FAS-PASS Maths

JUNE 2017 CSEC MATHEMATICS PAPER 2

SECTION 1

1. (a) Using a calculator, or otherwise, calculate the EXACT value of

(i) $\left(4\frac{1}{3}-1\frac{2}{5}\right)\div\frac{4}{15}$

SOLUTION:

Required to calculate: $\left(4\frac{1}{3}-1\frac{2}{5}\right)\div\frac{4}{15}$ in exact form

Calculation:

$$4\frac{1}{3}-1\frac{2}{5}=\frac{13}{3}-\frac{7}{5}$$
$$=\frac{5(13)-3(7)}{15}$$
$$=\frac{65-21}{15}$$
$$=\frac{44}{15}$$

So, now we have

$$\left(4\frac{1}{3}-1\frac{2}{5}\right)\div\frac{4}{15}=\frac{44}{15}\div\frac{4}{15}$$
$$=\frac{^{11}\cancel{44}}{\cancel{15}}\times\frac{\cancel{15}}{\cancel{4}_1}$$
$$=11 \text{ (in exact form)}$$

(ii) $\frac{(3.1-1.15)^2}{0.005}$

SOLUTION:

Required to calculate: $\frac{(3.1-1.15)^2}{0.005}$ in exact form

Calculation:

$$\frac{(3.1-1.15)^2}{0.005}=\frac{(1.95)^2}{0.005} \quad \text{(By calculator)}$$
$$=\frac{3.8025}{0.005}$$
$$=760.5 \text{ (in exact form)}$$

(b) A store is promoting a new mobile phone under two plans. Plan A and Plan B. The plans are advertised as shown in the table below.

	Plan A	**Plan B**
Deposit	\$400	\$600
Monthly installment	\$65	\$80
Number of months to repay	12	6
Tax on ALL payments	0%	5%

(i) Calculate the TOTAL cost of a phone under Plan A.

SOLUTION:
Data: Table showing two mobile phone plans as advertised by a store.

Required to calculate: The total cost of a phone under Plan A

Calculation:
Total cost under Plan A

$$= \text{The deposit} + (\text{Monthly installments} \times \text{No. of months to repay}) + \text{Tax}$$
$$= \$400 + (\$65 \times 12) + \$0$$
$$= \$(400 + 780)$$
$$= \$1180$$

(ii) Determine which of the two plans, A or B, is the better deal. Justify your answer.

SOLUTION:
Required to find: The better deal of the two plans
Solution:
Total cost under Plan B

$$= \text{The deposit} + (\text{Monthly installments} \times \text{No. of months to repay}) + \text{Tax}$$
$$= \$600 + (\$80 \times 6) + \frac{5}{100}(\$600 + (\$80 \times 6))$$
$$= \$(600 + 480) + \frac{5}{100}(\$600 + \$480)$$
$$= 1.05 \times \$(600 + 480)$$
$$= \$1134$$

If the better deal is supposed to mean the plan that has a lesser cost, then Plan B is the better deal as there is a savings of $\$1180 - \$1134 = \$46$.

(c) John's monthly electricity bill is based on the number of kWh of electricity that he consumes for that month. He is charged $5.10 per kWh of electricity consumed. For the month of March 2016, two meter readings are displayed in the table below.

	Meter Readings (kWh)
Beginning 01 March	0 3 0 1 1
Ending 31 March	0 3 3 0 7

(i) Calculate the TOTAL amount that John pays for electricity consumption for the month of March 2016.

SOLUTION:
Data: Table showing John's electricity meter readings, in kWh, at the beginning and end of March 2016. John is charged $5.10 per kWh for electricity used.

Required to calculate: The total amount John pays for electricity for March 2016

Calculation:
Number of kWh used
$= \text{End reading on 31 March} - \text{Beginning reading on 01 March}$

$$= 03307 - 03011 = 296$$

The cost at $5.10 per kWh $= 296 \times \$5.10 = \$1509.60$

$\therefore$ If John pays the full amount that is required, then he would pay $1 509.60.

(ii) For the next month, April 2016, John pays $2 351.10 for electricity consumption. Determine his meter reading at the end of April 2016.

SOLUTION:
Data: John pays $2 351.10 for electricity in April 2016.

Required to calculate: John's meter reading at the end of April 2016
Calculation:
For the month of April John pays $2 351.10.

At $5.10 per kWh, the number of kWh used in April $= \dfrac{\$2351.10}{\$5.10}$

$= 461\text{kWh}$

Hence, the meter reading at the end of April should be the reading at the end of March + 461

$$
\begin{array}{r}
= 0\ 3\ 3\ 0\ 7\ + \\
4\ 6\ 1 \\
\hline
0\ 3\ 7\ 6\ 8 \\
\hline
\end{array}
$$

2. (a) Factorise the following expressions completely.

(i) $6y^2 - 18xy$

SOLUTION:
Required to factorise: $6y^2 - 18xy$
Solution:

$$
\begin{aligned}
6y^2 - 18xy &= 6y \times y - 6y \times 3x \\
&= 6y(y - 3x)
\end{aligned}
$$

(ii) $4m^2 - 1$

SOLUTION:
Required to factorise: $4m^2 - 1$
Solution:

$4m^2 - 1 = (2m)^2 - (1)^2$

This is now in the form of the difference of two squares.

So, $4m^2 - 1 = (2m - 1)(2m + 1)$

(iii) $2t^2 - 3t - 2$

SOLUTION:
Required to factorise: $2t^2 - 3t - 2$
Solution:

$2t^2 - 3t - 2 = (2t + 1)(t - 2)$

$\begin{bmatrix} 2t^2 - 4t + t - 2 \\ 2t^2 - 3t - 2 \end{bmatrix}$ We may even expand to confirm our result

(b) Write as a single fraction and simplify

$$\frac{5p+2}{3} - \frac{3p-1}{4}$$

SOLUTION:

Required to write: $\frac{5p+2}{3}-\frac{3p-1}{4}$ as a simplified single fraction.

Solution:

$$\frac{5p+2}{3}-\frac{3p-1}{4}$$

$$\frac{4(5p+2)-3(3p-1)}{12}=\frac{20p+8-9p+3}{12}$$

$$=\frac{20p-9p+8+3}{12}$$

$$=\frac{11p+11}{12}$$

$$=\frac{11(p+1)}{12}$$

(c) A formula is given as $d=\sqrt{\frac{4h}{5}}$.

(i) Determine the value of d when $h=29$. Give your answer correct to 3 significant figures.

SOLUTION:

Data: $d=\sqrt{\frac{4h}{5}}$ and $h=29$

Required to calculate: d

Calculation:

When $h=29$

$$d=\sqrt{\frac{4(29)}{5}}$$

$$=\sqrt{\frac{116}{5}}$$

$$=\sqrt{23.2} \qquad \text{(using the calculator)}$$

$$=4.81\underline{\underline{6}}$$

$$=4.82 \text{ (correct to 3 significant figures)}$$

(ii) Make h the subject of the formula.

SOLUTION:

Required to make: h the subject of the formula

Solution:

$$d = \sqrt{\frac{4h}{5}}$$

Squaring both sides to remove the hindrance of the root sign

$$d^2 = \frac{4h}{5}$$

$$\therefore 5 \times d^2 = 4h$$

$$4h = 5d^2$$

$$h = \frac{5d^2}{4}$$

3. (a) The Universal set, U, is defined as follows:

$$U = \{x : x \in N,\ 2 < x < 12\}$$

The sets M and R are subsets of U such that

$$M = \{\text{odd numbers}\}$$
$$R = \{\text{square numbers}\}$$

(i) List the members of the subset M.

SOLUTION:
Data: $U = \{x : x \in N,\ 2 < x < 12\}$ and M and R are subsets of U where $M = \{\text{odd numbers}\}$ and $R = \{\text{square numbers}\}$.
Required to list: The members of the subset M.
Solution:

$$U = \{3, 4, 5, 6, 7, 8, 9, 10, 11\}$$
$$\therefore M = \{3, 5, 7, 9, 11\}$$

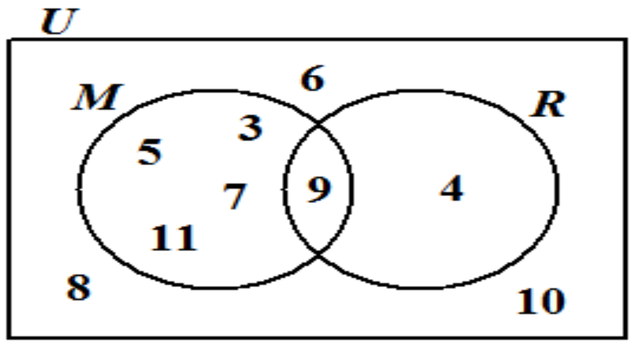
(ii) List the members of the subset R.

SOLUTION:
Required to list: The members of the subset R
Solution:

$$R = \{4, 9\}$$

(iii) Draw a Venn diagram that represents the relationship among the defined subsets of U.

