FAS-PASS Maths

CXC MATHEMATICS MAY-JUNE 2013 PAPER 2

SECTION I

1. (a) (i) **Required to calculate:** $\dfrac{1\frac{4}{5}-\frac{1}{3}}{2\frac{2}{5}}$.

Solution:
Working the numerator first

$$1\frac{4}{5}-\frac{1}{3}$$
$$=1\frac{3(4)-5(1)}{15}$$
$$=1\frac{12-5}{15}$$
$$=1\frac{7}{15}$$

The question is now reduced to: $\dfrac{1\frac{7}{15}}{2\frac{2}{5}}=\dfrac{\frac{22}{15}}{\frac{12}{5}}$

For this division, we multiply the numerator by the inverted denominator to get:

$$=\frac{22}{15}\times\frac{5}{12}$$
$$=\frac{11}{18}\text{(in exact form)}$$

(ii) **Required To Calculate:** $\sqrt{1.5625}+(0.32)^2$

Solution:
This might prove difficult by the methods of arithmetic, so it is best we use the calculator

$$\sqrt{1.5625}+(0.32)^2=1.25+0.1024\text{ (by the calculator)}$$
$$=1.3524\text{ (in exact form)}$$

(b) **Data:** Table which shows the carton size and the corresponding cost of 'Smiley Orange Juice'.
Required to find: The carton size which proves to be the 'better buy'
Solution:
This can be done by calculating the 'unit price' in each case.

If 350 ml is sold for \$4.20.

Then 1 ml will cost the buyer $\frac{\$4.20}{350} = \$0.012$ per ml

If 450 ml is sold for \$5.13

Then 1 ml will cost the buyer $\frac{\$5.13}{450} = \$0.0114$ per ml

\$0.0114 is a lesser value than \$0.012. Hence, if a 'better buy' is meant to be the item that 'costs less', then a 450 ml carton costing \$5.13 is a better buy that a 350 ml carton costing \$4.20

(c) **Data:** Faye borrows \$9 600 at 8% per annum compound interest.

(i) **Required to calculate:** The interest on the loan for the first year.
Solution:
Interest for the first year = 8% of \$9 600

$$= \frac{8}{100} \times \$9\,600$$
$$= \$768$$

(ii) **Data:** Faye repaid \$4 368 at the end of the first year.
Required to calculate: The amount owed at the start of the second year
Solution:
At the end of the first, the amount owed
= The sum borrowed + The interest acquired
= \$9 600 + \$768
= \$10 368

Since Faye repays \$4 368, the amount now owed at the end of the first year, that is, at the beginning of the second year
= The total amount that was owed at the end of the first year – The amount that was re-paid
= \$10 368 - \$4 368
= \$6 000

(iii) **Required to calculate:** The interest on the remaining balance for the second year.
Solution:
The interest rate = 8% per annum on the remaining balance
$\therefore$ The amount of interest on the remaining balance = 8% of \$6 000

$$= \frac{8}{100} \times \$6\,000$$
$$= \$480$$

2. (a) (i) **Required to factorise:** $2x^3 - 8x$

Solution:

$$2x^3 - 8x = 2x\left(x^2 - 4\right)$$
$$= 2x\left\{(x)^2 - (2)^2\right\}$$

We restructure the terms within the curly brackets and notice that they are the difference of two squares. They can be further factorised and in so doing factorise the given expression completely to give
$2x\ (x-2)\ (x+2)$.

(ii) **Required to factorise:** $3x^2 - 5x - 2$

Solution:

$3x^2 - 5x - 2 = (3x + 1)\ (x - 2)$

(b) (i) **Data:** $F = \frac{9}{5}C + 32$

Required to make: C the subject of the formula

Solution:

We leave all terms involving only C on one side of the equation and then simplify to get

$$F = \frac{9}{5}C + 32$$
$$F - 32 = \frac{9}{5}C$$
$$\frac{9}{5}C = F - 32 \quad (\times 5)$$
$$9C = 5(F - 32) \quad (\div 9)$$
$$C = \frac{5(F - 32)}{9}$$

(ii) **Data:** $F = 113$

Required to calculate: C, when $F = 113$

Solution:

We substitute $F = 113$ in the equation for C obtained in (i)

Recall: $C = \frac{5(F - 32)}{9}$

When $F = 113$

$$C = \frac{5(113-32)}{9}$$
$$= \frac{5(81)}{9}$$
$$= 45$$

(c) **Data:** 500 tickets were sold at a concert.
x tickets were sold at \$6 each and the rest at \$10 each.

(i) a) **Required to find:** The number of tickets sold at \$10 each.
Solution:
The number of tickets sold = 500
The number of tickets sold at \$6 each = x
Hence, the remainder, which was sold at \$10 each $= 500 - x$

b) **Required to find:** The amount of money collected for the sale of the 500 tickets.
Solution:
We find the cost of the tickets sold at \$6 and the cost of the tickets sold at \$10 each. Then we total these two costs.

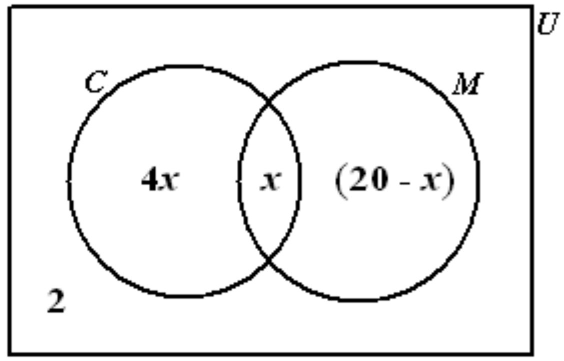
The cost of x tickets at \$6 each = 6 x x
= \$ $6x$
The cost of $(500 - x)$ tickets at \$10 each = \$10 x $(500 - x)$
= \$10 $(500 - x)$
= \$5 000 – $10x$
Hence, the total amount of money collected on the sale of all 500 tickets
$= \$(6x + 5\,000 - 10x)$
$= \$(5\,000 - 4x)$

(ii) **Data:** The sum of money collected = \$4 108
Required to calculate: The number of \$6 tickets sold
Solution:
The amount of money collected = \$ (5 000 – $4x$)
Hence,

$$5\,000 - 4x = 4108$$
$$5\,000 - 4108 = 4x$$
$$4x = 892$$
$$x = \frac{892}{4}$$
$$x = 223$$

$\therefore$ The number of tickets sold for \$6 each is x and which is 223.

3. (a) **Data:** The numbers of students who use cameras or phones to take photographs.
(i) **Required to complete:** The Venn diagram based on the data.
Solution:



(ii) **Required to find:** An expression for the total number of students in the survey.

Solution:

The total number of students in each of the subsets of the Universal set is

$= 2 + 4x + x + (20 - x)$

$= 2 + 20 + 4x + x - x$

$= 4x + 22$

in a simplified form

(iii) **Required to calculate:** The number of students who use only cameras.

Solution:

The total number of students in the class is 30.

So,

$4x + 22 = 30$

$\therefore 4x = 30 - 22$

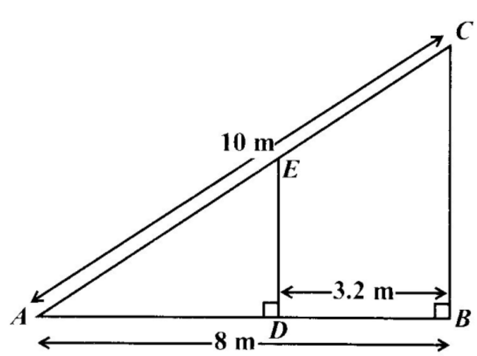
$4x = 8$

$x = \frac{8}{4}$

$x = 2$

The number of students who used cameras only $= 4x = \$(4 \times 2) = 8$

(b) **Data:**



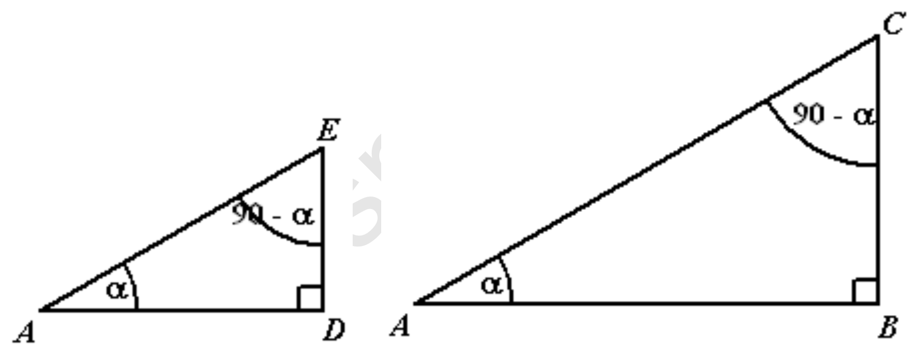
(i) **Required to calculate:** The length of *BC*.
Solution:
Consider the triangle *ABC*

