FAS-PASS
Maths

CSEC MATHEMATICS JANUARY 2016
SECTION I

1. (a) Using a calculator, or otherwise, calculate the EXACT value of

$$\left(3.6+\sqrt{51.84}\right)\div 3.75$$

SOLUTION:

Required to calculate: The exact value of $\left(3.6+\sqrt{51.84}\right)\div 3.75$.

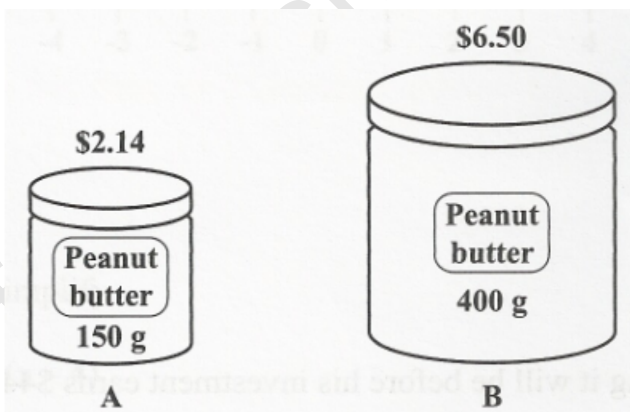
Calculation:

$\left(3.6+\sqrt{51.84}\right)\div 3.75$

Taking the positive root of $\sqrt{51.84}$.

$$\left(3.6+\sqrt{51.84}\right)\div 3.75=\left(3.6+7.2\right)\div 3.75 \quad \text{(By the calculator)}$$
$$=10.8\div 3.75$$
$$=2.88 \text{ (exactly)}$$

(b) The diagram below, **not drawn to scale**, shows two jars of peanut butter of the same brand.



Which of the jars shown above is the BETTER buy?
Show ALL working to support your answer.

SOLUTION:

Data: Diagram showing Jar A containing 150g of peanut butter costing $2.14 and Jar B containing 400g of peanut butter costing $6.50.

Required to determine: Which one of Jar A or Jar B is the better buy
Calculation:
In Jar A, 150g of peanut butter costs $2.14.

$\therefore \text{Price per gram} = \frac{\$2.14}{150}$
$= \$0.014 \text{ (correct to 3 decimal places)}$

In Jar B, 400g of peanut butter costs $6.50.

$\therefore \text{Price per gram} = \frac{\$6.50}{400}$
$= \$0.016 \text{ (correct to 3 decimal places)}$

$\$0.014 < \$0.016$

Hence, if better is supposed to mean costing less, then the buying of peanut butter in Jar A is a better buy than buying the peanut butter in Jar B.

(c) Thomas invested $1 498 at 6% simple interest per annum.

Calculate:

(i) The interest, in dollars, earned after six months.

SOLUTION:

Data: Thomas invested $1498 at 6% simple interest per annum.
Required to calculate: The interest, in dollars, earned after 6 months
Calculation:

$\text{Principal} = \$1498$

$\text{Rate} = 6\% \text{ per annum}$

$\text{Time} = \frac{6}{12}$
$= \frac{1}{2} \text{ year}$

$$\text{Simple Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$
$$= \frac{\$1498 \times 6 \times \frac{1}{2}}{100}$$
$$= \$44.94$$

(ii) The TOTAL amount of money in his account after 3 years.

SOLUTION:

Required to calculate: The total amount of money in the account after 3 years.
Calculation:

$$\text{Interest earned after 3 years} = \frac{\$1498 \times 6 \times 3}{100}$$
$$= \$269.64$$

Hence, the total amount in Thomas' account after 3 years
$= \text{Principal} + \text{Interest earned}$ (Assuming no withdrawals are made)
$= \$1498 + \$269.64$
$= \$1767.64$

(iii) How long will it be before his investment earns \$449.40?

SOLUTION:
Required to calculate: The time before the investment to earn an interest of \$449.40
Calculation:

$$\$449.40 = \frac{\$1498 \times 6 \times T}{100} \quad \text{(where } T \text{ is the time taken)}$$

$$\text{Time}, T = \frac{\$449.40 \times 100}{\$1498 \times 6}$$
$$= 5$$

$\therefore$ The investment earns \$449.40 in 5 years.

2. (a) (i) Solve for x, where x is a real number.

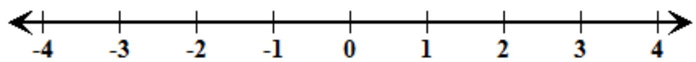
$$8 - x \leq 5x + 2$$

SOLUTION:

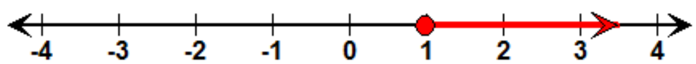
Data: $8 - x \leq 5x + 2$
Required to calculate: x
Calculation:

$8 - x \leq 5x + 2$
$8 - 2 \leq 5x + x$
$6 \leq 6x$
$\therefore 6x \geq 6$
$\div 6$
$x \geq 1$

(ii) Show your solution to (a) (i) on the number line below.



SOLUTION:
Required to show: The result on the number line
Solution:



(b) Expand and simplify

$$2x(x+5)-3(x-4)$$

SOLUTION:
Required to expand: And simplify, $2x(x+5)-3(x-4)$.
Solution:

$$2x(x+5)-3(x-4)=2x^2+10x-3x+12$$
$$=2x^2+7x+12$$

(c) Simplify

$$\frac{3x^2\times 4x^3}{2x}$$

SOLUTION:
Required to simplify: $\frac{3x^2\times 4x^3}{2x}$
Solution:

$$\frac{3x^2\times 4x^3}{2x}=\frac{3\times 4\times x^{2+3}}{2x}$$
$$=\frac{3\times 4}{2}x^{2+3-1}$$
$$=6x^4$$

(d) Write as a single fraction, in its lowest terms,

$$\frac{x+1}{2}+\frac{5-x}{5}$$

SOLUTION:

Required to write: $\frac{x+1}{2}+\frac{5-x}{5}$ as a fraction in its lowest terms.

Solution:

$$\frac{x+1}{2}+\frac{5-x}{5}$$

$$\frac{5(x+1)+2(5-x)}{10}=\frac{5x+5+10-2x}{10}$$

$$=\frac{(5x-2x)+(5+10)}{10}$$

$$=\frac{3x+15}{10}$$

$$=\frac{3(x+5)}{10}$$ as a fraction in its lowest terms.

(e) Factorise completely

$$4x^2-4$$

SOLUTION:

Required to factorise: $4x^2-4$

Solution:

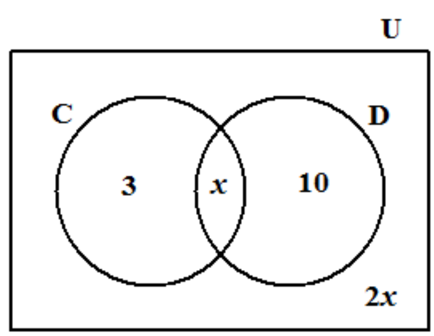
$$4x^2-4=4(x^2-1)$$

$$=4\{(x)^2-(1)^2\}$$

$(x)^2-(1)^2$ is a difference of two squares and factorises to $(x-1)(x+1)$

Hence, $4x^2-1=4(x-1)(x+1)$

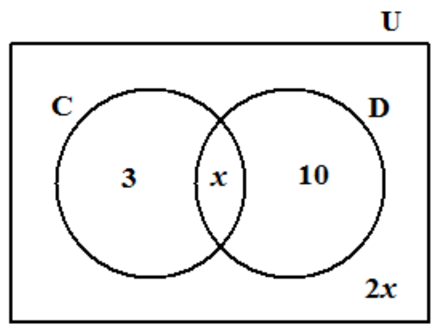
3. (a) The Venn diagram below shows the number of students in Form 5A who have visited Canada (C) or Dominica (D).



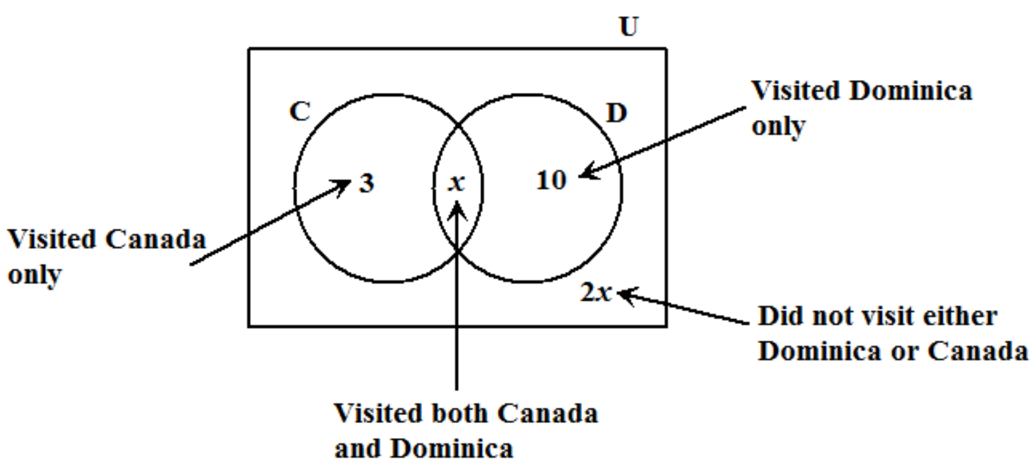
(i) How many students have visited Dominica ONLY?

SOLUTION:

Data: Venn diagram showing the number of students in Form 5A who have visited Canada or Dominica.
D = {Students who visited Dominica}
C = {Students who visited Canada}



Required to find: The number of students who visited Dominica only
Solution:
First we identify the students in each subset of the universal set.



The number of students who visited Dominica only, is 10.

(ii) Write an **expression**, in terms of x, to represent the TOTAL number of students who have visited Canada.

SOLUTION:

Required to write: An expression, in terms of x, for the total number of students who visited Canada

Solution:
The total number of students who visited Canada $=3+x$

(iii) Given that there are 25 students in Form 5A, calculate the value of x.

SOLUTION:

Data: There are 25 students in Form 5A.
Required to calculate: The value of x
Calculation:
The total number of students in all of the subsets = 25

$$3+x+10+2x=25$$
$$3x+13=25$$
$$3x=25-13$$
$$3x=12$$
$$\div 3$$
$$x=4$$

(iv) Hence write down the number of students in each of the following subsets:

- $C \cup D$
- $C \cap D$
- $(C \cup D)'$

SOLUTION:

Required to write: The number of students in the sets, $C \cup D$, $C \cap D$ and $(C \cup D)'$.
Solution:
The number of students in:

- $C \cup D = 3+x+10$
  $= 3+4+10$ (Recalling that $x = 4$)
  $= 17$
- $C \cap D = x$
  $= 4$
- $(C \cup D)' = 2x$
  $= 2(4)$
  $= 8$

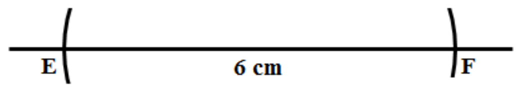
(b) **(i)** Using **a ruler, a pencil and pair of compasses**, construct accurately, the **square** EFGH where EF = 6 cm.

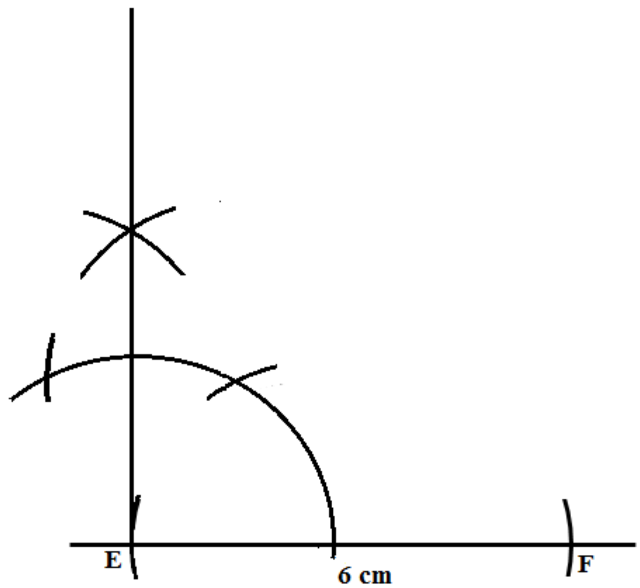
(Show ALL construction lines and curves.)