SOLUTION:
Required to draw: A Venn diagram to illustrate the information given about the sets U, M and R
Solution:

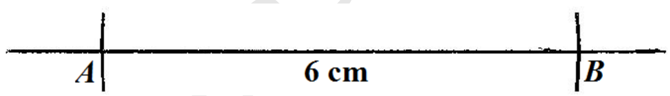


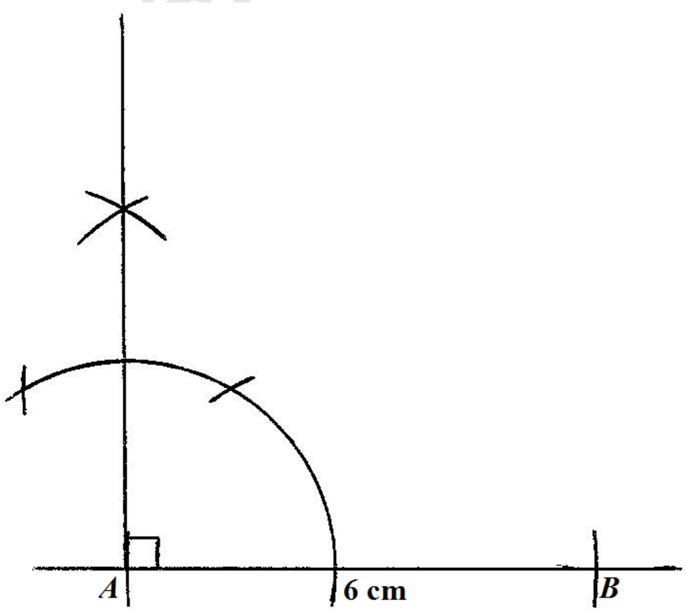
(b) Using **a ruler, a pencil and a pair of compasses**,

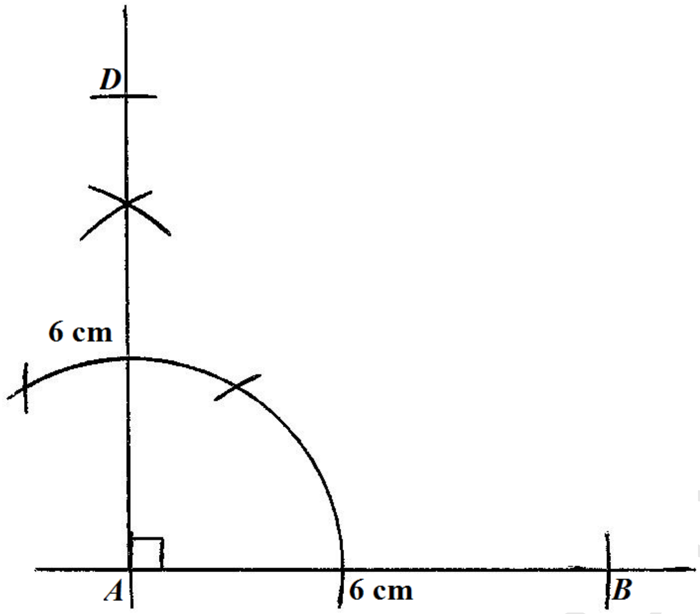
(i) construct accurately, the square $ABCD$, with sides 6 cm.

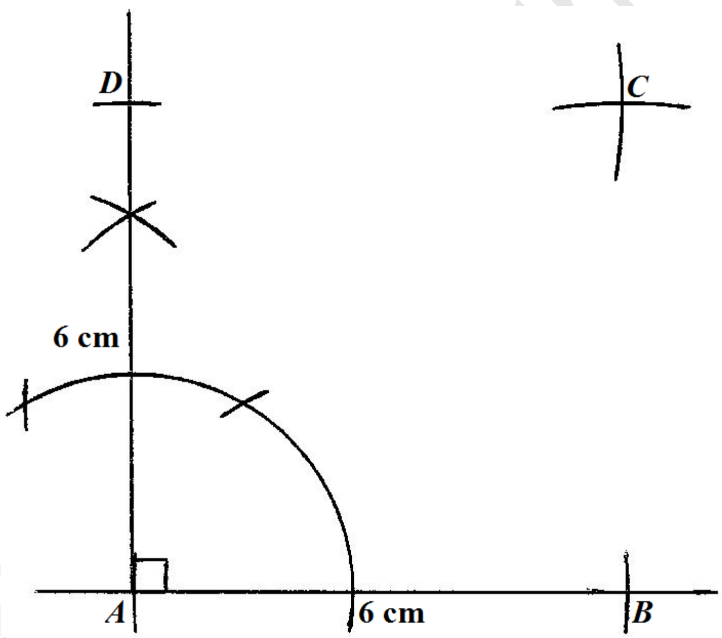
SOLUTION:
Required to construct: A square $ABCD$ with sides 6 cm.

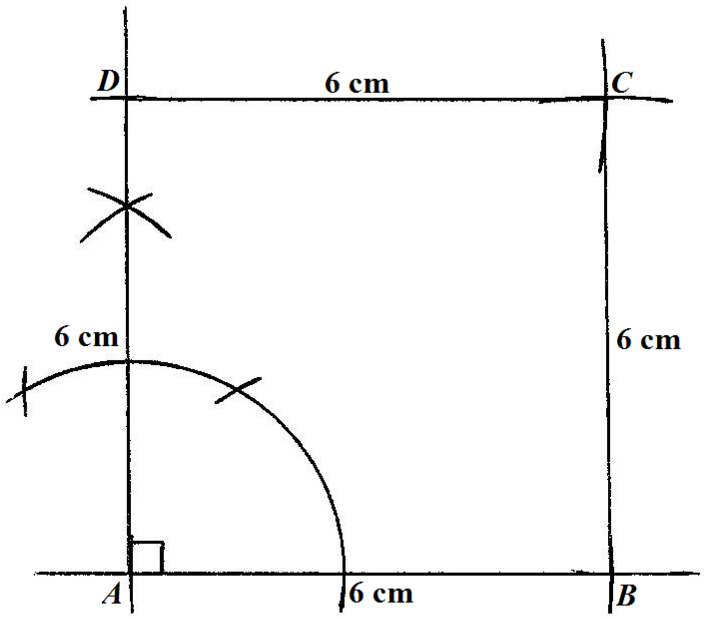
Solution:






D
6 cm
A
6 cm
B

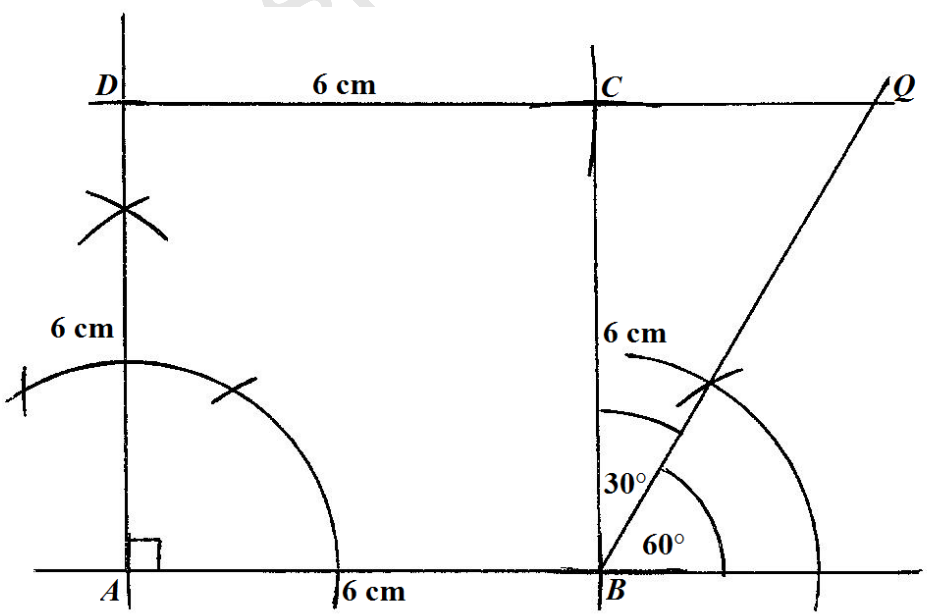

D
C
6 cm
A
6 cm
B



(ii) construct, as an extension of your diagram in (b) (i), the trapezium *DABQ* so that $\angle ABQ = 120°$.

[Note: Credit will be given for clearly drawn construction lines.]

SOLUTION:
Required To Construct: An extension of the diagram previously down trapezium *DABQ* with the $\angle ABQ = 120°$.
Solution:



(iii) Hence, measure and state the length of BQ.

SOLUTION:
Required to state: The length of BQ, by measurement
Solution:
$BQ = 6.9$ cm (by measurement with a ruler)

4. (a) The function f is defined as $f(x) = \frac{1}{3}x - 2$.

(i) Find the value of $f(3) + f(-3)$.

SOLUTION:
Data: $f(x) = \frac{1}{3}x - 2$
Required to find: $f(3) + f(-3)$
Solution:

$$f(3) = \frac{1}{3}(3) - 2 = 1 - 2 = -1$$

$$f(-3) = \frac{1}{3}(-3) - 2 = -1 - 2 = -3$$

So,

$$f(3) + f(-3) = -1 + (-3) = -1 - 3 = -4$$

(ii) Calculate the value of x for which $f(x) = 5$.