$$AB^2 + BC^2 = AC^2 \quad \text{(Pythagoras' Theorem)}$$

$$\therefore BC^2 = AC^2 - AB^2$$
$$= (10)^2 - (8)^2$$
$$= 100 - 64$$
$$= 36$$

Therefore $BC = \sqrt{36} = 6$ m

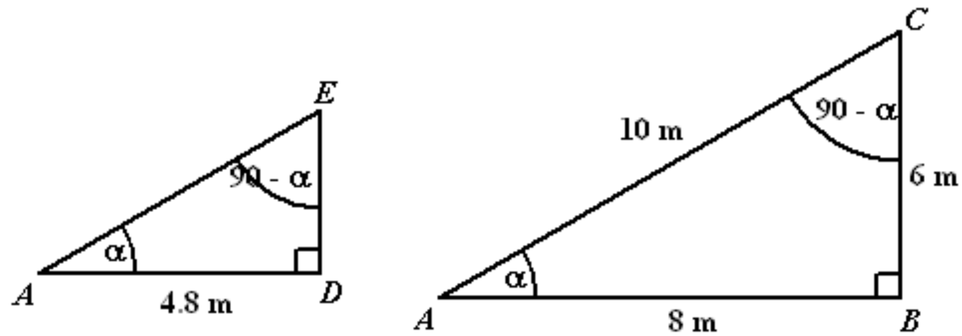
(ii) **Required to explain:** Why triangles *ABC* and *ADE* are similar.
Solution:
Looking at the triangles ΔADE and ΔABC separately.



Angle D = Angle B = 90° (data)
$\hat{A}$ is common to both triangles so the size of angle A is unimportant
Now we let $\hat{A} = \alpha$
Therefore, angle E = angle C = (90 -α)
The two triangles, *ADE* and *ABC*, have the same size of angles and are equiangular.
However, none of their sides are equal
$AD \neq AB$
$DE \neq BC$
$AE \neq AC$
So the two triangles are equiangular but not congruent. Therefore, the two triangles *ADE* and *ABC* are similar. They possess the same shape, but they differ in size.

(iii) **Required to calculate:** The length of *DE*.
Solution:
If two triangles (or any figures) are similar, then the ratio of their corresponding sides are equal.

$$AD = (8 - 3.2)\text{ m}$$
$$= 4.8\text{ m}$$



Hence, using this theorem we deduce that

$$\frac{AD}{AB} = \frac{DE}{BC}$$
$$\frac{4.8}{8} = \frac{DE}{6}$$
$$DE = \frac{6 \times 4.8}{8}$$
$$= 3.6\text{ m}$$

OR

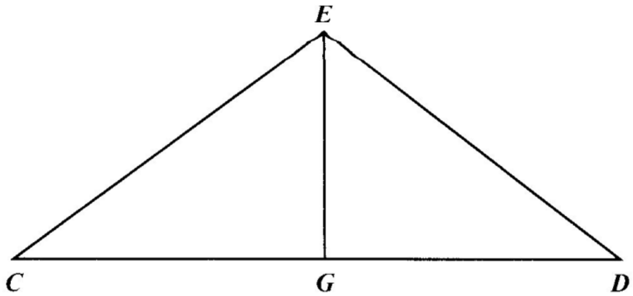
Consider ΔABC

$$\hat{A} = \tan^{-1}\left(\frac{6}{8}\right)$$
$$\hat{A} = 36.87° \text{ (to 2 decimal places)}$$

Now we consider the right–angled ADE.

$$\tan(36.87°) = \frac{DE}{4.8}$$
$$DE = 4.8 \times \tan(36.87°)$$
$$= 3.6\text{ m}$$

4. (a) **Data:** Isosceles triangles *CDE*, with *G* the midpoint of *CD*.



(i) **Required to measure:** *DE*
Solution:
$DE = 5.0$ cm (by measurement with a ruler)

(ii) **Required to measure:** $E\hat{C}D$
Solution:
$E\hat{C}D = 37°$ (by measurement with a protractor)

(iii) **Required to determine:** The perimeter of the triangle *CDE*
Solution:
ED = *EC* = 5.0 cm and *CD* = 8.0 cm (by measurement with the ruler).
The perimeter of triangle CDE = (5.0+5.0+8.0) cm
= 18.0 cm

(iv) **Required to calculate:** The area of triangle *CDE*
Solution:
EG = 3.0 cm by measurement.
Since *G* is the midpoint of *CD* and ΔCDE is isosceles, then EG is perpendicular to *CD*.

$$\text{The area of } \Delta CDE = \frac{\text{Base} \times \perp \text{Height}}{2}$$
$$= \frac{8 \times 3}{2}$$
$$= 12 \text{ cm}^2$$

OR

Since we know the lengths of all three sides of the triangle, then we can use Heron's formula

Let s = $\frac{1}{2}(5 + 5 + 8)$ cm
= 9 cm (and which is half the perimeter of the triangle).

Area = $\sqrt{9\,(9-5)(9-5)(9-8)}$ (Heron's Formula)
$= \sqrt{9 \times 4 \times 4 \times 1}$
$= 12 \text{ cm}^2$

OR

We also know two sides and the included angle

$$\text{Area } = \frac{1}{2}(\text{Side})(\text{Side})\sin(\text{included angle})$$

$$= \frac{1}{2}(EC)(CD)\sin E\hat{C}D$$

$$= \frac{1}{2}(5)(8)\sin(37°)$$

$$= 12.0 \text{ cm}^2 \text{ (to 1 decimal place)}$$

(b) Data: $A = (-1, 4)$ and $B = (3, 2)$

(i) **Required to calculate:** The gradient of AB
Solution:

$$\text{Gradient of } AB = \frac{2-4}{3-(-1)}$$

$$= -\frac{1}{2}$$

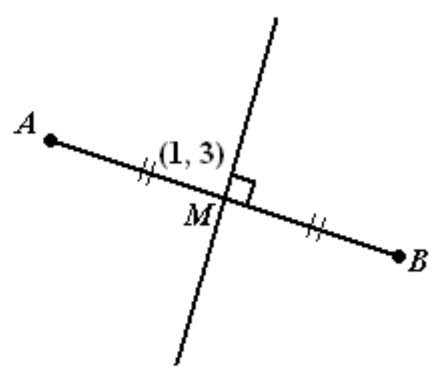
(ii) **Required To Calculate:** The coordinates of the midpoint of AB
Solution:
Let the midpoint of AB be M.

Using the midpoint formula, we get

$$M = \left(\frac{-1+3}{2}, \frac{4+2}{2}\right)$$

$$= (1, 3)$$

(iii) **Required to determine:** The equation of the perpendicular bisector of AB
Solution:



The gradient of the perpendicular bisector or any perpendicular for that matter is $\frac{-1}{-\frac{1}{2}} = 2$.
(The product of the gradients of perpendicular lines = -1).

The midpoint, M lies on AB as well as on the perpendicular bisector.
Using the coordinates of M as a point on the perpendicular bisector of AB
The equation of the perpendicular bisector of AB is now calculated by using

$$\frac{y-3}{x-1} = 2$$
$$y-3 = 2(x-1)$$
$$y-3 = 2x-2$$
$$y = 2x+1 \text{ or any other equivalent form}$$

5. (a) **Data:** Incomplete table showing values of a direct proportion between the variables A and R.

(i) **Required to express:** A in terms of R and a constant k.
Solution:
A proportion sign can be replaced by an equal sign with the introduction of a constant.
$$A \propto R^2$$
$$\therefore A = kR^2 \quad (k \text{ is the constant of proportion})$$

(ii) **Required to calculate:** Value of k.
Solution:
From the table of values, A = 36 when R = 3. Substituting will give
$$\therefore 36 = k(3)^2$$
$$36 = 9k$$
$$k = \frac{36}{9}$$
$$k = 4$$
$$\therefore A = 4R^2$$
Answer $k = 4$

(iii) **Required to complete:** The table given.
Solution:
$A = 4R^2$
When $R = 5$

$$A = 4(5)^2$$
$$= 4(25)$$
$$= 100$$

When $A = 196$

$$196 = 4R^2$$
$$\frac{196}{4} = R^2$$
$$R^2 = 49$$
$$R = \sqrt{49}$$
$$R = 7$$

Mathematically speaking, $R = 7$ or -7, when we take the root of 49, but we take the positive value, assuming that R takes only positive values from the appearance of values in the table.