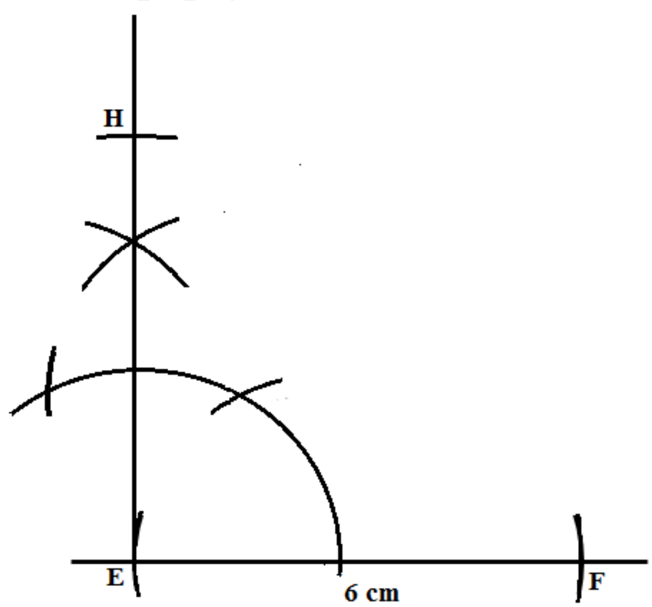
SOLUTION:

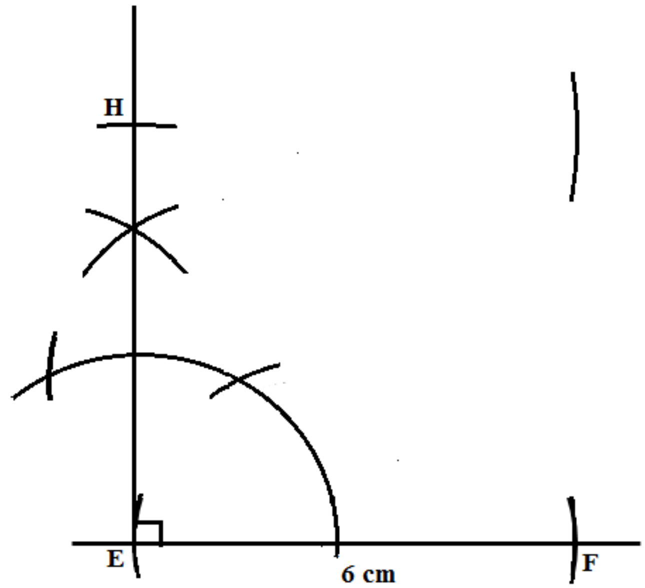
Required to construct: The square EFGH where EF = 6 cm.

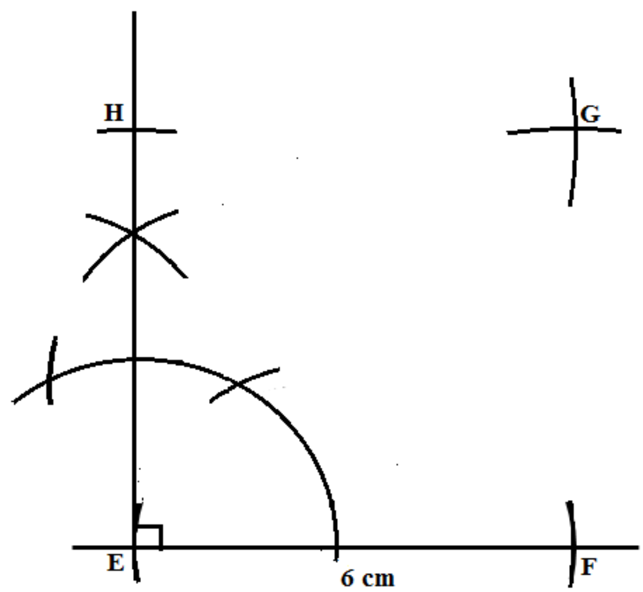
Solution:

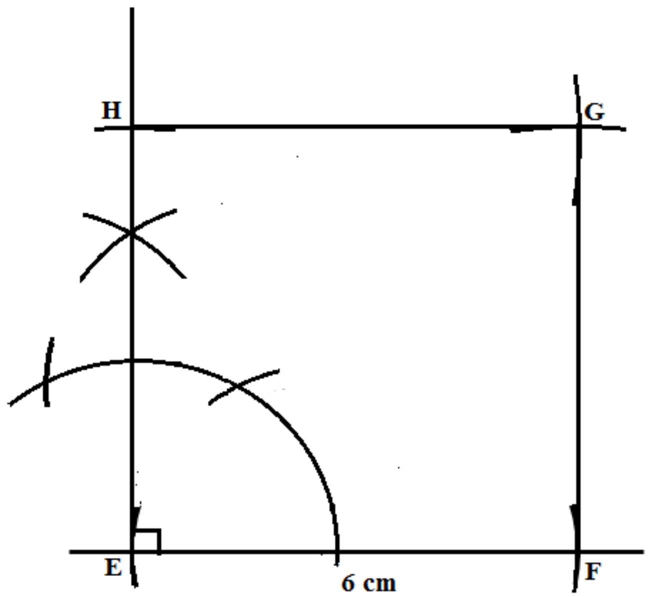







H
E
6 cm
F

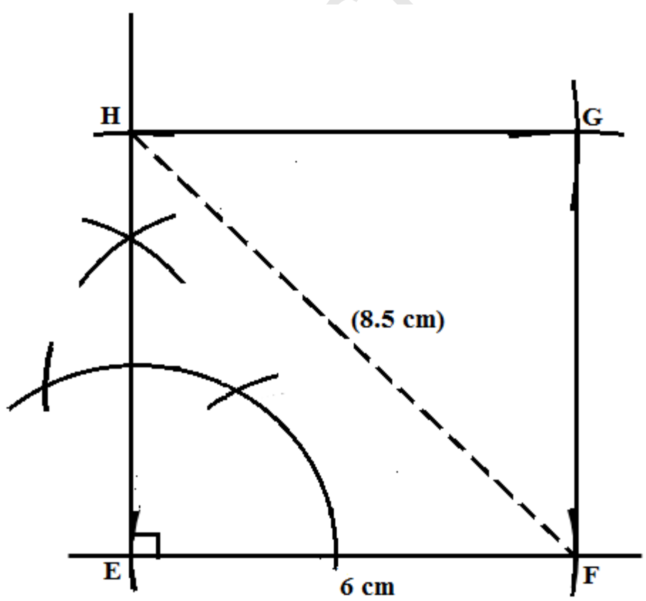

H
G
E
6 cm
F



(ii) Measure, and state in centimetres, the length of the diagonal FH

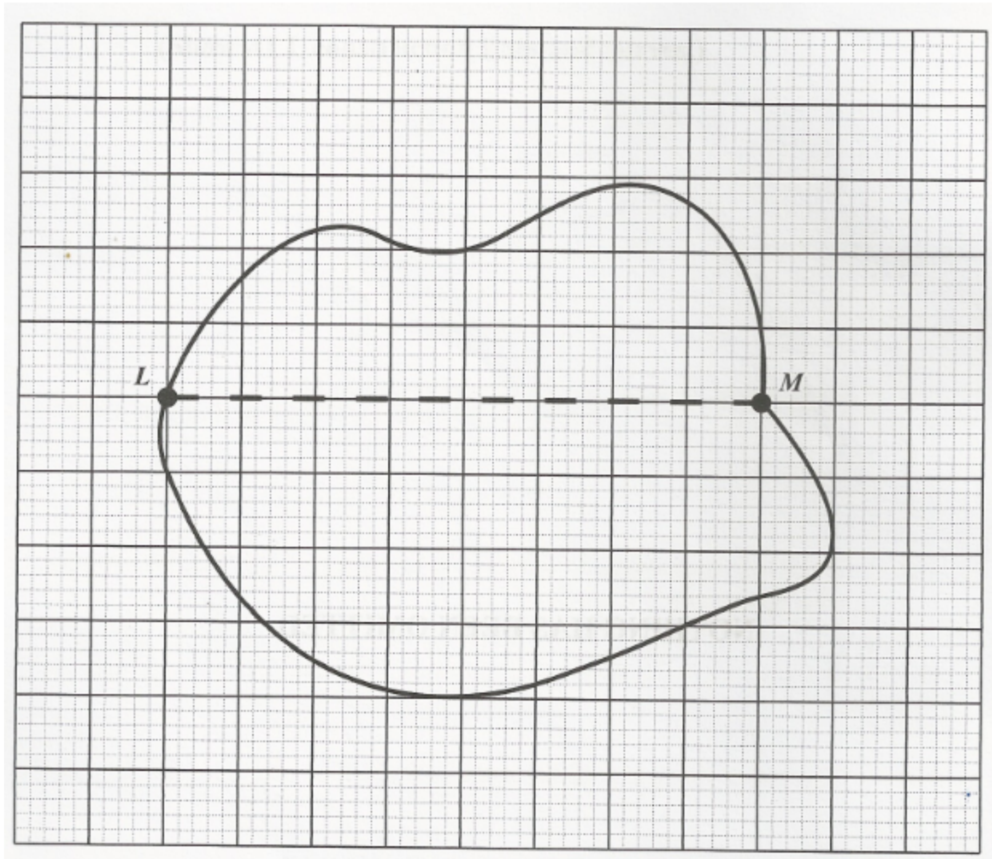
SOLUTION:

Required to measure: And state the length of diagonal FH.
Solution:



FH = 8.5 cm (correct to 1 decimal place)

4. (a) The diagram below shows the map of an island drawn on a grid of 1 cm squares.



(i) State, in cm, the length of *LM* as shown in the diagram.

SOLUTION:

Data: Map of an island drawn on a grid of 1 cm squares.
Required to state: The length of *LM* as shown in the diagram
Solution:
The length of *LM* shown in the diagram = 8 cm (by measurement)

(ii) Estimate, by counting squares, the area of the map shown in the diagram.

SOLUTION:
Required to estimate: The area of the map shown in the diagram, by counting squares
Solution:

An area that occupies 'more than $\frac{1}{2}$ square' is considered as 1 square.

An area that occupies 'less than $\frac{1}{2}$ square' is ignored.

We count the whole squares and record the number in each row.

The map occupies 7 rows.

Row	Area of more than $\frac{1}{2}$ square	Number of whole squares	Total
1	2	0	2
2	2	5	7
3	1	7	8
4	0	8	8
5	2	7	9
6	2	5	7
7	4	0	4
		Total	45

Each square is of area 1 cm^2.
$\therefore$ The area of the map = $(1 \times 45)\ cm^2$
$= 45\ cm^2$

(iii) On the island, the actual distance LM is 20 km. Complete the following statement:

On the map, 1 cm represents ……………………….. km.

SOLUTION:

Data: Actual distance of LM is 20 km.
Required to complete: The statement given
Solution:
LM is actually 20 km long.
$\therefore 8$ cm on the map $\equiv$ 20 km in actual distance

$$\therefore 1 \text{ cm represents } \frac{20}{8} \text{ km} = 2\frac{1}{2} \text{ km}$$

On the map, 1 cm represents $2\frac{1}{2}$ km.

(iv) Write the scale of the map in the form $1 : x$.

SOLUTION:

Required to write: The scale of the map in the form $1 : x$.
Solution:

If 1 cm represents $2\frac{1}{2}$ km, then 1 cm represents $2\frac{1}{2}\times 100\times 1000$ cm.

$\therefore$ The scale is, $1:2.5\times 100\times 1000=1:250\,000$, which is of the form $1:x$, where $x=250\,000$.

(v) What distance on the island will be 3 cm on the map?

SOLUTION:

Required to calculate: The actual distance that is represented by 3 cm on the map

Calculation:
If a distance on the map is 3 cm long, then its actual distance
$=3\times 250\,000$ cm =750 000 cm = 7.5 km

OR

If we recall the scale, 1 cm represent $2\frac{1}{2}$ km, then 3 cm will represent a distance of $3\times 2\frac{1}{2}$ km $=7\frac{1}{2}$ km.

(vi) What area on the island will be represented by 3 cm^2 on the map?

SOLUTION:

Required to calculate: The area on the island that is represented by 3 cm^2 on the map.

Calculation:
1 cm on the map represents an actual length of $2\frac{1}{2}$ km.

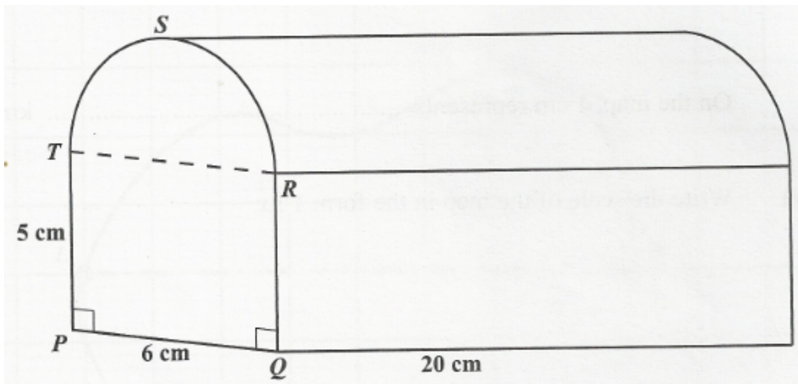
$\therefore 1\text{cm}^2$ will represent an area of $\left(2\frac{1}{2}\times 2\frac{1}{2}\right)$ km^2.

So, 3 cm^2 will represent an area of

$$\left(2\frac{1}{2}\times 2\frac{1}{2}\times 3\right)\text{km}^2=\left(\frac{5}{2}\times\frac{5}{2}\times 3\right)\text{km}^2$$
$$=\frac{75}{4}\text{ km}^2$$
$$=18\frac{3}{4}\text{ km}^2$$

(b) The diagram below, **not drawn to scale**, shows a prism with cross section *PQRST* and length 20 cm. *PQRST* is made up of a rectangle *PQRT* and a semicircle *RST* such that $PQ = 6$ cm and $QR = 5$ cm.

