$$

$\therefore 370$ completed discs can be made.

(b) A globe is a scaled spherical representation of the earth. The actual length of the equator (*LL*) is $40\,000$km and is represented on the globe by a piece of string of length 160 cm.

(i) What length of string would represent an actual distance of 500 km on the globe?

SOLUTION:
Data: Length of the equator $= 40\,000$ km
Equator is represented by a string of length 160 cm
Required to calculate: The length of string required to represent 500 km
Calculation:
$40\,000$ km is represented by 160 cm

$\therefore 1$ km will be represented by $\frac{160}{40\,000}$ cm

500 km will be represented by $\frac{160}{40\,000} \times 500 = 2$ cm

(ii) The distance between Palmyra (*P*) and Quintec (*Q*) is represented on the globe by a string of length 25 cm. Calculate the value of *PQ*, the actual distance, in km, between *P* and *Q*.

SOLUTION:
Data: Distance of the globe from *P* to *Q* is represented by 25 cm
Required to calculate: The actual distance *PQ*
Calculation:
160 cm represents $40\,000$km

1 cm represents $\frac{40\,000}{160}$ km

25 cm represents $\frac{40\,000}{160} \times 25 \text{ km} = 6\,250 \text{ km}$

7. A sequence of figures is made from squares of unit length. The first three figures in the sequence are shown below.

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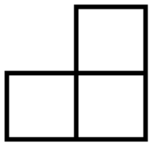


Figure 1

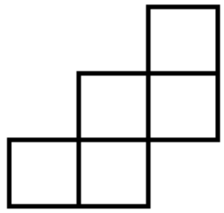


Figure 2

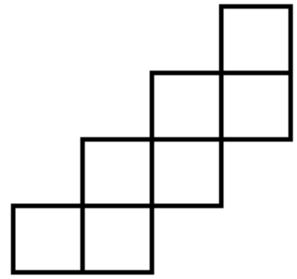
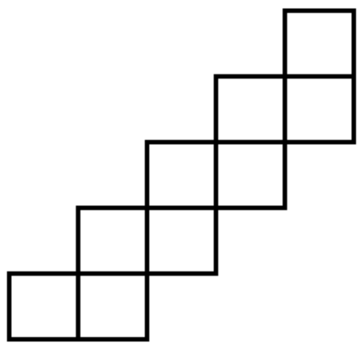


Figure 3

(a) Draw Figure 4 of the sequence.

SOLUTION:
Data: Diagrams showing a sequence of figures made of squares of unit length.
Required To Draw: The 4th figure in the sequence
Solution:



(b) Study the pattern of numbers in each row of the table below. Each row relates to one of the figures in the sequence of figures. Some rows have not been included in the table.

Complete the rows numbered (i), (ii) and (iii).

	Figure	**Number of Squares in Figure (S)**	**Perimeter of Figure (P)**
	1	3	8
	2	5	12
	3	7	16
(i)	4		
	⋮	⋮	⋮
(ii)		43	
(iii)	n		

SOLUTION:
Data: Table showing the relationship between the number of squares in the figure and the perimeter of the figure.

Required To Complete: The table given
Solution:
Let us concentrate on the table values of *n* and *S*.
S increases by 2.
Hence, $S = 2n \pm$ a constant
When $S = 3,\ n = 1$

$\therefore 3 = 2(1) + 1$ The constant appears to be 1.

So we suspect that, $S = 2n + 1$

Let's check: $n = 1$ $S = 2(1) + 1 = 3$ Correct

$n = 2$ $S = 2(2) + 1 = 5$ Correct

$n = 3$ $S = 2(3) + 1 = 7$ Correct

Hence, we confirm the relation that $S = 2n + 1$

Let us look at the table values of *P* and *n*.
P increases by 4.
So, $P = 4n \pm$ a constant
When $n = 1$ $P = 8$ $= 4(1) + 4$
The constant appears to be 4.
So we suspect that $P = 4n + 4$

Let's check: $n = 1$ $P = 4(1) + 4 = 8$ Correct

$n = 2$ $P = 4(2) + 4 = 12$ Correct

$n = 3$ $P = 4(3) + 4 = 16$ Correct

So, we confirm the relation that $P = 4n + 4$

We now have:

	Figure	Number of Squares in Figure (*S*)	Perimeter of Figure (*P*)
	1	3	8
	2	5	12
	3	7	16
(i)	4	$S = 2(4) + 1 = 9$	$P = 4(4) + 4 = 20$
	$\vdots$	$\vdots$	$\vdots$
(ii)	21	43 $\therefore 2n + 1 = 43$ $2n = 42$ $n = 21$	$P = 4(21) + 4$ $= 84 + 4$ $= 88$
(iii)	n	$S = 2n + 1$	$P = 4n + 4$

SECTION II
Answer ALL questions in this section.

RELATIONS, FUNCTIONS AND GRAPHS

8. (a) The diagram shows six points of the function $y = 3x + \frac{1}{x}$. The coordinates of these six points are given in the table below.

x	0.1	0.2	0.5	1	1.5	2	2.2	2.5
y	10.3	5.6		4	5.2		7.1	7.9

(i) Complete the table above by calculating and inserting the missing values of *x*.

SOLUTION:

Data: $y = 3x + \frac{1}{x}$

Required to calculate: y when $x = 0.5$ and $x = 2$

Calculation:

When $x = 0.5$

$$y = 3(0.5) + \frac{1}{0.5}$$
$$= 1.5 + 2$$
$$= 3.5$$

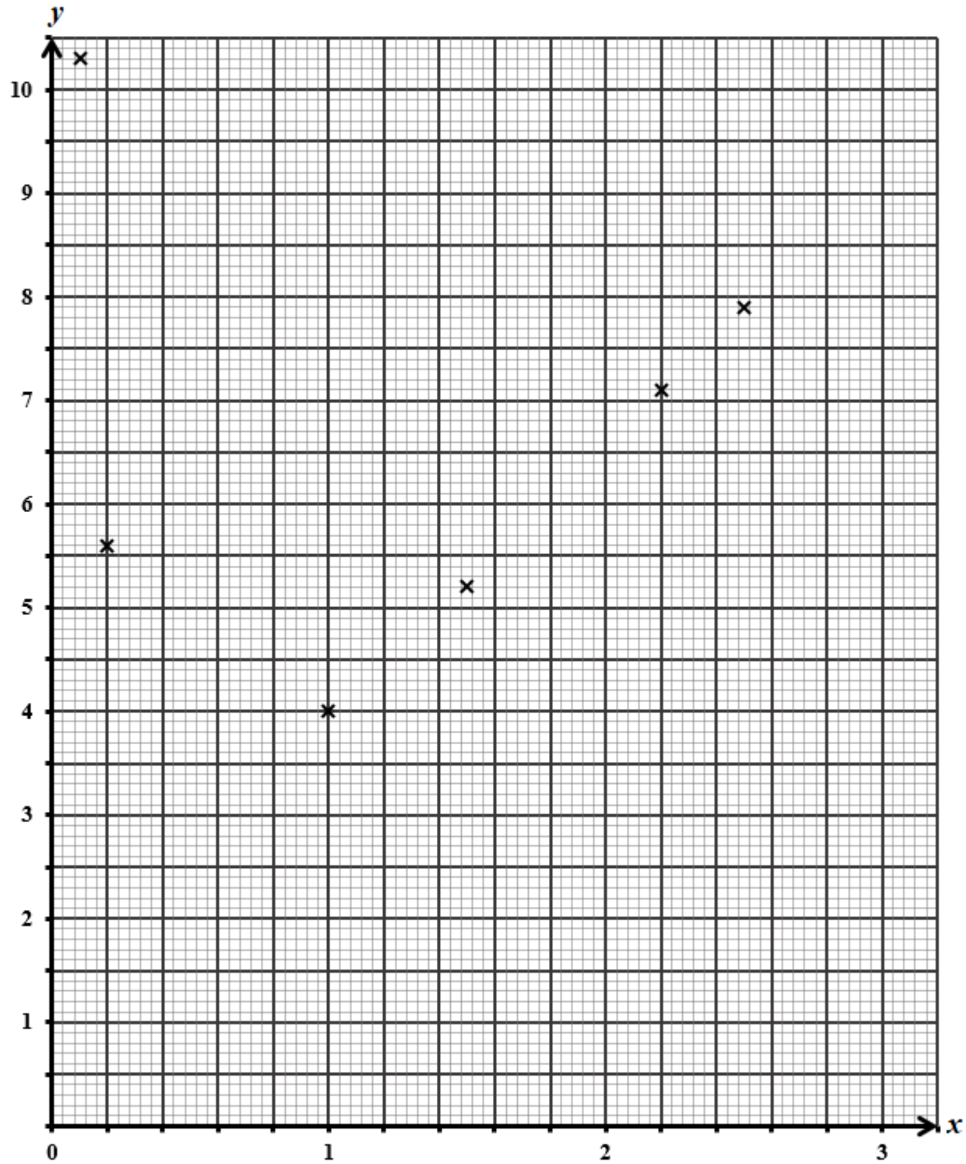
When $x = 2$

$$y = 3(2) + \frac{1}{2}$$
$$= 6 + \frac{1}{2}$$
$$= 6.5$$

The completed table looks like:

x	0.1	0.2	0.5	1	1.5	2	2.2	2.5
y	10.3	5.6	3.5	4	5.2	6.5	7.1	7.9

(ii) On the diagram given, the ordered pairs shown in the table have been plotted except for the missing ones. Using your answers in (a) (i), plot the missing points and connect all the points with a smooth curve.



SOLUTION:
Required To Plot: The missing points and the draw the graph of
$y = 3x + \frac{1}{x}$
Solution:

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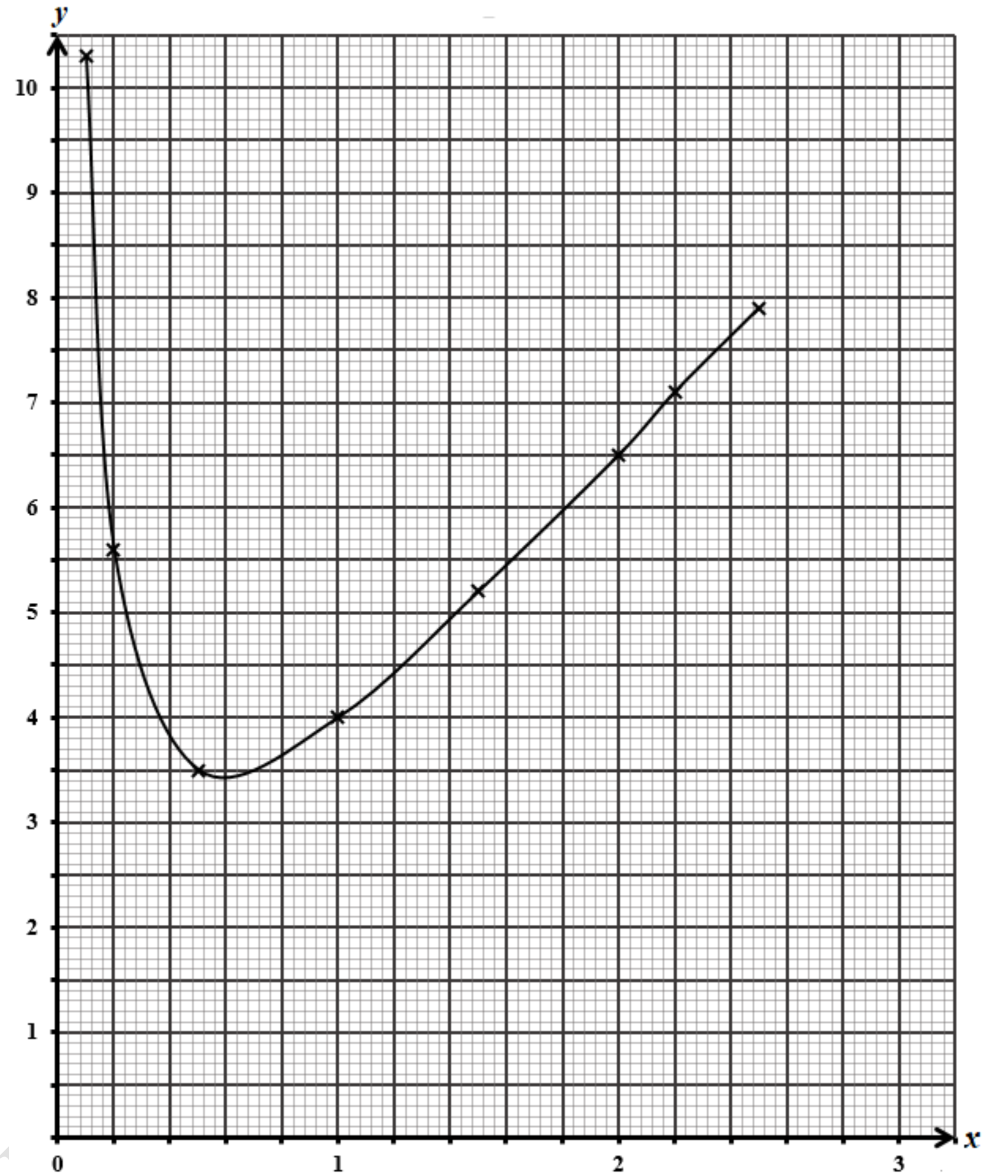