$\therefore$ The completed table now looks like:

A	36	**100**	196
R	3	5	**7**

(b) **Data:** $f(x) = \frac{2x+1}{3}$ and $g(x) = 4x + 5$

(i) **Required to calculate:** $fg(2)$

Solution:

First we find $g(2)$ by substituting $x = 2$ in $g(x)$.

$$g(2) = 4(2) + 5$$
$$= 8 + 5$$
$$= 13$$
$$\therefore fg(2) = f(13)$$
$$= \frac{2(13)+1}{3}$$
$$= \frac{26+1}{3}$$
$$= \frac{27}{3}$$
$$= 9$$

OR

We could find the composite function *fg*

$$fg(x) = \frac{2(4x+5)+1}{3}$$
$$= \frac{8x+10+1}{3}$$
$$= \frac{8x+11}{3}$$
$$fg(2) = \frac{8(2)+11}{3}$$
$$= \frac{16+11}{3}$$
$$= \frac{27}{3}$$
$$= 9$$

(ii) **Required to determine:** $f^{-1}(3)$

Solution:

Let $y = f(x)$

Make x the subject and replace y by x to obtain the inverse.

$$y = \frac{2x+1}{3}$$
$$3y = 2x+1$$
$$2x = 3y-1$$
$$x = \frac{3y-1}{2}$$

Now we replace y by x to get:

$$f^{-1}(x) = \frac{3x-1}{3}$$

$$f^{-1}(3) = \frac{3(3)-1}{2}$$
$$= \frac{8}{2}$$
$$= 4$$

6. (a) **Data:** Car travelling at 54 kmh^{-1} covers a distance in 20 seconds.

(i) **Required to calculate:** Speed of the car in m/s

Solution:

Recall 1 km = 1 000 m and 1 h = 3 600 s.

So,

$$54 \text{ kmh}^{-1} = \frac{54 \times 1000}{3600} \text{ ms}^{-1}$$
$$= 15 \text{ ms}^{-1}$$

(It is useful to note that a car CANNOT TRAVEL and it would have been better to say that a car moves at a speed of 54 kmh^{-1} …)

(ii) **Required to calculate:** The distance in metres between the 2 sign posts
Solution:

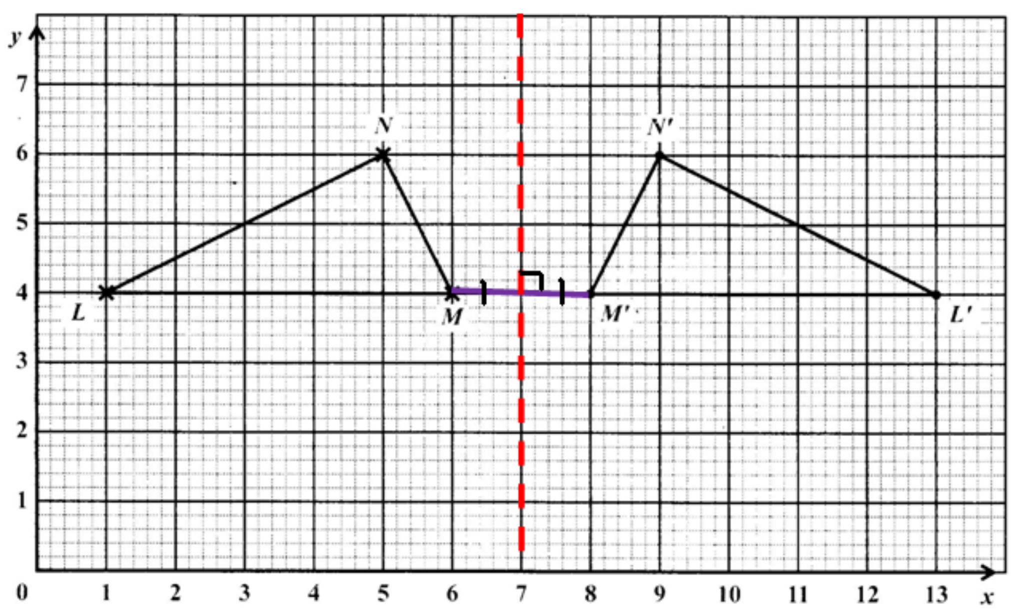
$$\text{Distance} = \text{Constant Speed} \times \text{Time}$$
$$= 15 \text{ ms}^{-1} \times 20 \text{ s}$$
$$= 300 \text{ m}$$

(b) **Data:** ΔLMN is mapped onto $\Delta L'M'N'$ under a single transformation

(i) **Required to describe:** The transformation
Solution:
By observation,
(i)The image and the object are congruent.
(ii)The image is laterally inverted (flipped).

The conclusion is that the transformation is a reflection.

To find the line of reflection:
The perpendicular bisector of the line joining any object point to its corresponding image point will be the line of reflection.
So,to obtain the line of reflection, we can, for example, join M to M' and determine the perpendicular bisector. This is shown in the diagram, below.



The perpendicular bisector is the vertical line with the equation $x = 7$.
Hence, the transformation is a reflection in the line $x = 7$.

We could also have used either of the remaining two object points, *L* or *N* and their corresponding image points in a like manner to obtain the same result.

(ii) **Data:** $LMN \xrightarrow{\begin{pmatrix} 0 \\ -3 \end{pmatrix}} L'M'N'$
Required to draw: $\Delta L''M''N''$
Solution:
We can find the images of *L, M* and *N* by calculation

$$L \xrightarrow{\begin{pmatrix} 0 \\ -3 \end{pmatrix}} L''$$

$$\begin{pmatrix} 1 \\ 4 \end{pmatrix} \xrightarrow{\begin{pmatrix} 0 \\ -3 \end{pmatrix}} \begin{pmatrix} 1+0 \\ 4-3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$\therefore L'' = (1, 1)$

$$M \xrightarrow{\begin{pmatrix} 0 \\ -3 \end{pmatrix}} M''$$

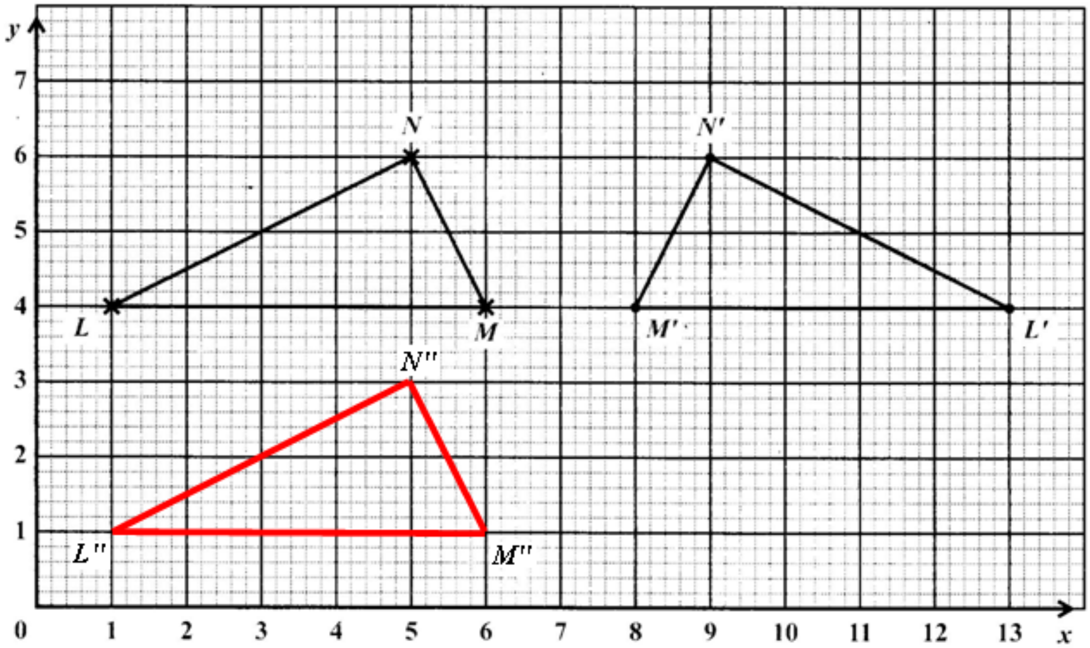
$$\begin{pmatrix} 6 \\ 4 \end{pmatrix} \xrightarrow{\begin{pmatrix} 0 \\ -3 \end{pmatrix}} \begin{pmatrix} 6+0 \\ 4-3 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

$\therefore M'' = (6, 1)$

$$N \xrightarrow{\begin{pmatrix} 0 \\ -3 \end{pmatrix}} N''$$

$$\begin{pmatrix} 5 \\ 6 \end{pmatrix} \xrightarrow{\begin{pmatrix} 0 \\ -3 \end{pmatrix}} \begin{pmatrix} 5+0 \\ 6-3 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$\therefore N'' = (5, 3)$
The image is drawn in red on the grid below.



OR
We may simply shift each vertex of ΔLMN by 3 units vertically downwards to obtain $\Delta L''M''N''$ since there is no horizontal shift and the vertical shift is 3 units downwards.