SOLUTION:
Required to calculate: x when $f(x) = 5$
Calculation:

$$f(x) = 5$$

$$\therefore \frac{1}{3}x - 2 = 5$$

$$\frac{1}{3}x = 5 + 2 = 7$$

$$x = 7 \times 3 \qquad (\times 3)$$

$$x = 21$$

(iii) Determine the inverse function, $f^{-1}(x)$

SOLUTION:
Required to find: $f^{-1}(x)$
Solution:

$f(x)=\frac{1}{3}x-2$

Let $y=\frac{1}{3}x-2$

Make x the subject of the formula:

$y+2=\frac{1}{3}x$

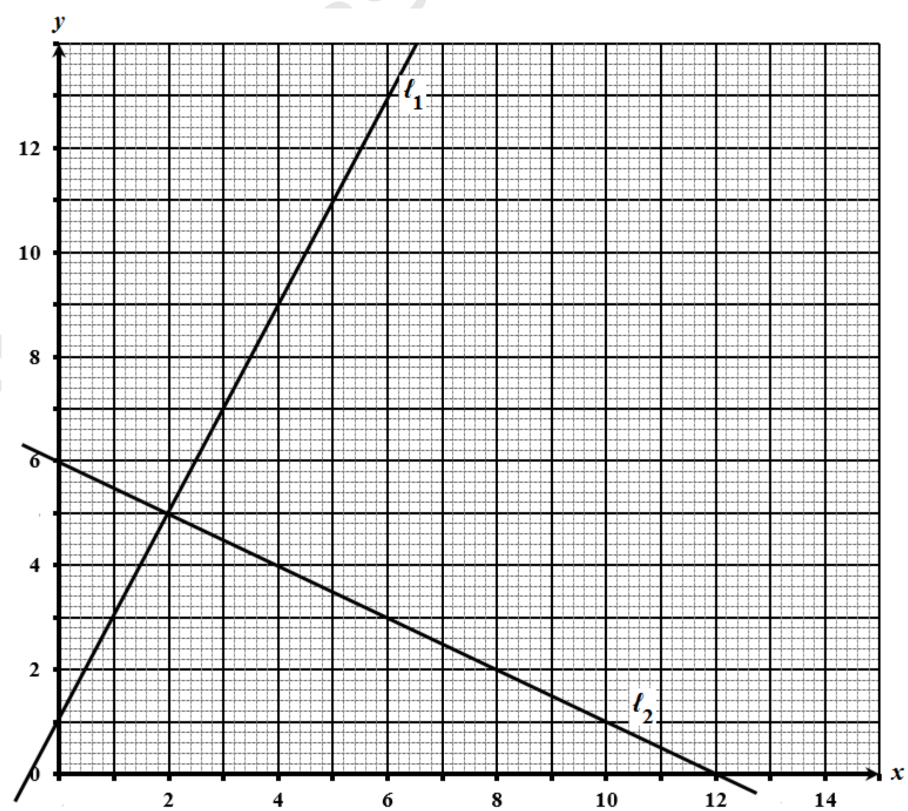
$3(y+2)=x$

$x=3(y+2)$

Replace y by x to obtain:

$f^{-1}(x)=3(x+2)$

(b) The graph below shows two straight lines, ℓ_1 and ℓ_2. Line ℓ_1 intercepts the y – axis at $(0, 1)$. Line ℓ_2 intercepts the x and y axes at $(12, 0)$ and $(0, 6)$ respectively.



(i) Calculate the gradient of the lines ℓ_1 and ℓ_2.

SOLUTION:
Data: Diagram showing two lines ℓ_1 and ℓ_2. The y – intercept of ℓ_1 is $(0, 1)$. The y – intercept of ℓ_2 is $(0, 6)$ and the x – intercept is $(12, 0)$.
Required to calculate: The gradient of ℓ_1 and ℓ_2
Calculation:
Two points on ℓ_1 are $(0, 1)$ and $(2, 5)$which is the point of intersection of ℓ_1 and ℓ_2. Each was obtained by a read-off.

$$\therefore \text{Gradient of } \ell_1 = \frac{5-1}{2-0} = \frac{4}{2} = 2$$

Two points on ℓ_2 are $(12, 0)$ and $(0, 6)$.

$$\therefore \text{Gradient of } \ell_2 = \frac{6-0}{0-12} = -\frac{1}{2}$$

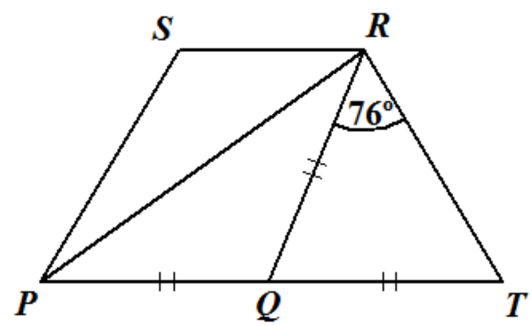
(ii) Determine the equation of the line ℓ_1

SOLUTION:
Required to find: The equation of the line ℓ_1
Solution:
The general equation of a straight line is of the form $y = mx + c$, where m is the gradient and c is the intercept on the y – axis. In this case, we have already found $m = 2$ and noted that $c = 1$.
$\therefore$ The equation of the line ℓ_1 is $y = 2x + 1$.

(iii) What is the relationship between ℓ_1 and ℓ_2? Give a reason for your answer.

SOLUTION:
Required to find: The relationship between ℓ_1 and ℓ_2.
Solution:
Gradient of $\ell_1 = 2$
Gradient of $\ell_2 = -\frac{1}{2}$
Gradient of ℓ_1 $\times$Gradient of $\ell_2 = 2 \times -\frac{1}{2} = -1$
Hence, ℓ_1 is perpendicular to ℓ_2 since the product of the gradients of perpendicular lines is -1.

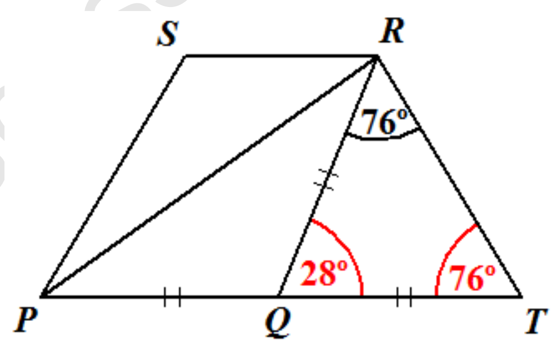
5. (a) *PTRS*, **not drawn to scale**, is a quadrilateral. *Q* is a point on *PT* such that $QT = QR = QP$. Angle $QRT = 76°$.



Determine, given a reason for each step of your answer, the measure of

(i) angle *RQT*

SOLUTION:
Data: Diagram showing a quadrilateral PTRS such that $QT = QR = QP$ and angle $QRT = 76°$.
Required to find:
$R\hat{Q}T$
Solution:



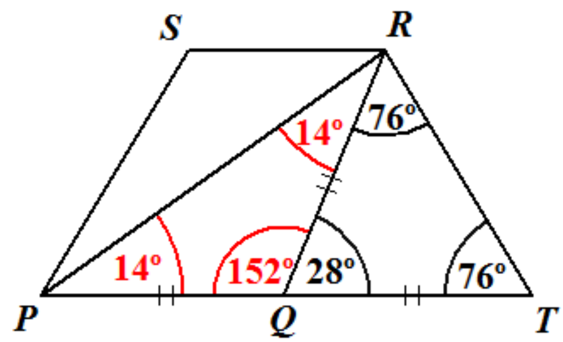
Angle $RTQ = 76°$
(The base angles of the isosceles triangle *RQT* are equal)

$\therefore$ Angle $RQT = 180° - (76° + 76°)$
$= 28°$ (The sum of interior angles in a triangle $= 180°$)

(ii) angle *PRT*

SOLUTION:

Required to find: angle *PRT*
Solution:



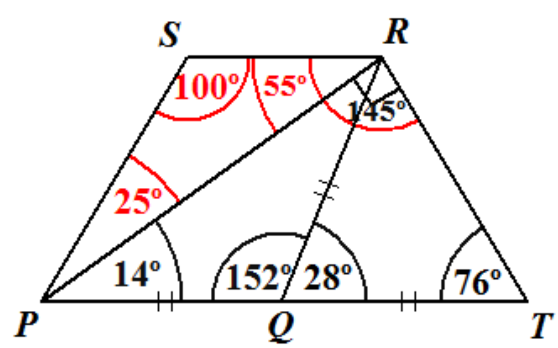
Angle $QPR =$ Angle QRP
(The base angles of the isosceles triangle RQP are equal)

Angle $QPR +$ Angle $QRP = 28°$
(The exterior angle of a triangle is equal to the sum of the interior opposite angles)

$\therefore$ Angle $QRP = \dfrac{28°}{2} = 14°$
Hence, angle $PRT = 76° + 14° = 90°$

(iii) angle SPT, given that angle $SRT = 145°$ and angle $PSR = 100°$.

SOLUTION:
Data: Angle $SRT = 145°$ and angle $PSR = 100°$
Required to find: angle SPT
Solution:

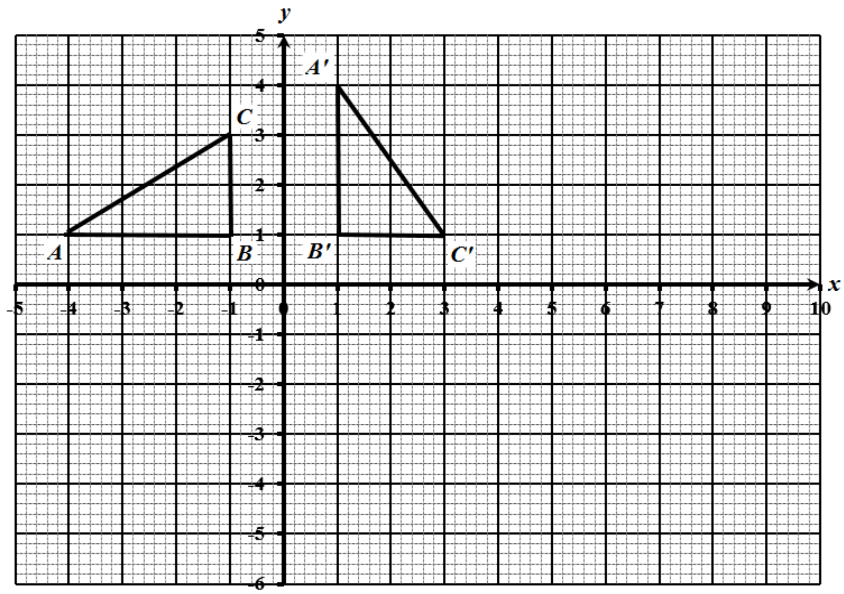


Angle $SRP = 145° - 90°$
$= 55°$

Angle $SPR = 180° - (100° + 55°)$
$= 25°$ (Sum of the interior angles in a triangle $= 180°$)