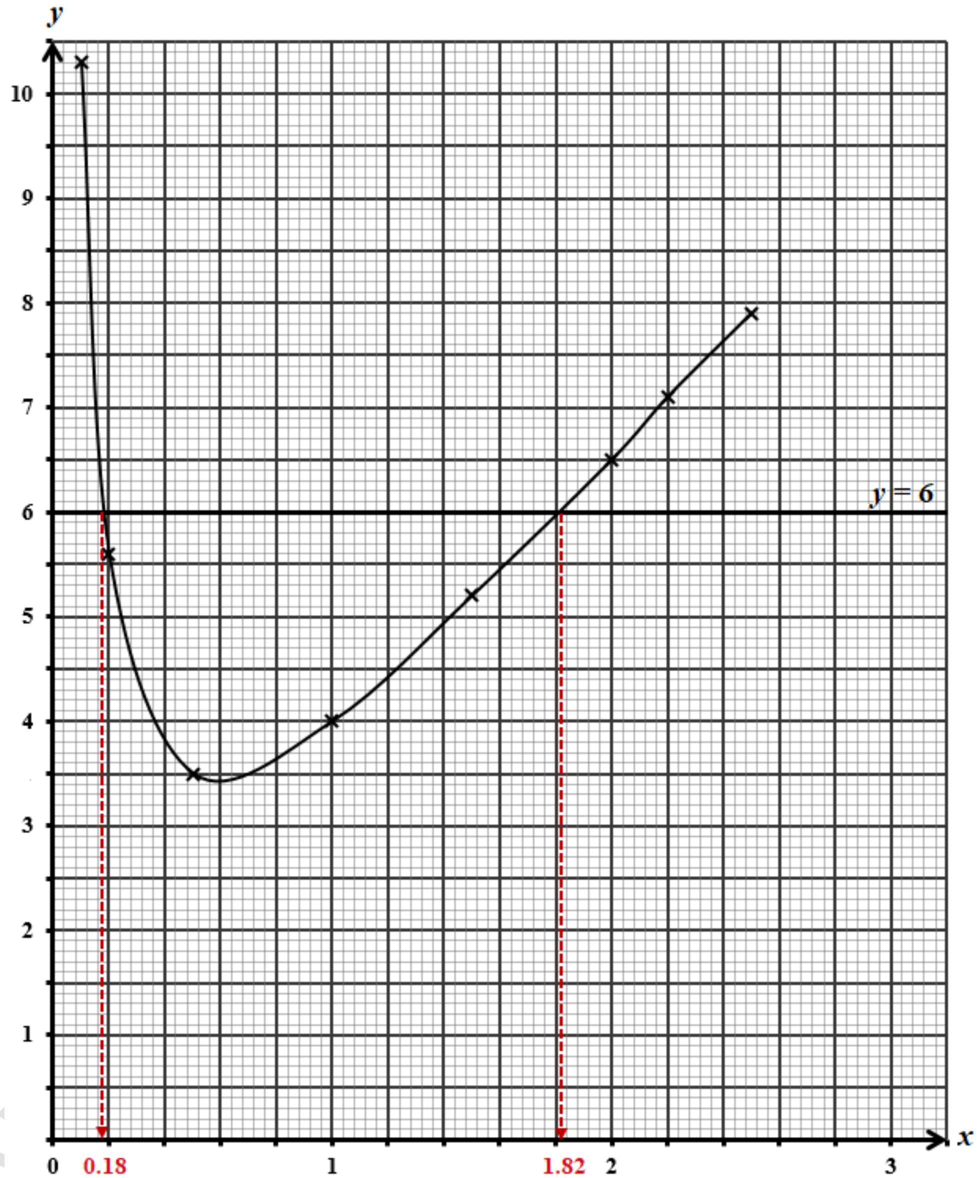
(iii) By drawing an appropriate straight line on the diagram on page 25, find the approximate solutions to the equation

$$3x + \frac{1}{x} = 6$$

SOLUTION:

Required To Find: x when $3x + \frac{1}{x} = 6$, using the graph

Solution:



From the graph, when $3x+\frac{1}{x}=6$, $x=1.82$ and 0.18

(b) The speed-time graph below shows information on the first 60 seconds of a car's journey.