(iii) **Required to name:** A combination of two transformations which may be used to map $\Delta L''M''N''$ onto $\Delta L'M'N'$.
Solution:
$\Delta L''M''N''$, could first be translated by the opposite vector which is $-\begin{pmatrix} 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ thereby occupying the same position as ΔLMN. Next, the image must be reflected in the line $x = 7$ so as to be mapped onto $\Delta L'M'N'$.

It is important to realise that the two transformations may be done in the reverse order to obtain the same result. That is, the $\Delta L''M''N''$ can be first reflected in the vertical line $x = 7$ and afterwards translated by the opposite vector $-\begin{pmatrix} 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ to obtain the same final image.

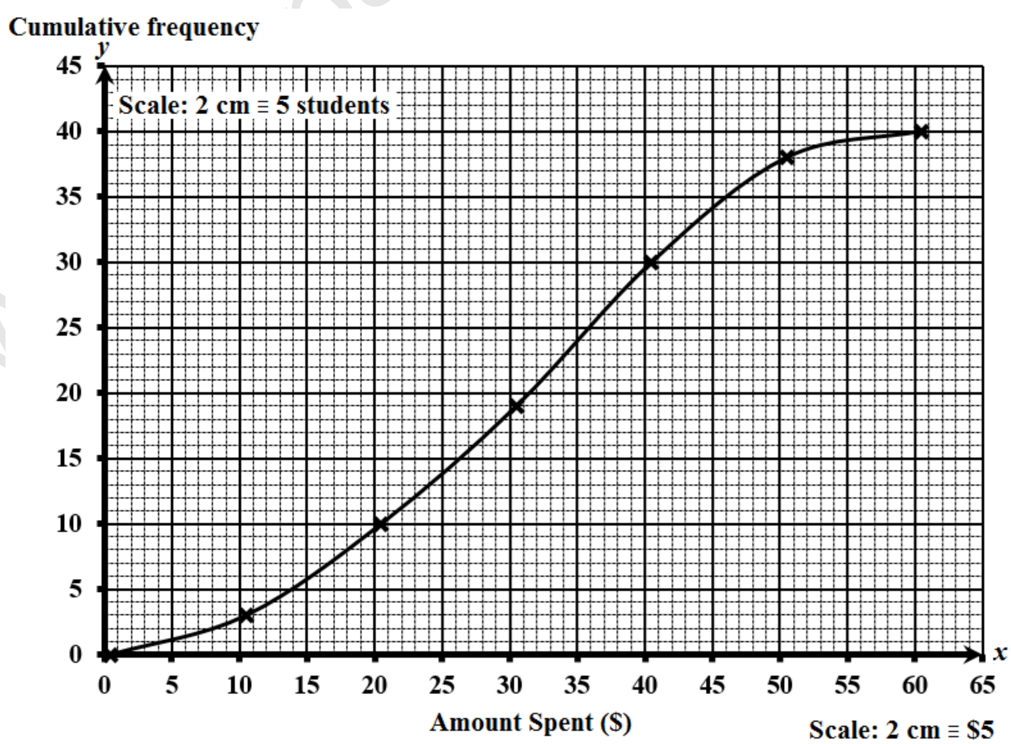
7. **Data:** Table showing the amount of money spent by 40 students for a week, at the School's canteen.
   (a) **Required to complete:** The table to show the cumulative frequency.
   Solution:

The table is modified to show the lower and upper class boundaries (LCB and UCB) and the points to be plotted for the cumulative frequency curve. The data is a continuous variable and the amount spent being expressed to the nearest dollar. We therefore add in a column to show the Lower and Upper class boundaries (**L.C.B** and **U.C.B**) and another to show the points that are to be plotted.

L.C.L-Lower class limit
U.C.L-Upper class limit

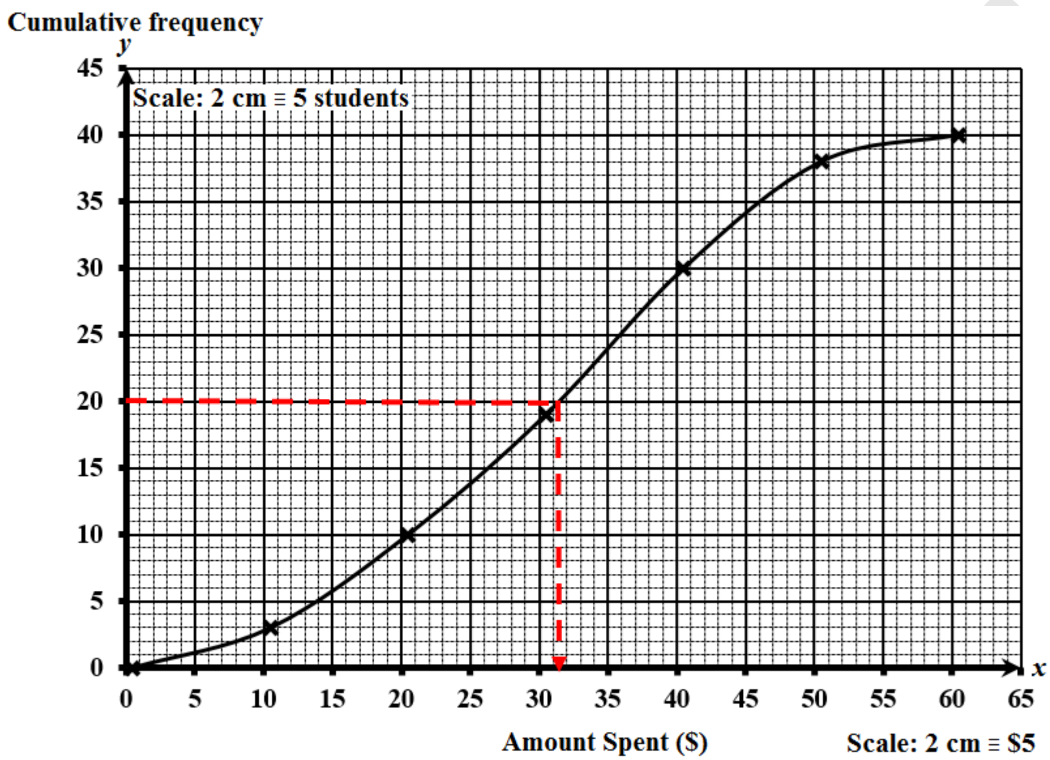
Amount Spent (x) L.C.L-U.C.L.	**L.C.B U.C.B**	**No. of Students Frequency (f)**	**Cumulative Frequency**	**Points to be Plotted**
				$(0.5, 0)$
1 – 10	$0.5 \leq x < 10.5$	3	3	$(10.5, 3)$
11 – 20	$10.5 \leq x < 20.5$	7	$7 + 3 = 10$	$(20.5, 10)$
21 – 30	$20.5 \leq x < 30.5$	9	$10 + 9 = 19$	$(30.5, 19)$
31 – 40	$30.5 \leq x < 40.5$	11	$19 + 11 = 30$	$(40.5, 30)$
41 – 50	$40.5 \leq x < 50.5$	8	$30 + 8 = 38$	$(50.5, 38)$
51 – 60	$50.5 \leq x < 60.5$	2	$38 + 2 = 40$	$(60.5, 40)$

Note that the point coordinates $(0.5, 0)$ is obtained by checking backwards so to ensure that the cumulative frequency curve starts from the horizontal axis.



