FAS-PASS Maths

CSEC MATHEMATICS MAY-JUNE 2014

SECTION I

1. (a) (i) **Required to calculate:** The exact value of $5.25 \div 0.015$.
Calculation:

$$5.25 \div 0.015 = \frac{5.25}{0.015}$$
$$= \frac{5250}{15}$$
$$= \frac{1050}{3}$$
$$= 350$$

This is the result in exact form

OR

We can obtain the same answer of 350 by using the calculator.

(ii) **Required to calculate:** The exact value of $\sqrt{6.5025}$
Calculation:

$\sqrt{6.5025} = 2.55$ (by calculator)
This question might be difficult to solve by the arithmetic method, so the calculator usage would be best.

(iii) **Required to calculate:** The exact value of, 3.142×2.236^2.
Calculation:

$$3.142 \times 2.236^2 = 3.142 \times 2.236 \times 2.236$$ (using the calculator)
$$= 15.7\underline{\underline{0}}9$$

The underlined digit, which is the 4th digit, counting from left to right, is less than 5 and so this underlined digit, which is referred to as the deciding digit and all of the digits written to its right are ignored, to express the answer to 3 s.f . In this case, the third digit remains unaltered as 7.

Answer = 15.7 (when expressed to 3 significant figures)

(b) **Data:** Ratio of cement, sand and gravel is 1:4:6.
(i) **Required to calculate:** The number of buckets of gravel needed for 4 buckets of cement.
Calculation:

Cement	Sand	Gravel
1	4	6

According to the table shown above, 1 bucket of cement requires 6 buckets of gravel.
Hence, 4 buckets of cement would require $6 \times 4 = 24$ buckets of gravel.
Answer = 24 buckets of gravel

(ii) **Data:** 20 buckets of sand are used.

a) **Required to calculate:** The number of buckets of cement needed
Calculation:

Cement	Sand	Gravel
1 1×5 5	4 4×5 20	6 6×5 30

4 buckets of sand is to be mixed with 1 bucket of cement
So, 1 bucket of sand will be mixed with ¼ bucket of cement
And 20 buckets of sand will be used with ¼ × 20 = 5 buckets of cement.
Hence, the amount of cement to be used with 20 buckets of sand will be 5 buckets.

Answer = 5 buckets

b) **Required to calculate:** The number of buckets of gravel needed.
Calculation:
1 bucket of cement is to be mixed with 6 buckets of gravel
Hence 5 buckets of cement will be mixed with 5 × 6 = 30 buckets of gravel.
This is also illustrated in the above table, the amount of gravel needed will be 30 buckets.
Answer = 30 buckets

(c) **Data:** The cash price of a laptop = $1299.
The hire purchase plan requires $350 deposit and 10 equal monthly payments of $120.

(i) **Required to calculate:** The hire purchase price of the laptop
Calculation:
The cost of the laptop, using the hire purchase plan will be the deposit added to the total amount that is to be paid in the 10 monthly payments.

$= \$350 + (10 \times \$120)$

$= \$350 + \$1200$

$= \$1550$

(ii) **Required to calculate:** The amount of money saved by buying the laptop for cash.

Calculation:

The hire purchase plan costs more than the cash price

The amount saved by paying cash = The hire purchase price – The cash price

$= \$1550 - \$1299$

$= \$251$

2. (a) **Required to write:** $\dfrac{x-2}{3}+\dfrac{x+1}{4}$ as a single fraction in its lowest terms.

Solution:

The LCM of 3 and 4 is 12

So,

$$\frac{x-2}{3}+\frac{x+1}{4}$$

$$\frac{4(x-2)+3(x+1)}{12}=\frac{4x-8+3x+3}{12}$$

$$=\frac{7x-5}{12} \text{ (as a single fraction in its lowest terms)}$$

(b) (i) **Required to write:** An equation to represent that statement given.

Solution:

When 4 is added to a certain number, the result is the same as halving the number and adding 10.

Let the unknown number be written as x.

4 added to x is $4 + x$

Halving the number is $\frac{1}{2}x$

Then adding to 10 gives $\frac{1}{2}x + 10$

Hence, $4+x$ is the same as $\frac{1}{2}(x)+10$ (data)

$$\therefore 4+x=\frac{1}{2}x+10$$

$$4+x-\frac{1}{2}x-10=0$$

$$\frac{1}{2}x-6=0$$

when expressed in a simplified form.

(ii) **Required to write:** An equation to represent that statement given.

Solution:

Squaring a number and subtracting 6 gives the same result as doubling the number and adding 9.

Let the number be y.

The square of y is $y \times y = y^2$

Now subtracting will give $y^2 - 6$

Doubling y gives $y \times 2 = 2y$

Then adding 9 gives $2y + 9$

$(y)^2 - 6$ is the same as $2 \times y + 9$ (data)

$$y^2 - 6 = 2y + 9$$

$$y^2 - 6 - 2y - 9 = 0$$

$$y^2 - 2y - 15 = 0$$

when expressed in a simplified form

(c) **Data:**

$$\boxed{\text{Input } x} \rightarrow \boxed{\text{Multiply by } 3} \rightarrow \boxed{\text{Add } 5} \rightarrow \boxed{\text{Output } y}$$

(i) **Required to write:** A formula for y in terms of x.

Solution:

$$\boxed{\text{Input } x} \rightarrow \boxed{\text{Multiply by } 3} \rightarrow \boxed{\text{Add } 5} \rightarrow \boxed{\text{Output } y}$$

$$\underbrace{x \times 3}_{= 3x}$$

$$\underbrace{\qquad\qquad}_{3x + 5}$$

$$\underbrace{\qquad\qquad}_{3x + 5 = y}$$

or

$$y = 3x + 5$$

(ii) **Required to find:** The number that would be the output if the number 4 is the input.

Solution:

If the input, x, is 4, then the output y could be calculated as

$$y = 3(4) + 5$$
$$= 12 + 5$$
$$= 17$$

$\therefore$ The output number is 17 as shown above.

(iii) **Required to find:** The input number if the output number is 8.
Solution:
If the output is 8 then the input, which is denoted by x could be calculated by saying

$$8 = 3x + 5$$
$$8 - 5 = 3x$$
$$3x = 3$$
$$x = \frac{3}{3}$$
$$x = 1$$

$\therefore$ The input number is 1 when the output is 8.

(iv) **Required to reverse:** The formula in (c) (i) to write x in terms of y.

Solution:
We would therefore be trying to make x the subject in the equation

$$y = 3x + 5$$
$$3x + 5 = y$$
$$3x = y - 5$$
$$x = \frac{y-5}{3}$$

(d) **Data:** $2x + 3y = 9$ and $3x - y = 8$
Required to solve: The pair of simultaneous equations given
Solution:
Let
$2x + 3y = 9$ …❶ and
$3x - y = 8$ …❷
Using the method of substitution we can say,
From equation ❷
$3x - y = 8$
$3x - 8 = y$

Substituting the expression for $y = 3x - 8$ in equation ❶, we get

$$
\begin{aligned}
2x+3(3x-8)&=9\\
2x+9x-24&=9\\
2x+9x&=9+24\\
11x&=33\\
x&=\frac{33}{11}\\
x&=3
\end{aligned}
$$

Substituting $x=3$ in the expression, $y=3x-8$, we get

$$
\begin{aligned}
y&=3(3)-8\\
&=9-8\\
&=1
\end{aligned}
$$

$\therefore x=3$ and $y=1$.

We could also have substituted $x = 1$ in either of the equations to find y.

Alternative Method:
Using the method of elimination,
Let
$2x+3y=9 \quad \ldots$❶ and
$3x-y=8 \quad \ldots$❷

Equation ❷ $\times 3$
$9x-3y=24 \quad \ldots$❸

Equation ❶ + Equation ❸ will eliminate y

$$
\begin{array}{r}
2x+3y=9 \quad + \\
\underline{9x-3y=24} \\
\underline{11x \qquad =33}
\end{array}
$$

$$
\begin{aligned}
x&=\frac{33}{11}\\
&=3
\end{aligned}
$$

Now, substituting $x=3$ into equation ❶, we get

$$2(3)+3y=9$$
$$3y=9-6$$
$$3y=3$$
$$y=\frac{3}{3}$$
$$=1$$

$\therefore x=3$ and $y=1$.

(We could have substituted $x=3$ in the other equation, that is equation (2) to obtain $y=1$)

Alternative Method:

Using the matrix method

Let

$2x+3y=9$ …❶ and

$3x-y=8$ …❷

Writing the equations in matrix form by using the coefficients of *x* and *y* in the given equations we obtain

$$\begin{pmatrix} 2 & 3 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 9 \\ 8 \end{pmatrix}\text{…matrix equation}$$

Let $A=\begin{pmatrix} 2 & 3 \\ 3 & -1 \end{pmatrix}$ the 2 x 2 matrix in the matrix equation.

We now find the inverse of *A* denoted by A^{-1}

$$\det A=(2\times -1)-(3\times 3)$$
$$=-2-9$$
$$=-11$$

$$A^{-1}=-\frac{1}{11}\begin{pmatrix} -1 & -(3) \\ -(3) & 2 \end{pmatrix}$$
$$=\begin{pmatrix} \frac{1}{11} & \frac{3}{11} \\ \frac{3}{11} & -\frac{2}{11} \end{pmatrix}$$

Recall a matrix multiplied by its inverse gives the identity matrix and the identity matrix multiplied by any matrix gives back the same matrix.

Multiplying the matrix equation by A^{-1}, we obtain

$$\begin{pmatrix} \frac{1}{11} & \frac{3}{11} \\ \frac{3}{11} & -\frac{2}{11} \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{11} & \frac{3}{11} \\ \frac{3}{11} & -\frac{2}{11} \end{pmatrix}\begin{pmatrix} 9 \\ 8 \end{pmatrix}$$

$$I\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \left(\frac{1}{11}\times 9\right)+\left(\frac{3}{11}\times 8\right) \\ \left(\frac{3}{11}\times 9\right)+\left(-\frac{2}{11}\times 8\right) \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{9}{11}+\frac{24}{11} \\ \frac{27}{11}-\frac{16}{11} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{33}{11} \\ \frac{11}{11} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

Both sides of the equation are now equal 2 × 1 matrices.
So, equating corresponding elements we can find x and y
$\therefore x = 3$ and $y = 1$.