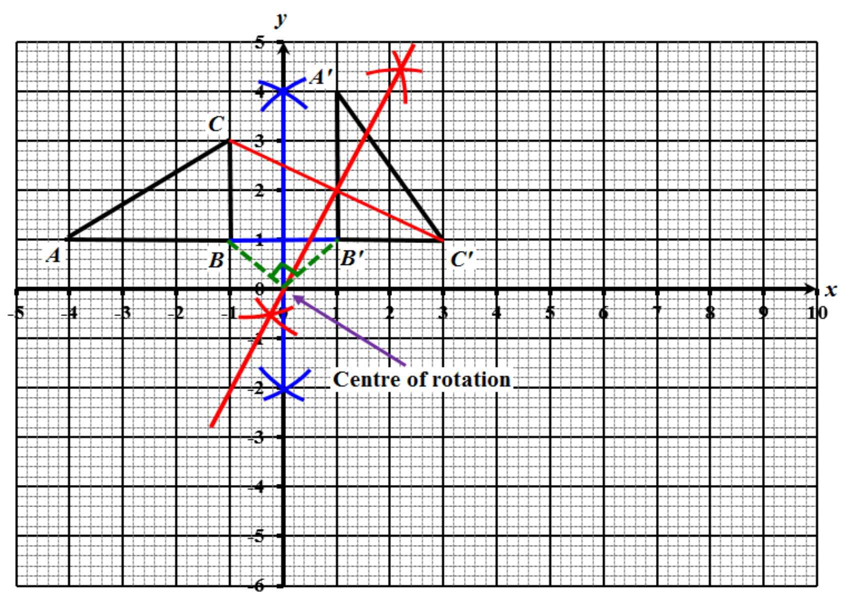
Hence, angle $SPT = 25° + 14°$
$= 39°$

(b) The diagram below shows a triangle ABC and its image, $A'B'C'$, under a single transformation.



(i) Describe completely the transformation that maps ΔABC to $\Delta A'B'C'$.

SOLUTION:
Data: Diagram showing triangle ABC and its image $A'B'C'$ after a single transformation.
Required To Describe: The transformation that maps ΔABC unto $\Delta A'B'C'$.
Solution:



$A'B'C'$ is congruent to ABC and is re-oriented. Hence, the transformation is a rotation. The perpendicular bisectors of BB' and AA' meet at O. Hence, O is the center of rotation. $B\hat{O}B' = 90°$ and the 'sweep' from OB to OB' is clockwise. The transformation is therefore a 90° clockwise rotation about O.

(ii) The translation vector $T = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ maps $\Delta A'B'C'$ to $\Delta A''B''C''$. On the diagram above, drawn the $\Delta A''B''C''$.

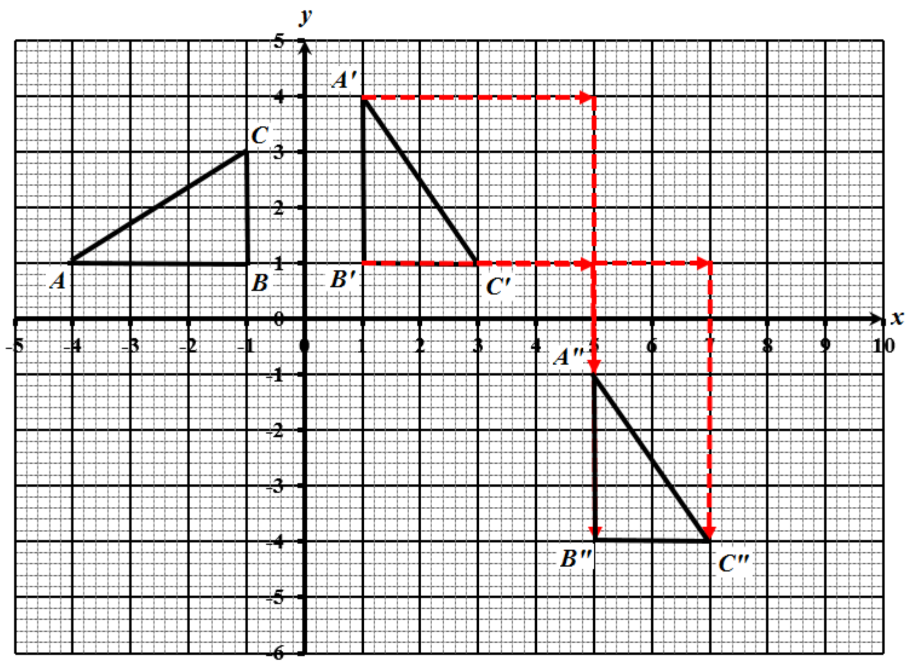
SOLUTION:

Data: $\Delta A'B'C'$ is mapped to $\Delta A''B''C''$ by $T = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$.

Required To Draw: $\Delta A''B''C''$

Solution:

See graph, $A' = (1,\ 4)$, $B' = (1,\ 1)$ and $C' = (3,\ 1)$



$$\Delta A'B'C' \xrightarrow{T = \begin{pmatrix} 4 \\ -5 \end{pmatrix}} \Delta A''B''C''$$

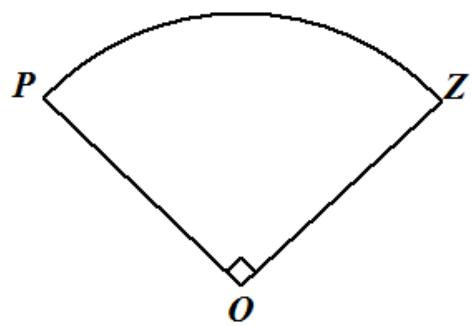
$$A'\ (1,\ 4) \xrightarrow{\begin{pmatrix} 4 \\ -5 \end{pmatrix}} A''\ (5,\ -1)$$

$$B'\ (1,\ 1) \xrightarrow{\begin{pmatrix} 4 \\ -5 \end{pmatrix}} B''\ (5,\ -4)$$

$$C'\ (3,\ 1) \xrightarrow{\begin{pmatrix} 4 \\ -5 \end{pmatrix}} C''\ (7,\ -4)$$

6. (a) **In this problem take π to be $\frac{22}{7}$.**

The diagram below, **not drawn to scale**, shows a field in the shape of a sector of a circle with center *O* and diameter 28 m. Angle *POZ* is 90°.



Calculate:

(i) the area of the field

SOLUTION:
Data: Diagram showing a field *POZ* in the shape of a sector of a circle with center *O* and diameter 20 m. Angle *POZ* is 90°.
Required to calculate: The area of the field
Calculation:

$$\text{Radius} = \frac{\text{Diameter}}{2} = \frac{28\ \text{m}}{2} = 14\ \text{m}$$

$$\text{Area of the field} = \frac{90^\circ}{360^\circ} \times \pi (14)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$$

$$= 154\ \text{m}^2$$

(ii) the perimeter of the field

SOLUTION:
Required to calculate: The perimeter of the field
Calculation:
The perimeter of the field

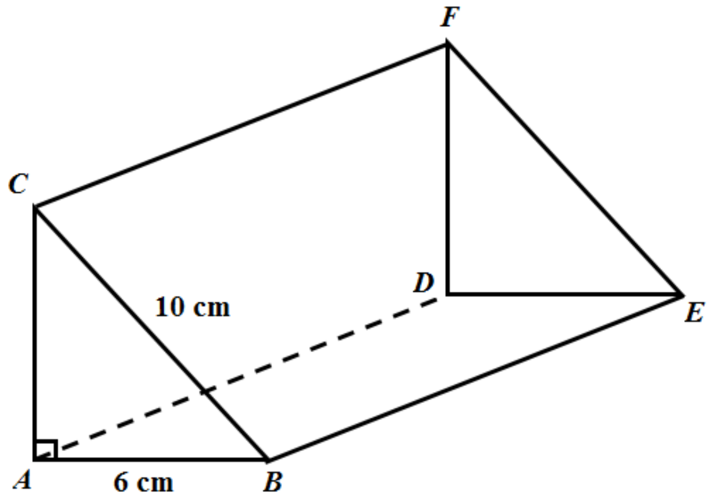
$$= \text{Length of radius } OP + \text{Length of arc } PZ + \text{Length of radius } ZO$$

$$= 14 + \frac{90^\circ}{360^\circ}(2 \times \pi \times 14) + 14$$

$$= 14 + \left(\frac{1}{4} \times 2 \times \frac{22}{7} \times 14\right) + 14$$

$$= 50\ \text{m}$$

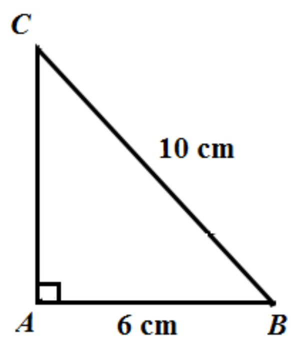
(b) The diagram below, **not drawn to scale**, shows a triangular prism *ABCDEF*. The cross-section is the right-angled triangle, *ABC*, where $AB = 6\text{ cm}$ and $BC = 10\text{ cm}$.