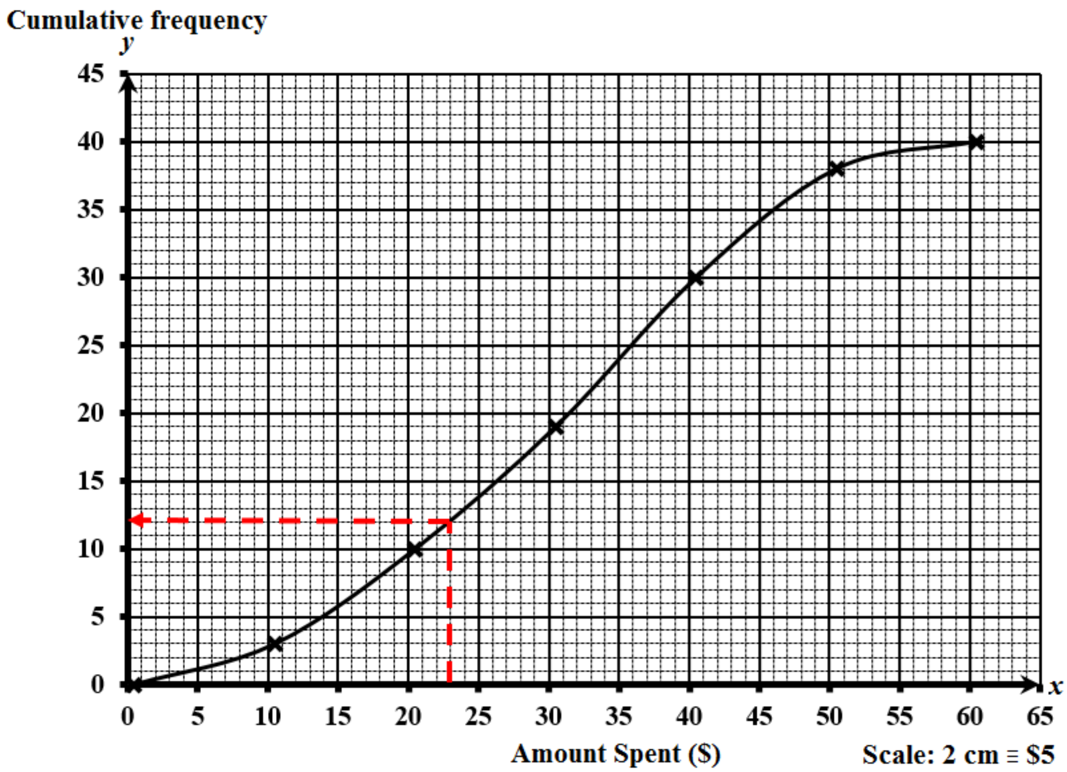
(c) (i) **Required to estimate:** The median amount of money spent.
Solution:
$\frac{1}{2}$ of $40 = 20$

We draw a horizontal from the mark of 20 on the vertical axis and then drop the vertical from the point where this horizontal line meets the curve. We now read-off the money value to obtain the median amount of money spent. Both lines are shown in red on the diagram below.



The median amount of money spent = $31.50 obtained by a read-off

(ii) **Required to estimate:** The probability that a student spends less than $23 dollars in a week.
Solution:

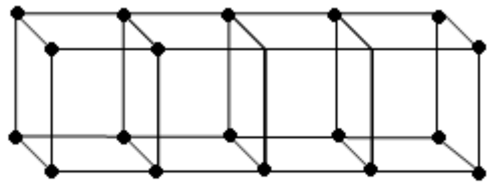


From the monetary value of \$23, we draw a vertical to meet the curve. At the point where this vertical meets the curve, we now draw the horizontal to obtain the number of students who spent less than \$ 23. We find this value to be 12. Both lines are shown in red on the diagram.

$$P(\text{Student spends less than } \$23)$$
$$= \frac{\text{No. of students who spends less than } \$23}{\text{Total no. of students}}$$
$$= \frac{12}{40}$$
$$= \frac{3}{10}$$

This may also be expressed as 30% or 0.3

8. **Data:** Drawings showing the first three diagrams in a sequence.
   (a) **Required to draw:** The fourth diagram in the sequence.
   Solution: The sketch of diagram 4 is to be done on the answer sheet that was provided and would look like that shown below. It basically was found obtained by attaching figure 1 to figure 3.



(b) **Required to complete:** The missing values in the table given.
Solution:

	Name of Diagram (N)	No. of Wires (W)	No. of Balls (B)
	1	12	8
	2	20	12
	3	28	16
(i)	4		
(ii)	20		

Notice that as the value of N increases by 1, the value of W increases by 8. For example,

$N = 1$ $\quad W = 12$

$N = 2$ $\quad W = 12 + 8 = 20$

$N = 3$ $\quad W = 20 + 8 = 28$

Hence, when $N = 4$ $\quad W = 28 + 8 = 36$

Notice that as N increases by 1, B increases by 4.

$N = 1$ $\quad B = 8$

$N = 2$ $\quad B = 8 + 4 = 12$

$N = 3$ $\quad B = 12 + 4 = 16$

Hence, when $N = 4$ $\quad B = 16 + 4 = 20$

$\therefore$ The row for (i) should be

(i)	4	**36**	**20**

We seek a relationship between N and W.

N	1	2	3
W	12	20	28

Since W increases by 8, when N increases by 1 then
$W = 8N + ?$

$12 = 8(1) + 4$

$20 = 8(2) + 4$

$28 = 8(3) + 4$

$\therefore W = 8N + 4$

When $N = 20$
$W = 8(20) + 4$
$= 164$

Similarly, we seek a relationship between N and B.

N	1	2	3
B	8	12	16

As N increases by 1, B increases by 4.
So,
$B = 4N + ?$

$8 = 4(1) + 4$

$12 = 4(2) + 4$

$16 = 4(3) + 4$

$\therefore B = 4N + 4$

When $N = 20$
$B = 4(20) + 4$
$= 84$

The row for (ii) should now read:

(ii)	20	164	84

The completed table should therefore be:

	Name of Diagram (N)	No. of Wires (W)	No. of Balls (B)
	1	12	8
	2	20	12
	3	28	16
(i)	4	**36**	**20**
(ii)	20	**164**	**84**

(c) (i) **Required to write:** The rule to find W.
Solution:
$W = 8N + 4$ (found from before)

(ii) **Required to write:** The rule to find B.
Solution:
$B = 4N + 4$ (found from before)

SECTION II

9. (a) (i) **Data:** Trish wishes to buy x oranges and y mangoes. Her bag can only hold 6 fruits.
Required to write: An inequality to represent this information
Solution:
Number of oranges $= x$ (data)
Number of mangoes $= y$ (data)
The total number of fruits will be $= x + y$
The bag has space for only 6 fruits.
$\therefore$ **The total number of oranges and mangoes must not exceed** 6.
This is written algebraically as $x + y \not> 6$
Hence, $x + y \leq 6 \quad \ldots(1)$

(ii) **Data:** Trish must buy at least 2 mangoes.
Required to write: An equality to represent this information
Solution:
Trish must buy at least 2 mangoes.
Hence $y \geq 2 \quad \ldots(2)$

(iii) **Data:** More information about the number of oranges and mangoes is represented by $y \leq 2x$.
Required to write: Information represented by this inequality.
Solution:

$$\underset{\text{Number of mangoes}}{y} \quad \leq \quad \underset{\text{Twice the number of oranges}}{2x}$$

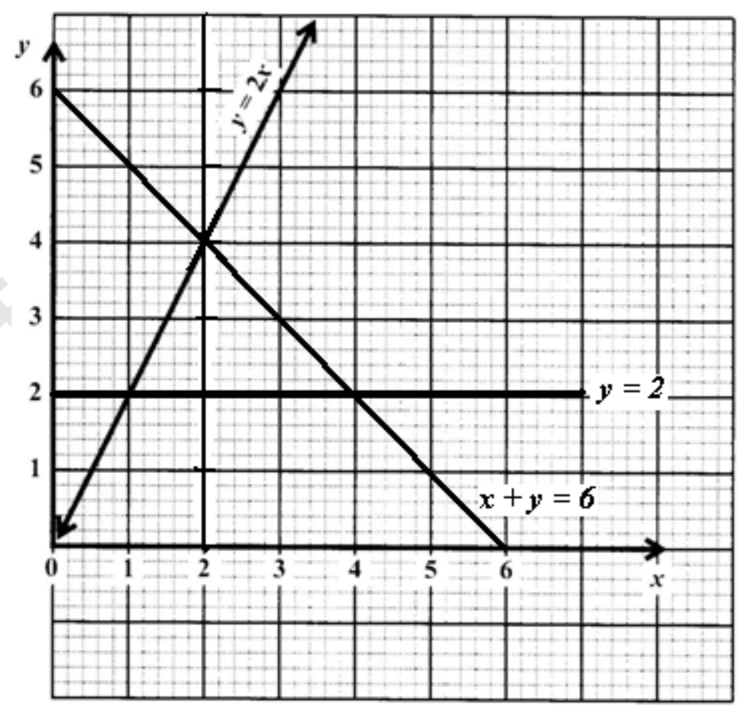
$\therefore$ The number of mangoes (y) is less than or equal to twice the number (2 x x) of oranges.

OR

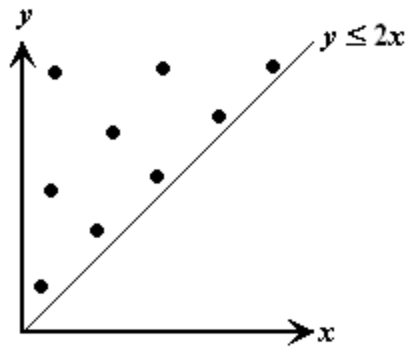
The number of mangoes must not be more than twice the number of oranges.