Alternative Method:
The graphical method is being illustrated
We treat each equation as a straight line and by first finding two points on each line we draw each line on the same axes.

$2x + 3y = 9$

x	y
0	3
$4\frac{1}{2}$	0

We obtain the coordinates of any two points on this line.

When $x = 0$

$$2(0) + 3y = 9$$
$$3y = 9$$
$$y = 3$$

$3x - y = 8$

x	y
0	-8
$2\frac{2}{3}$	0

We obtain the coordinates of any two points on this line.

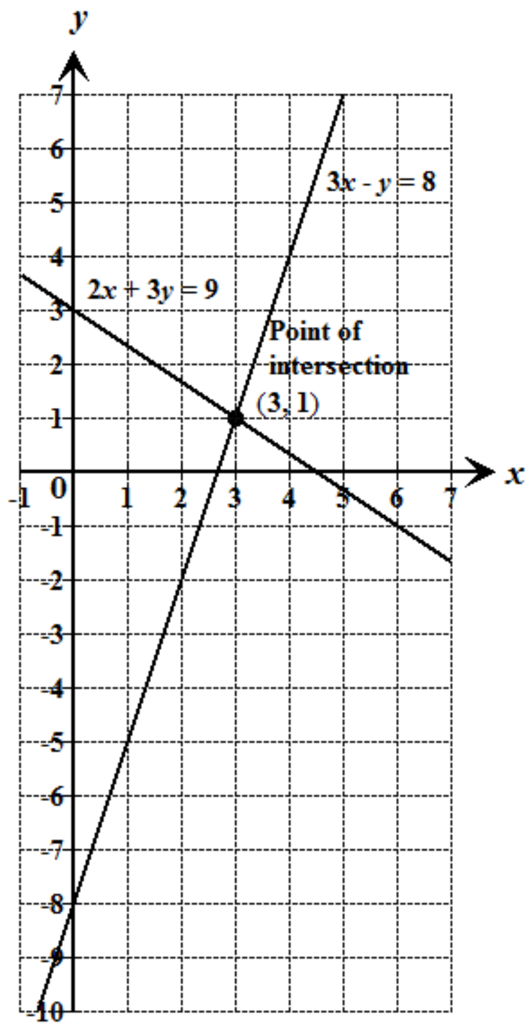
When $x = 0$

$$3(0) - y = 8$$
$$y = -8$$

When $y=0$
$2x+3(0)=9$
$2x=9$
$x=\frac{9}{2}$ or $4\frac{1}{2}$

When $y=0$
$3x-0=8$
$x=\frac{8}{3}$ or $2\frac{2}{3}$

We draw both lines on the same coordinate axes and extend either or both, only if necessary, so that their point of intersection is obtained. This is shown on the diagram below.



The point of intersection is (3,1) obtained by a read-off. The x coordinate of this point is 3 and the y coordinate is 1. This gives the solution to the equations as

$x=3$ and $y=1$.

3. (a) **Data:** $U=\{\text{Integers between 11 and 26}\}$, $A=\{\text{Even numbers}\}$ and $B=\{\text{Multiples of 3}\}$.

(i) **Required to calculate:** The number of members in the universal set, U.
Calculation:

$U = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26\}$

$\therefore n(U) = 16$ (found by checking)

Alternative Method:
We could have said that

$n(U) = 26 - 11 + 1$

$\quad = 16$

(ii) **Required to list:** The members of the set A which are the set of even numbers from 11 to 26

Solution:

$A = \{12, 14, 16, 18, 20, 22, 24, 26\}$

(iii) **Required to list:** The members of the subset B.

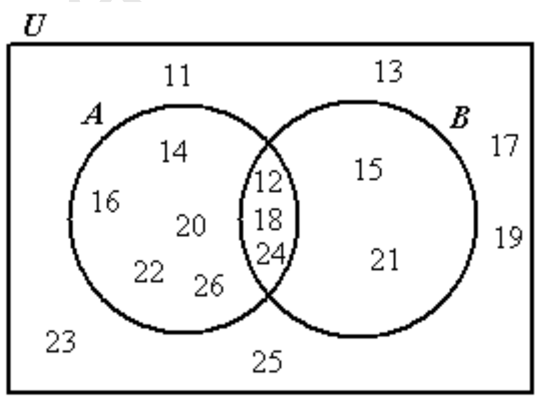
Solution:

These are the multiples of 3 from 11 to 26

$B = \{12, 15, 18, 21, 24\}$

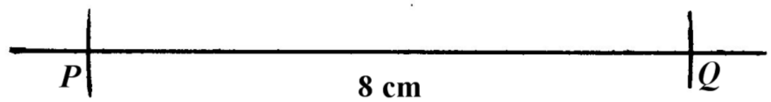
(iv) **Required to draw:** A Venn diagram to represent the relationships among A, B and U.

Solution:

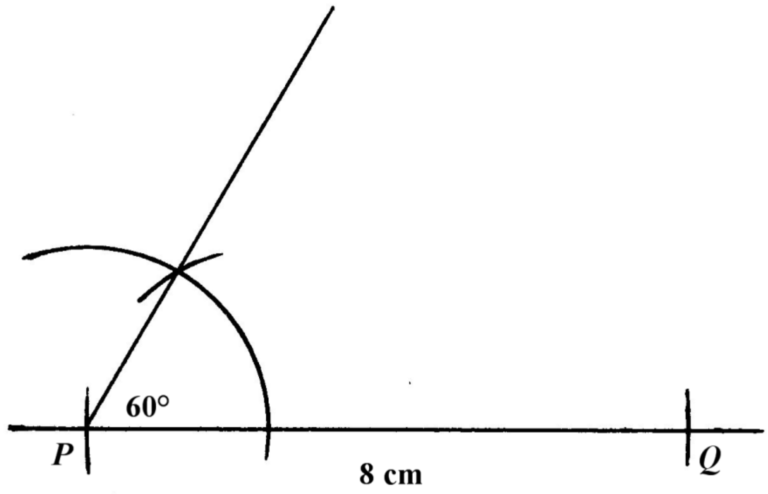


(b) (i) a) **Required to construct:** Triangle PQR in which PQ = 8 cm, PR = 6 cm and angle P = 60°.

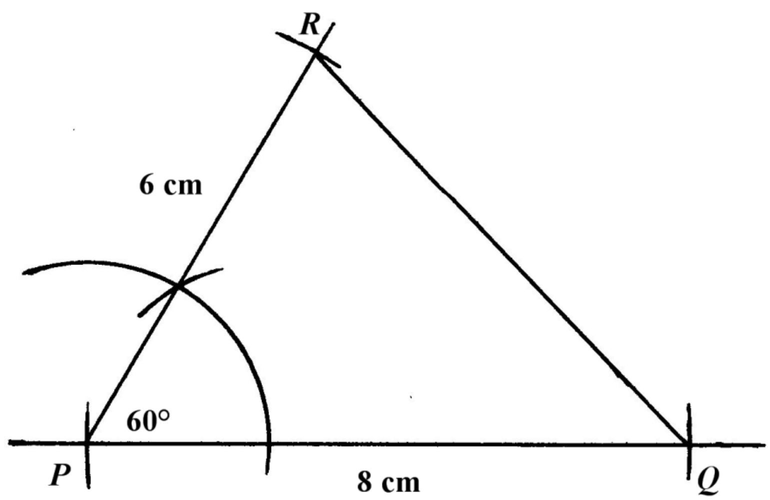
Construction: We first draw a straight line longer that 8 cm and with the pair of compasses, we cut off a segment that is 8 cm in length. The arcs must be clearly shown.



At the point, P, we construct an angle of 60^0



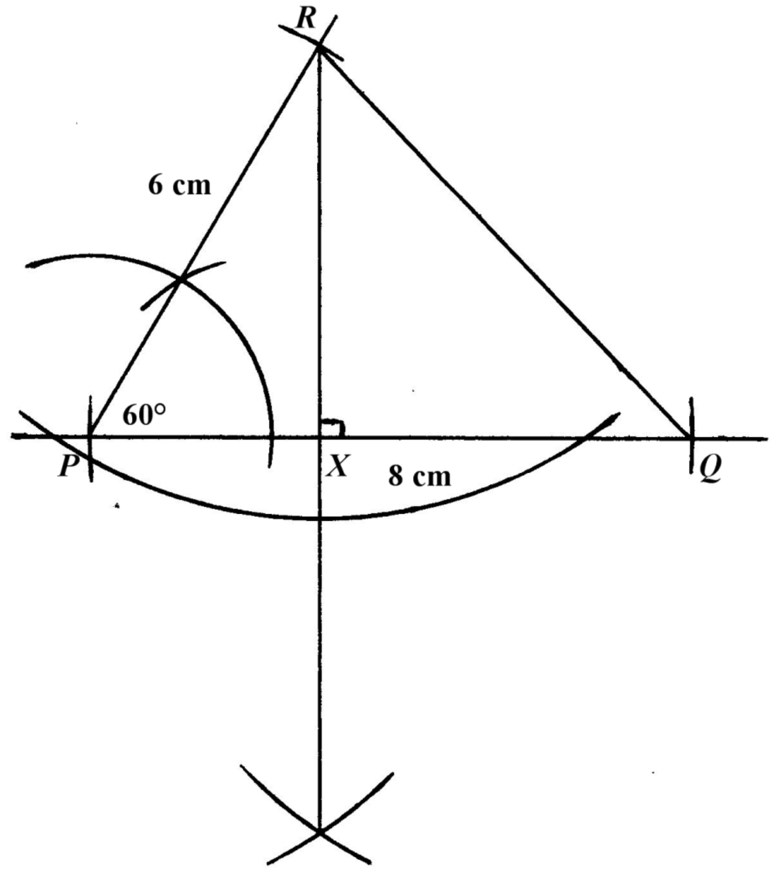
The line is extended, if necessary, so that we can cut off the segment that is 6 cm long. This gives the point *R* and so the triangle *PQR* can now be drawn by joining *R* to *Q*.



b) **Required to construct:** The line segment *RX* which is perpendicular to *PQ* and meets *PQ* at *X*.