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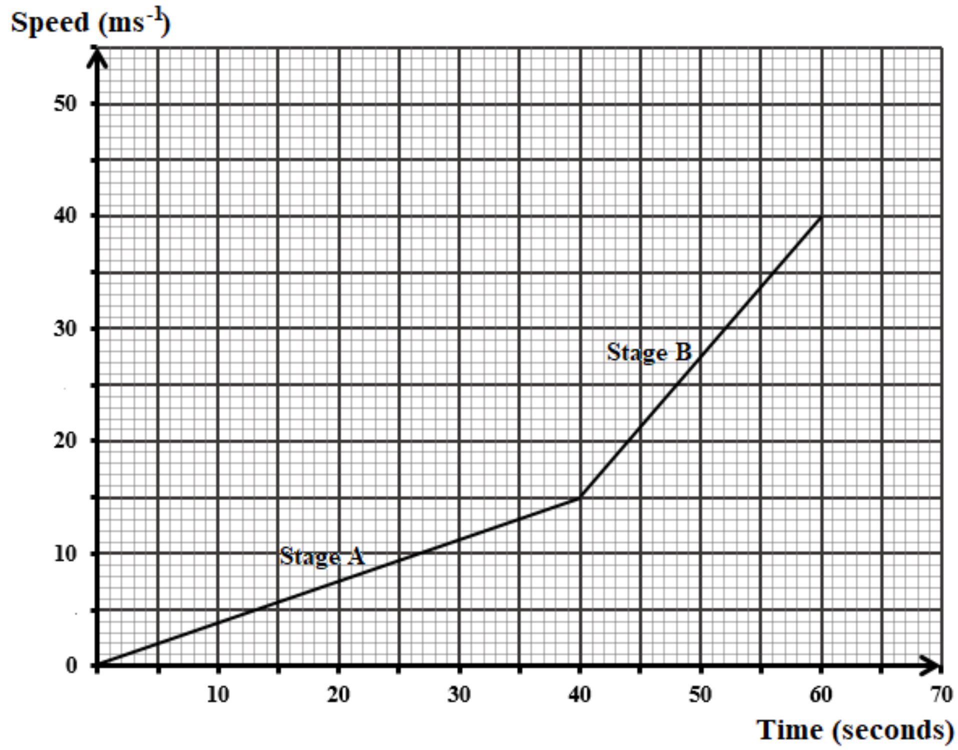
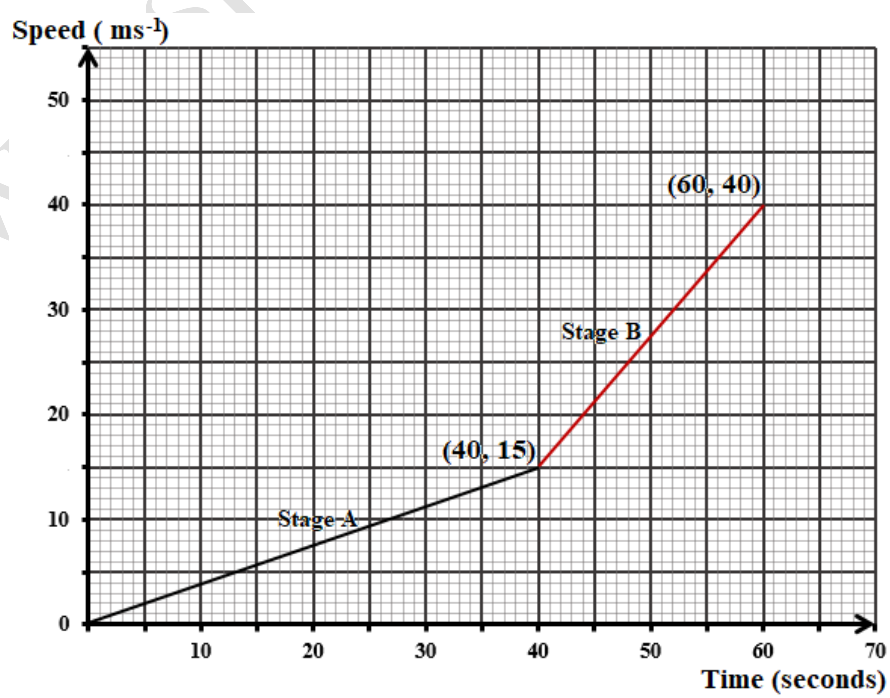


(i) Calculate the acceleration, in ms^{-2}, of the car during Stage B.

Data: Speed-time graph showing a car's journey in 60 seconds.
Required to calculate: The car's acceleration in Stage B.
Calculation:

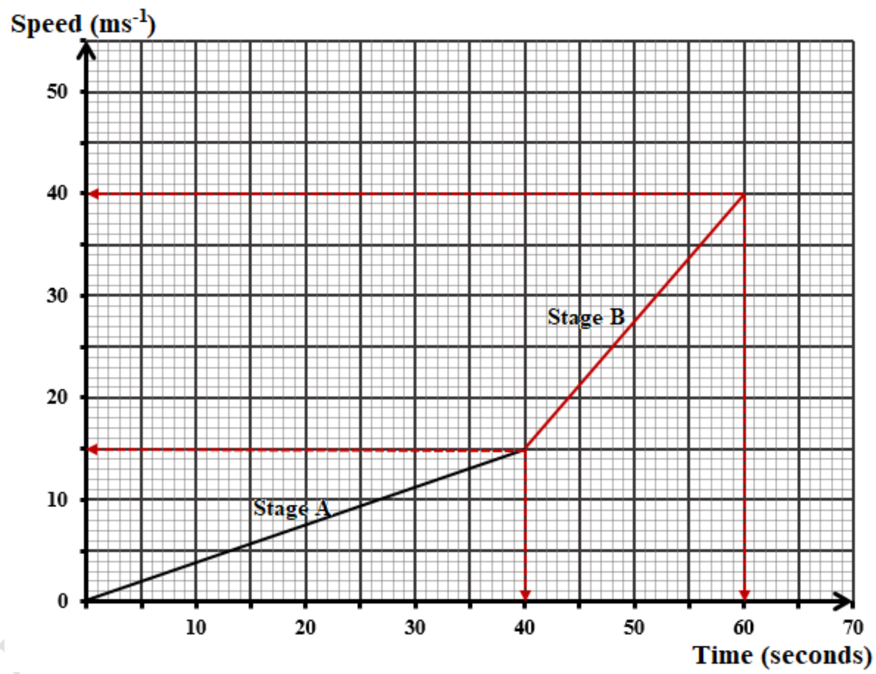


The gradient of the branch will give the acceleration.

$$\begin{aligned}\text{Gradient } &= \frac{40-15}{60-40}\\ &= \frac{25}{20}\\ &= 1.25 \text{ ms}^{-2}\end{aligned}$$

(ii) Calculate the average speed of the car during Stage B.