Calculate

(i) the area of the triangle *ABC*

SOLUTION:
Data: Diagram showing the triangular prism *ABCDEF* which cross-section is a right-angled triangle. $AB = 6\text{ cm}$ and $BC = 10\text{ cm}$.
Required to calculate: The area of triangle *ABC*
Calculation:



$BC^2 = AC^2 + AB^2$ (Pythagoras' Theorem)

$(10)^2 = AC^2 + (6)^2$

$AC^2 = 100 - 36$

$AC^2 = 64$

$AC = \sqrt{64}$

$= 8\text{ cm}$ (taking the positive value of the root)

$$\text{Area of } \Delta ABC = \frac{6 \times 8}{2}\text{ cm}^2$$

$$= \frac{48}{2}\text{ cm}^2$$

$$= 24\text{ cm}^2$$

(ii) the length of the prism, if the volume is 540 cm^3

SOLUTION:
Data: The volume of the prism is 540 cm^3.
Required to calculate: The length of the prism.
Calculation:

$$\text{Volume of prism} = \text{Area of cross-section} \times \text{Length of prism}$$
$$\therefore 540 = 24 \times \text{Length of prism}$$
$$\text{Length of prism} = \frac{540}{24}$$
$$= 22.5 \text{ cm}$$

(iii) the surface area of the prism.

SOLUTION:
Required to calculate: The surface area of the prism.
Calculation:
Surface area of the prism $=$ Sum of the area of all five sides

$$\text{Area of } \Delta ABC = \frac{6 \times 8}{2} = 24 \text{ cm}^2$$
$$\text{Area of } \Delta DEF = \frac{6 \times 8}{2} = 24 \text{ cm}^2$$

Area of rectangle $ABED = 6 \times 22.5 = 135 \text{ cm}^2$
Area of rectangle $ADFE = 8 \times 22.5 = 180 \text{ cm}^2$
Area of rectangle $BEFC = 10 \times 22.5 = 225 \text{ cm}^2$
Hence, the surface area of the prism $= 24 + 24 + 135 + 180 + 225$
$= 588 \text{ cm}^2$

7. The table below shows the speeds, to the nearest kmh^{-1}, of 90 vehicles that pass a checkpoint.

Speed (in kmh^{-1})	**Frequency**	**Cumulative Frequency**
0 – 19	5	5
20 – 39	11	16
40 – 59	26	
60 – 79	37	
80 – 99	9	
100 – 119	2	

(a) For the class interval 20 – 39, as written in the table above, complete the following sentences.

(i) The upper class limit is ………………………………………………………….

(ii) The class width is ……………………………………………………………..

(iii) Sixteen vehicles passed a checkpoint at no more than ……………....kmh^{-1}

SOLUTION:
Data: Cumulative frequency table showing the speeds, in kmh^{-1}, of 90 vehicles passing a certain checkpoint.

Required to complete: The sentences given for the class interval 20 – 39
Solution:
(See the modified table done)
(i) The upper class limit is 39.

(ii) The class width is $39.5-19.5=20$.

(iii) Sixteen vehicles passed a checkpoint at no more than 39.5 kmh^{-1}.

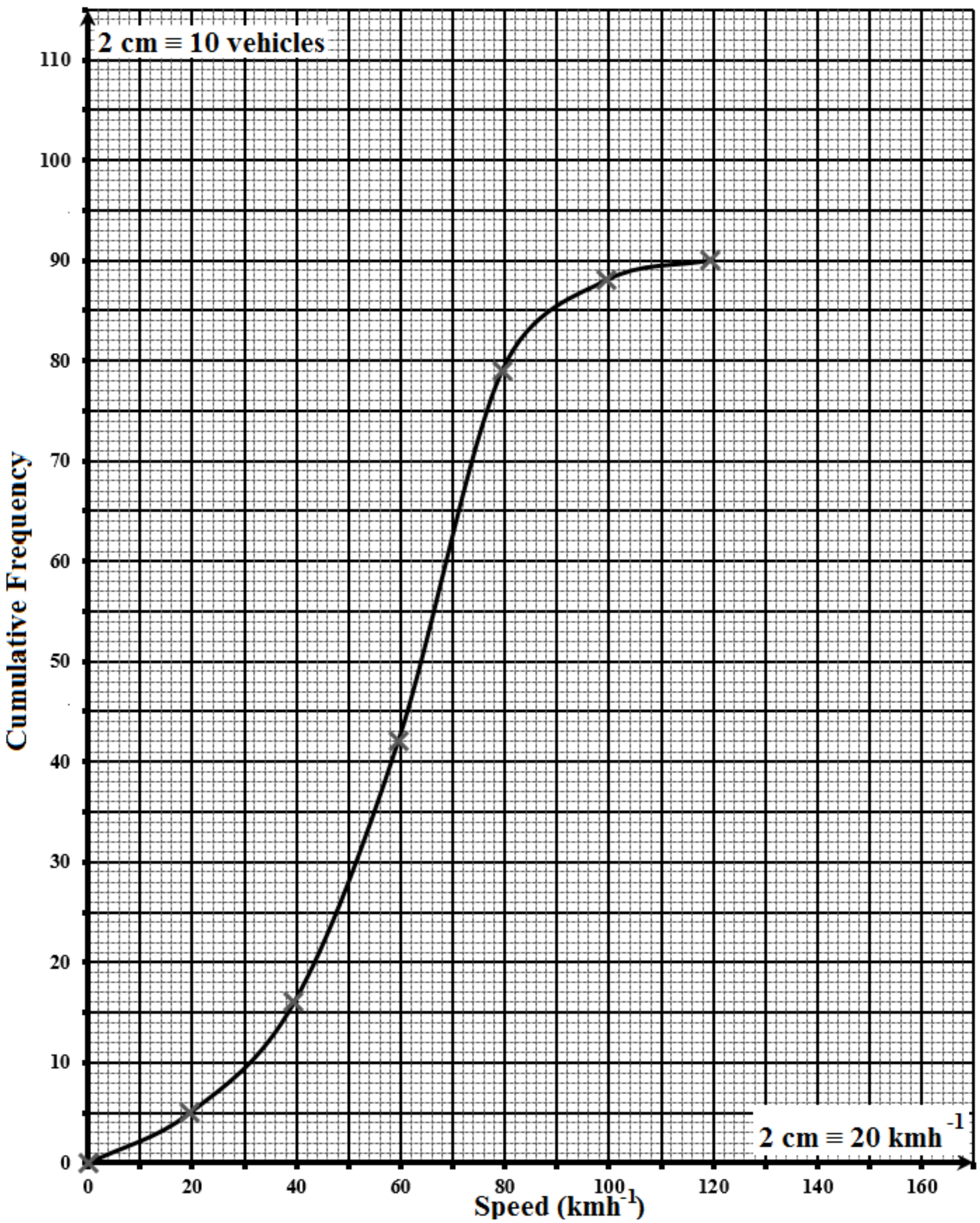
(b) Complete the table shown above by inserting the missing values for the cumulative frequency column.

SOLUTION:
Required to complete: The cumulative frequency give.
Solution:
L.C.L-lower class limit, L.C.B-lower class boundary
U.C.L-upper class limit, U.C.B-upper class boundary

Speed, *x*, (in kmh^{-1}) L.C.L U.C.L	**Class Boundaries L.C.B U.C.B.**	**Frequency**	**Cumulative Frequency**	**Points to Plot (U.C.B., CF)**
				$(0, 0)$
0 – 19	$0 \le x < 19.5$	5	5	$(19.5, 5)$
20 – 39	$19.5 \le x < 39.5$	11	16	$(39.5, 16)$
40 – 59	$39.5 \le x < 59.5$	26	$26+16=42$	$(59.5, 42)$
60 – 79	$59.5 \le x < 79.5$	37	$37+42=79$	$(79.5, 79)$
80 – 99	$79.5 \le x < 99.5$	9	$9+79=88$	$(99.5, 88)$
100 – 119	$99.5 \le x < 119.5$	2	$2+88=90$	$(119.5, 90)$

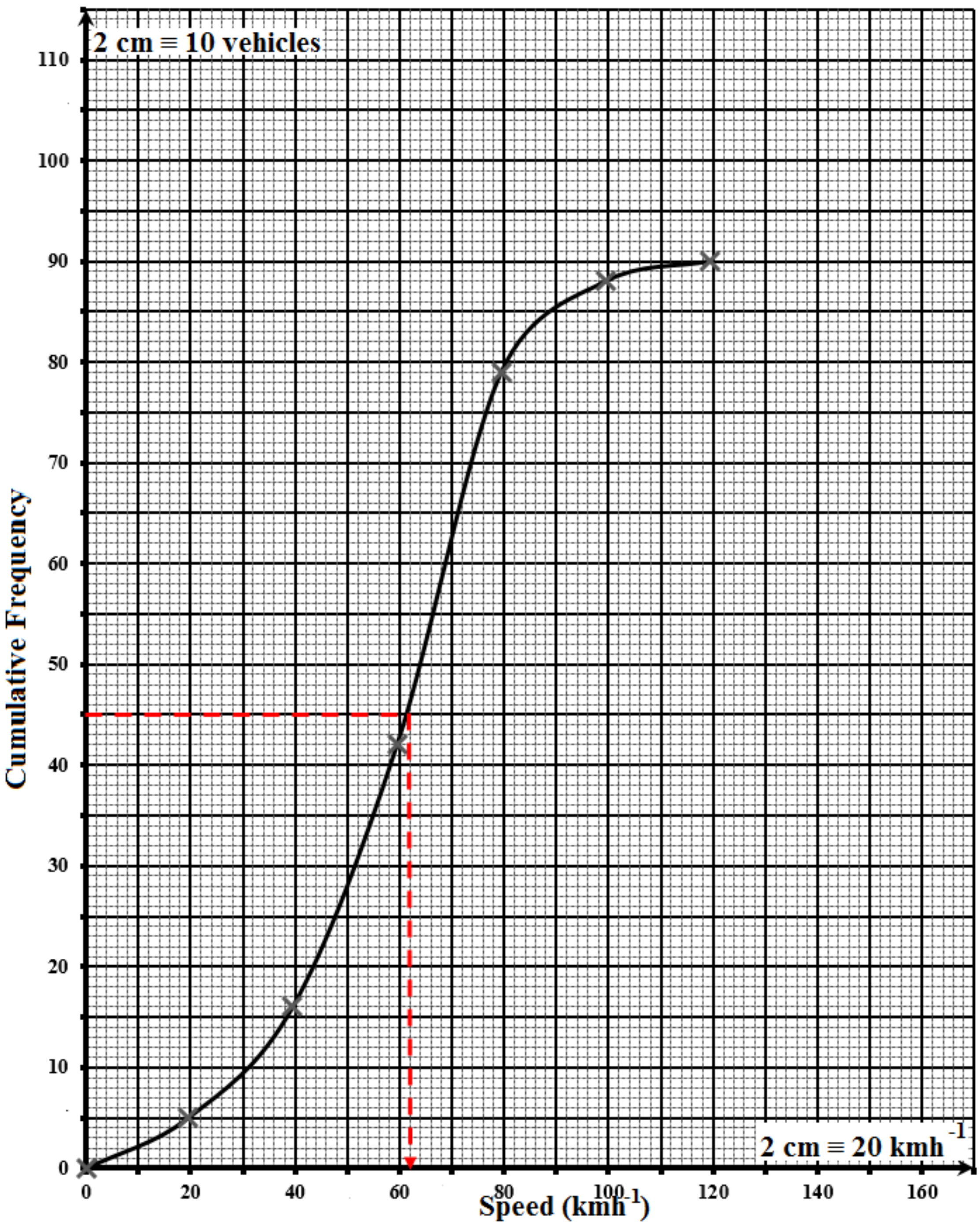
(c) On the grid provided, using a scale of 2 cm to represent 20 kmh^{-1} on the x – axis and 2 cm to represent 10 vehicles on the y – axis, draw the cumulative frequency curve to represent the information in the table.