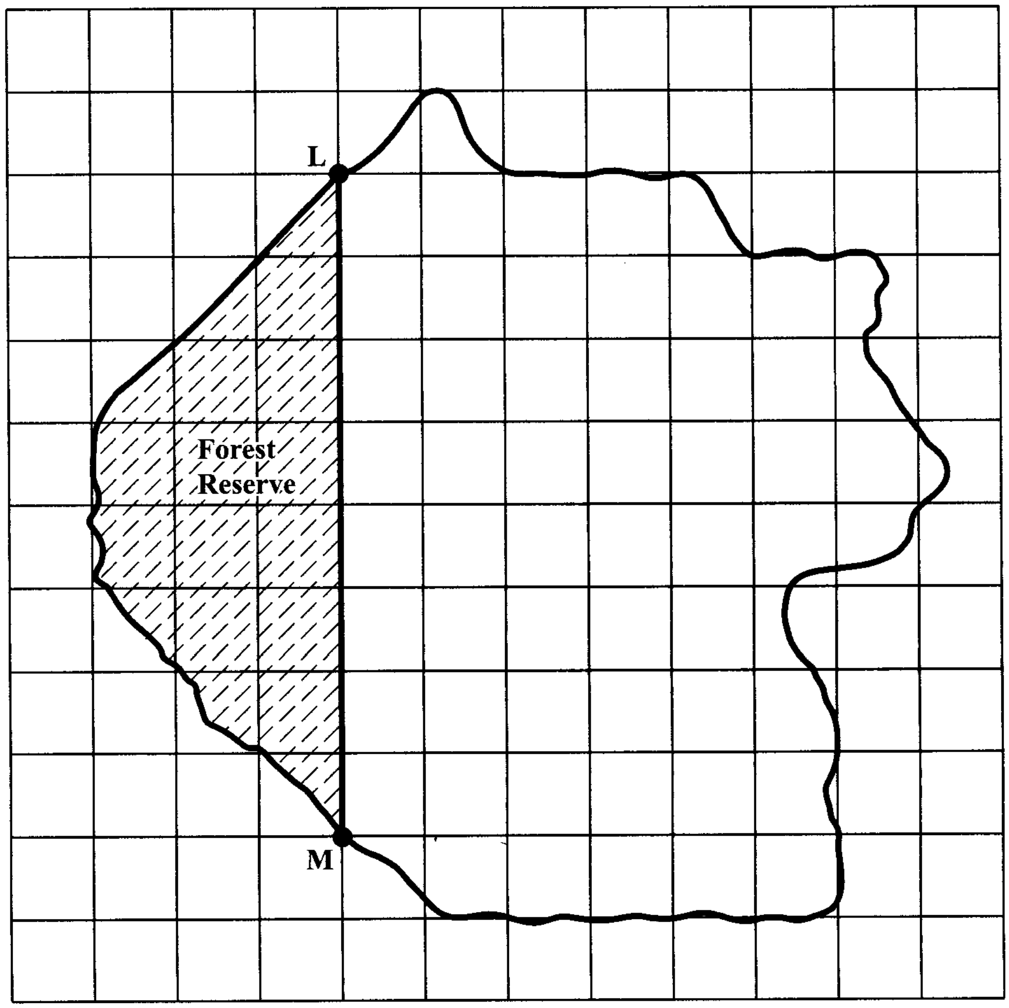
Construction: From *R*, we construct the straight line, perpendicular to *PQ* so as to meet *PQ* at *X*.
From *R*, an arc is drawn to cut the line *PQ*.

From these two points and with the same radii two arcs are drawn to intersect. The perpendicular connects R to this point of intersection, meeting PQ at X, as shown.



(ii) **Required to measure:** The size of angle QRX.
Solution:
The size of $Q\hat{R}X$, found by measurement and using the protractor, is 44°.

4. Data: Map of an island on a grid of 1 cm squares. The scale on the map is 1:50 000.



(a) (i)

Required to complete: The sentence given

Solution:

1 cm on the map represents 50 000 cm on the island.

1 metre = 100 cm

So, 50 000 cm in metres is

$$= \frac{50\,000}{100} \text{ m}$$

$$= 500 \text{ m}$$

$$= \frac{500}{1000} \text{ km}$$

$$= \frac{1}{2} \text{ km}$$

(ii) **Required to complete:** The sentence given
Solution:
An area of 1 cm^2 on the map actually represents an area of
$(50\,000 \times 50\,000)\ \text{cm}^2 = 2\,500\,000\,000\ \text{cm}^2$

$$\equiv 250\,000\ \text{m}^2 \qquad (1\ \text{km}^2 = 1\,000 \times 1000\ \text{m}^2)$$

$$\equiv \frac{1}{4}\ \text{km}^2$$

(iii) **Required to complete:** The sentence given
Solution:
1 cm on map = 50 000 cm

$$= \frac{50\,000}{100\,000}\ \text{km}$$

$$= \frac{1}{2}\ \text{km (as seen in part (i))}$$

(b) (i) **Required to state:** The distance, in centimetres, of *LM.*
Solution:
The distance *L* to *M,* show by the line *LM* = 8 cm
(by measurement or found by counting squares).

(ii) **Required to calculate:** The actual distance, in kilometres, from *L* to *M*.
Calculation:

Since $1\ \text{cm} \equiv \frac{1}{2}\ \text{km}$

Then the actual distance $LM = 8 \times \frac{1}{2}\ \text{km}$

$$= 4\ \text{km}$$

(c) (i) **Required to estimate:** The area of the forest reserve as shown on the map
Solution:
The estimated area of the forest reserve formed by counting squares
= The number of squares × the area of 1 square

Checking the number of squares in each row

Row 1 $\approx \frac{1}{2}$

Row 2 $\approx 1\frac{1}{2}$

Row 3 $\approx 2\frac{1}{2}$

Row 4 ≈ 3

Row 5 ≈ 3

Row 6 $\approx 2\frac{1}{2}$

Row 7 $\approx 1\frac{1}{2}$

Row 8 $\approx \frac{1}{2}$

Hence, the total number of squares is approximately 15

The estimated area of the forest reserve on the map is 15 x 1 = 15 cm^2.

(ii) **Required to calculate:** The actual area of the forest reserve, in km^2.
Calculation:

The actual area of forest reserve $= 15 \times \left(\frac{1}{2} \times \frac{1}{2}\right)$ km^2 (1 cm^2 $= \frac{1}{2} \times \frac{1}{2}$ km^2)

$= 3\frac{3}{4}$ km^2

5. (a) **Data:** The triangle *ABC* with coordinates, $A(1, 2)$, $B(4, 2)$ and $C(1, 0)$.

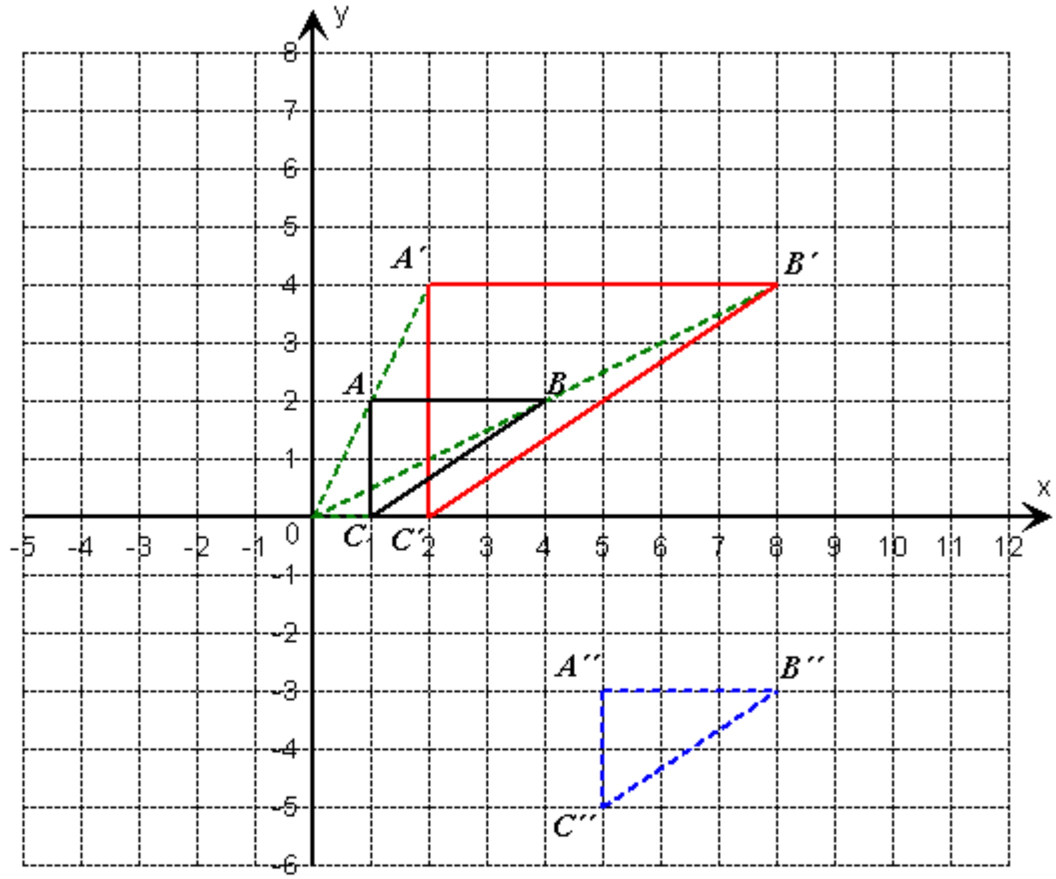
(i) **Required to draw:** The triangle, $A'B'C'$, which is the image of triangle *ABC* under an enlargement, centre *O* and scale factor 2.
Solution:

We extend the line *OA* to *OA′* so that *OA′* is 2 × the length of *OA*. This is because the scale factor of the enlargement is 2. The coordinates of *A′* will be (2, 4).
The procedure is repeated to obtain the images of *B* and *C*.