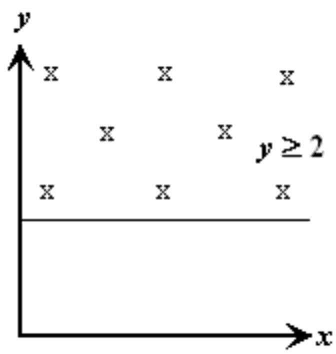
(iv) **Required to draw:** The lines associated with the two inequalities obtained in (i) and (ii).
Solution:
Let $x + y = 6$
We find the coordinates of two points on the straight line

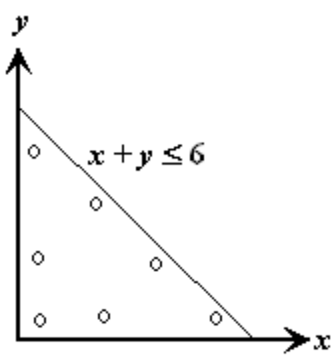
x	y
0	6
6	0



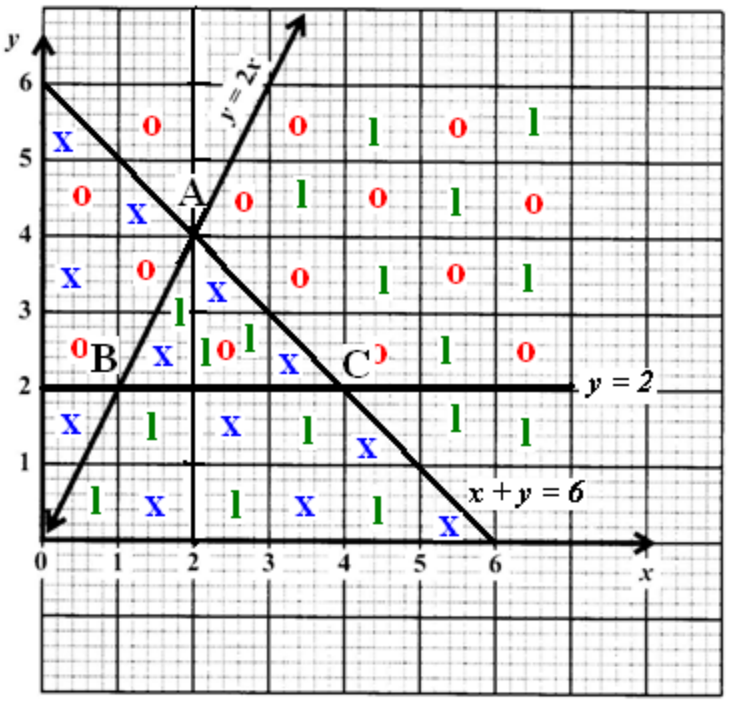
(v) **Required to shade**: The region that satisfies all the inequalities.
Solution:





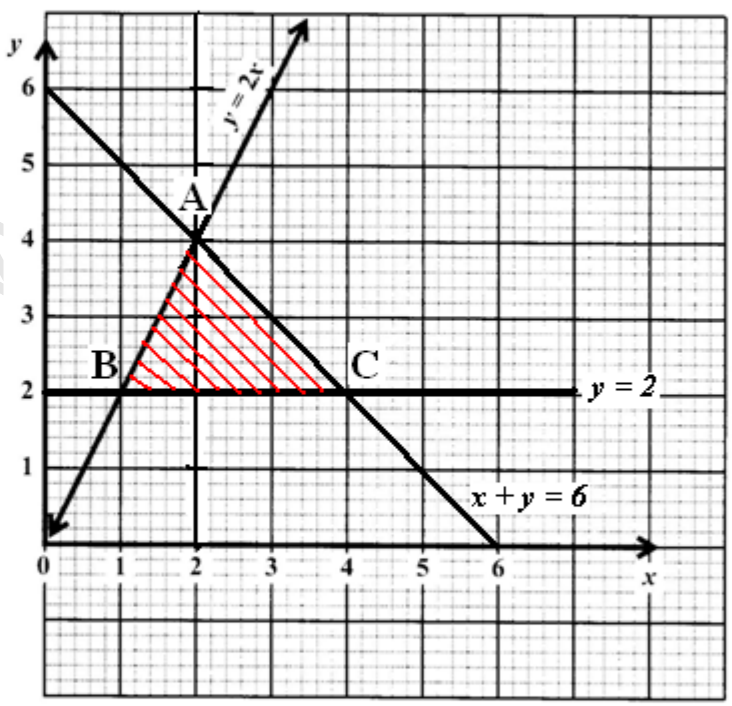


We now shade the regions occupied by all three inequalities, using the accompanying key shown above to obtain:



$$\left.\begin{aligned} x + y &\leq 6 \\ y &\geq 2 \\ y &\leq 2x \end{aligned}\right\}$$ Identified by the region ABC(the feasible region)

This is shown shaded in the diagram below. Any point on or within ABC will satisfy all three inequalities.



(b) (i) **Required to write:** $3x^2-12x+8$ in the form $a(x+h)^2+k$.

Solution:

We can do this by expanding the required form and equating coefficients with the given equation.

$$\begin{aligned}3x^2-12x+8&=a(x+h)^2+k\\&=a(x+h)(x+h)+k\\&=a\left\{x^2+hx+hx+h^2\right\}+k\\&=ax^2+2ahx+ah^2+k\end{aligned}$$

Equating the coefficients of x^2

$a=3$

Equating the coefficients of x and recalling $a=3$

$$2ah=-12$$
$$\therefore 2(3)h=-12$$
$$h=-2$$

Equating the constant term and recalling $a=3$ and $h=-2$

$$ah^2+k=8$$
$$\therefore 3(-2)^2+k=8$$
$$12+k=8$$
$$k=-4$$

$$\therefore 3x^2-12x+8=3(x-2)^2-4$$

OR

$$\begin{aligned}3x^2-12x+8&=3\left(x^2-4x\right)+8\\&=3(x-2)^2+*\end{aligned}$$

$$\begin{aligned}3(x-2)^2&=3(x-2)(x-2)\\&=3\left(x^2-4x+4\right)\\&=3x^2-12x+12\end{aligned}$$

$$\begin{array}{r}-\ 4\\ \hline 8\\ \hline\end{array}$$

$$\therefore *=4$$

Hence, $3x^2 - 12x + 8 = 3(x-2)^2 - 4$ and is of the form $a(x+h)^2 + k$, where $a = 3$, $h = -2$ and $k = -4$, and a, h and k are constants.

(ii) **Required to sketch:** The graph of $y = 3x^2 - 12x + 8$, showing the intercept on the y – axis and the coordinates of the minimum turning point.

Solution:

When $x = 0$

$y = 3(0)^2 - 12(0) + 8$

$= 8$

$\therefore$ The curve cuts the y – axis at the point $(0, 8)$.

$3x^2 - 12x + 8 = 3(x-2)^2 - 4$

Any term to be squared is greater than or equal to 0. So 0 is its lowest value.

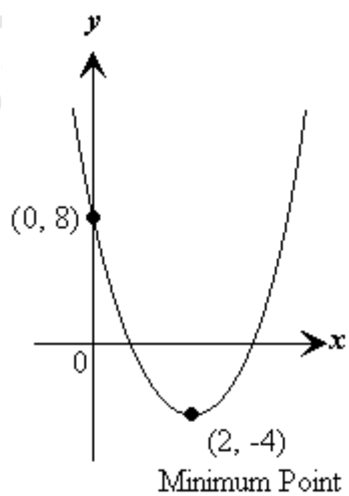
$3(x-2)^2$ is greater than or equal to 0 for all values of x.

Hence, $y_{\text{minimum}} = 0 - 4 = -4$

When $3(x-2)^2 = 0$ and y is a minimum, $x = 2$

Therefor (2, -4) is the minimum point on the curve.

The sketch of $y = 3x^2 - 12x + 8$ would look like:



The minimum point could have been found by other methods such as:

The vertical axis of symmetry of the quadratic graph has equation

$$x = \frac{-(-12)}{2(3)} = 2$$

(The quadratic graph has a minimum point since the coefficient of x^2 is positive)

The x – coordinate of the minimum point is the same x value as in the equation of the axis of symmetry since the axis of symmetry passes through the minimum point.

When $x = 2$

$$y = 3(2)^2 - 12(2) + 8$$
$$= -4$$

$\therefore (2, -4)$ is the minimum turning point.