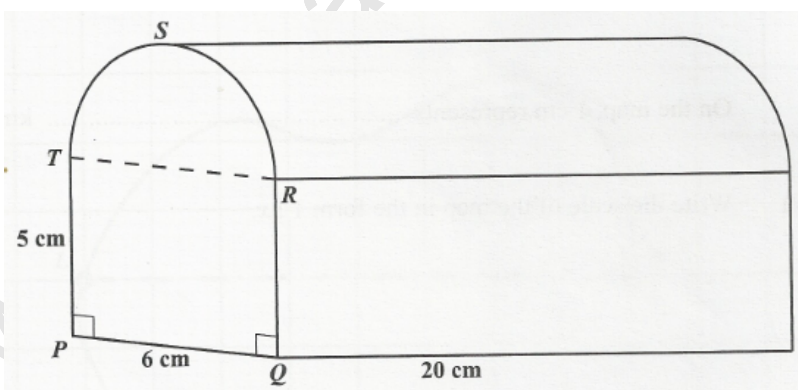
Use $\pi = 3.14$.



(i) Calculate the area of the cross section *PQRST*.

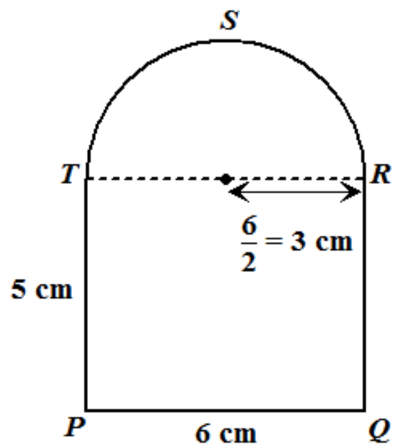
SOLUTION:

Data: Diagram showing a prism with cross section *PQRST* and length 20 cm. *PQRST* is made up of a rectangle *PQRT* and a semicircle *RST* such that $PQ = 6$ cm and $QR = 5$ cm.



Required to calculate: The area of the cross section *PQRST*

Calculation:



The area of the cross section *PQRST* = Area of rectangle *PQRT* + Area of the semicircle *RST*

$= (5\times 6)\ \text{cm}^2 + \frac{1}{2}\pi(3)^2\ \text{cm}^2$

$= (30+14.13)\ \text{cm}^2$

$= 44.13\ \text{cm}^2$

(ii) An engineer needs a similar prism whose volume is NOT more than 900 cm^3. Estimate, in cm, the length of the longest prism he can use.

SOLUTION:

Data: An engineer requires a similar prism whose volume is not more than 900 cm^3.
Required to calculate: The longest prism he can use
Calculation:
The volume of the required prism must not exceed 900 cm^3.
Volume of prism = Cross sectional area of the prism $\times$ the length
Let the length be x cm.
$\therefore 44.13\times x \ngtr 900$

Hence,
$44.13x \leq 900$

$x \leq \frac{900}{44.13}$

$x \leq 20.394$

$x \leq 20.39$ cm (correct to 2 decimal places)

$\therefore$ The prism should not be longer than 20.39 cm (correct to 2 decimal places).

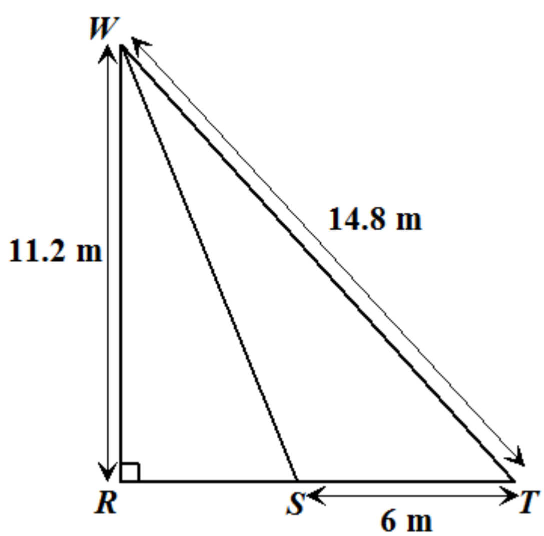
NOTE: Similar figures, in mathematics, clearly implies that they are to be of the same shape but differ in size. Therefore, they cannot have the same dimensions of cross section and possess a different length. Hence, it is best if the question had been stated-

A prism with the same cross-sectional area is to be made and with a volume not exceeding 900 cm^3…

In fact, if another prism has the same cross-sectional area and is either longer or shorter, then the two prisms will definitely not be similar.

ALSO:
What is the purpose of giving the prism as 20 cm long in the diagram? This serves no purpose in any of the required calculations whatsoever and unused numerical data can be very misleading, if not confusing.

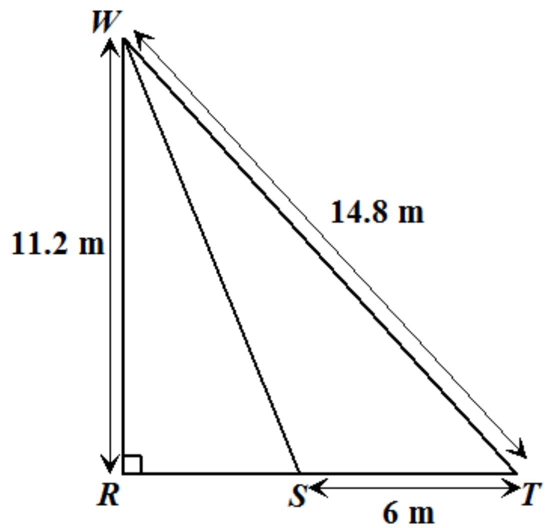
5. (a) In the diagram below, **not drawn to scale**, $ST = 6$ m, $WR = 11.2$ m, $WT = 14.8$ m and angle $WRS = 90°$.



Calculate, giving your answer to 1 decimal place

(i) The length *RS*.

SOLUTION:
Data: Diagram showing, $ST = 6$ m, $WR = 11.2$ m, $WT = 14.8$ m and angle $WRS = 90°$.



Required to calculate: The length of *RS*.
Calculation:

$(11.2)^2 + (RT)^2 = (14.8)^2$ (Pythagoras' Theorem)

$\therefore RT^2 = (14.8)^2 - (11.2)^2$

$RT^2 = 93.6$

$RT = 9.67 \text{ m}$

Length of *RS* = Length of *RT* – Length of *ST*

$= 9.67 - 6$

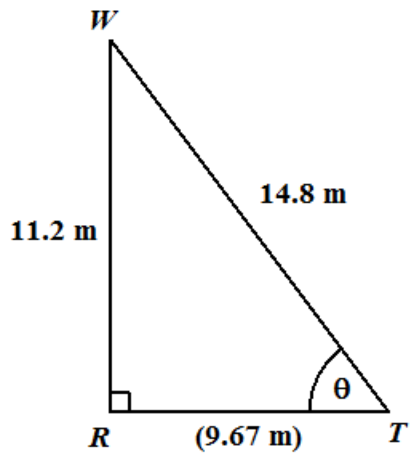
$= 3.6\underline{7} \text{ m}$

$= 3.7 \text{ m}$ (correct to 1 decimal place)

(ii) The measure of angle *RTW*.

SOLUTION:
Required to calculate: The measure of $R\hat{T}W$.

Calculation:



Let $R\hat{T}W$ be θ.

$$\therefore \sin\theta = \frac{11.2}{14.8}$$

$$\theta = \sin^{-1}\left(\frac{11.2}{14.8}\right)$$

$$= 49.1\underline{\underline{7}}^\circ$$

$$= 49.2^\circ \text{ (correct to 1 decimal place)}$$

OR

$$\cos\theta = \frac{9.67}{14.8}$$

$$\theta = \cos^{-1}\left(\frac{9.67}{14.8}\right)$$

$$= 49.2^\circ \text{ (correct to 1 decimal place)}$$

OR

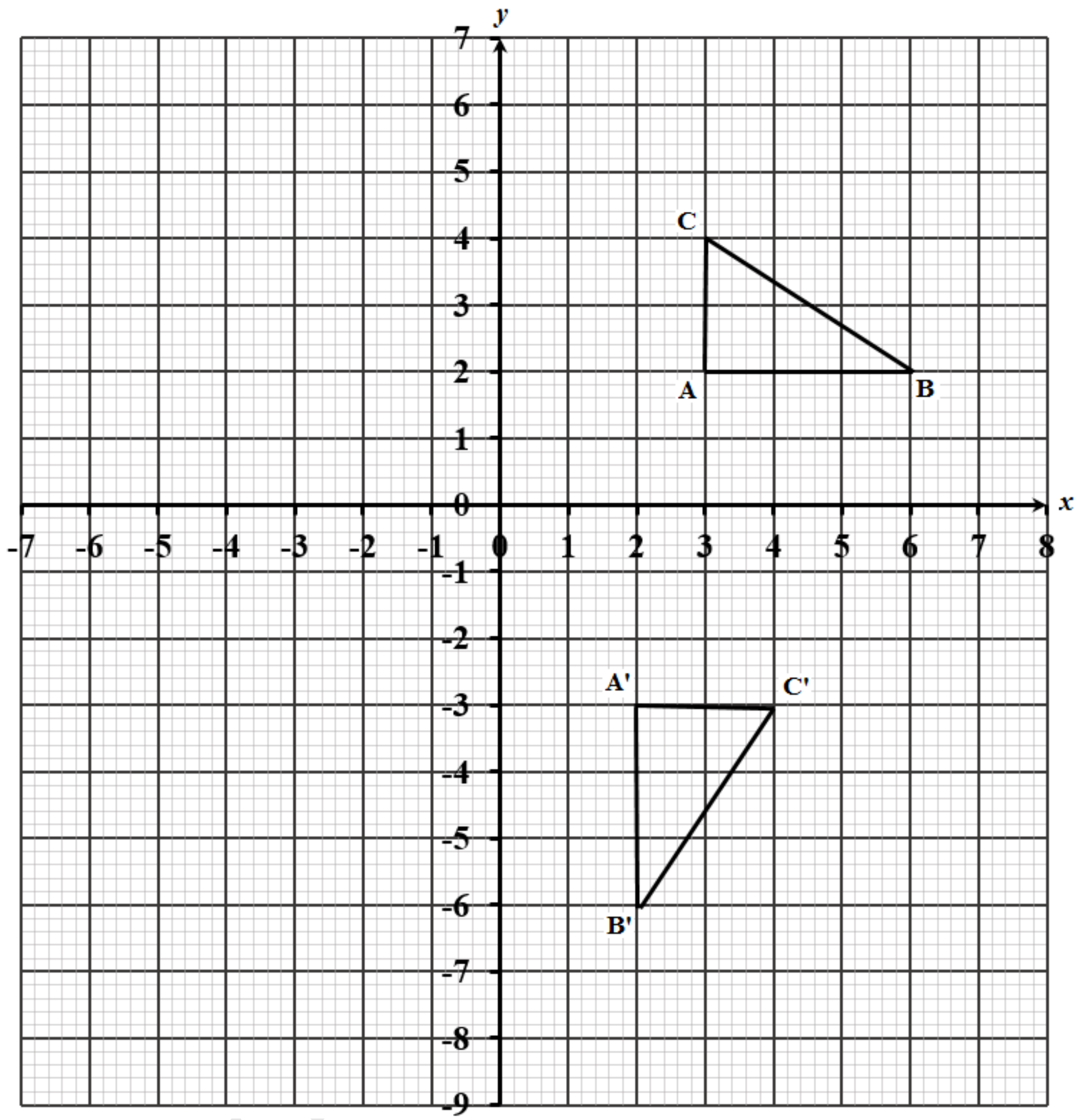
$$\tan\theta = \frac{11.2}{9.67}$$

$$\theta = \tan^{-1}\left(\frac{11.2}{9.67}\right)$$

$$= 49.2^\circ$$

(correct to 1 decimal place)

(b) The graph below shows triangle ABC and its image $A'B'C'$ after undergoing a single transformation.



(i) Write down the coordinates of the vertices of $\Delta A'B'C'$ ΔABC.

SOLUTION:

Data: Graph showing triangle ABC and its image $A'B'C'$ after undergoing a single transformation.
Required to write: The coordinates of the vertices of ΔABC
Solution:
From the given diagram, the vertices of ΔABC are:
A $(3, 2)$
B $(6, 2)$
C $(3, 4)$

(ii) Write down the coordinates of the vertices of $\Delta A'B'C'$.

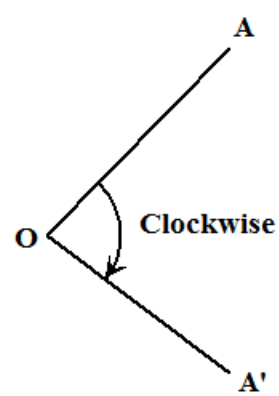
SOLUTION:
Required to write: The coordinates of the vertices of $\Delta A'B'C'$.
Solution:
From the diagram given, the vertices of $\Delta A'B'C'$ are:
$A'(2, -3)$
$B'(2, -6)$
$C'(4, -3)$

(iii) Describe FULLY the transformation that maps triangle ABC onto triangle $A'B'C'$.