We extend the line *OB* to *OB′* so that *OB′* is 2 × the length of *OB*, since the scale factor of the enlargement is 2. The coordinates of *B′* will be (8, 4).

We extend the line *OC* to *OC′* so that *OC′* is 2 × the length of *OC*, since the scale factor of the enlargement is 2. The coordinates of *C′* will be (0, 2).

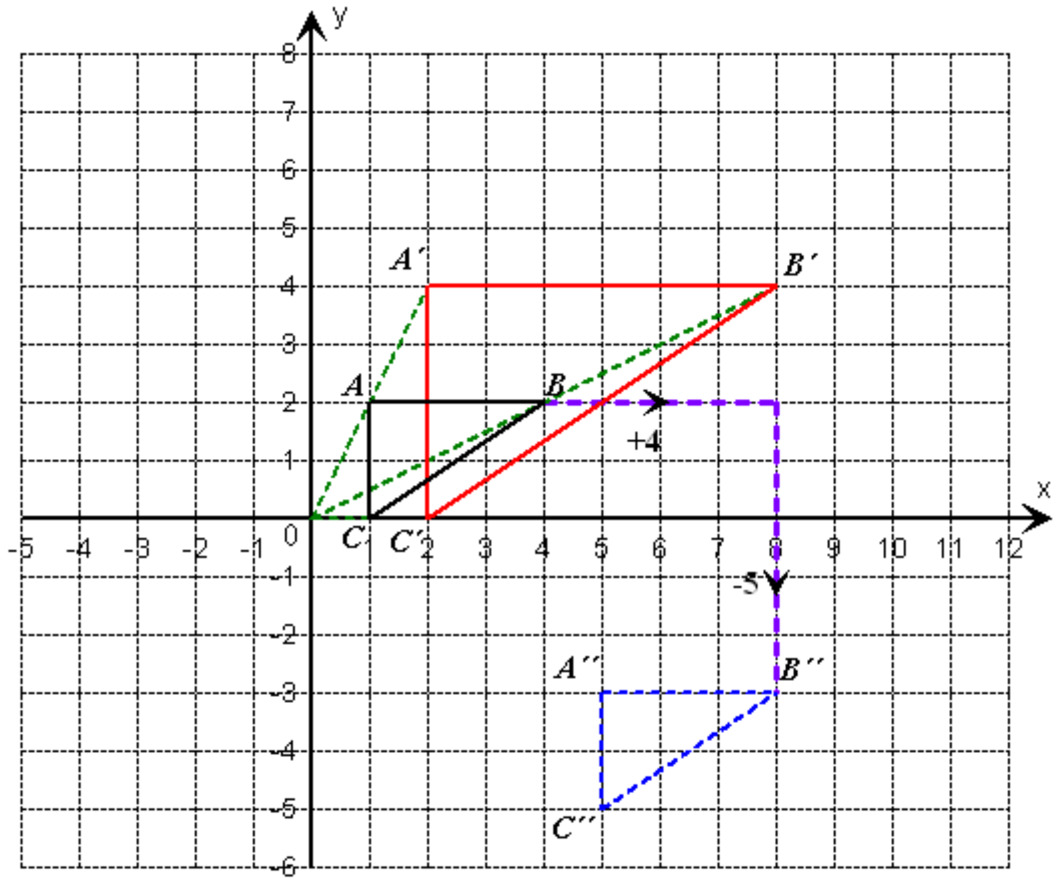
This is shown in the diagram below.



(ii) **Data:** Triangle $A''B''C''$ is the image of triangle ABC under a transformation M.
Required to describe: The transformation M completely.

Solution: The image, $A''B''C''$ is observed to be congruent to the object ABC and is in the same orientation as the object. There is no flipping or lateral inversion. Hence the transformation is a translation.

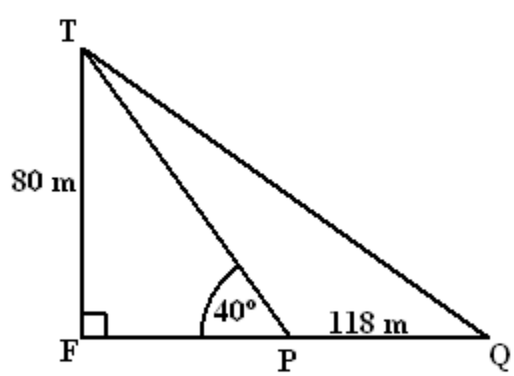
We can determine the transformation matrix by observing an object point and its corresponding image point. For example, if we choose to observe the object point, B, then its corresponding image point will be B'. We would notice that B is shifted 4 units horizontally to the right and 5 units vertically downwards.
(This would be the same for each point on the object, A, B and C, which is the triangle ABC.)



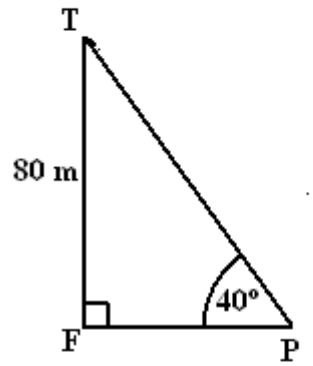
The triangle ABC is hence mapped onto triangle $A''B''C''$ by a shift of 4 units horizontally to the right and 5 units vertically downwards and is a translation.

$\therefore M$ is a translation, represented by the matrix, $M = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$.

(b) **Data:**

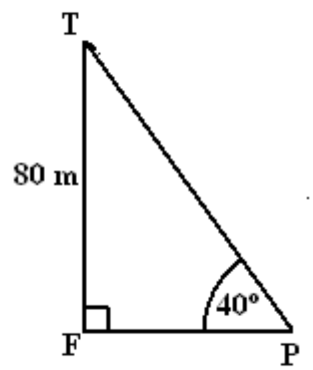


(i) **Required to determine:** The angle of elevation of T from P.
Solution:



This is shown by the angle $F\hat{P}T = 40°$ and illustrated in the diagram above.

(ii) **Required to determine:** The length of *FP*.
Solution:



$$\tan 40° = \frac{80}{FP}$$

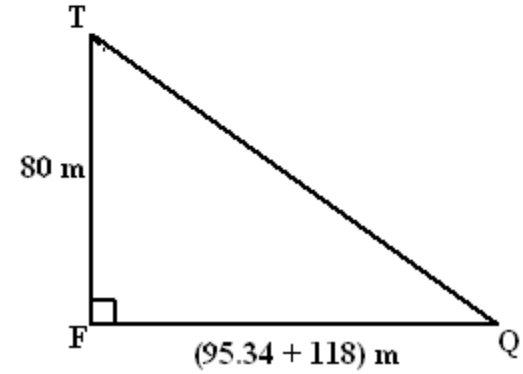
$$\therefore FP = \frac{80}{\tan 40°}$$

$$= 95.34\underline{\underline{0}} \text{ m}$$

$$= 95.34 \text{ m}$$

(iii) **Required to determine:** The angle of elevation of *T* from *Q*.

Solution:



The required angle is shown as $T\hat{Q}F$ in the above diagram

$$\tan T\hat{Q}F = \frac{80}{95.34+118}$$

$$\tan T\hat{Q}F = 0.375$$

$$T\hat{Q}F = \tan^{-1}(0.375)$$

$$= 20.5\underline{\underline{5}}°$$

$$= 20.6° \text{ (to the nearest } 0.1°)$$

6. Data: Diagram of the quadratic graph $y = x^2$ for $-4 \le x \le 4$, $M(-1, y)$ and $N(x, 9)$.

(a) (i) **Required to determine:** The value of x.

Solution:

$N(x, 9)$

$y = x^2$ (data)

$\therefore 9 = x^2$

$\therefore x = \sqrt{9}$

$= \pm 3$

At N, x is observed as positive, therefore $x = 3$.

(ii) **Required to determine:** The value of y.

Solution:

$M(-1, y)$

$y = x^2$

So, $y = (-1)^2$

$y = 1$

(b) (i) **Required to determine:** The gradient of the line *MN*.