SOLUTION:
Required to calculate: Average speed of the car in Stage B.
Calculation:
Distance covered $=$ Area under the graph



The region whose area is required is a trapezium

$$\begin{aligned}\text{Area } &= \frac{1}{2}(15+40)\times(60-40)\\ &= \frac{1}{2}(55)\times 20\\ &= 550 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Time taken } &= 60-40\\ &= 20 \text{ s}\end{aligned}$$

Average speed = total distance covered ÷ total time taken

$$\text{Average speed } = \frac{550 \text{ m}}{20 \text{ s}}$$

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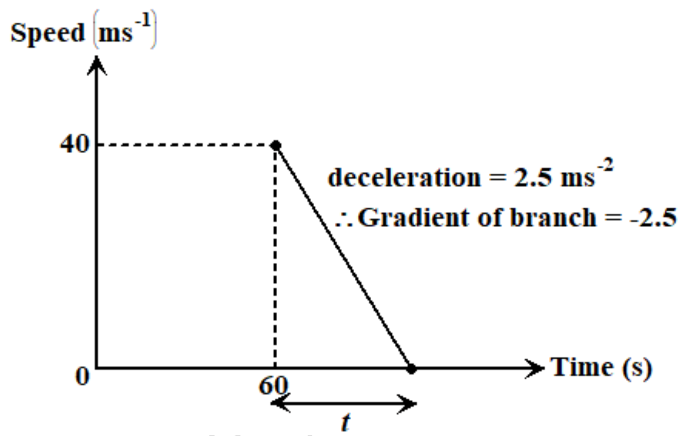
$= 27.5 \text{ ms}^{-1}$

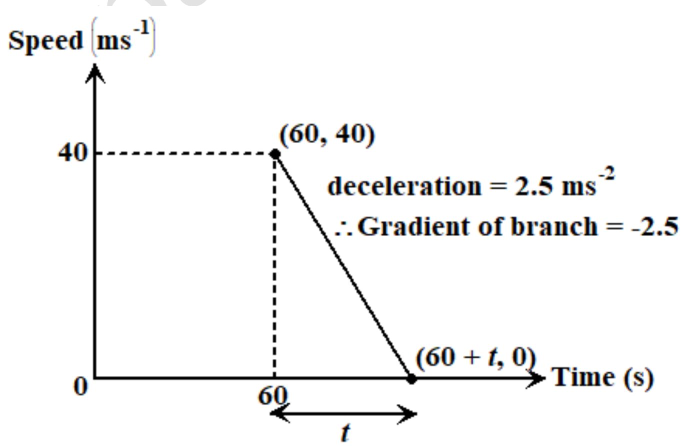
(iii) At time $t = 60$ seconds, the car starts to slow down with a uniform deceleration of 2.5 ms^{-2}.

Determine how long it will take the car to come rest.

SOLUTION:
Data: At $t = 60$, the car decelerates uniformly to rest
Required to find: Time taken for car to come to rest
Solution:



Let the time taken be t.



$$\therefore \frac{40-0}{60-(60+t)}=-2.5$$

$$40=-t(-2.5)$$

$$40=2.5t$$

$$t=\frac{40}{2.5}$$

$$=16$$

So, the car comes to rest after a total of $60+16=76$ s from the start.

Alternative Method which may be used by students of kinematics

Let initial velocity, $u = 40\ ms^{-1}$

final velocity, $v=0$ ms^{-1}

acceleration, $a=-2.5$ ms^{-2}

$$v = u + at$$

$$0 = 40 + (-2.5t)$$

$$2.5t = 40$$

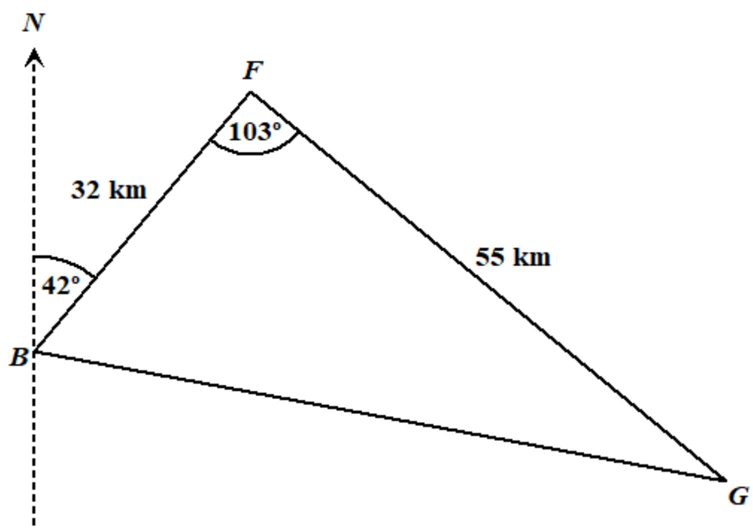
$$t = \frac{40}{2.5}$$

$$t = 16$$

So, the car comes to rest after $60+16=76$ s from the start.

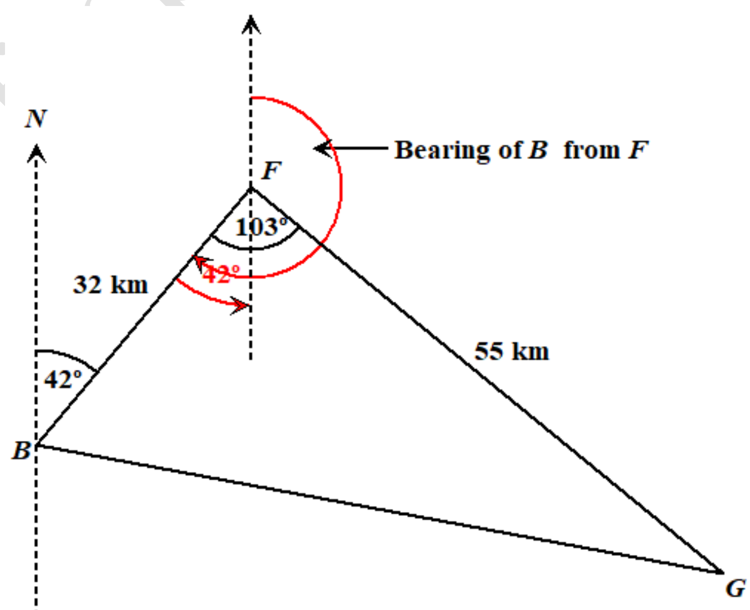
GEOMETRY AND TRIGONOMETRY

9. (a) The diagram below, not drawn to scale, shows the relative positions of three reservoirs *B*, *F* and *G*, all on ground level. The distance $BF = 32$ km, $FG = 55$ km, $\angle BFG$ is 103° and *F* is on a bearing of 042° from *B*.