SOLUTION:

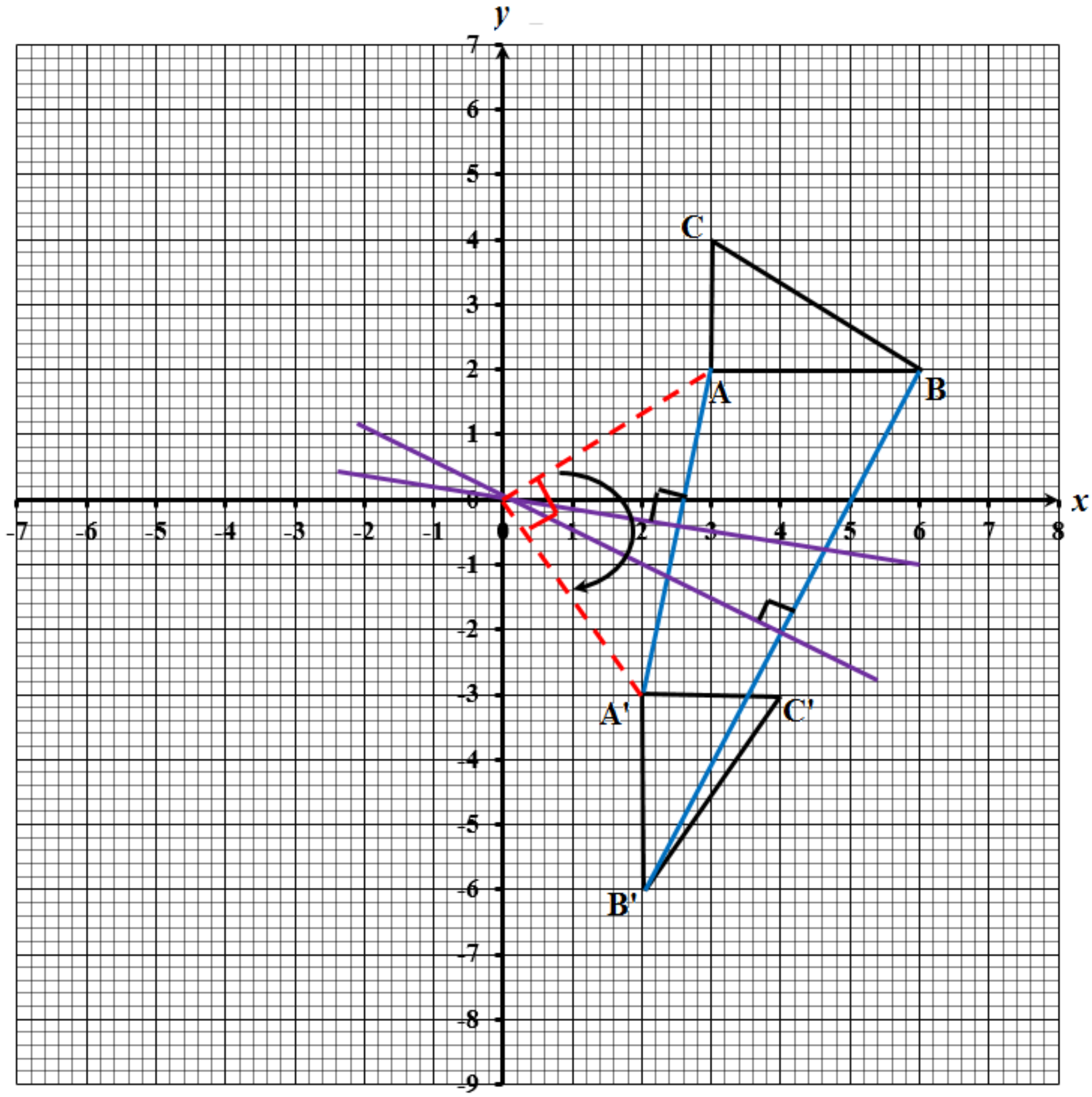
Required to describe: The transformation that maps triangle ABC onto triangle, $A'B'C'$.
Solution:
(i) ΔABC and $\Delta A'B'C'$ are congruent and (ii) The image of $\Delta A'B'C'$ is re-oriented with respect to the object ΔABC. Therefore, the transformation is a rotation.

Join A to A' and construct the perpendicular bisector of AA'.
Join B to B' and construct the perpendicular bisector of BB'.
Extend the two perpendicular bisectors so that they meet. This occurs at O. Therefore, O is the center of rotation. (The perpendicular bisector of CC' would have also passed through O).
Angle AOA' = 90° (same for angle BOB' and angle COC')



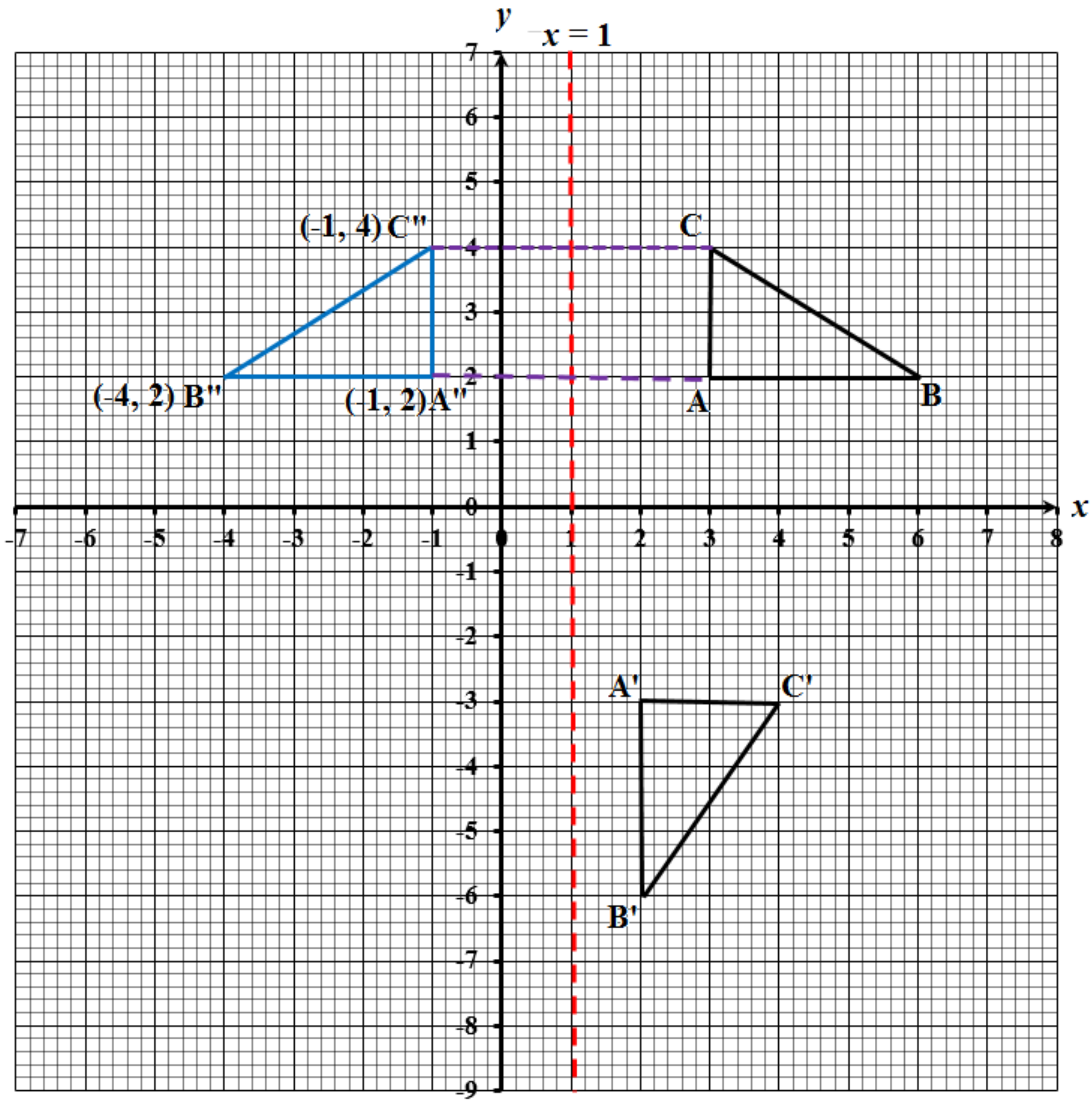
The movement from OA to OA' is in the clockwise direction, as illustrated.

Hence, the transformation which maps ΔABC onto $\Delta A'B'C'$ is a 90° clockwise rotation about O.



(iv) On the graph on page 16, draw the line $x = 1$ AND triangle $A''B''C''$, the image of triangle ABC after a reflection in the line $x = 1$.

SOLUTION:
Required to draw: The line $x = 1$ and triangle $A''B''C''$, the image of triangle ABC after a reflection in the line $x = 1$.
Solution:



The vertices of triangle $A''B''C''$ are $A''(-1, 2)$, $B''(-4, 2)$ and $C''(-1, 4)$.

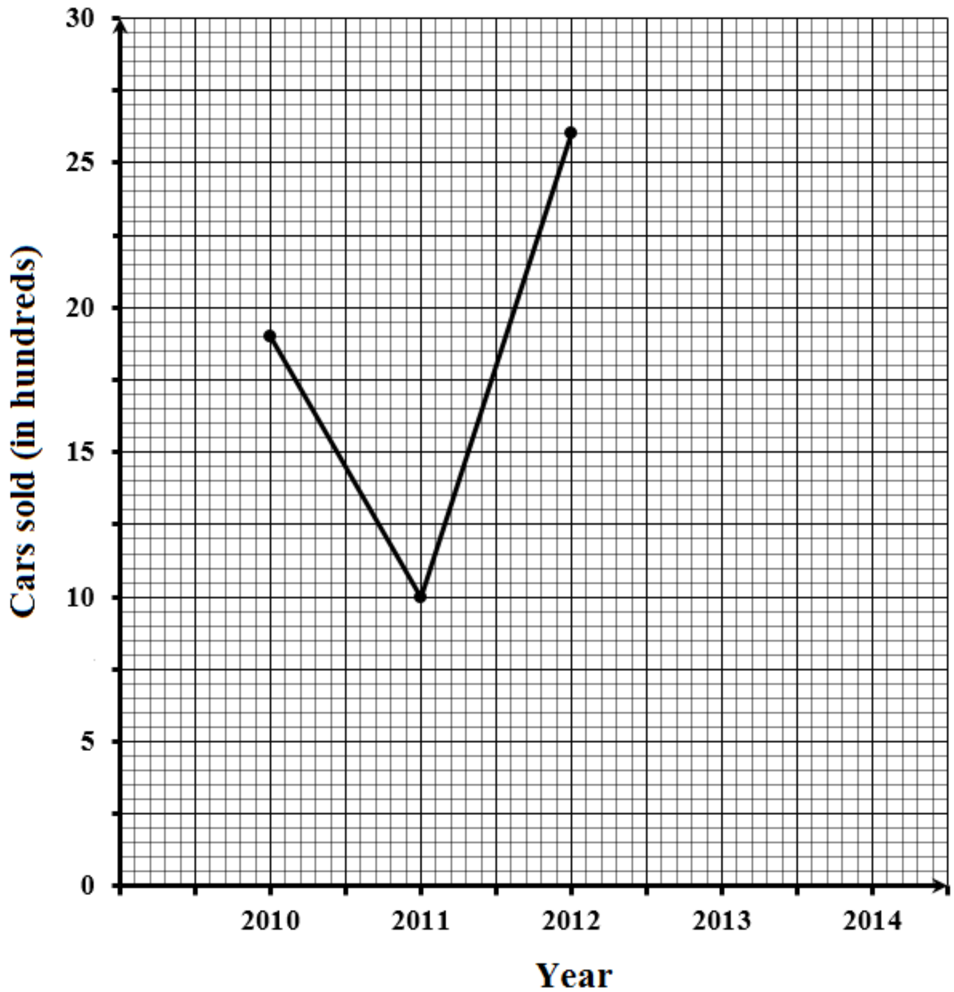
(v) State ONE geometrical relationship among ΔABC, $\Delta A'B'C'$ and $\Delta A''B''C''$.

SOLUTION:
Required to state: A geometrical relationship among ΔABC, $\Delta A'B'C'$ and $\Delta A''B''C''$.
Solution:
The triangles ABC, $A'B'C'$ and $A''B''C''$ are all congruent.

Also, they are all right angled triangles with shorter sides of lengths 2 units and 3 units and hypotenuse of length $\sqrt{13}$ units.

6. (a) The table below gives the number of cars sold in a country, in hundreds, from 2010 to 2014.