Solution:
$M=(-1, 1)$ and $N=(3, 9)$

Gradient of $MN=\dfrac{9-1}{3-(-1)}$ (using the gradient formula)

$=\dfrac{8}{4}$

$=2$

(ii) **Required to determine:** The equation of the line *MN*.
Solution:
Gradient of $MN=2$

Choosing $N=(3, 9)$ and using the formula $\dfrac{y-y_1}{x-x_1}=m$, where (x_1, y_1) is a point on the straight line with gradient *m*.
The equation of *MN* is

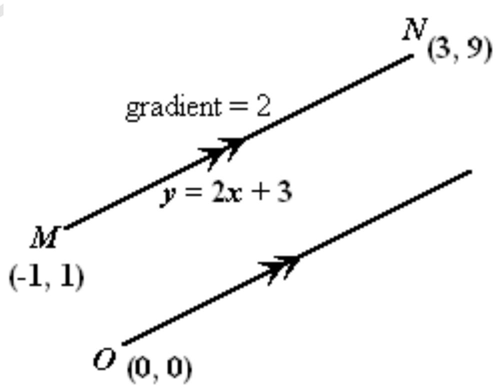
$\dfrac{y-9}{x-3}=2$

$y-9=2(x-3)$

$y-9=2x-6$

$y=2x+3$ or any other equivalent form

(iii) **Required to determine:** The equation of the line parallel to MN and passing through the origin.
Solution:



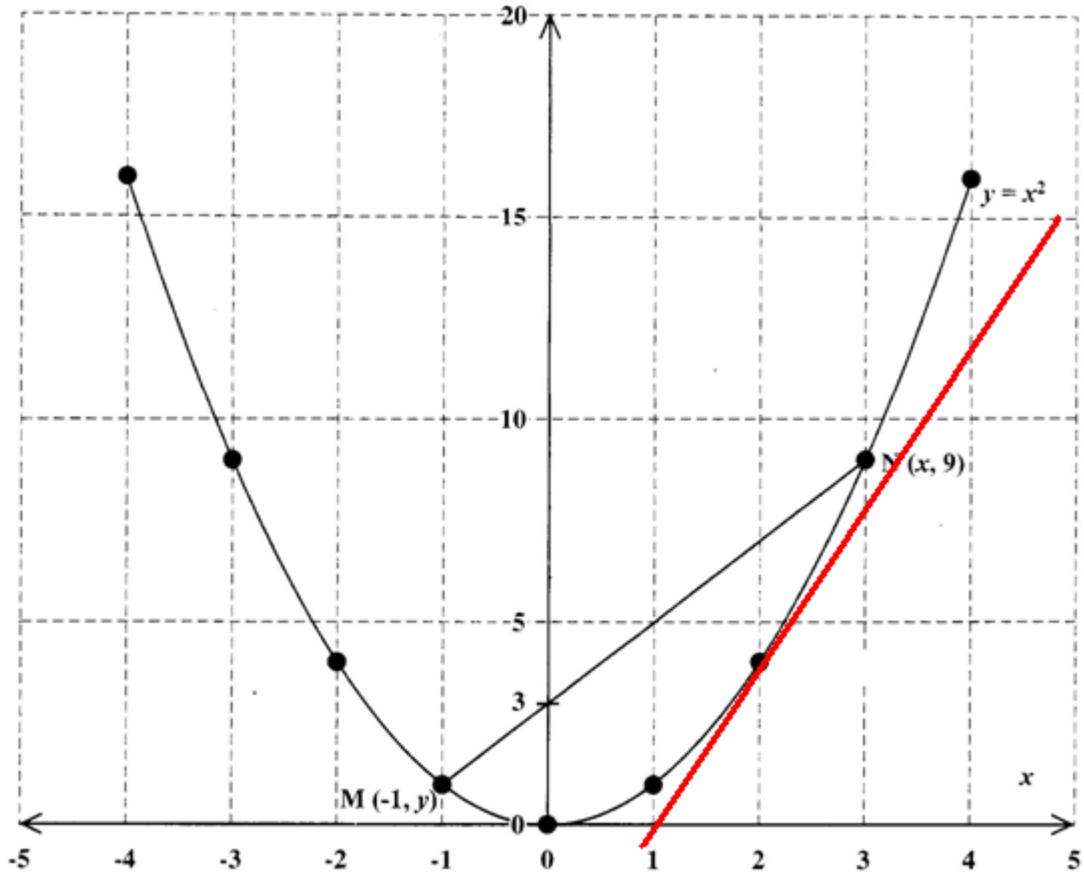
The required line has the same gradient as *MN*, which is 2, since parallel lines have the same gradient. Since we have the coordinates of a point on the line as well as its gradient, we can find the equation.

$\therefore$ Equation of the required line is $\dfrac{y-0}{x-0}=2$

$$y = 2x$$

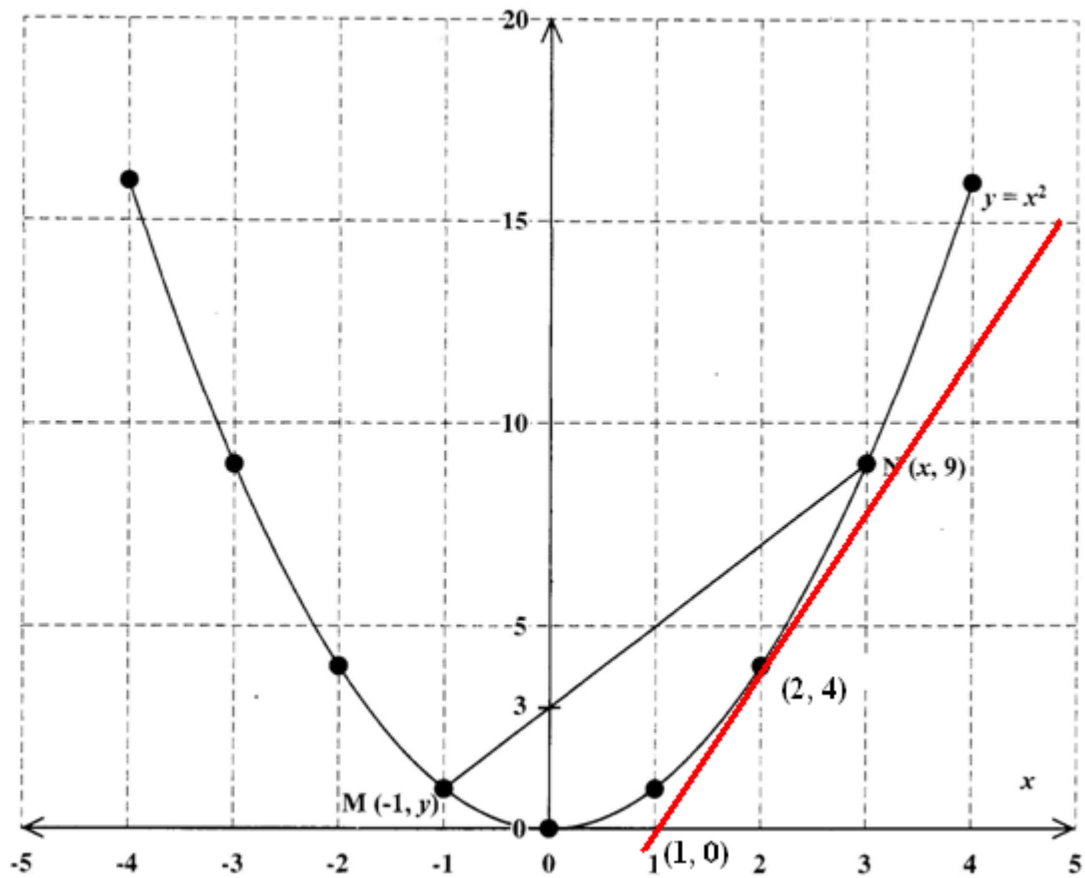
(c) **Required to draw:** The tangent to the graph $y = x^2$ at the point $(2, 4)$.

Solution:



The tangent is shown in red.

(d) **Required to estimate:** The gradient of the curve at $(2, 4)$.
Solution:



Choosing two points on the tangent to the curve $y = x^2$ at the point $(2, 4)$.
We may choose, say the point $(2, 4)$, the point at which the tangent is drawn and the point $(1, 0)$ where the tangent appears to cut the x-axis, as shown on the diagram above.

$$\text{The gradient of the tangent } = \frac{4-0}{2-1} = \frac{4}{1} = 4$$

7. Data: Raw data showing the number of books in the bags of 30 students.
(a) **Required to copy and complete:** the frequency table for the data shown.
Solution: From the raw data we complete the frequency table as shown:-

Number of Books (x)	Tally	Frequency (f)	$f \times x$
1	$\vert\vert$	2	2
2	$\vert\vert\vert$	3	6
3	$\cancel{\vert\vert\vert\vert}$	5	$5 \times 3 = 15$
4	$\cancel{\vert\vert\vert\vert}\ \vert$	6	$6 \times 4 = 24$
5	$\cancel{\vert\vert\vert\vert}\ \vert\vert$	7	$7 \times 5 = 35$
6	$\vert\vert\vert\vert$	4	$4 \times 6 = 24$
7	$\vert\vert\vert$	3	$3 \times 7 = 21$
		$\sum f = 30$	

(b) **Required to state:** The modal number of books in the bags of the sample of students.
Solution:
The highest frequency (which is 7) occurs with the score of 5 books and this can be observed from the frequency table shown above.
$\therefore$ The modal number of books is therefore 5.

(c) (i) **Required to calculate:** The total number of books.
Calculation:
The total number of books is $\sum fx$, that is the sum of the $\boldsymbol{f\,x}$ values

$$\sum fx = 2 + 6 + 15 + 24 + 35 + 24 + 21$$
$$= 127$$

(ii) **Required to calculate:** The mean number of books per bag.
Calculation:
The mean number of books per bag is $\bar{x} = \dfrac{\sum fx}{\sum f}$.

$$\bar{x} = \frac{\sum fx}{\sum f}$$
$$= \frac{127}{30}$$
$$= 4.2\underline{\underline{3}}$$
$$= 4.2 \text{ (to 1 decimal place)}$$

(d) **Required to determine:** The probability that a student chosen at random has less than 4 books in their bag.

Solution:

$$P(\text{Student has less than 4 books}) = \frac{\text{No. of students who have less than 4 books}}{\text{Total number of students}}$$

$$= \frac{2+3+5}{30}$$

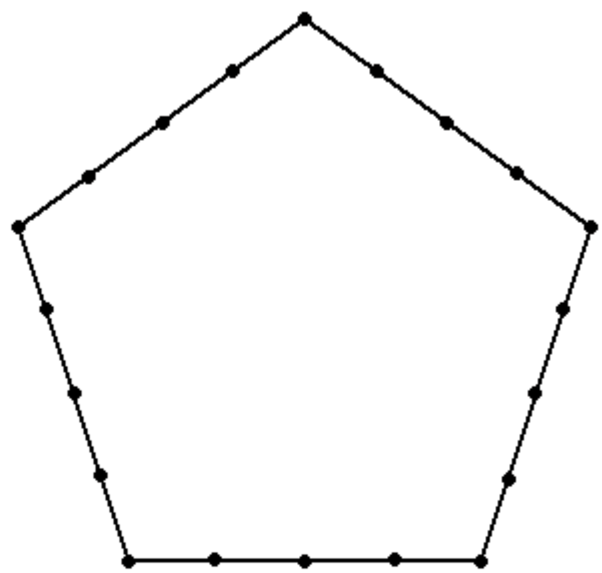
$$= \frac{10}{30}$$

$$= \frac{1}{3}$$

(This probability can also be expressed as a percentage of $33\frac{1}{3}$% or as a decimal equivalent; though in this case it would not be advisable since 1/3 does not have an exact decimal equivalent).