SOLUTION:
Required to draw: The cumulative frequency curve to illustrate the information given
Solution:



(d) **(i)** On your graph, draw reference lines to estimate the speed at which no more than 50% of the vehicles drove as they passed the check point.

SOLUTION:
Required to show: The speed at which no more than 50% of the vehicles drove past the checkpoint using reference lines
Solution:



(ii) What is the estimated speed?

SOLUTION:
Required to state: The estimated speed at which no more than 50% of the vehicles passed the checkpoint
Solution:
The estimated speed is 62 kmh^{-1}

8. The first four figures in a sequence are shown below. Figure 1 is a single black dot, while each of the others consist of black dots arranged in an equilateral manner.

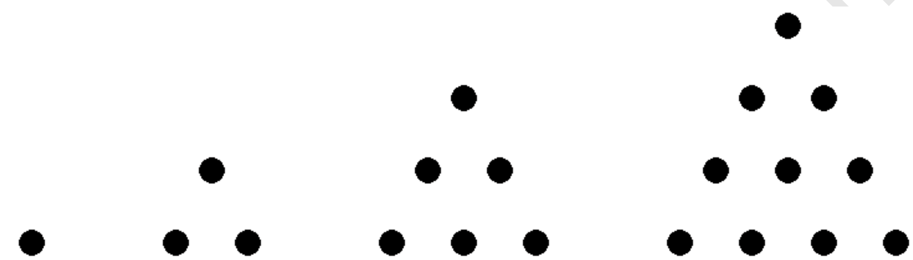
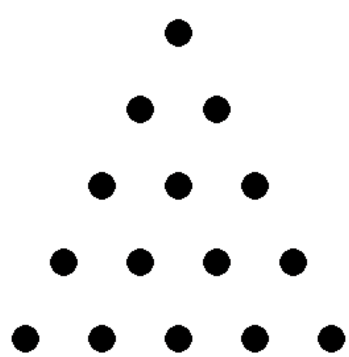


Figure 1 **Figure 2** **Figure 3** **Figure 4**

(a) Draw Figure 5 of the sequence in the space below.

SOLUTION:
Data: Figures showing a sequence of black dots arranged in an equilateral manner.
Required to draw: The fifth figure in the sequence

Solution:



(b) How many dots would be in Figure 6?

SOLUTION:
Required to state: The number of dots in Figure 6
Solution:

Figure	Number of Dots
1	1
2	3
3	6
4	10
5	15

By observation, the number of dots in the figure appear to be,
½ × (the number of the figure) × (1 added to the number of the figure)

$\therefore$ Figure 6 should have $\frac{1}{2}6(6+1)=21$ dots

The table below refers to the figures and the number of dots in each figure. Study the patterns below.

Figure, *n*	**Number of Dots, *d*, in terms *n***	**Number of Dots Used, *d***
1	$\frac{1}{2}\times1\times(1+1)$	1
2	$\frac{1}{2}\times2\times(2+1)$	3
3	$\frac{1}{2}\times3\times(3+1)$	6
11		
n		

(c) Complete the row which corresponds to Figure 11 in the table above.

SOLUTION:
Data: Table showing the pattern of the number of dots used in the sequence of figures.
Required to complete: The row in the table that corresponds to Figure 11

Solution:

Figure, n	Number of Dots, d, in terms n	Number of Dots Used, d
1	$\frac{1}{2}\times 1\times(1+1)$	1
2	$\frac{1}{2}\times 2\times(2+1)$	3
3	$\frac{1}{2}\times 3\times(3+1)$	6
11	$\frac{1}{2}\times 11\times(11+1)$	66
n		

(d) Determine which figure in the sequence has 210 dots.

SOLUTION:
Required to find: The figure that has 210 dots in the sequence
Solution:

$$\frac{1}{2}n(n+1)=210$$

$$n(n+1)=420$$

$$=20\times(20+1)$$

$$\therefore n=20$$

$\therefore$ Figure 20 has 210 dots.

(e) Write a simplified algebraic expression for the number of dots, d, in the Figure n.

SOLUTION:
Required to state: An algebraic expression for the number of dots, d, in Figure n
Solution:

$$\frac{1}{2}\times n\times(n+1)=\frac{1}{2}n(n+1)$$

The completed table looks like:

Figure, *n*	Number of Dots, *d*, in terms *n*	Number of Dots Used, *d*
1	$\frac{1}{2}\times 1\times(1+1)$	1
2	$\frac{1}{2}\times 2\times(2+1)$	3
3	$\frac{1}{2}\times 3\times(3+1)$	6
11	$\frac{1}{2}\times 11\times(11+1)$	66
n	$\frac{1}{2}\times n\times(n+1)$	$\frac{1}{2}n(n+1)$

(f) Show that there is no diagram that has exactly 1 000 dots.

SOLUTION:
Required To Show: No diagram in the sequence has 1 000 dots.
Solution:

Let $\frac{1}{2}n(n+1)=1000$

$$n(n+1)=2000$$

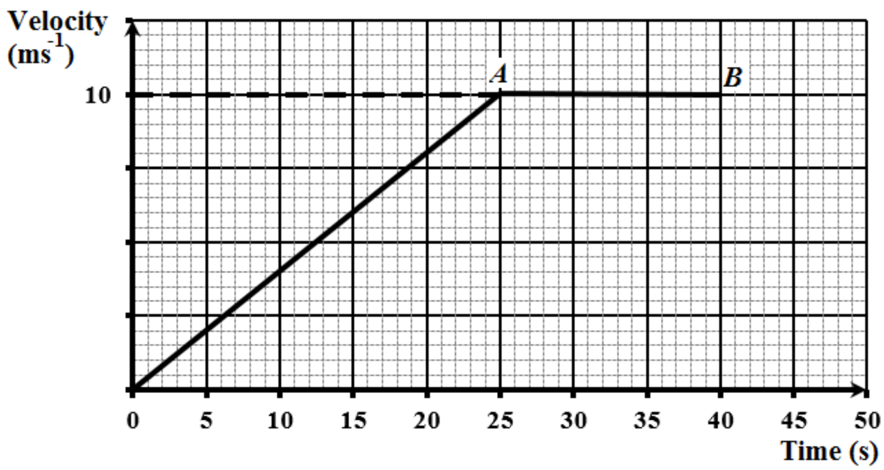
There are no two consecutive integers, *n* and *n* +1 being consecutive integers, whose product is exactly 2 000. Therefore, no diagram has 1 000 dots.

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) The velocity – time graph below shows the motion of a cyclist over a period of 40 seconds.



(i) Calculate the gradient of

a) *OA*

SOLUTION:
Data: A velocity – time graph showing the motion of a cyclist for a 40 second period.
Required to find: The gradient of *OA*.
Solution:
$O=(0,\ 0)$ and $A=(25,\ 10)$

Gradient of $OA=\dfrac{10-0}{25-0}=\dfrac{2}{5}$

b) *AB*

SOLUTION:

Required to find: The gradient of *AB*.
Solution:
$A=(25,\ 10)$ and $B=(40,\ 10)$

Gradient of $AB=\dfrac{10-10}{40-25}=0$ (as expected for a horizontal line)

(ii) Complete the following statements.

The cyclist started from rest, where his velocity was ……………….. ms^{-1}, and steadily increased his velocity by ……………….. ms^{-1} each second during the first 25 seconds. During the next 15 seconds, his velocity remained constant, that is, his acceleration was …………….. ms^{-2}.

SOLUTION:
Required to complete: The sentences given.
Solution:
The cyclist started from rest, where his velocity was **0** ms^{-1}, and steadily increased his velocity by $\frac{2}{5}$ ms^{-1} each second during the first 25 seconds. During the next 15 seconds, his velocity remained constant, that is, his acceleration was **0** ms^{-2}.

(iii) Determine the average speed of the cyclist over the 40-second period.