Year	2010	2011	2012	2013	2014
Cars sold (in hundreds)	19	10	26	16	30

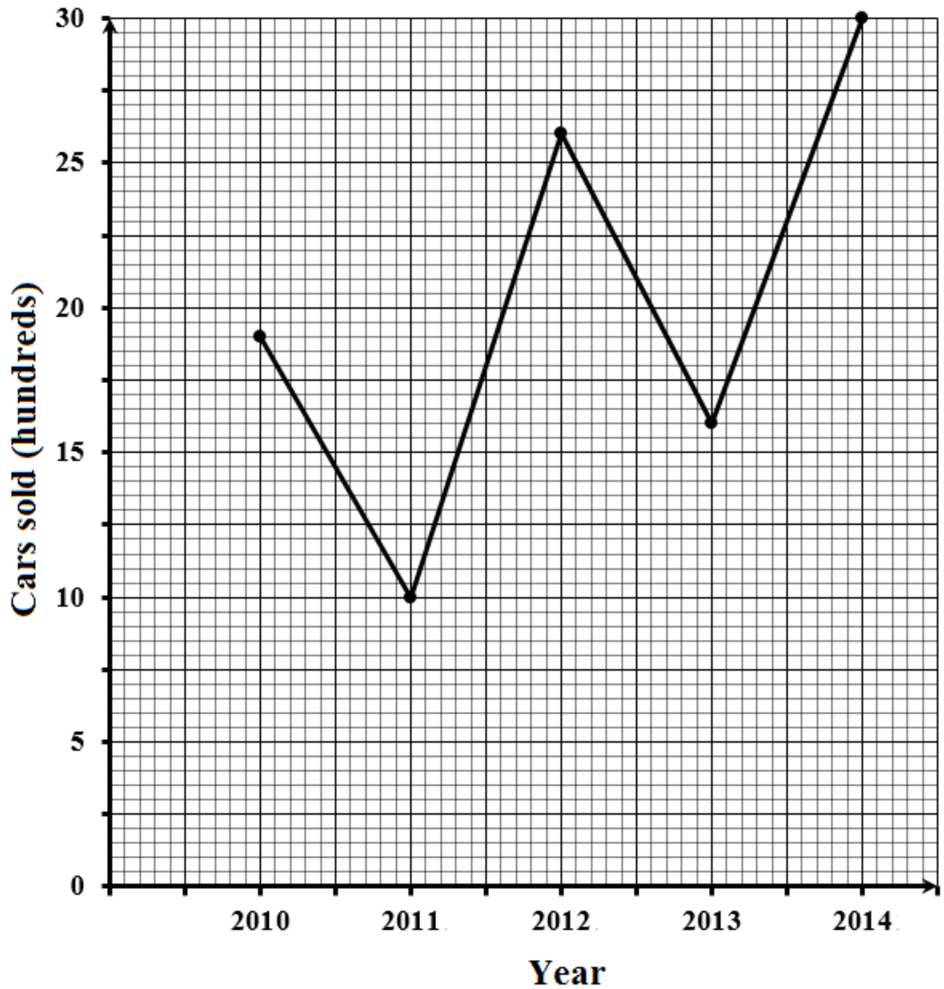


(i) Complete the graph on page 18 to represent the given information.

SOLUTION:

Data: Table showing the number of cars sold in a country, in hundreds, from 2010 to 2014 and the incomplete line graph representing some information from the table.
Required to complete: The line graph to represent the information given

Solution:



(ii) Between which two consecutive years was there the GREATEST increase in the number of cars sold?

SOLUTION:

Required to state: Between which two consecutive years was the greatest increase in the sale of cars?
Solution:

Period in years	**Increase in number of cars sold (Positive implies increase; negative implies decrease, from the previous year)**
2010 – 2011	$1000 - 1900 = -900$
2011 – 2012	$2600 - 1000 = +1600$
2012 – 2013	$1600 - 2600 = -1000$
2013 – 2014	$3000 - 1600 = +1400$

Hence, the greatest increase in the number of cars sold occurs between 2011 and 2012.

(iii) What was the TOTAL number of cars sold in the five year period between 2010 to 2014?

SOLUTION:

Required to calculate: The total number of cars sold in the period between 2010 to 2014
Calculation:
The total number of cars sold in the five year period
$= 1900 + 1000 + 2600 + 1600 + 3000$
$= 10100$

(iv) **Data:** The mean number of cars sold from **2010** to **2015** was 22.5 hundred. How many cars were sold in 2015?

SOLUTION:
Data: The mean number of cars sold from 2010 to 2015 is 22.5 hundred.
Required to calculate: The number of cars sold in 2015.
Calculation:
The mean number of cars sold from 2010 to 2015 is 22.5 hundred
$= 22.5 \times 100$
$= 2250$ cars

Hence, the total number of cars sold in the period 2010 to 2015 = 2250 × 6
$= 13500$

$\therefore$ Number of cars sold in 2015
= Total number of cars sold between 2010 to 2015 (inclusive)– Total number of cars sold between 2010 and 2014 (inclusive)
$= 13500 - 10100$
$= 3400$

(b) **(i)** A line JK has equation $2y = 5x + 6$. Determine the gradient of JK.

SOLUTION:
Data: The line JK has equation, $2y = 5x + 6$.
Required to find: The gradient of the line JK.
Solution:
The equation of the line JK is $2y = 5x + 6$.
$2y = 5x + 6$
$\div 2$

$y = \frac{5}{2}x + 3$ which is of the form $y = mx + c$, where $m = \frac{5}{2}$ is the gradient.

The gradient of the line JK is $\frac{5}{2}$

Another line GH, is perpendicular to JK and passes through the point, $(5, -1)$.

(ii) State the gradient of the line GH.

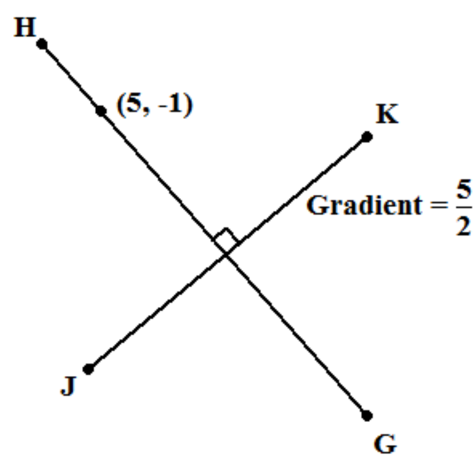
SOLUTION:

Data: The line GH is perpendicular to JK and passes through the point $(5, -1)$.

Required to state: The gradient of the line GH.

Solution:

GH is perpendicular to JK.



$$\text{Gradient of GH} = \frac{-1}{\frac{5}{2}}$$
$$= -\frac{2}{5}$$

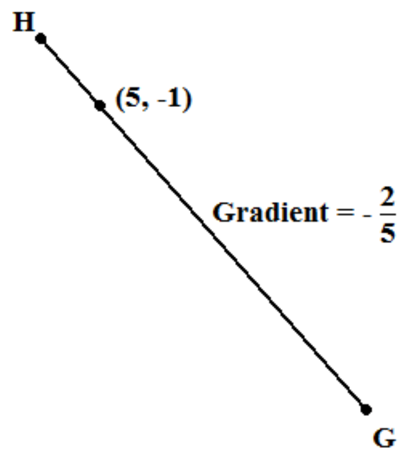
(The product of the gradients of perpendicular lines is -1).

(iii) Determine the equation of line GH.

SOLUTION:

Required to find: The equation of the line GH

Solution:



Equation of GH is $\dfrac{y-(-1)}{x-5} = -\dfrac{2}{5}$

$$5(y+1) = -2(x-5)$$

$$5y+5 = -2x+10$$

$5y = -2x+5$ or any other equivalent form.

7. The table below shows how the minutes taken by all students to complete a science experiment were recorded and grouped.

Time (minutes)	Number of Students who Completed (Frequency)	Cumulative Frequency
1 – 5	1	1
6 – 10	2	3
11 – 15	5	
16 – 20	7	
21 – 25	10	
26 – 30	15	
31 – 35	8	
36 – 40	2	

(a) Complete the cumulative frequency column in the table.

SOLUTION:

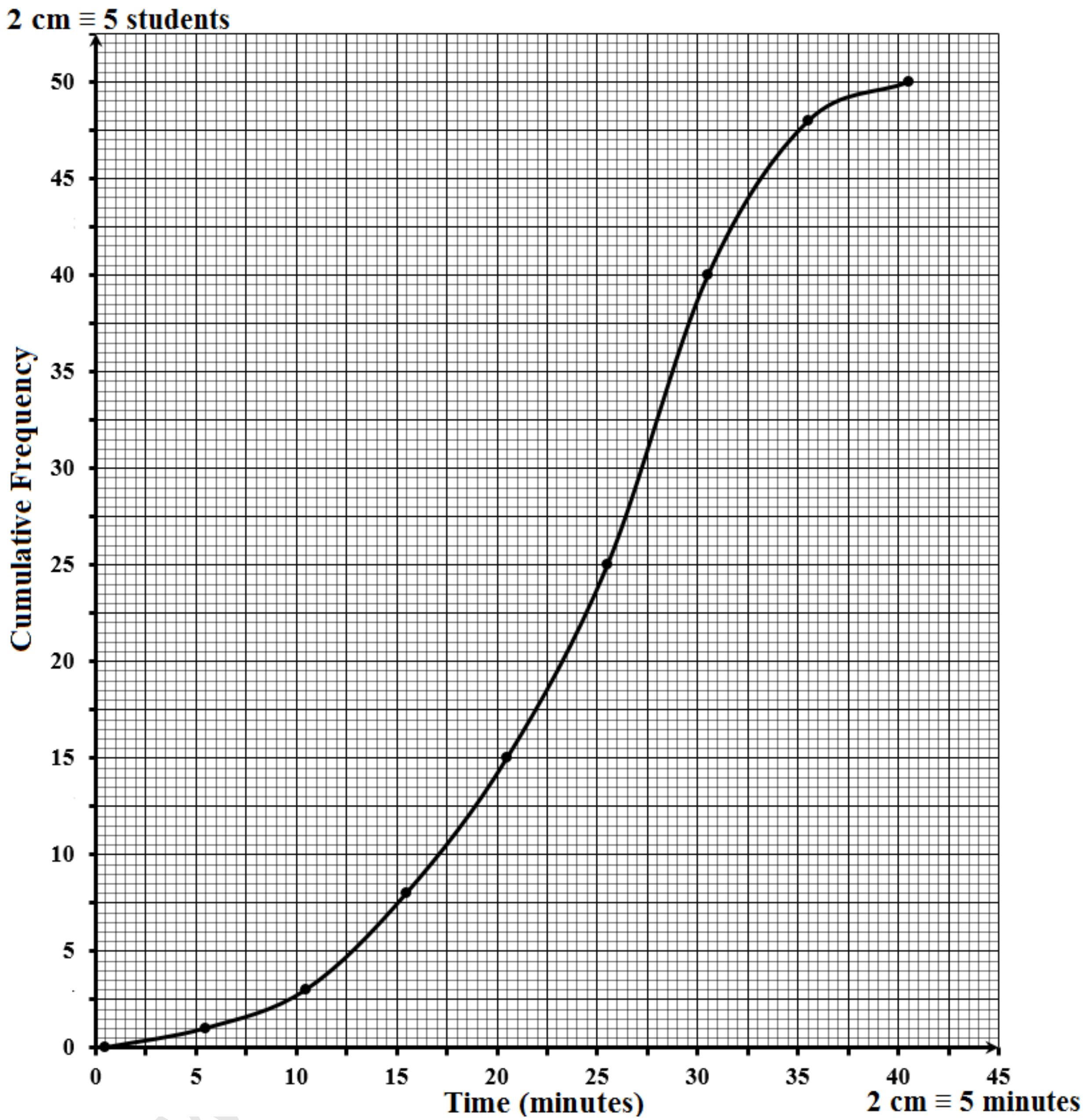
Data: Table showing how the minutes taken by all students to complete a science experiment were recorded and grouped.
Required to complete: The cumulative frequency column in the table
Solution:
The data is continuous. The table is modified to show:

Time, t, in minutes L.C.L. U.C.L	L.C.B U.C.B	No. of Students (f)	Cumulative Frequency	Points to be Plotted (U.C.B, C.F)
				$(0.5, 0)$
1 – 5	$0.5 \le t < 5.5$	1	1	$(5.5, 1)$
6 – 10	$5.5 \le t < 10.5$	2	$2+1=3$	$(10.5, 3)$
11 – 15	$10.5 \le t < 15.5$	5	$3+5=8$	$(15.5, 8)$
16 – 20	$15.5 \le t < 20.5$	7	$8+7=15$	$(20.5, 15)$
21 – 25	$20.5 \le t < 25.5$	10	$15+10=25$	$(25.5, 25)$
26 – 30	$25.5 \le t < 30.5$	15	$25+15=40$	$(30.5, 40)$
31 – 35	$30.5 \le t < 35.5$	8	$40+8=48$	$(35.5, 48)$
36 – 40	$35.5 \le t < 40.5$	2	$48+2=50$	$(40.5, 50)$

(b) On the grid on page 23, using a scale of **2 cm to represent 5 minutes on the *x* – axis** and **2 cm to represent 5 students on the *y* – axis**, draw a cumulative frequency curve to represent the information in the table.

SOLUTION:
Required to draw: a cumulative frequency curve to represent the information in the table, using a scale of 2 cm to represent 5 minutes on the x – axis and 2 cm to represent 5 students on the y – axis.
Solution:

Checking backwards, we obtain the starting point $(0.5, 0)$. This is done so that the curve starts from the horizontal axis.