8. Data: A sequence of pentagons increasing in size.

(a) **Required to draw:** The next figure in the given sequence.
Solution:



(b) **Required to complete:** The missing information in the table below.
Solution:

Figure (f)	Total Number of Dots: Formula	Total Number of Dots: Number (n)
1	$5\times2-5$	5
2	$5\times3-5$	10
3	$5\times4-5$	15
4	Not Required	Not Required
5		
6		

We notice, by observation, that in the column listed for formula, the expression is always
$5\times$(No. of the figure + 1) – 5
This can be generalised to
Number of dots, $n = 5 \times f + 5 - 5 = 5f - 5 + 5 = 5f$

Hence, $5f = n$, the numerical value of which is calculated in the third column.
Hence,

Figure (*f*)	**Total Number of Dots**	
	Formula	**Number (*n*)**
1	$5\times2-5$	5
2	$5\times3-5$	10
3	$5\times4-5$	15
4	Not Required	Not Required
5	$5\times(5+1)-5$	25
6	$5\times(6+1)-5$	30

(c) **Required to write:** An expression in f for the number (n) of dots used in drawing the f th figure.
Solution:
The number of dots, n, in the f th figure is given by $5f = n$, (as shown above in (b)).

(d) **Required to determine:** The figure in the sequence containing 145 dots.
Solution:
The number of dots $n = 145$
From the formula deduced we substitute to obtain
$145 = 5f$
$f = 145 \div 5$
$=29$

Hence, there are 145 dots in the 29th figure.

SECTION II

9. (a) **Data:** $g(x) = 4x + 3$ and $f(x) = \dfrac{2x+7}{x+1}$

(i) **Required to state:** The value of x for which $f(x)$ is undefined.

Solution:

$$f(x) = \frac{2x+7}{x+1}$$

Let us look at the denominator of the expression
When $x = -1$

$$f(x) = \frac{2(-1)+7}{-1+1}$$
$$= \frac{5}{0}$$
$$= \infty$$

and which is called infinity
$\therefore f(x)$ is undefined (OR NOT VALID) for $x = -1$.

(ii) **Required to calculate:** The value of $gf(5)$.

Calculation:
We first find $f(5)$

$$f(5) = \frac{2(5)+7}{5+1}$$
$$= \frac{17}{6}$$

Now we find $gf(5)$ using the value of $f(5)$ obtained

$$gf(5) = g\left(\frac{17}{6}\right)$$
$$= 4\left(\frac{17}{6}\right) + 3$$
$$= 11\frac{1}{3} + 3$$
$$= 14\frac{1}{3}$$

Alternative method:
We can, instead, find the expression for $gf(x)$

$$gf(x) = 4\left(\frac{2x+7}{x+1}\right) + 3$$
$$= \frac{8x+28}{x+1} + 3$$

We need not simplify in order to substitute $x = 5$

$$gf(5) = \frac{8(5)+28}{5+1} + 3$$
$$= \frac{68}{6} + 3$$
$$= 11\frac{1}{3} + 3$$
$$= 14\frac{1}{3}$$

(iii) **Required to find:** $f^{-1}(x)$

Solution:

$$f(x) = \frac{2x+7}{x+1}$$

We let $y = f(x)$ and make x the subject. Then we replace y by x to find the inverse of f.

$$y = \frac{2x+7}{x+1}$$
$$y(x+1) = 2x+7$$
$$xy + y = 2x + 7$$
$$xy - 2x = 7 - y$$
$$x(y-2) = 7 - y$$
$$x = \frac{7-y}{y-2}$$

Replace y by x we get $f^{-1}(x) = \dfrac{7-x}{x-2}$

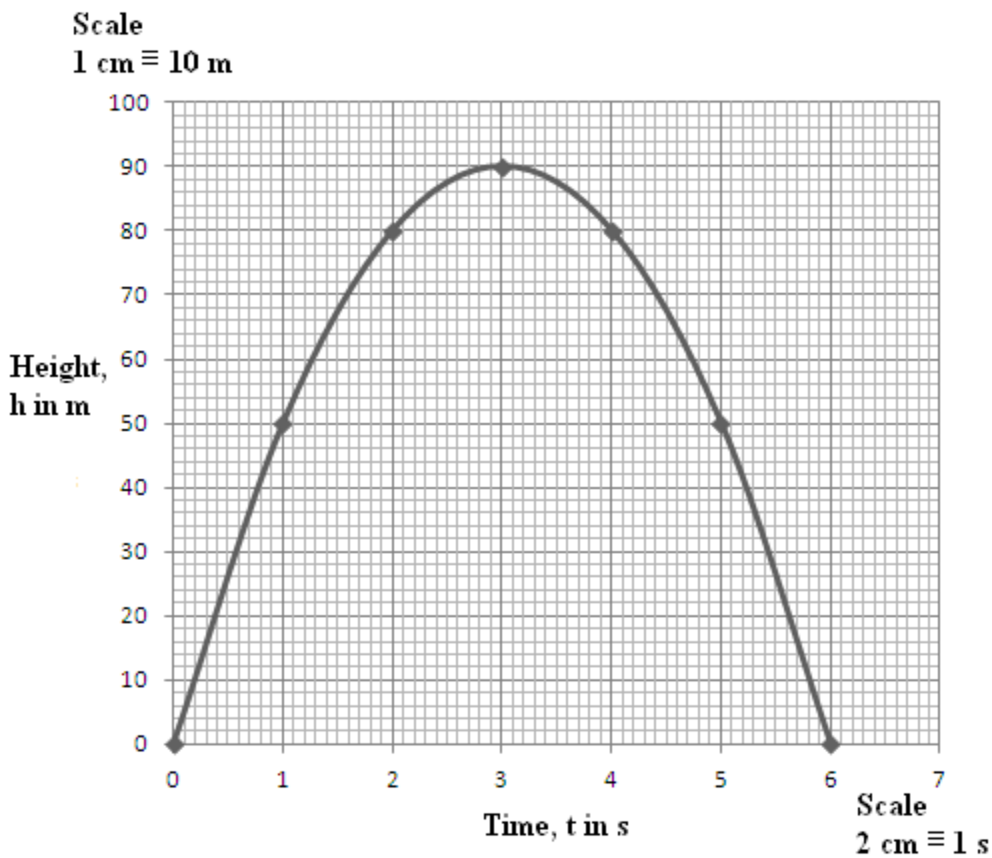
Notice when x approaches 2, the denominator approaches 0 and $f(x)$ is ∞. So, $f(x)$ is not valid for $x = 2$.

(b) **Data:** Table showing height (h) in metres vs time (t) in seconds for a ball thrown vertically upwards.

***t* (s)**	0	1	2	3	4	5	6
***h* (m)**	0	50	80	90	80	50	0

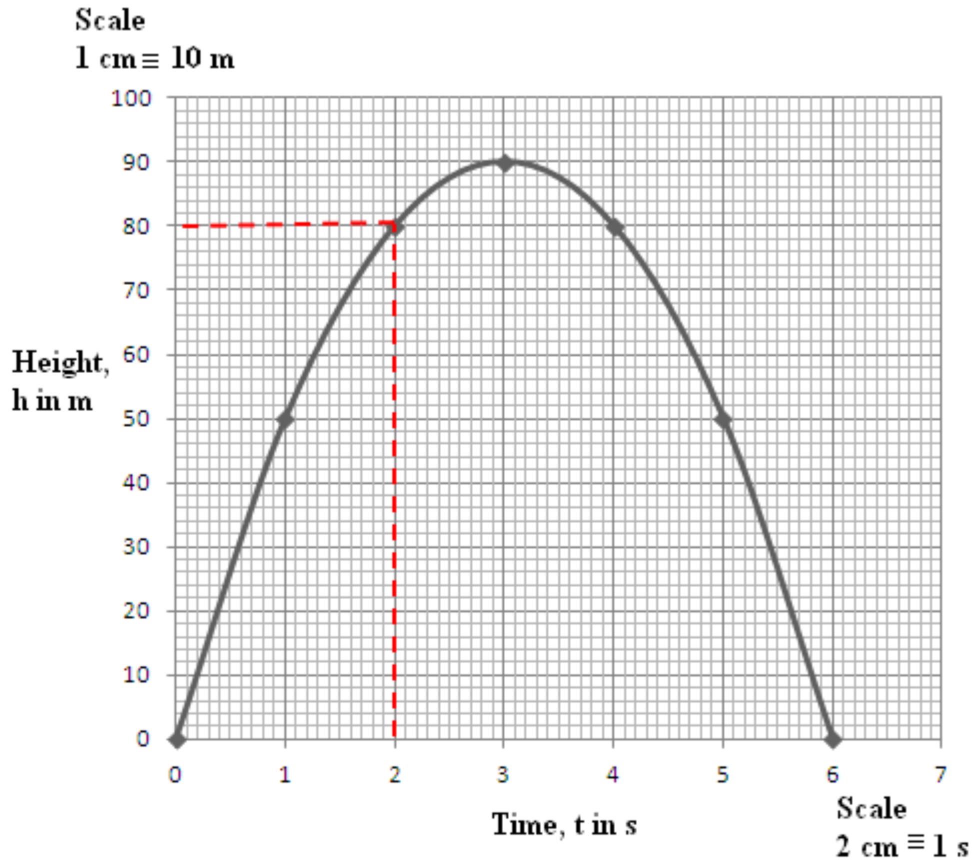
(i) **Required to plot:** The graph to show h vs t using the scales given.