(i) Determine the bearing of *B* from *F*.

SOLUTION:
Data: Diagram showing the relative positions of three reservoirs
Required to determine: The bearing of *B* from *F*
Solution:

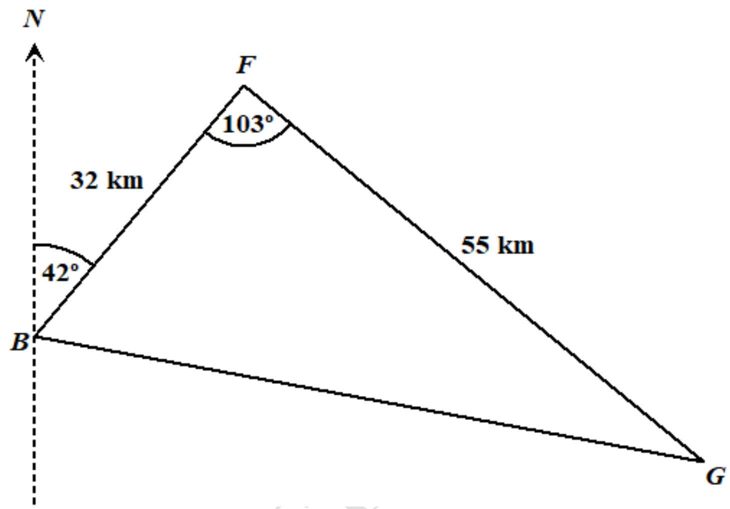


The angle made by *BF* and the South line is 42^0 (alternate)

The bearing of B from F $= 180° + 42°$ (indicated by the red arrow)
$= 222°$

(ii) Calculate the distance, BG, giving your answer to one decimal place.

SOLUTION:
Required to calculate: BG
Calculation:

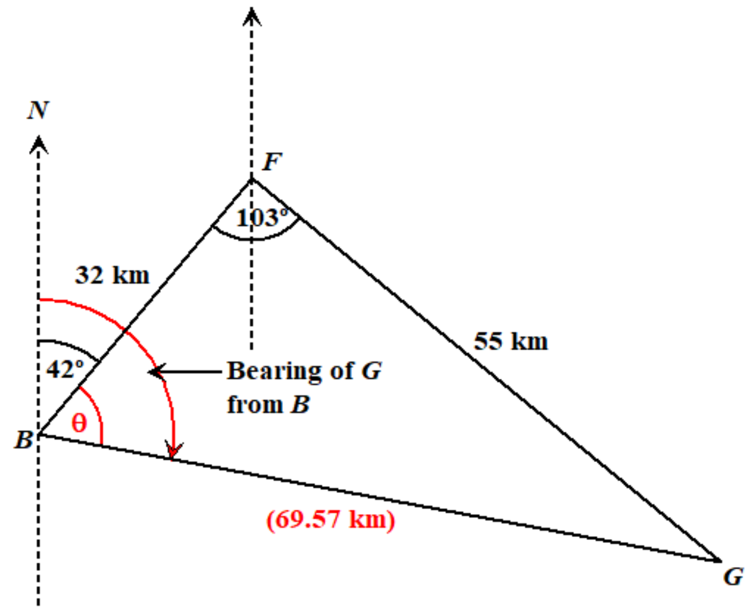


Let us consider the triangle BFG

$$BG^2 = (32)^2 + (55)^2 - 2(32)(55)\cos 103° \quad \text{(Cosine law)}$$
$$BG^2 = 1024 + 3025 - 3520(-0.22495)$$
$$= 4049 + 791.828$$
$$= 4840.828$$
$$BG = 69.58$$
$$= 69.6 \text{ km (correct to 1 decimal place)}$$

(iii) Calculate, to the nearest degree, the bearing of G from B.

SOLUTION:
Required to calculate: The bearing of G from B
Calculation:



The bearing of G from B will be the angle $N\hat{B}G$ as shown $= \theta + 42°$

$$\frac{69.58}{\sin 103°} = \frac{55}{\sin \theta} \qquad \text{(Sine rule)}$$

$$\therefore \sin \theta = \frac{55 \times \sin 103°}{69.58}$$

$$= 0.7702$$

$$\theta = \sin^{-1}(0.7702)$$

$$= 50.4°$$

$$N\hat{B}Q = 42° + 50.4°$$

$$= 92.4°$$

So, the bearing of G from B is 092°(to the nearest degree).