SOLUTION:
Required to find: Average speed of the cyclist over the 40 second period.
Solution:

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$\begin{aligned}\text{Area under the graph} &= \text{Distance covered}\\ &= \frac{1}{2}\{(40-25)+(40-0)\}\times 10\\ &= \frac{1}{2}(15+40)\times 10\\ &= 275 \text{ m}\end{aligned}$$

$$\therefore \text{Average speed} = \frac{275 \text{ m}}{40 \text{ s}}$$

$$= 6.875 \text{ ms}^{-1} \text{ or } 6\frac{7}{8} \text{ ms}^{-1}$$

(b) Consider the following pair of simultaneous equations:

$$x^2 + 2xy = 5$$
$$x + y = 3$$

(i) WITHOUT solving, show that $(1, 2)$ is a solution for the pair of simultaneous equations.

SOLUTION:
Data: The pair of simultaneous equations $x^2 + 2xy = 5$ and $x + y = 3$.
Required to show: $(1, 2)$ is a solution for the pair of simultaneous equations without solving
Solution:
Substitute $x = 1$ and $y = 2$ into both equations to test for 'truth or falsity'
When $x = 1$ and $y = 2$

$$x^2 + 2xy = 5$$
$$(1)^2 + 2(1)(2) = 5$$
$$1 + 4 = 5$$
$$5 = 5 \text{ (True)}$$

When $x = 1$ and $y = 2$
$x + y = 3$
$1 + 2 = 3$
$3 = 3$ (True)
$x = 1$ and $y = 2$ satisfy both equations
$\therefore (1, 2)$ is a solution for the pair of simultaneous equations.

(ii) Solve the pair of simultaneous equations above to determine the **other** solution.

SOLUTION:
Required to find: The other solution for the pair of simultaneous equations.
Solution:
Let $x^2 + 2xy = 5$ …❶
$x + y = 3$ …❷

From equation ❷
$y = 3 - x$ …❸
Substitute equation ❸ into equation ❶

$$x^2 + 2x(3 - x) = 5$$
$$x^2 + 6x - 2x^2 = 5$$
$$-x^2 + 6x - 5 = 0$$
$\times -1$
$$x^2 - 6x + 5 = 0$$
$$(x - 1)(x - 5) = 0$$

$x-1=0$ $\qquad$ $x-5=0$

$x=1$ $\qquad$ $x=5$

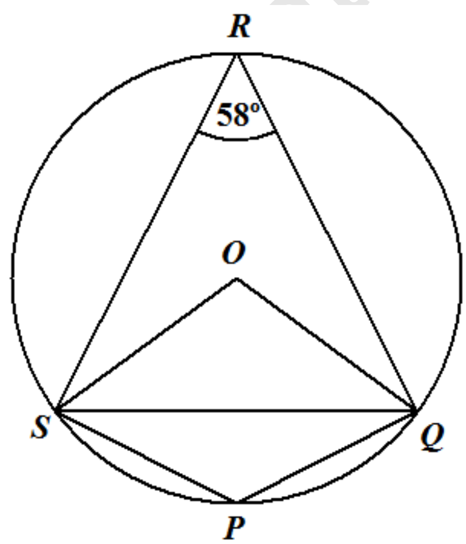
We already know that $x=1$ is part of one set of solution. We consider the other.
When $x=5$

$$y=3-x$$
$$=3-5$$
$$=-2$$

$\therefore$ The other solution is $(5,\ -2)$.

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

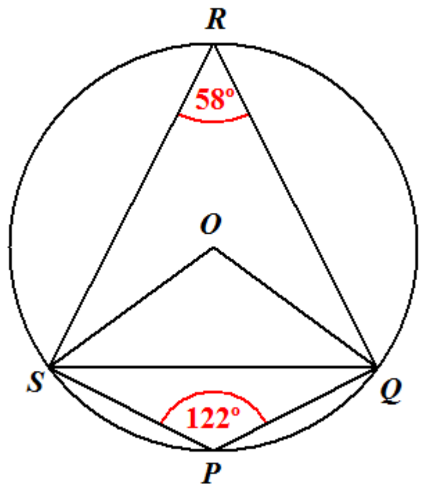
10. (a) P, Q, R and S are four points on the circumference of the circle shown below. Angle $QRS = 58°$.



Using the geometrical properties of a circle to give reasons for each step of your answer, determine the measure of

(i) $\angle SPQ$

SOLUTION:
Data: Diagram of a circle where P, Q, R and S lie on its circumference and angle $QRS = 58°$.
Required to calculate: $\angle SPQ$
Calculation:



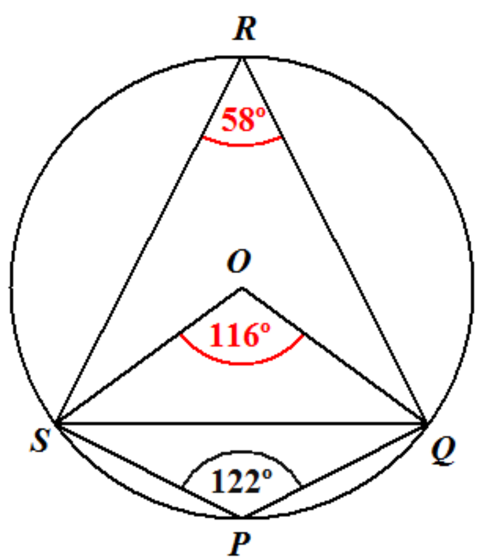
$\angle SPQ = 180° - 58°$

$= 122°$

(The opposite angles of a cyclic quadrilateral are supplementary.)

(ii) $\angle OQS$

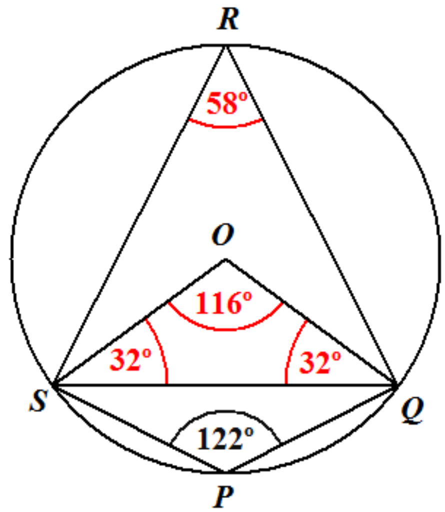
SOLUTION:
Required to calculate: $\angle OQS$
Calculation:



$\angle SOQ = 2(58°)$

$= 116°$

(The angle subtended by a chord at the center of a circle is twice the angle that the chord subtends at the circumference, standing on the same arc.)



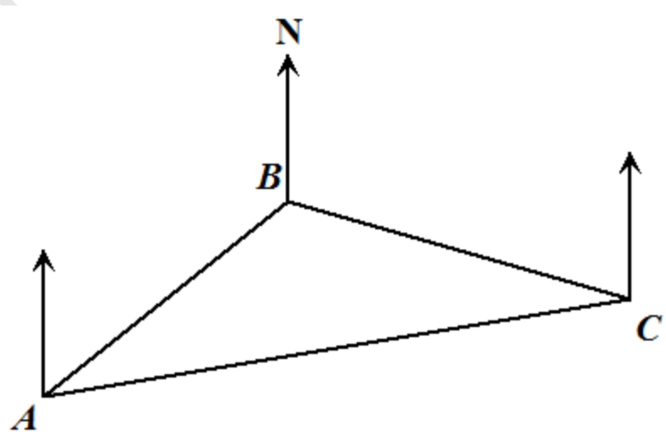
$\angle OSQ = \angle OQS$ (The base angles of an isosceles triangle are equal)

$$\angle OSQ + \angle OQS = 180° - 116°$$
$$= 64°$$

$$\therefore \angle OQS = \frac{1}{2}(64°)$$
$$= 32°$$

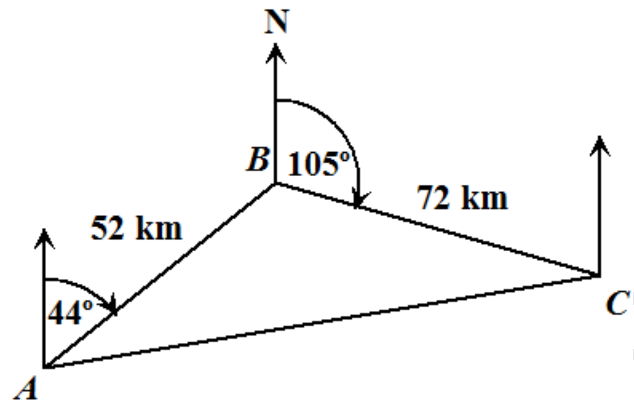
(b) A ship leaves Port A and sails 52 km on a bearing of 044° to Port B. The ship then changes course to sail to Port C, 72 km away, on a bearing of 105°.

(i) On the diagram below, **not drawn to scale**, label the known distances travelled and the known angles.



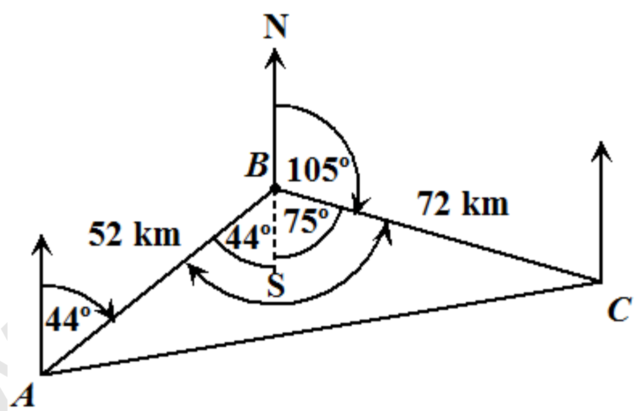
SOLUTION:

Data: A ship leaves Port A and sails on a bearing of 044° to Port B, 52 km away. It then sails to Port C on a bearing of 105°, 72 km away.
Required to label: The sides and angles given on the diagram
Solution:



(ii) Determine the measure of $\angle ABC$.