Solution: Using the scales given, we obtain the curve,



(ii) a) **Required to determine:** The average speed of the ball during the 2 seconds.

Solution:



We find the height attained after 2 seconds. This is shown in red.
When $t = 2,\ h = 80$ (as shown by the read-off)

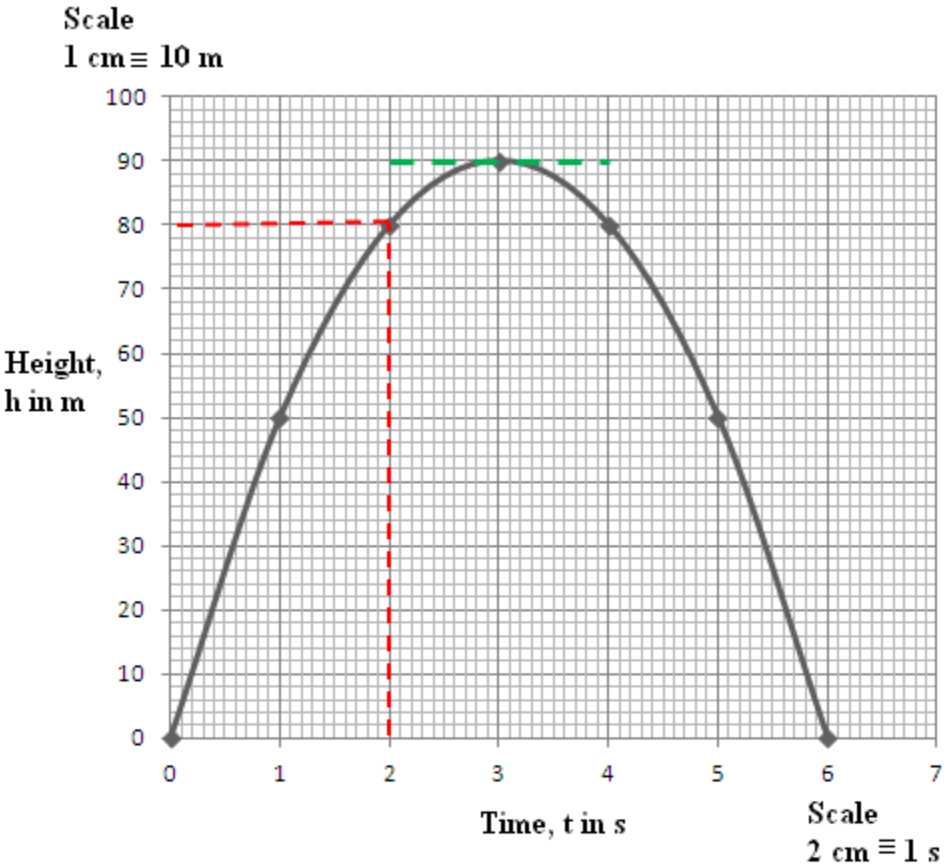
$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{80 \text{ m}}{2 \text{ s}}$$

$$= 40 \text{ ms}^{-1}$$

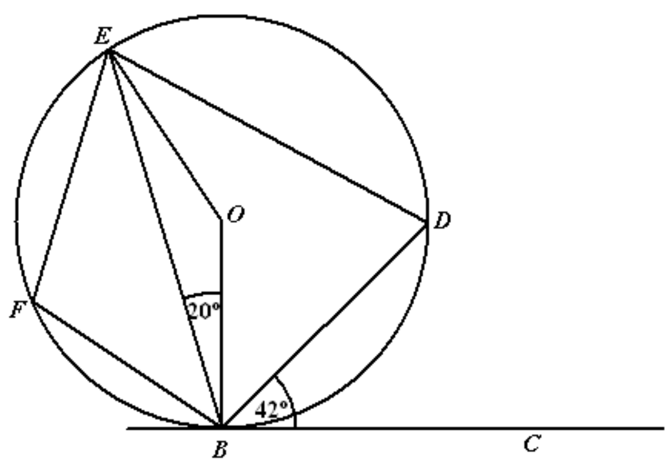
b) **Required to determine:** The speed of the ball when $t = 3$
(NOT $t = 3$ seconds, since the unit of t is given).

Solution:



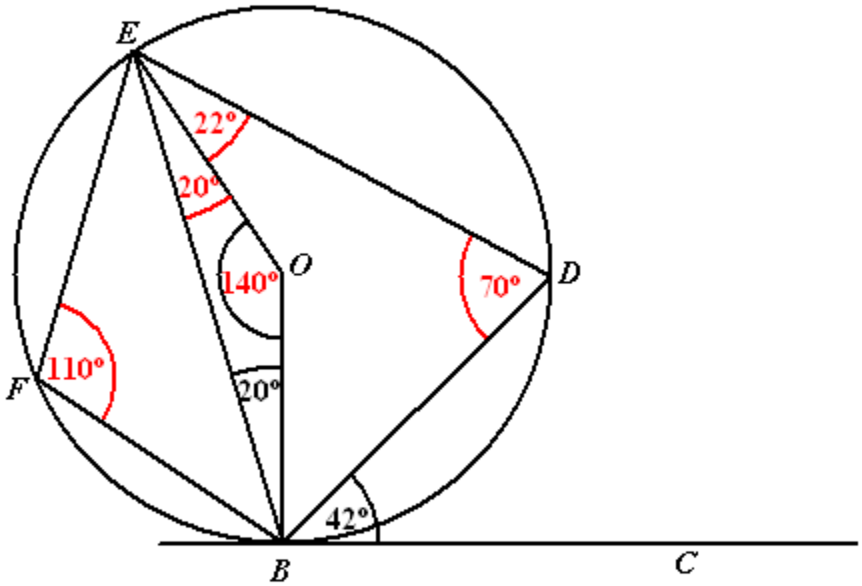
The gradient of the tangent to the graph at any point on a distance-time graph, gives the speed at that time. The tangent to the graph when $t = 3$ is a horizontal line. The gradient of any horizontal line = 0. Therefore, the speed of the ball when $t = 3$ is 0 ms^{-1}.

10. (a) **Data:**



In the diagram, O is the center of the circle, BC is the tangent at B, $O\hat{B}E = 20°$ and $C\hat{B}D = 42°$.

(i) **Required to calculate:** $B\hat{O}E$
Calculation:



$OB = OE$ (radii of the same circle)

$O\hat{E}B = 20°$ (The base angles of an isosceles triangle are equal)

$B\hat{O}E = 180° - (20° + 20°)$

$= 140°$

(The sum of the interior angles of any triangle = 180°)

(ii) **Required to calculate:** $O\hat{E}D$
Calculation:

$B\hat{E}D = 42°$ (The angle made by a tangent to a circle and any chord, at the point of contact, is equal to the angle in the alternate segment of the circle).

$\therefore O\hat{E}D = 42° - 20°$

$= 22°$

(iii) **Required to calculate:** $B\hat{F}E$
Calculation:

$E\hat{D}B = \frac{1}{2}(140°)$

$= 70°$

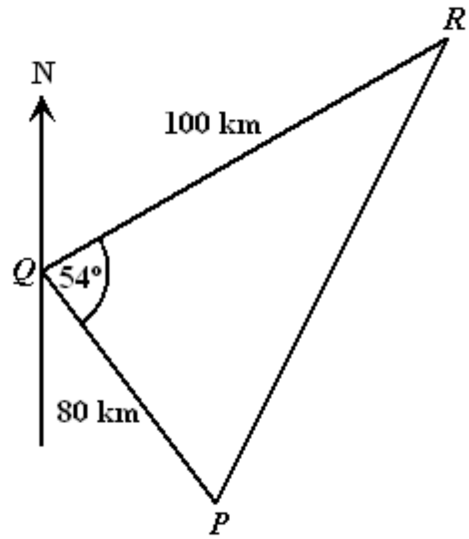
(The angle subtended by a chord at the center of a circle is twice the angle that the chord subtends at the circumference, standing on the same arc).

$B\hat{F}E = 180° - 70°$

$= 110°$

(The opposite angles of a cyclic quadrilateral are supplementary i.e total 180^0. The converse of this statement is also true).

(b) **Data:** Diagram showing the position of three points *P*, *Q* and *R*.



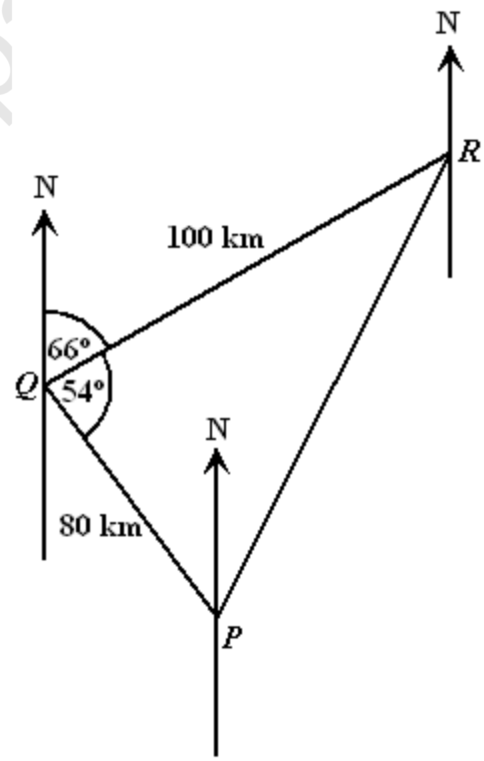
The bearing of *R* from *Q* is 066°.

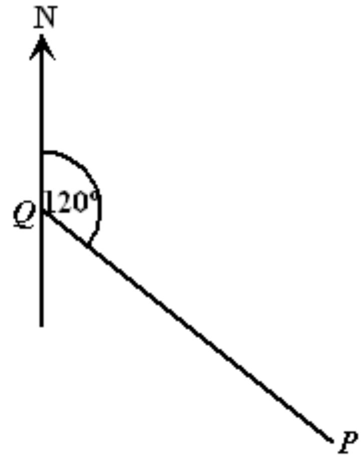
(i) **Required to calculate:** The bearing of *P* from *Q*.