Using the graph, estimate

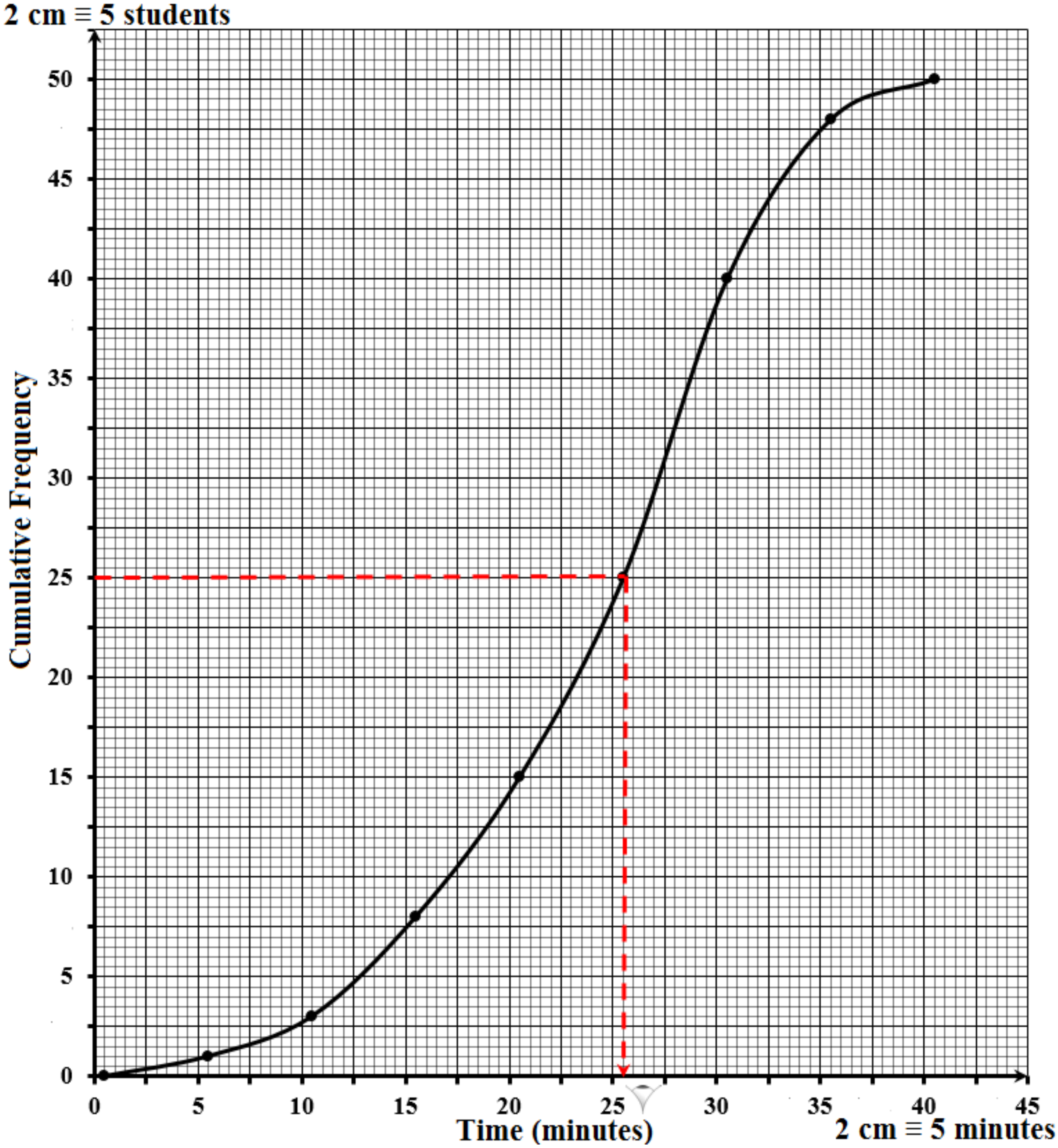
(i) the median time taken to complete the experiment

SOLUTION:

Required to find: The median time taken to complete the experiment from the graph

Solution:

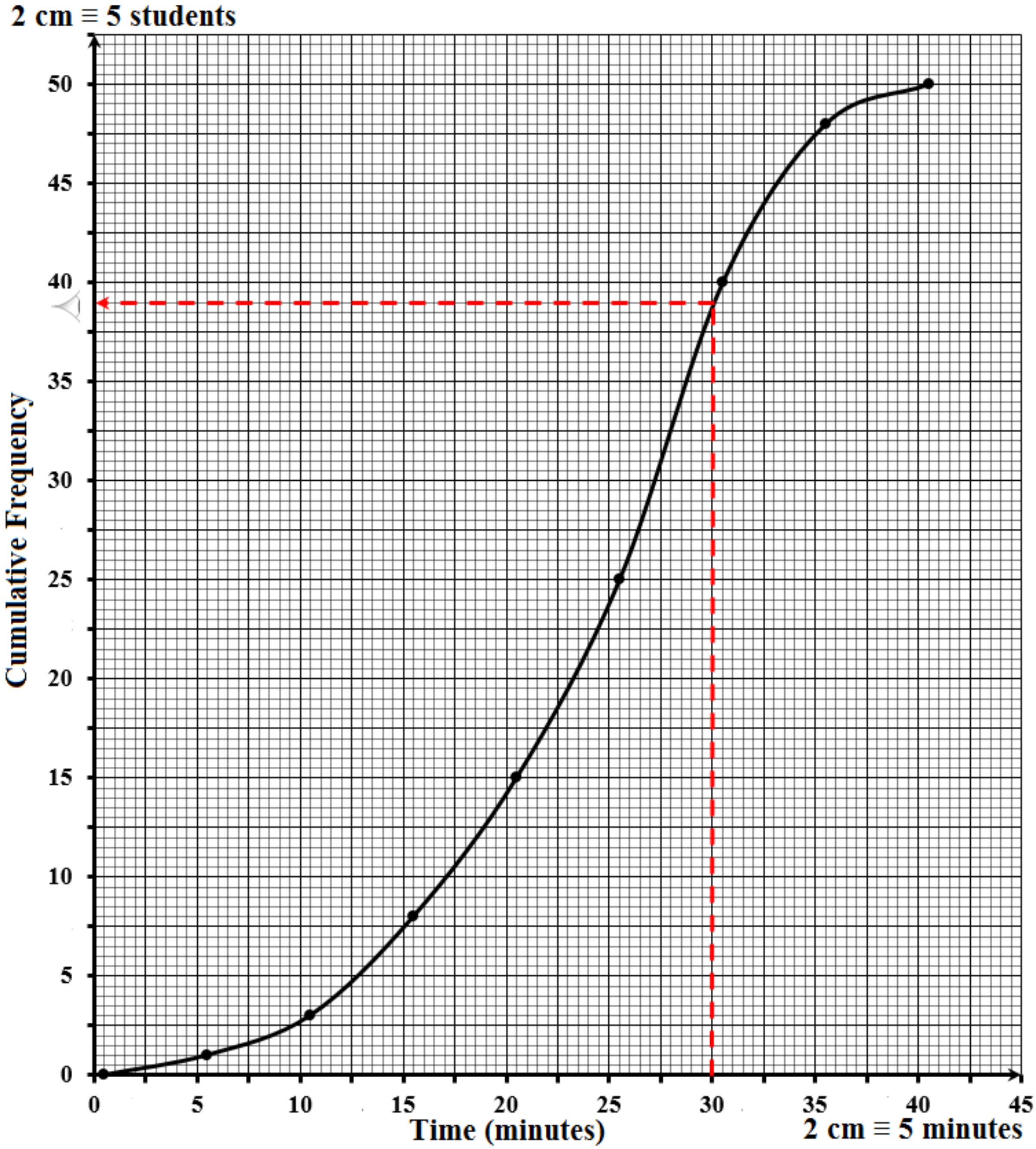
In order to find the median, a horizontal line at cumulative frequency $\frac{1}{2}(50) = 25$ is drawn to meet the curve and a vertical drawn from this point.



$\therefore$ The median time taken is 25.5 minutes.

(ii) The probability that a student, chosen at random, took **30 minutes or less** to complete the experiment

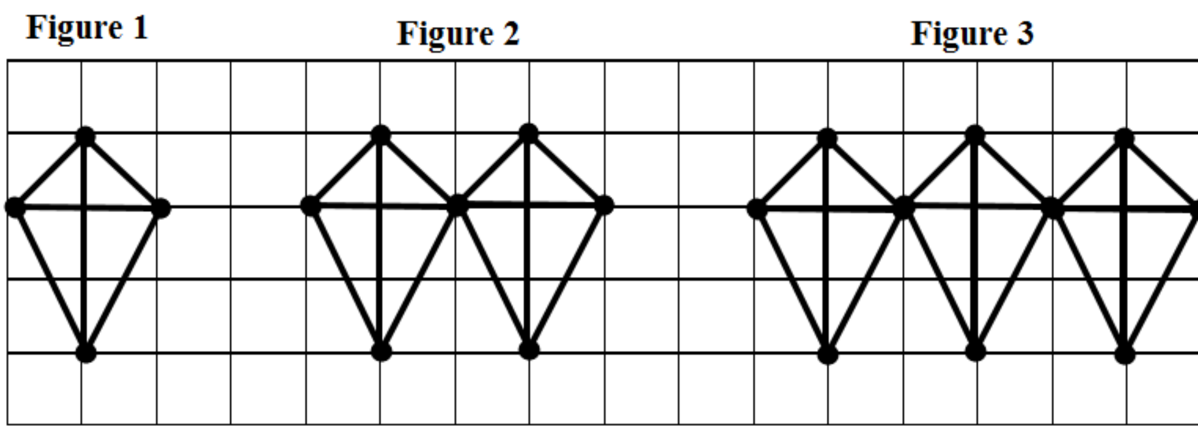
SOLUTION:
Required to find: The probability that a student, chosen at random, took 30 minutes or less to complete the experiment, using the graph.
Solution:



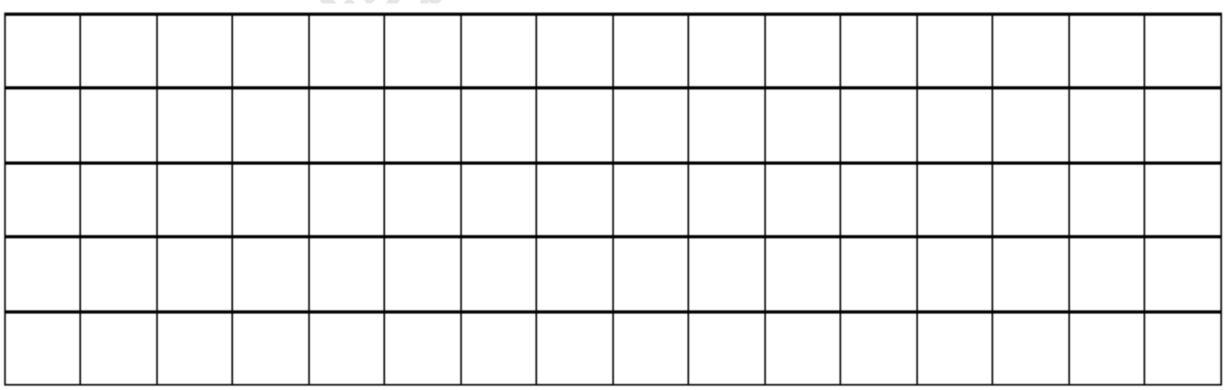
From the curve, the number of students who took 30 minutes or less = 39

$$\therefore \text{P(Student took 30 minutes or less)}$$
$$= \frac{\text{No. of students who took 30 minutes or less}}{\text{Total no. of students}}$$
$$= \frac{39}{50}$$
$$= 0.78$$

8. The diagram below shows the first three figures in a sequence of figures.

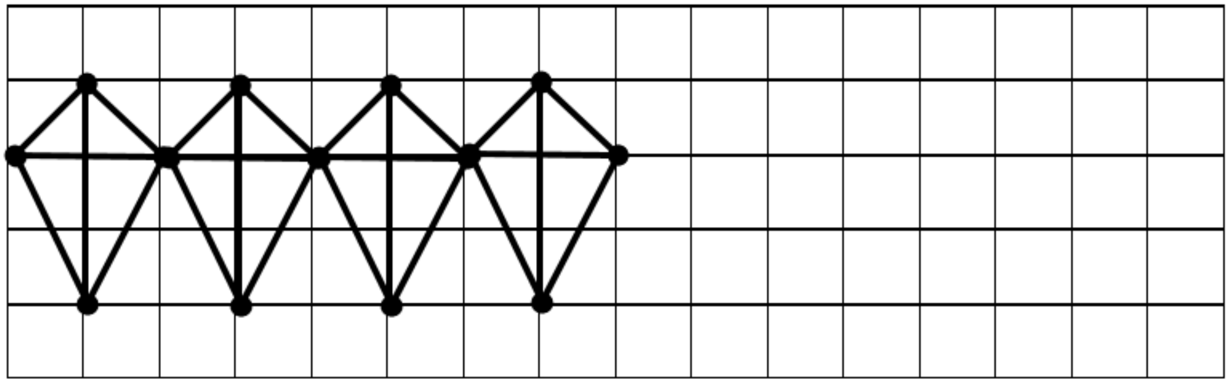


(a) Draw the fourth figure in the sequence.



SOLUTION:

Data: Diagrams showing the first three figures in a sequence of figures.
Required to draw: The fourth diagram in the sequence.
Solution:



(b) The table below shows the number of dots and lines in each figure. Study the pattern in the table and complete the table by inserting the missing values in the rows numbered (i), (ii), (iii) and (iv).

	Figure	Number of Dots	Number of Lines
	1	4	6
	2	7	11
	3	10	16
(i)	4	…………………	………………
		Entries omitted for Figures 5 – 9	
(ii)	10	…………………	………………
		Entries omitted for some Figures	
(iii)	……..	49	………………
		Entries omitted for some Figures	
(iv)	*N*	…………………	………………

SOLUTION:

Data: An incomplete table showing the number of lines and dots in each figure in the sequence.
Required to complete: The table given.
Solution:
Looking at the table to obtain a pattern:

Figure N	No. of Dots D	No. of Lines L
1	4	6
2	7	11
3	10	16
	Since D increases by 3, then $D = 3N + 1$	Since L increases by 5, then $L = 5N + 1$

ALTERNATIVELY,

N	No. of Dots	Pattern
1	4	4 + 0(3)
2	7	4 + 1(3)
3	10	4 + 2(3)
n		$4 + (n - 1)(3)$ $= 4 + 3n - 3$ $= 3n + 1$

N	No. of Lines	Pattern
1	6	6 + 0(5)
2	11	6 + 1(5)
3	16	6 + 2(5)
n		$6 + (n - 1)(5)$ $= 6 + 5n - 5$ $= 5n + 1$

Hence, all the values for (i) to (iv) can now be calculated.