SOLUTION:
Required to find: $\angle ABC$
Solution: Let the line NB be extended to S, as shown.



$\angle ABS = 44°$ (Alternate angles are equal)

$$\angle CBS = 180° - 105°$$
$$= 75°$$
(two angles which make a straight line are supplementary)

$$\therefore \angle ABC = 44° + 75°$$
$$= 119°$$

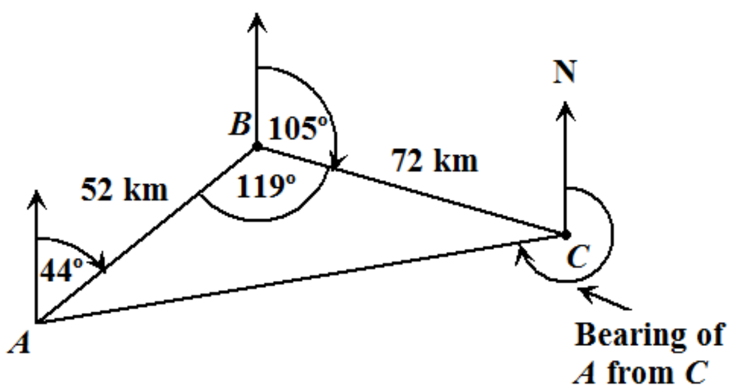
(iii) Calculate, to the nearest km, the distance AC.

SOLUTION:
Required to calculate: *AC*
Calculation:
By the cosine rule:

$$AC^2 = (52)^2 + (72)^2 - 2(52)(72)\cos(119°)$$
$$= 2704 + 5184 - (-3630.25)$$
$$= 11518.25$$
$$AC = \sqrt{11518.25}$$
$$= 107.\underline{\underline{3}} \text{ km}$$
$$= 107 \text{ km (to the nearest km)}$$

(iv) Show that the bearing of *A* from *C*, to the nearest degree, is 260°.

SOLUTION:
Required to show: The bearing of *A* from *C* is 260°
Solution:



$\angle BCN = 75°$ (Co-interior angles are supplementary)

By the sine rule:

$$\frac{52}{\sin A\hat{C}B} = \frac{107.3}{\sin 119°}$$
$$\sin A\hat{C}B = \frac{52 \sin 119°}{107.3}$$
$$= 0.4238$$
$$A\hat{C}B = \sin^{-1}(0.4238)$$
$$= 25.07°$$

The bearing of *A* from $C = 360° - (75° + 25.07°) = 259.9^0$

$= 260°$ (correct to the nearest degree

Q.E.D.

11. (a) Matrices A and B are such that

$$A=\begin{pmatrix}3 & 2\\ 5 & 4\end{pmatrix} \text{ and } B=\begin{pmatrix}4 & 0\\ 3 & -1\end{pmatrix}.$$

(i) Show by multiplying A and B, that $AB \neq BA$.

SOLUTION:

Data: $A=\begin{pmatrix}3 & 2\\ 5 & 4\end{pmatrix}$ and $B=\begin{pmatrix}4 & 0\\ 3 & -1\end{pmatrix}$

Required to show: $AB \neq BA$

Proof:

$$A\times B=\underset{2\times 2}{\begin{pmatrix}3 & 2\\ 5 & 4\end{pmatrix}}\underset{\times\ 2\times 2=2\times 2}{\begin{pmatrix}4 & 0\\ 3 & -1\end{pmatrix}}$$

$$=\begin{pmatrix}e_{11} & e_{12}\\ e_{21} & e_{22}\end{pmatrix}$$

$$\begin{aligned}e_{11}&=(3\times 4)+(2\times 3)\\ &=12+6\\ &=18\end{aligned}$$

$$B\times A=\underset{2\times 2}{\begin{pmatrix}4 & 0\\ 3 & -1\end{pmatrix}}\underset{\times\ 2\times 2=2\times 2}{\begin{pmatrix}3 & 2\\ 5 & 4\end{pmatrix}}$$

$$=\begin{pmatrix}e_{11} & e_{12}\\ e_{21} & e_{22}\end{pmatrix}$$

$$\begin{aligned}e_{11}&=(4\times 3)+(0\times 5)\\ &=12+0\\ &=12\end{aligned}$$

e_{11} of $AB \neq e_{11}$ of BA. There is no need for further working since, regardless of whether the remaining three corresponding entries of AB are equal or not to those of BA, all the entries of AB will never be equal to the entries of BA. $\therefore AB \neq BA$.

Q.E.D.

(ii) Find A^{-1}, the inverse of A.

SOLUTION:
Required to find: A^{-1}
Solution:

$$\begin{aligned}\det A &= (3\times 4)-(2\times 5)\\ &= 12-10\\ &= 2\end{aligned}$$

$$\begin{aligned}A^{-1} &= \frac{1}{2}\begin{pmatrix} 4 & -(2) \\ -(5) & 3 \end{pmatrix}\\ &= \begin{pmatrix} 2 & -1 \\ -\frac{5}{2} & \frac{3}{2} \end{pmatrix}\end{aligned}$$

(iii) Write down the 2×2 matrix representing the matrix product AA^{-1}.

SOLUTION:
Required to write: The 2×2 matrix AA^{-1}.
Solution:

$$\underset{2\times 2}{A} \times \underset{2\times 2}{A^{-1}} = \underset{2\times 2}{I}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(b) **(i)** Write the following pair of simultaneous equations as a matrix equation.

$$\begin{aligned}3x+2y&=1\\ 5x+4y&=5\end{aligned}$$

SOLUTION:
Data: $3x+2y=1$ and $5x+4y=5$
Required to write: The given pair of simultaneous equations as a matrix equation
Solution:

$$\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

(ii) Write the solution of your matrix equation in (b) (i) as a product of two matrices.

SOLUTION:

Required to write: The solution of the matrix equation as a product of two matrices

Solution:

$$\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$$A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$\times A^{-1}$

$$A \times A^{-1}\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

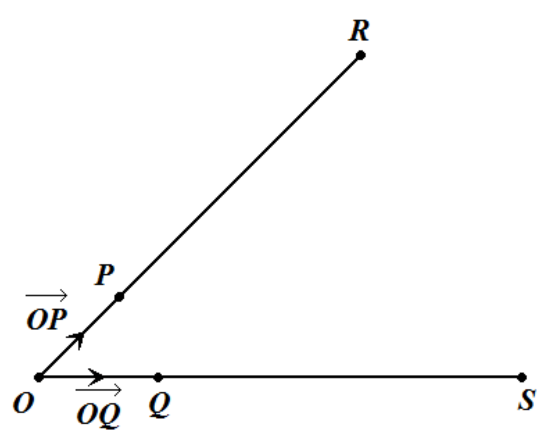
$$I\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -\frac{5}{2} & \frac{3}{2} \end{pmatrix}\begin{pmatrix} 1 \\ 5 \end{pmatrix} \text{ as a product of two matrices.}$$

If calculated, (which was not required according to the question) the right hand side would give $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ and equating corresponding entries gives $x = -3$ and $y = 5$.

(c) The position vectors of the points P and Q relative to an origin O, are $OP = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $OQ = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ respectively.

The diagram below shows that $PR = 3OP$ and $QS = 3OQ$.



(i) Express in the form $\begin{pmatrix} x \\ y \end{pmatrix}$, vector

- OS

SOLUTION:
Data: Diagram showing position of vectors of the points P and Q, where $OP = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $OQ = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ and $PR = 3OP$ and $QS = 3OQ$.

Required to express: OS in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:

$$OQ = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

$$QS = 3\begin{pmatrix} 5 \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} 15 \\ 0 \end{pmatrix}$$

$$OS = OQ + QS$$
$$= \begin{pmatrix} 5 \\ 0 \end{pmatrix} + \begin{pmatrix} 15 \\ 0 \end{pmatrix}$$
$= \begin{pmatrix} 20 \\ 0 \end{pmatrix}$ and is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x = 20$ and $y = 0$.

- PQ

SOLUTION:
Required to express: PQ in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:

$$PQ = PO + OQ$$
$$= -\begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$
$= \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ and is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x = 1$ and $y = -3$.

- *RS*

SOLUTION:

Required to express: RS in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:

$$PR = 3OP$$
$$= 3\begin{pmatrix} 4 \\ 3 \end{pmatrix}$$
$$= \begin{pmatrix} 12 \\ 9 \end{pmatrix}$$

$$OR = OP + PR$$
$$= \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 12 \\ 9 \end{pmatrix}$$
$$= \begin{pmatrix} 16 \\ 12 \end{pmatrix}$$

$$RS = RO + OS$$
$$= -\begin{pmatrix} 16 \\ 12 \end{pmatrix} + \begin{pmatrix} 20 \\ 0 \end{pmatrix}$$

$= \begin{pmatrix} 4 \\ -12 \end{pmatrix}$ and is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x = 4$ and $y = -12$.

(ii) State TWO geometrical relationships between *PQ* and *RS*.

SOLUTION:
Required to state: Two geometrical relationships between *PQ* and *RS*.
Solution:

$$PQ = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$RS = \begin{pmatrix} 4 \\ -12 \end{pmatrix}$$
$$= 4\begin{pmatrix} 1 \\ -3 \end{pmatrix}$$
$$= 4PQ$$

Hence, RS is a scalar multiple (which is 4) of PQ. Therefore, RS and PQ are parallel.

$$RS = 4\begin{pmatrix} -1 \\ 3 \end{pmatrix}$$
$$= 4 \times PQ$$
$$|RS| = 4|PQ|$$

That is to say, the length of RS is 4 times the length of PQ.