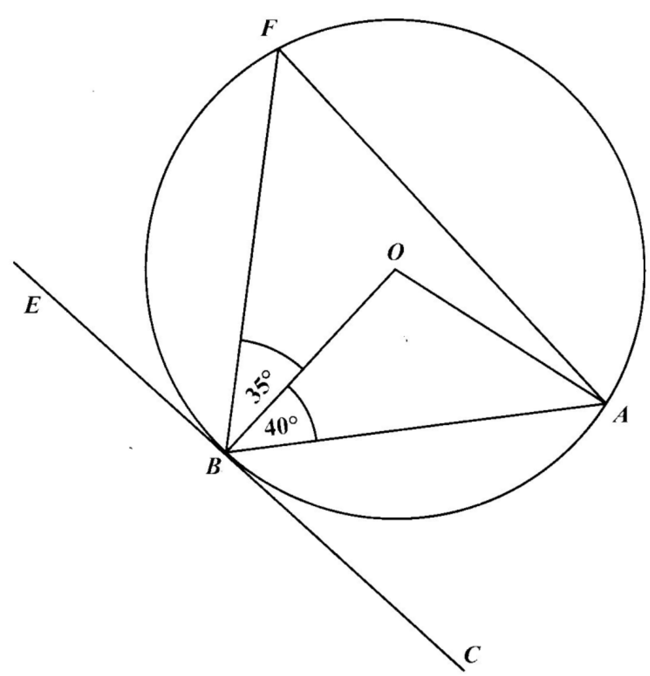
Some advanced students of mathematics may also have used differential calculus, since the question does not specify any particular method.
The gradient function,

$$\frac{dy}{dx} = 3(2x) - 12$$
$$= 6x - 12$$

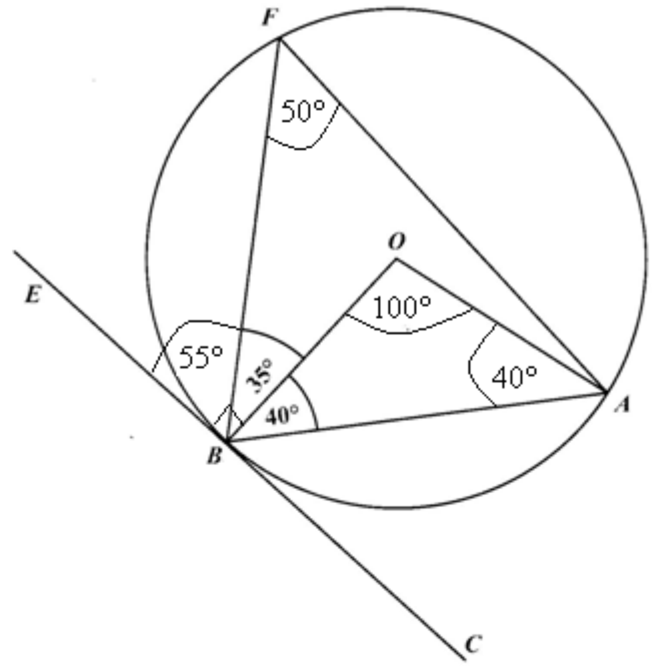
When $\frac{dy}{dx} = 0$, $x = 2$ and $y = -4$

$\frac{d^2y}{dx^2} = 6 > 0 \Rightarrow (2, -4)$ is a minimum stationary point.

10. (a) **Data:** Diagram showing a circle, center O. EBC is a tangent to the circle. $O\hat{B}A = 40°$ and $O\hat{B}F = 35°$.



(i) **Required to calculate:** $E\hat{B}F$
Solution:



$O\hat{B}E = 90°$ (The angle made by a tangent to a circle and a radius, at the point of contact, is 90°)
OB is the radius and *EBC* is the tangent. The point of contact is *B*.

$\therefore E\hat{B}F = 90° - 35°$
$= 55°$

(ii) **Required to calculate:** $B\hat{O}A$
Solution:
$OB = OA$ (radii of the same circle)
$\therefore \Delta OAB$ is an isosceles triangle with $O\hat{B}A = O\hat{A}B = 40°$
(The base angles of an isosceles triangle are equal).

So, $B\hat{O}A = 180° - (40° + 40°)$
= 100° (The sum of the three angles of a triangle = 180°)

(iii) **Required to calculate:** $A\hat{F}B$
Solution:
$$A\hat{F}B = \frac{1}{2}(100°)$$
$$= 50°$$

(The angle subtended by a chord at the center of a circle is twice the angle that the chord subtends at the circumference, standing on the same arc.)

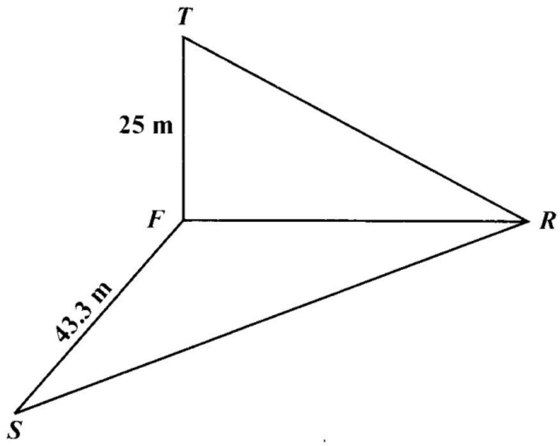
(iv) **Required to calculate:** $O\hat{A}F$

Solution:

$$O\hat{A}F = 180° - \{50° + 35° + 40° + 40°\}$$
$$= 15°$$

(The sum of the three angles of a triangle = 180°)

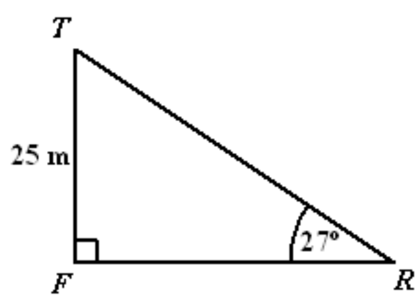
(b) **Data:** Tower *FT* is 25 m high. *R* is East of *F* and the angle of elevation of *T* from *R* = 27°. *S* is South of *F*. *SF* = 43.3 m.

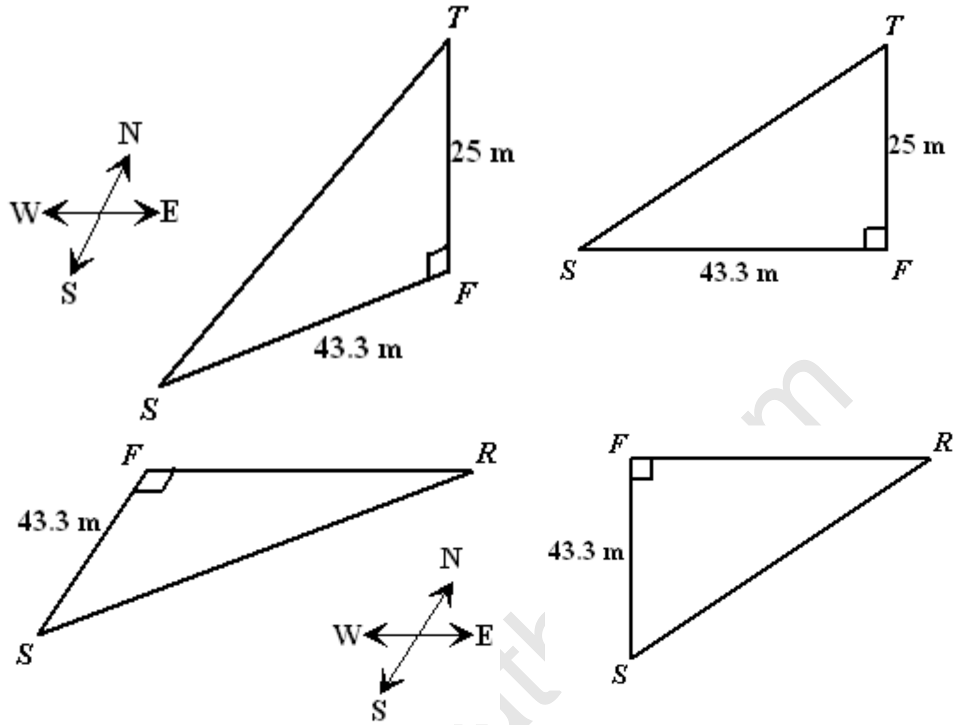


(i) **Required to sketch:** Separate diagrams of triangles *RFT*, *TFS* and *SFR*, marking in all sides and angles.

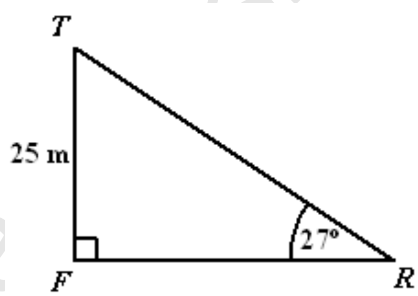
Solution:

All the triangles are right angled but appear slanting as the figure is 3-d. We draw each one as they appear in the diagram and as they actually are.