Calculation:

We find it helpful to modify the diagram to illustrate the bearing of *R* from *Q* and the North line at all three points, *P*, *Q* and *R*.

This is shown below

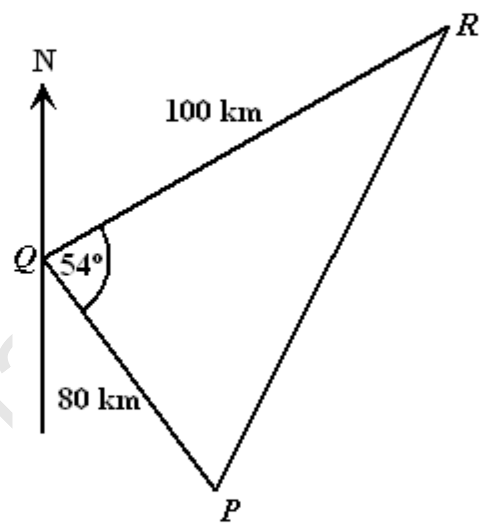




The bearing of P from $Q = 66° + 54°$
$= 120°$
(The direction measured from N, in a clockwise direction)

(ii) **Required to calculate:** The distance PR correct to 2 decimal places.

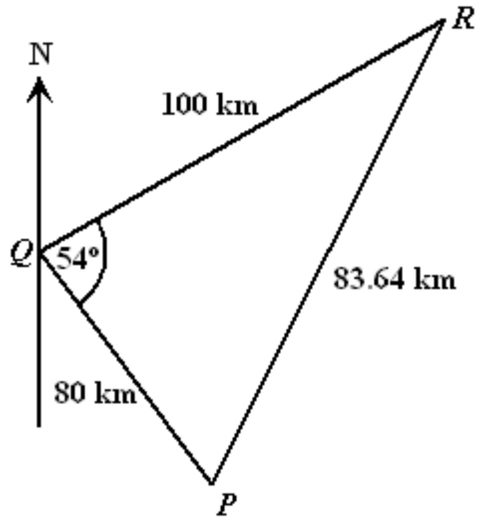
Calculation:



It is convenient to use the cosine rule to give
$PR^2 = (100)^2 + (80)^2 - 2(100)(80) \cos 54^0$
$PR^2 = 6995.436$
$PR = \sqrt{(6995.436)}$
$PR = 83.638$ km
$PR = 83.64$ km (to 2 dp)

(iii) **Required to calculate:** $Q\hat{P}R$ to the nearest degree.

Calculation:



It is convenient to apply the sine rule to triangle PQR to give

$$\frac{83.64}{\sin 54°} = \frac{100}{\sin Q\hat{P}R}$$

$$\sin Q\hat{P}R = \frac{100 \sin 54°}{83.64}$$

$$= 0.9673$$

$$\therefore Q\hat{P}R = \sin^{-1}(0.9673)$$

$$= 75.\underline{\underline{3}}°$$

$$= 75° \text{(to the nearest degree)}$$

11. (a) **Data:** $M = \begin{pmatrix} 7 & 2 \\ p & -1 \end{pmatrix}$

Required to determine: The value of p for which M does not have an inverse
Solution:
If det $M = 0$, then M is deemed as singular and will not have an inverse.

$\det M = (7 \times -1) - (2 \times p)$ (from definition)

Let $\det M = 0$.

$-7 - 2p = 0$

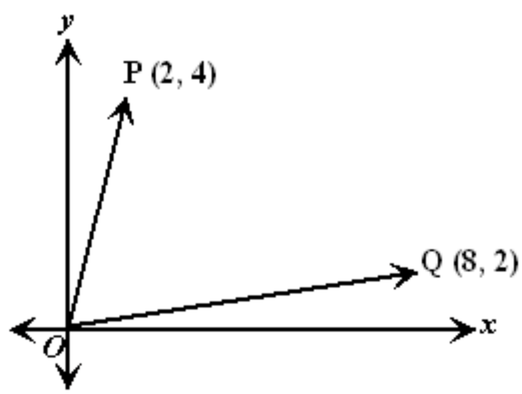
$2p = -7$

$p = -\frac{7}{2}$ or $-3\frac{1}{2}$

(b) **Data:** $4x - 2y = 0$ and $2x + 3y = 4$

Required to express: The given equations in the form $AX = B$, where A, X and B are matrices.
Solution:
$4x - 2y = 0$
$2x + 3y = 4$
Considering the coefficients of the terms in x and in y we obtain
$\therefore \begin{pmatrix} 4 & -2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ is of the form $AX = B$, where $A = \begin{pmatrix} 4 & -2 \\ 2 & 3 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$.

(c) **Data:**



(i) **Required to state:** The term used to describe OP.
Solution:
OP is the position vector from O to P. O is the fixed point from which the vector is drawn.

(ii) a) **Required to write:** OP in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

Solution:
$P\,(2, 4)$
$\therefore OP = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and is expressed in the form $\begin{pmatrix} a \\ b \end{pmatrix}$, where $a = 2$ and $b = 4$.

b) **Required to write:** OQ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

Solution:
$Q = (8, 2)$

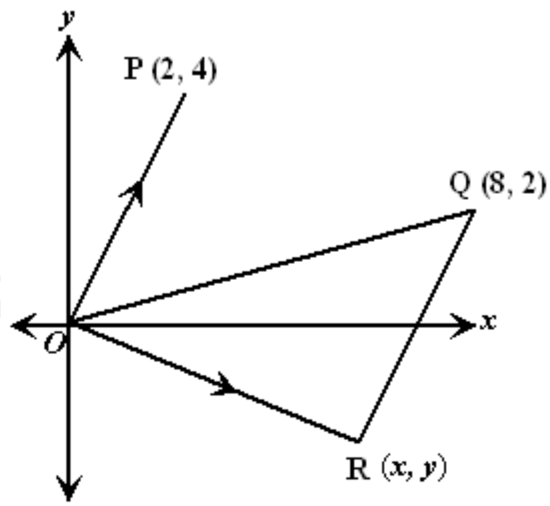
$\therefore OQ = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$ and is expressed in the form $\begin{pmatrix} a \\ b \end{pmatrix}$, where $a = 8$ and $b = 2$.

c) **Required to write:** PQ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

Solution:
Using the vector triangle law we get

$$PQ = PO + OQ$$
$$= -\begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 8 \\ 2 \end{pmatrix}$$
$$= \begin{pmatrix} 6 \\ -2 \end{pmatrix} \text{ is of the form } \begin{pmatrix} a \\ b \end{pmatrix}, \text{ where } a = 6 \text{ and } b = -2.$$

(iii) **Data:** $OP = RQ$
Required to determine: The coordinates of the point, R
Solution:



We let R be the point (x, y). So, $OR = \begin{pmatrix} x \\ y \end{pmatrix}$

By the vector triangle law

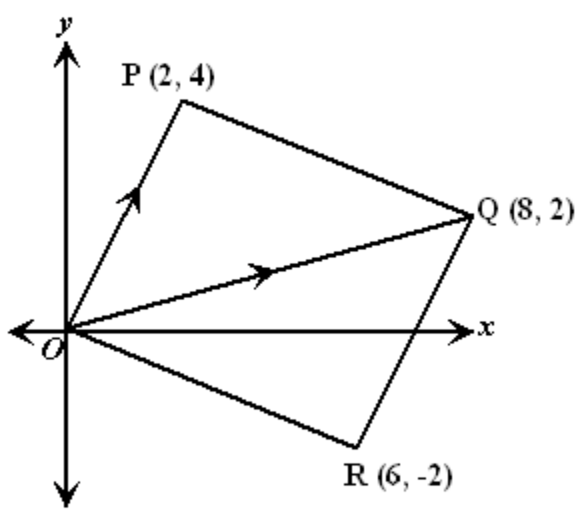
$$RQ = RO + OQ$$
$$= -\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 8 \\ 2 \end{pmatrix}$$
$$= \begin{pmatrix} -x+8 \\ -y+2 \end{pmatrix}$$

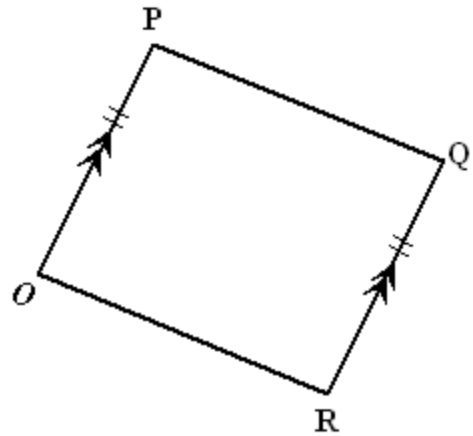
Since $OP = RQ$, then
$$\begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -x+8 \\ -y+2 \end{pmatrix}$$
Equating corresponding components since the vectors are equivalent.
This gives
$$2 = -x + 8$$
$$x = 8 - 2$$
$$x = 6$$

$$4 = -y + 2$$
$$y = 2 - 4$$
$$y = -2$$

$\therefore OR = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ and so the coordinates of R will be $(6, -2)$.

(iv) **Required to state:** The type of quadrilateral formed by $PQRO$.
Solution:





In the quadrilateral *PQRO* we have observed that one pair of opposite sides (*OP* and *RQ*) are both parallel and equal. This is one of the necessary and sufficient conditions needed to conclude that *PQRO* is a parallelogram.

(Rule: If one pair of opposite sides of a quadrilateral are both parallel and equal, then the quadrilateral is a parallelogram – law of geometry).

We also could have shown:

The opposite sides are equal in length

OR

The opposite sides are parallel

OR

The diagonals have the same midpoint which indicates that the diagonals bisect each other.