(i) $N = 4$

$D = 3(4) + 1 = 13$

$L = 5(4) + 1 = 21$

(ii) $N = 10$

$D = 3(10) + 1 = 31$

$L = 5(10) + 1 = 51$

(iii) $D = 49$

$49 = 3N + 1$

$3N = 48$

$N = 16$

$L = 5(16) + 1 = 81$

(iv) Already done

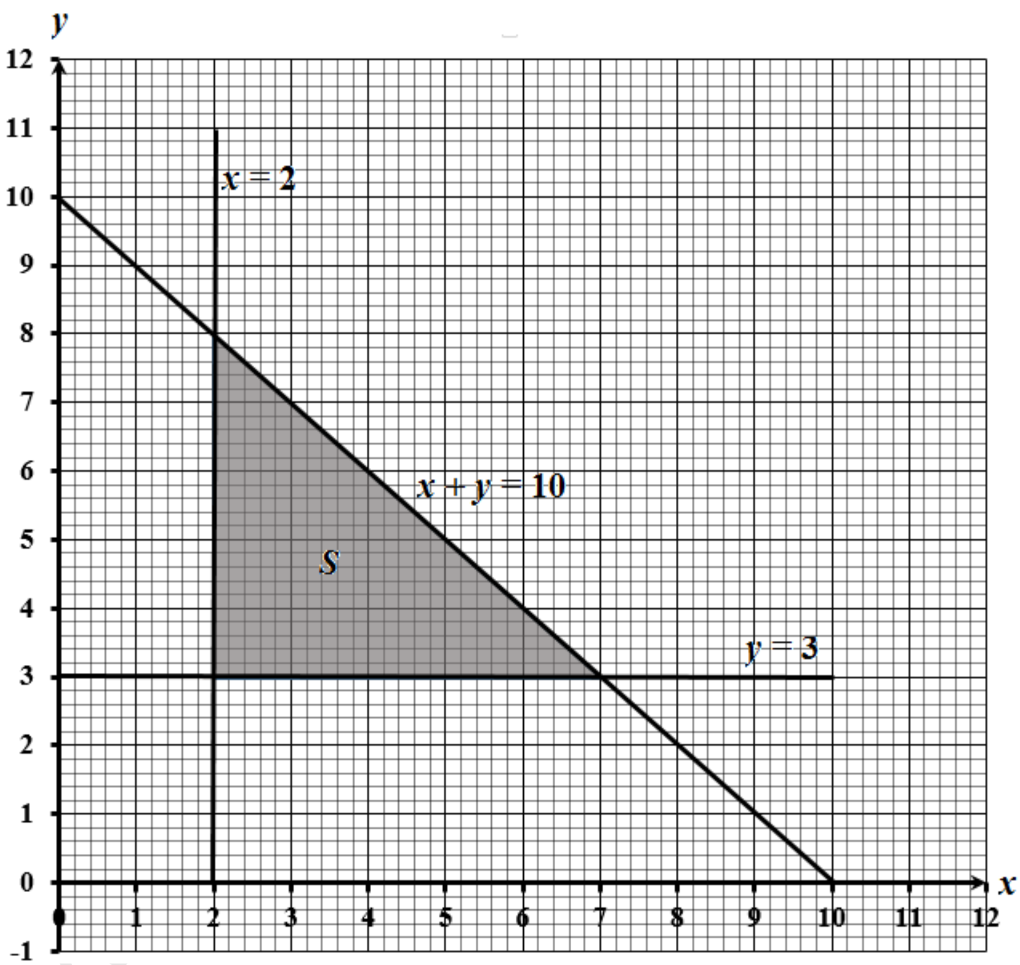
The completed table is:

	Figure	Number of Dots	Number of Lines
	1	4	6
	2	7	11
	3	10	16
(i)	4	13	21
		Entries omitted for Figures 5 – 9	
(ii)	10	31	51
		Entries omitted for some Figures	
(iii)	16	49	81
		Entries omitted for some Figures	
(iv)	N	$3N+1$	$5N+1$

SECTION II
ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) The diagram below shows the graph of three lines and a shaded region, *S*, defined by three inequalities associated with these lines.

The inequality associated with the line $y = 3$ is $y \geq 3$.



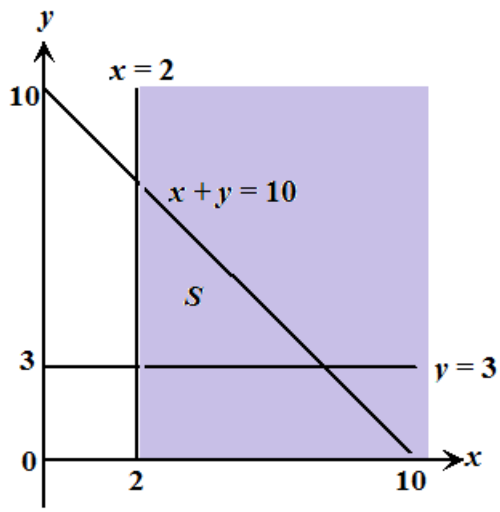
(i) State the other TWO inequalities which define the shaded region.

SOLUTION:

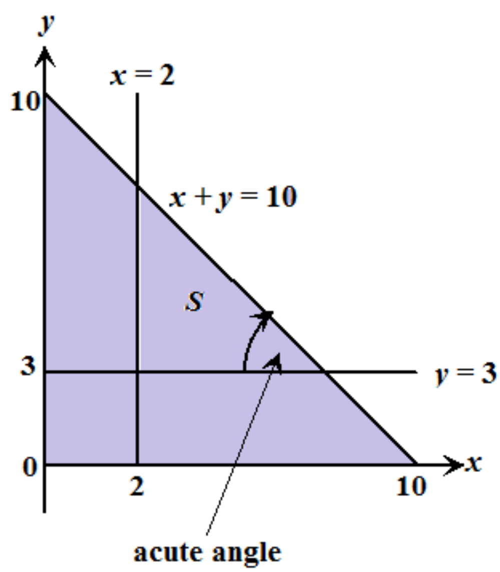
Data: Diagram showing the graph of three lines and a shaded region, *S*, defined by three inequalities associated with these lines. The inequality associated with the line $y = 3$ is $y \geq 3$.

Required to state: The other two inequalities which define the shaded region

Solution:



The right side of the vertical $x = 2$ is shown shaded. This identifies $x \geq 2$ (the line being included).



The side that makes an acute angle with the horizontal is shown shaded. This identifies $x + y \leq 10$ (the line being included).

The function $\mathrm{P} = 5x + 2y - 3$ satisfies the solution set represented by the closed triangular region.

(ii) Identity the three pairs of $(x,\ y)$ values for which P has a maximum or a minimum value.

SOLUTION:
Data: The function $P = 5x + 2y - 3$ satisfies the solution set represented by the closed triangular region.
Required to state: The three pairs of $(x,\ y)$ values for which P has a maximum or a minimum value
Solution:
$P = 5x + 2y - 3$ satisfies the solution set, *S*, represented by the closed triangle. This is *S* and identifies $x \geq 2,\ x + y \leq 10$ and $y \geq 3$. This region is also called the feasible region.

The three pairs of $(x,\ y)$ values for which P has either a maximum or minimum value are from two of the three vertices of the triangle S. The coordinates of the three vertices are $(2,\ 3)$, $(2,\ 8)$ and $(7, 3)$. One set of coordinates will correspond to the maximum value of P and another set of coordinates will correspond to the minimum value of P.

(iii) Which pair of $(x,\ y)$ values make P a **maximum**?

Justify your answer.

SOLUTION:
Required to find: The pair of $(x,\ y)$ values that makes P a maximum
Solution:
Testing each point by substituting in $P = 5x + 2y - 3$.
Taking (2,3)
When $x = 2,\ y = 3$
$P = 5(2) + 2(3) - 3$
$= 13$

Taking (2,8)
When $x = 2,\ y = 8$
$P = 5(2) + 2(8) - 3$
$= 23$

Taking (7,3)
When $x = 7,\ y = 3$
$P = 5(7) + 2(3) - 3$
$= 38$

$\therefore$ Maximum P = 38 and P has a maximum value at the point $(7,\ 3)$.

(b) The functions $f(x)$ and $g(x)$ are defined as follows:

$$f(x)=\frac{3}{2x+1} \text{ and } g(x)=x^2$$

(i) Evaluate EACH of the following:

- $g\left(-\frac{1}{2}\right)$
- $fg\left(-\frac{1}{2}\right)$

SOLUTION:

Data: $f(x)=\frac{3}{2x+1}$ and $g(x)=x^2$

Required to evaluate: $g\left(-\frac{1}{2}\right)$ and $fg\left(-\frac{1}{2}\right)$.

Solution:

- $g(x)=x^2$

$$g\left(-\frac{1}{2}\right)=\left(-\frac{1}{2}\right)^2$$
$$=\frac{1}{4}$$

- $fg\left(-\frac{1}{2}\right)=f\left(\frac{1}{4}\right)$ (recalling that g (1/2) = ¼)

$$=\frac{3}{2\left(\frac{1}{4}\right)+1}$$
$$=\frac{3}{1\frac{1}{2}}$$
$$=2$$

(ii) Write an expression in *x* for $f^{-1}(x)$.

SOLUTION:

Required to find: $f^{-1}(x)$ in terms of *x*.

Solution:

$$\text{Let } y = \frac{3}{2x+1}$$

$$(2x+1)y = 3$$

$$2x+1 = \frac{3}{y}$$

$$2x = \frac{3}{y} - 1$$

$$x = \frac{\frac{3}{y} - 1}{2}$$

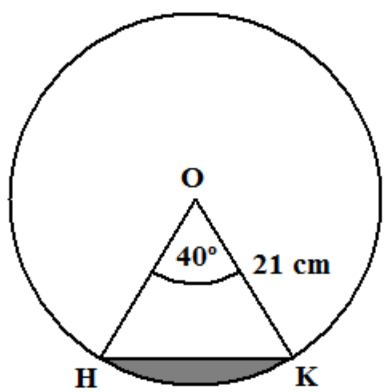
$$x = \frac{3}{2y} - \frac{1}{2}$$

Replace y by x to obtain:

$$f^{-1}(x) = \frac{3}{2x} - \frac{1}{2},\ x \neq 0$$ **OR** $$f^{-1}(x) = \frac{3-x}{2x}, x \neq 0$$

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) The figure below, **not drawn to scale**, shows a circle with center O. The radius of the circle is 21 cm and the angle $\text{HOK} = 40°$.



Use $\pi = \frac{22}{7}$

Determine

(i) the area of the minor sector HOK.

SOLUTION:

Data: A figure showing a circle with center O. The radius of the circle is 21 cm and the angle $\text{HOK} = 40°$.

Required to calculate: Area of the minor sector HOK
Calculation:

$$\text{Area of the minor sector HOK} = \frac{40°}{360°} \times \frac{22}{7}(21)^2$$
$$= 154 \text{ cm}^2$$

(ii) the area of triangle HOK.

SOLUTION:

Required to calculate: Area of triangle HOK.
Calculation:

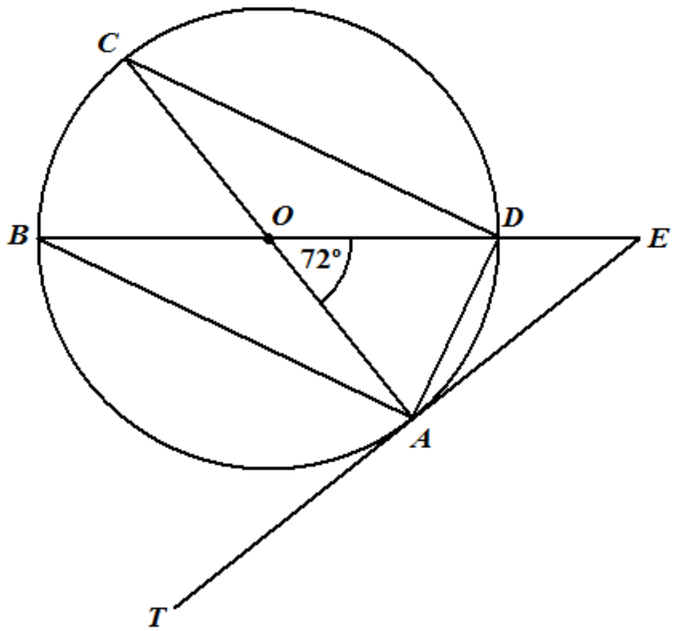
$$\text{Area of triangle HOK} = \frac{1}{2}(21)(21)\sin 40°$$
$$= 141.73\underset{=}{4} \text{ cm}^2$$
$$= 141.73 \text{cm}^2 \text{ (correct to 2 decimal places)}$$

(iii) the area of the shaded segment.

SOLUTION:

Required to calculate: The area of the shaded segment
Calculation:
The area of the shaded segment
= The area of minor sector HOK – The area of triangle HOK
$= 154 - 141.73$
$= 12.27 \text{ cm}^2$

(b) The diagram below, **not drawn to scale**, shows a circle with center *O*. *TAE* is a tangent to the circle at point *A* and angle $AOD = 72°$.