(ii) **Required to show:** $RF = 49.1$ m
Solution:



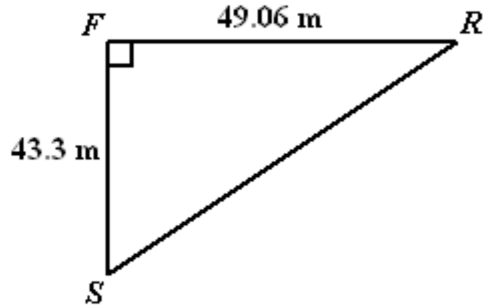
$$\tan 27° = \frac{25}{RF}$$

$$\therefore RF = \frac{25}{\tan 27°}$$

$$= 49.0\underset{=}{6} \text{ m}$$

$$= 49.1 \text{ m}$$

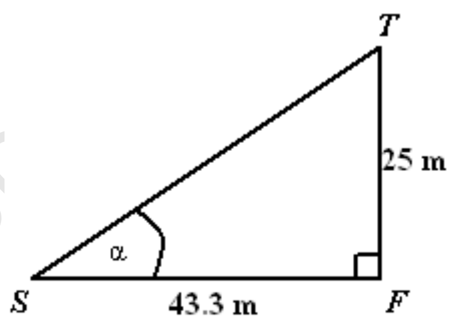
(iii) **Required to calculate:** *SR,* correct to 1 decimal place.
Solution:



The triangle is right-angled, so

$$SR^2 = (49.06)^2 + (43.3)^2 \quad \text{(Pythagoras' Theorem)}$$
$$= 2406.88 + 1874.89$$
$$= 4281.77$$
$$SR = \sqrt{4281.77}$$
$$= 65.4\underset{=}{3} \text{ m}$$
$$= 65.4 \text{ m correct to 1 decimal place}$$

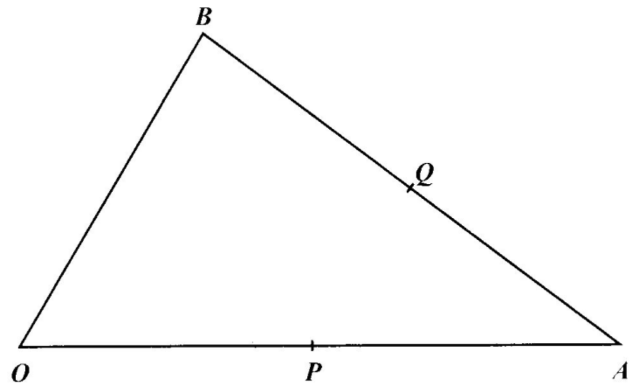
(iv) **Required to calculate:** The angle of elevation of the top of the tower, *T*, from *S*.
Solution:



Let the angle of elevation, *TSF*, be α as shown on the diagram.

$$\tan \alpha° = \frac{25}{43.3}$$
$$\alpha° = \tan^{-1}\left(\frac{25}{43.3}\right)$$
$$\alpha = 30.0\underset{=}{0}$$
$$= 30.0 \text{ to 1 decimal place}$$

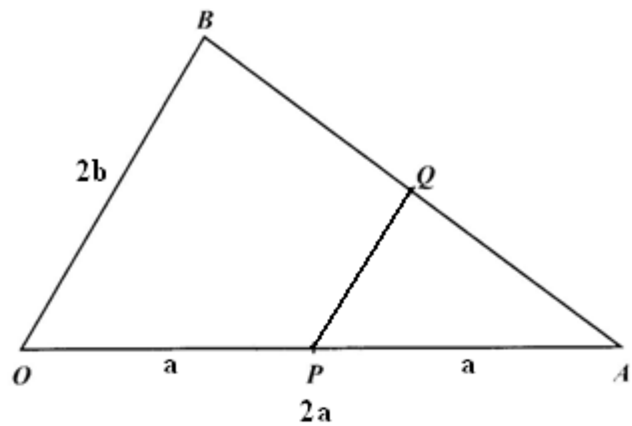
11. (a) **Data:**



P and Q are the midpoints of OA and AB respectively. $OA = 2\mathbf{a}$ and $OB = 2\mathbf{b}$.

(i) a) **Required to express:** AB in terms of $\mathbf{a}$ and $\mathbf{b}$.

Solution:



Applying the vector triangle law to get

$$AB = AO + OB$$
$$= -(2\mathbf{a}) + 2\mathbf{b}$$
$$= -2\mathbf{a} + 2\mathbf{b}$$

b) **Required to express:** PQ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Solution:

Since P is the midpoint of OA, then

$$OP = PA = \mathbf{a}$$

$$AQ = \frac{1}{2}AB$$

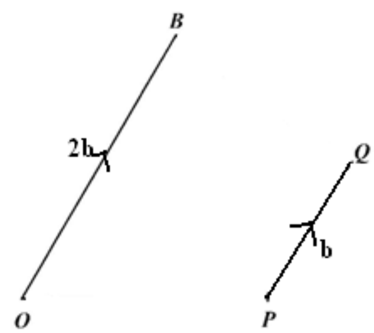
$$= \frac{1}{2}(-2\mathbf{a} + 2\mathbf{b})$$

$$PQ = PA + AQ$$

$$= \mathbf{a} + \frac{1}{2}(-2\mathbf{a} + 2\mathbf{b})$$

$$= \mathbf{b}$$

(ii) **Required to state:** Two geometrical relationships between OB and PQ.
Solution:



$OB = 2PQ$ (data)

Since PQ can be represented as a scalar multiple of AB, then OB and PQ are parallel.

$OB = 2PQ$

$\therefore |OB| = 2|PQ|$ The modulus sign denoting the length of the vector

That is, OB is twice the length of PQ or PQ is half the length of OB.

(b) **Data:** $M = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

(i) **Required to evaluate:** M^{-1}
Solution:
M^{-1} is the inverse of the matrix M

$$\det M = (2\times 3) - (1\times 4)$$

$$= 6 - 4$$

$$= 2$$

$$\therefore M^{-1} = \frac{1}{2}\begin{pmatrix} 3 & -(1) \\ -(4) & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{4}{2} & \frac{2}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1\frac{1}{2} & -\frac{1}{2} \\ -2 & 1 \end{pmatrix}$$

(ii) **Required to show:** $M^{-1}M = I$
Solution:
This is done by multiplying out the left hand side, M^{-1} M. The result Should be the right hand side = I and hence the proof.

$$M^{-1} \times M = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{4}{2} & \frac{2}{2} \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$$

$$2\times 2 \xrightarrow[2\times 2]{} 2\times 2$$

$$\begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$$

Working out each of the four entries we get,

$$e_{11} = \left(\frac{3}{2}\times 2\right) + \left(-\frac{1}{2}\times 4\right)$$
$$= 3 - 2$$
$$= 1$$

$$e_{12} = \left(\frac{3}{2}\times 1\right) + \left(-\frac{1}{2}\times 3\right)$$
$$= 1\frac{1}{2} - 1\frac{1}{2}$$
$$= 0$$

$$e_{21} = \left(-\frac{4}{2}\times 2\right) + \left(\frac{2}{2}\times 4\right)$$
$$= -4 + 4$$
$$= 0$$

$$e_{22} = \left(-\frac{4}{2}\times 1\right)+\left(\frac{2}{2}\times 3\right)$$
$$= -2+3$$
$$= 1$$

$$\therefore M^{-1}M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Q.E.D.

(iii) **Required to solve:** For r, s, t and u
Solution:
Let us multiply the given equation by M^{-1}.
Remember a matrix multiplied by its inverse gives the identity matrix.
And, a matrix by the identity matrix, gives the same matrix as the result.

$$M^{-1}\times M\times\begin{pmatrix} r & s \\ t & u \end{pmatrix} = M^{-1}\times\begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$$

$$\therefore I\times\begin{pmatrix} r & s \\ t & u \end{pmatrix} = \begin{pmatrix} 1\frac{1}{2} & -\frac{1}{2} \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$$

$$2\times 2 \xrightarrow[2\times 2]{} 2\times 2$$

$$\begin{pmatrix} r & s \\ t & u \end{pmatrix} = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$$

$$e_{11} = \left(1\frac{1}{2}\times 2\right)+\left(-\frac{1}{2}\times 4\right)$$
$$= 3-2$$
$$= 1$$

$$e_{12} = \left(1\frac{1}{2}\times 1\right)+\left(-\frac{1}{2}\times -1\right)$$
$$= 1\frac{1}{2}+\frac{1}{2}$$
$$= 2$$

$$e_{21} = (-2\times 2)+(1\times 4)$$
$$= -4+4$$
$$= 0$$

$$
\begin{aligned}
e_{22} &= (-2\times 1)+(1\times -1)\\
&= -2-1\\
&= -3
\end{aligned}
$$

$$\therefore \begin{pmatrix} r & s \\ t & u \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}$$

Both are 2 x 2 matrices and are equal.
Equating corresponding entries, we obtain
$r = 1$, $s = 2$, $t = 0$ and $u = -3$