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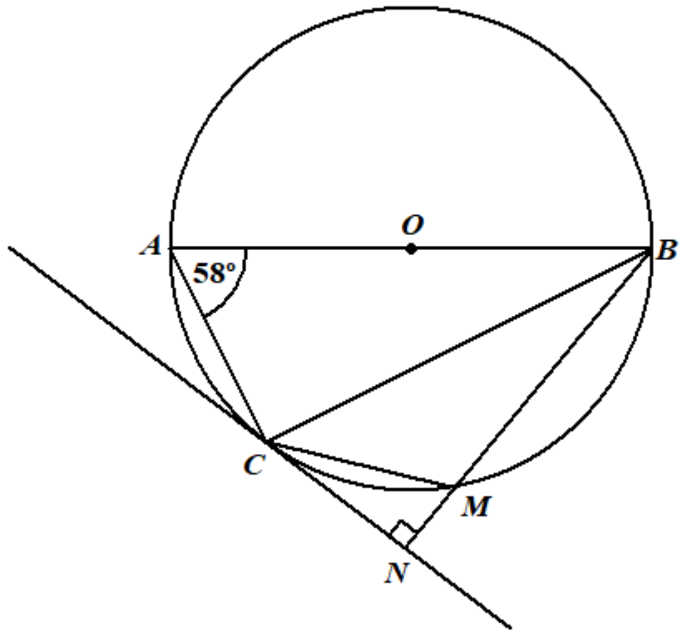
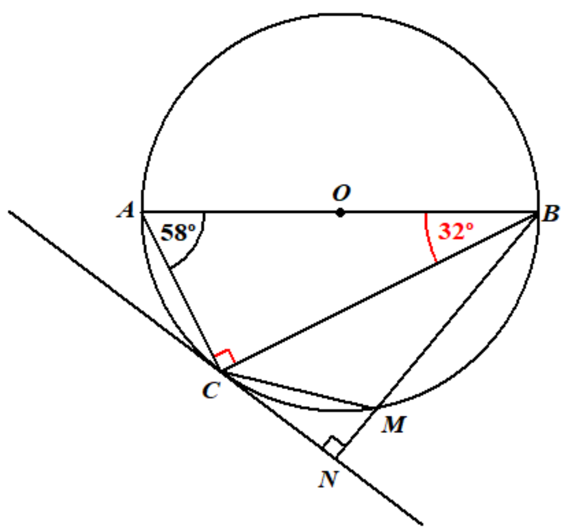
(b) The diagram below, **not drawn to scale**, shows a circle, with center *O*. The points *A*, *B*, *C* and *M* are on the circumference. The straight line *CN* is a tangent to the circle at the point *C* and is perpendicular to *BN*.



Determine, giving a reason for your answer,

(i) $A\hat{B}C$

SOLUTION:
Data: Diagram showing a circle, center *O*. *CN* is a tangent to the circle at the point *C* and is perpendicular to *BN*.
Required to find: $A\hat{B}C$
Solution:

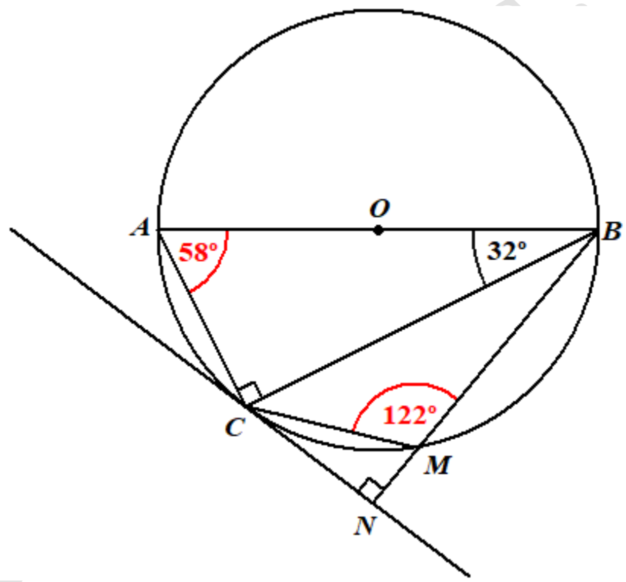


$A\hat{C}B = 90°$
(Angle in a semi-circle is a right angle.)

Let us consider ΔABC:
$A\hat{B}C = 180° - (90° + 58°)$
$= 32°$
(Sum of the angles in a triangle $= 180°$)

(ii) $C\hat{M}B$

SOLUTION:
Required to find: $C\hat{M}B$
Solution:



Consider the cyclic quadrilateral $ABMC$.
$C\hat{M}B = 180° - 58°$
$= 122°$
(The opposite angles in a cyclic quadrilateral are supplementary.)

(iii) $N\hat{C}M$

SOLUTION:
Required to find: $N\hat{C}M$
Solution:

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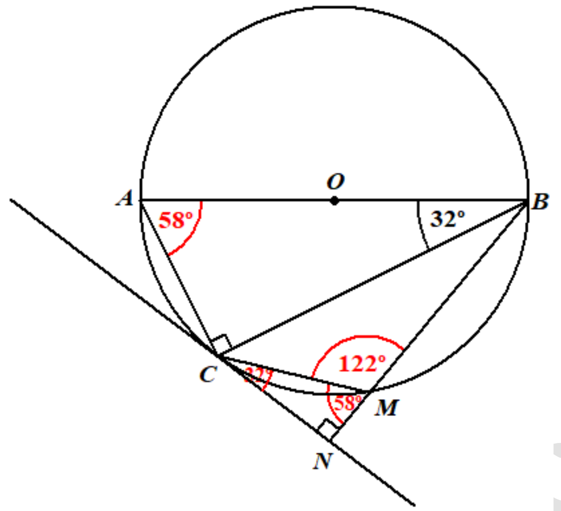


$$C\hat{M}N = 180° - 122°$$
$$= 58°$$
(Angles in a straight line total 180^0)

$$N\hat{C}M = 180° - (90° + 58°)$$
$$= 32°$$
(Sum of the angles in a triangle $= 180°$)

VECTORS AND MATRICES

10. (a) A transformation, *T*, is defined by the matrix

$$T = \begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}$$

The point $A\,(-2,\,3)$ is mapped on to the point $A'\,(a,\,b)$ under *T*.

(i) Find the value of *a* and of *b*.

SOLUTION:

Data: $T = \begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}$

$$A \xrightarrow{T} A'$$

$$\begin{pmatrix} -2 \\ 3 \end{pmatrix} \xrightarrow{\begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}} \begin{pmatrix} a \\ b \end{pmatrix}$$

Required to find: The value of *a* and of *b*

Solution:

$$\begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} (2\times -2)+(-1\times 3) \\ (2\times -2)+(0\times 3) \end{pmatrix}$$

$$= \begin{pmatrix} -4-3 \\ -4+0 \end{pmatrix}$$

$$= \begin{pmatrix} -7 \\ -4 \end{pmatrix}$$

Now, we equate corresponding entries
So, $a=-7$ and $b=-4$

(ii) Determine the transformation matrix that maps A' to A.