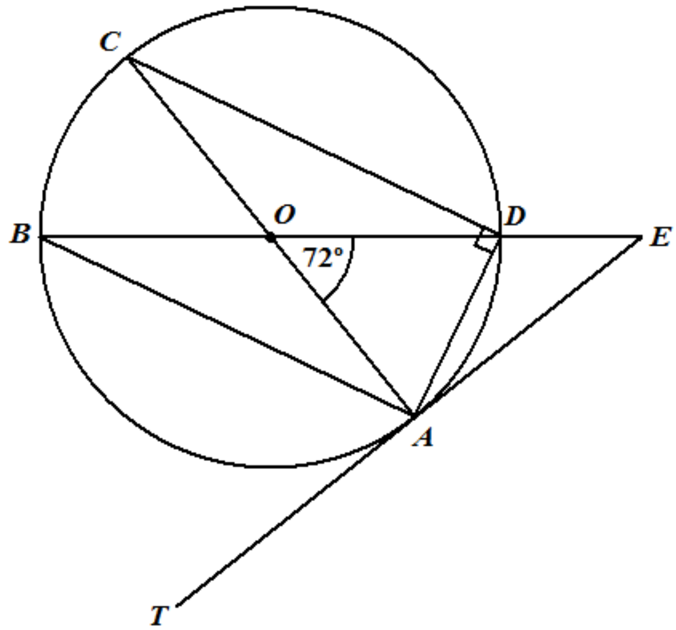


Calculate, giving the reason for each step of your answer, the measure of:

(i) $\angle ADC$

SOLUTION:

Data: Diagram showing a circle with center *O*. *TAE* is a tangent to the circle at point *A* and angle $AOD = 72°$.
Required to calculate: $\angle ADC$
Calculation:
$A\hat{D}C = 90°$
(Angle in a semi-circle = 90°)



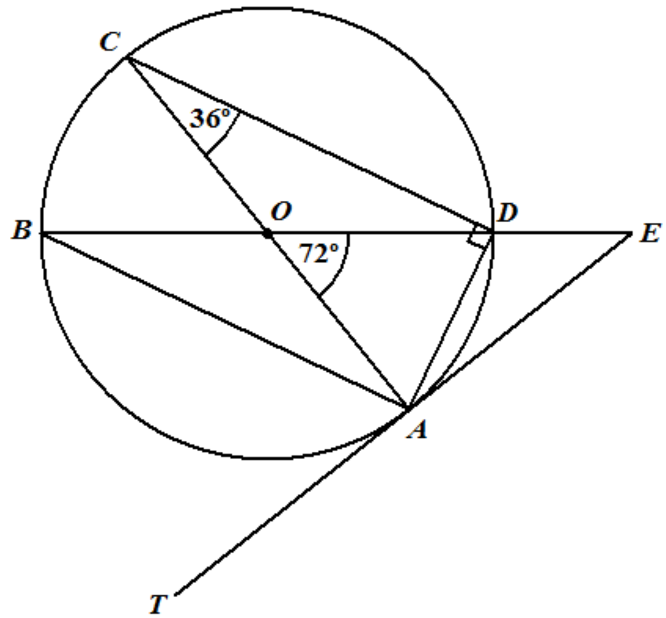
(ii) $\angle ACD$

SOLUTION:

Required to calculate: $\angle ACD$
Calculation:

$$A\hat{C}D = \frac{1}{2}(72°)$$
$$= 36°$$

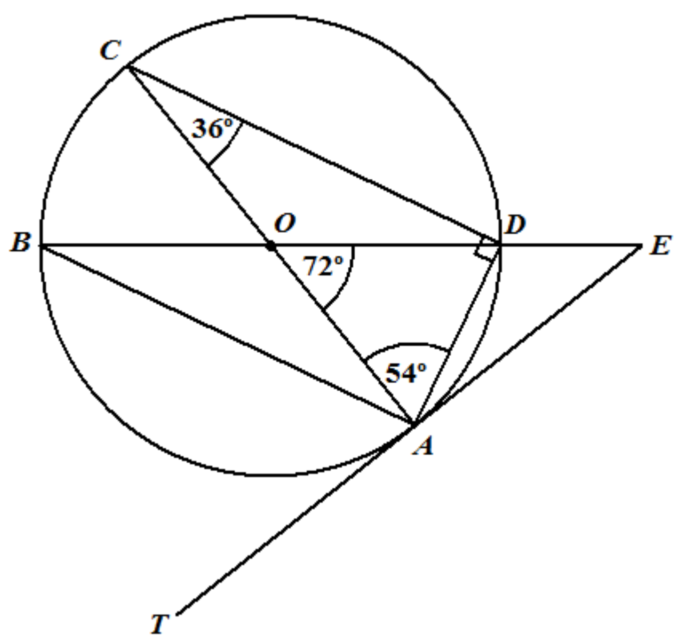
(The angle $A\hat{O}D$ subtended by a chord *AD* at the center of a circle, is twice the angle $A\hat{C}D$ that the chord subtends at the circumference, standing on the same arc.)



(iii) $\angle CAD$

SOLUTION:
Required to calculate: $\angle CAD$
Calculation:
Consider the triangle *CAD*

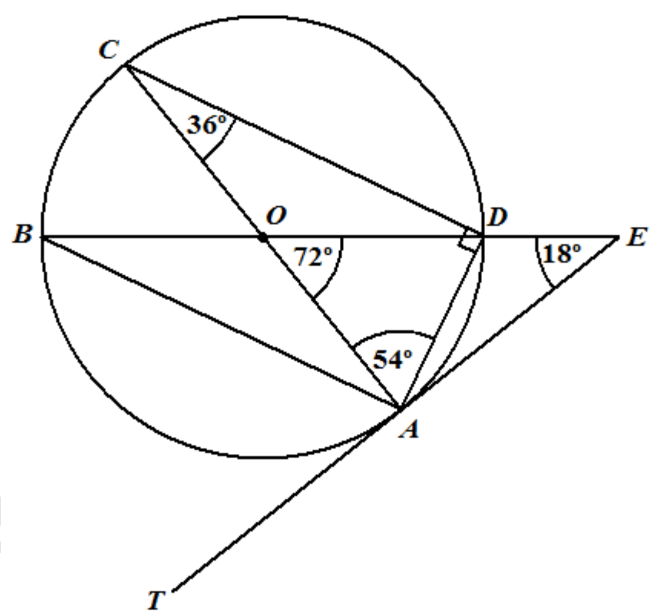
$$C\hat{A}D = 180° - (90° + 36°) \quad \text{(Sum of the interior angles in a triangle = 180°)}$$
$$= 54°$$



(iv) $\angle OEA$

SOLUTION:

Required to calculate: $\angle OEA$
Calculation:
$D\hat{A}E = 36°$
(Angle made by a tangent to a circle and a chord at the point of contact is equal to the angle in the alternate segment.)
Consider triangle *OEA*
Hence angle OEA = angle $DEA = (180^0 - (54^0 + 36^0 + 72^0)) = 18^0$
OR
Angle $OAE = 90^0$ (The angle made by a tangent to a circle and a radius at the point of contact, is a right angle)
Consider triangle *OAE*
Angle $OEA = 180^0 - (90^0 + 72^0) = 18^0$
(The sum of the interior angles of a triangle = 180°)



VECTORS AND MATRICES

11. (a) The points A, B and C have coordinates $A\,(-2, 8)$, $B\,(4, 2)$ and $C\,(0, 9)$. M is the midpoint of the line segment AB.

(i) Express EACH of the following in the form $\begin{pmatrix} x \\ y \end{pmatrix}$:

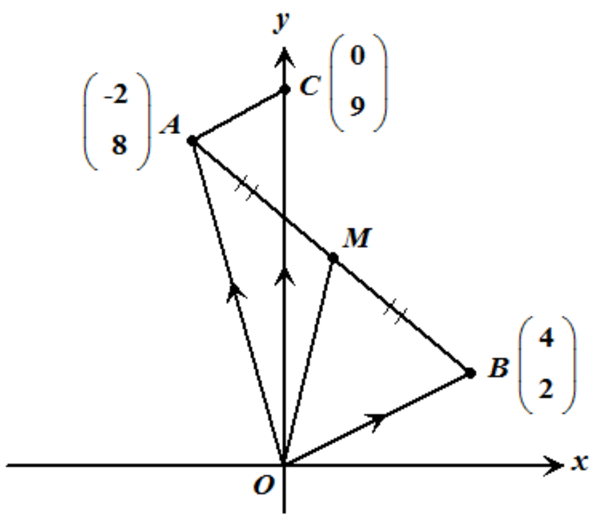
- OB
- AB
- OM

SOLUTION:

Data: The points A, B and C have coordinates $A(-2, 8)$, $B(4, 2)$ and $C(0, 9)$. M is the midpoint of the line segment AB.

Required to express: OB, AB and OM of the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:



If $B = (4, 2)$

then $OB = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x = 4$ and $y = 2$.

Similarly,

$A = (-2, 8)$

$OA = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$

$AB = AO + OB$

$= -\begin{pmatrix} -2 \\ 8 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -6 \end{pmatrix}$ is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x = 6$ and $y = -6$.

$$AM = \frac{1}{2}AB$$
$$= \frac{1}{2}\begin{pmatrix} 6 \\ -6 \end{pmatrix}$$
$$= \begin{pmatrix} 3 \\ -3 \end{pmatrix}$$

$$OM = OA + AM$$
$$= \begin{pmatrix} -2 \\ 8 \end{pmatrix} + \begin{pmatrix} 3 \\ -3 \end{pmatrix}$$
$$= \begin{pmatrix} 1 \\ 5 \end{pmatrix} \text{ is of the form } \begin{pmatrix} x \\ y \end{pmatrix}, \text{ where } x = 1 \text{ and } y = 5.$$

(ii) Using a vector method, show that AC and OB are parallel.

SOLUTION:
Required to prove: AC and OB are parallel

Proof:

$$AC = AO + OC$$
$$= -\begin{pmatrix} -2 \\ 8 \end{pmatrix} + \begin{pmatrix} 0 \\ 9 \end{pmatrix}$$
$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$AC = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$OB = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$
$$= 2\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$
$$= 2AC$$

$\therefore$ AC is a scalar multiple of OB and hence the two vectors AC and OB are parallel.

(b) The matrix M is defined as $M = \begin{pmatrix} 2p & -3 \\ 4 & 1 \end{pmatrix}$.

Determine the value of p for which the matrix M is singular.

SOLUTION:

Data: $M = \begin{pmatrix} 2p & -3 \\ 4 & 1 \end{pmatrix}$

Required to calculate: The value of p for which the matrix M is singular.

Calculation:

If M is singular then it does not have an inverse and $\det M = 0$.

$$\therefore (2p \times 1) - (-3 \times 4) = 0$$
$$2p + 12 = 0$$
$$2p = -12$$
$$p = -6$$

(c) A and B are two 2×2 matrices such that $A = \begin{pmatrix} 1 & 2 \\ -4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & -1 \\ 0 & 3 \end{pmatrix}$.

(i) Calculate $2A + B$.

SOLUTION:

Data: $A = \begin{pmatrix} 1 & 2 \\ -4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & -1 \\ 0 & 3 \end{pmatrix}$

Required to calculate: $2A + B$

Calculation:

$$2A + B = 2\begin{pmatrix} 1 & 2 \\ -4 & 3 \end{pmatrix} + \begin{pmatrix} 5 & -1 \\ 0 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} 2 & 4 \\ -8 & 6 \end{pmatrix} + \begin{pmatrix} 5 & -1 \\ 0 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} 2+5 & 4+-1 \\ -8+0 & 6+3 \end{pmatrix}$$
$$= \begin{pmatrix} 7 & 3 \\ -8 & 9 \end{pmatrix}$$

(ii) Determine B^{-1}, the inverse of B.

SOLUTION:

Required to find: B^{-1}
Solution:

$$\det B = (5\times 3) - (-1\times 0)$$
$$= 15$$

$$\therefore B^{-1} = \frac{1}{15}\begin{pmatrix} 3 & -(-1) \\ -(0) & 5 \end{pmatrix}$$
$$= \begin{pmatrix} \frac{3}{15} & \frac{1}{15} \\ 0 & \frac{5}{15} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{1}{5} & \frac{1}{15} \\ 0 & \frac{1}{3} \end{pmatrix}$$

(iii) Given that $\begin{pmatrix} 5 & -1 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$, calculate the values of x and y.

SOLUTION:

Data: $\begin{pmatrix} 5 & -1 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$
Required to calculate: The values of x and y.
Calculation:
Recall: $B = \begin{pmatrix} 5 & -1 \\ 0 & 3 \end{pmatrix}$
Pre-multiply the matric equation by B^{-1}.

$$B^{-1}\times B\times \begin{pmatrix} x \\ y \end{pmatrix} = B^{-1}\begin{pmatrix} 9 \\ 3 \end{pmatrix}$$

We know that

$$B^{-1}B = I$$

and, $I\times \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

Hence, the matrix equation can be simplified to:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & \frac{1}{15} \\ 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 9 \\ 3 \end{pmatrix}$$

$2 \times 2 \quad 2 \times 1$

$$= \begin{pmatrix} e_{11} \\ e_{21} \end{pmatrix}$$

$$e_{11} = \left(\frac{1}{5} \times 9\right) + \left(\frac{1}{15} \times 3\right) = 2$$

$$e_{21} = (0 \times 9) + \left(\frac{1}{3} \times 3\right) = 1$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Equating corresponding entries we obtain:
$\therefore x = 2$ and $y = 1$

NOTE: It is incorrect to say 'values of *x* and *y*,' since there is only one value for *x* and *y*. It should be, calculate or find the value of *x* and of *y*.