SOLUTION:
Required to find: The transformation matrix that maps A' to A
Solution:
If $A \xrightarrow{T} A'$
Then $A' \xrightarrow{T^{-1}} A$
Now we find T^{-1}:

$$T = \begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}$$

$$|T| = (2\times 0)-(-1\times 2)$$
$$= 2$$

$$T^{-1} = \frac{1}{2}\begin{pmatrix} 0 & -(-1) \\ -(2) & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{1}{2} \\ -1 & 1 \end{pmatrix}$$

(iii) Another transformation, *P*, is defined by the matrix $P = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$.

a) Find the single 2×2 matrix that represents the combined transformation of *T* followed by *P*.

SOLUTION:
Data: $P = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$

Required to find: The 2×2 matrix that represents the combined transformation of T followed by P

Solution:

T followed by P means that T is done first and followed by P. This is written as PT and the combined transformation is calculated as such.

$$PT = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$$

$$e_{11} = (0 \times 2) + (1 \times 2) = 2$$
$$e_{12} = (0 \times -1) + (1 \times 0) = 0$$
$$e_{21} = (1 \times 2) + (-2 \times 2) = -2$$
$$e_{22} = (1 \times -1) + (-2 \times 0) = -1$$

$\therefore$ The combined transformation T followed by P can be represented by the single 2×2 matrix $\begin{pmatrix} 2 & 0 \\ -2 & -1 \end{pmatrix}$.

b) Hence, find the image of the point $(1, 4)$ under this combined transformation.

SOLUTION:

Required to find: The image of $(1, 4)$ under the combined transformation

Solution:

$$\begin{pmatrix} 1 \\ 4 \end{pmatrix} \xrightarrow{\begin{pmatrix} 2 & 0 \\ -2 & -1 \end{pmatrix}} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} (2 \times 1) + (0 \times 4) \\ (-2 \times 1) + (-1 \times 4) \end{pmatrix}$$

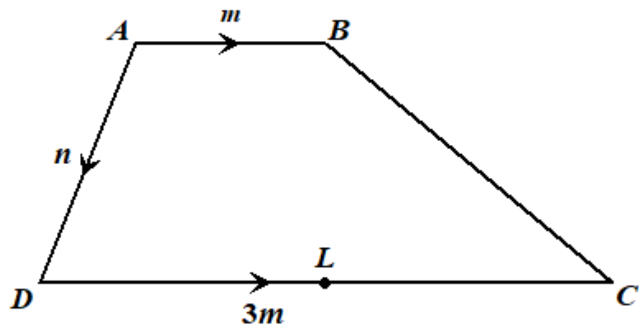
$$= \begin{pmatrix} 2 \\ -6 \end{pmatrix}$$

Equating corresponding entries.

$\therefore$ The image is $(2, -6)$.

(b) The diagram below, **not drawn to scale**, shows a quadrilateral *ABCD* in which

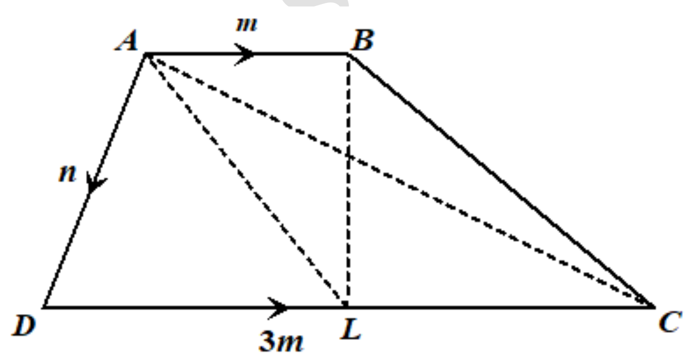
$\overrightarrow{AB} = m$, $\overrightarrow{DC} = 3m$ and $\overrightarrow{AD} = n$



(i) Complete the statement below on the geometric properties of the following vectors.

$\overrightarrow{AB}$ and $\overrightarrow{DC}$ are …………………………….. and $\left|\overrightarrow{AB}\right|$ is ………………………. times $\left|\overrightarrow{CD}\right|$.

SOLUTION:
Data:



Required to complete: The statement given.
Solution:

$\overrightarrow{AB} = m$
$\overrightarrow{DC} = 3m = 3 \times m$, where 3 is a scalar

Since $\overrightarrow{DC}$ is a scalar multiple of $\overrightarrow{AB}$ then $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are parallel.

$\overrightarrow{DC} = 3m$
$\overrightarrow{AB} = m$
$\therefore \left|\overrightarrow{AB}\right| = \frac{1}{3}\left|\overrightarrow{DC}\right|$

The completed statement is:

$\overrightarrow{AB}$ and $\overrightarrow{DC}$ are parallel and $\left|\overrightarrow{AB}\right|$ is $\frac{1}{3}$ times $\left|\overrightarrow{CD}\right|$.

(ii) Express $\overrightarrow{BC}$ in terms of m and n.

SOLUTION:
Required to express: $\overrightarrow{BC}$ in terms of m and n
Solution:

$$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} \qquad \text{(Triangle Law)}$$
$$= -(m) + \overrightarrow{AC}$$

Let us now find $\overrightarrow{AC}$:

$$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$$
$$= n + 3m$$

So, $\overrightarrow{BC} = -m + (n + 3m)$
$$= n + 2m$$

(iii) L is midpoint of $\overrightarrow{CD}$. Find $\overrightarrow{BL}$ in terms of m and n.

SOLUTION:
Data: L is midpoint of $\overrightarrow{CD}$
Required to find: $\overrightarrow{BL}$ in terms of m and n.
Solution:

$$\overrightarrow{BL} = \overrightarrow{BA} + \overrightarrow{AL}$$
$$= -(m) + \overrightarrow{AL}$$

Let us find $\overrightarrow{AL}$.

$$\overrightarrow{AL} = \overrightarrow{AD} + \overrightarrow{DL}$$
$$= \overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC}$$
$$= n + \frac{1}{2}(3m)$$

So, $\overrightarrow{BL} = -(m) + n + \frac{1}{2}(3m)$
$$= \frac{1}{2}m + n$$

We could also have used $\overrightarrow{BL} = \overrightarrow{BC} + \overrightarrow{CL}$ for the same result.

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