FAS-PASS
Maths

CSEC MATHEMATICS JANUARY 2013
PAPER 2
Section I

1. (a) **Required to calculate:** $(2.67\times 4.1)-1.3^2$

Calculation:

$$(2.67\times 4.1)-1.3^2 = 10.947-1.69 \text{ (by calculator)}$$
$$= 9.257 \text{ (exact)}$$

(b) **Data:** A list of holiday rates offered by Petty's Travel Club

(i) **Required to calculate:** The total cost of airfare and hotel accommodation for three nights.

Calculation:

The cost of the return airfare = $356 US (data)

The cost of the hotel accommodation for 3 nights at $97 US per night

$= \$97\times 3$

$= \$291$ US

Hence, total cost $= \$(356+291)$ US

$= \$647$ US

(ii) **Data:** Holiday rates as listed by Angie's Travel Club

Required to calculate: The cost in US dollars for a similar trip offered by Angie's Travel Club

Calculation:

EC$2.70 $\equiv$ US$1.00

The cost of a similar holiday by Angie's Travel Club= EC$1610

The cost in US dollars $= \dfrac{\$1610}{2.70}$

$= \text{US}\$596.29\underline{\underline{6}}$

$= \text{US}\$596.30$ (to the nearest cent)

(iii) **Required to state:** The travel club with the better offer.

Solution:

Angie's Travel Club's price of $596.30 is less than Petty's Travel Club's price of $647.

If 'better' in this context is supposed to mean cheaper, then we would choose Angie's Travel Club as having the better offer.

(iv) **Data:** The price of $1610 EC is inclusive of a 15% sales tax.

Required to calculate: The cost of the trip before the sales tax was added

Calculation:

If we consider the price before tax as, 100% then the cost of EC \$1610 is $(100+15)\% = 115\%$ on the pre-tax figure.

$\therefore$ 1% will be $\text{EC}\dfrac{\$1610}{115}$

Hence, the cost of the trip, before the tax is added, will be 100% and equal to

$$\text{EC } \frac{\$1\,610}{115} \times 100 = \$1\,400$$
$$= \text{EC } \$1\,400$$

2. (a) **Data:** $2(p+5)-7=4p$

Required to calculate: The value of *p*

Calculation:

We expand, simplify and put all terms involving *p* on one side of the equation.

$$2(p+5)-7=4p$$
$$2p+10-7=4p$$
$$10-7=4p-2p$$
$$3=2p$$
or
$$2p=3$$
$$p=\frac{3}{2} \text{ or } 1\frac{1}{2}$$

(b) (i) **Required to factorise:** $25m^2-1$

Solution:

We alter the given expression

$$25m^2-1=(5m)^2-(1)^2$$

This is now expressed as the difference of two squares and which is of a standard form. It factorises to give

$$\therefore 25m^2-1=(5m-1)(5m+1)$$

(ii) **Required to factorise:** $2n^2 - 3n - 20$

Solution:

$$2n^2-3n-20=(2n+5)(n-4)$$

We can confirm by expanding the result to give the original expression.

(c) **Data:** 1 lollipop has a mass of *x* grams.
1 toffee has a mass of *y* grams.
5 lollipops and 12 toffees have a mass of 61 grams.
10 lollipops and 13 toffees have a mass of 89 grams.

(i) **Required to write:** Two equations, in x and y, to represent the data given.
Solution:
If 5 lollipops at x grams each and 12 toffees at y grams have a total mass of 61 grams, then this can be expressed algebraically as

Then, $(5\times x)+(12\times y)=61$

$\therefore 5x+12y=61 \quad \ldots(1)$

Similarly, if 10 lollipops and 13 toffees have a total mass of 89 grams, then this can be expressed algebraically as

$(10\times x)+(13\times y)=89$

$\therefore 10x+13y=89 \quad \ldots(2)$

(ii) **Required to calculate:** The mass of one lollipop and one toffee.
Calculation:

We solve the two equations obtained above, to find the value for x and for y

$5x + 12y = 61 \quad \ldots(1)$
$10x + 13y = 89 \ldots(2)$

We solve simultaneously, using the method of substitution.
From equation (1)

$12y=61-5x$

$$y=\frac{61-5x}{12}$$

Substituting this expression in equation (2) we obtain

$$10x+13\left(\frac{61-5x}{12}\right)=89$$

$\times 12$

$$12(10x)+13(61-5x)=12(89)$$
$$120x+793-65x=1068$$
$$120x-65x=1068-793$$
$$55x=275$$
$$x=\frac{275}{55}$$
$$=5$$

Substituting $x=5$ into $y=\frac{61-5x}{12}$, we obtain

$$y = \frac{61-5(5)}{12}$$
$$= \frac{61-25}{12}$$
$$= \frac{36}{12}$$
$$= 3$$

(We could have substituted $x = 5$ in either equation (1) or (2) to solve for y)

Hence, $x = 5$ and $y = 3$ and

a) Mass of 1 lollipop = 5 grams

b) Mass of 1 toffee = 3 grams.

OR

We could have used the method of elimination to solve the two equations in x and y.

$5x + 12y = 61$ …(1)

$10x + 13y = 89$…(2)

We will try to eliminate the terms in x

Equation (1) $\times -2$

$-2(5x+12y) = -2(61)$

$-10x - 24y = -122$ ...(3)

Equation (3) + Equation (2)

$$\begin{array}{r} -10x-24y=-122 \quad + \\ 10x+13y= \quad 89 \\ \hline -11y=-\ 33 \\ \hline \end{array}$$

The terms in x are now eliminated upon addition to leave one equation in one unknown.

$\therefore y = \frac{-33}{-11}$

$= 3$

We now substitute $y = 3$ into Equation (1)

$$5x + 12(3) = 61$$
$$5x + 36 = 61$$
$$5x = 61 - 36$$
$$5x = 25$$
$$x = \frac{25}{5}$$
$$= 5$$

(or we could have substituted in equation (2))

Again we obtain,

a) The mass of 1 lollipop = 5 grams
and
b) The mass of 1 toffee = 3 grams.

OR

We could solve using the matrix method to solve for x and for y.

$5x + 12y = 61$ …(1)
$10x + 13y = 89$…(2)

We first write the equations in matrix form, by looking at the coefficients of x and y in both equations.

$$\begin{pmatrix} 5 & 12 \\ 10 & 13 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 61 \\ 89 \end{pmatrix} \text{ ... Matrix equation}$$

Let $A = \begin{pmatrix} 5 & 12 \\ 10 & 13 \end{pmatrix}$ the 2x2 matrix in the matrix equation

We find the inverse of A denoted by A^{-1}

$$\begin{aligned} \det A &= (5\times 13) - (12\times 10) \\ &= 65 - 120 \\ &= -55 \end{aligned}$$

$$\therefore A^{-1} = -\frac{1}{55}\begin{pmatrix} 13 & -(12) \\ -(10) & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{13}{55} & \frac{12}{55} \\ \frac{10}{55} & -\frac{5}{55} \end{pmatrix}$$

$\times$ The matrix equation by, A^{-1}

Recall

$A \times A^{-1} = I$, which is the identity matrix and

$I \times$ any matrix = same matrix

$$A^{-1} \times A \times \begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} \times \begin{pmatrix} 61 \\ 89 \end{pmatrix}$$

$$I \times \begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} \begin{pmatrix} 61 \\ 89 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{13}{55} & \frac{12}{55} \\ \frac{10}{55} & -\frac{5}{55} \end{pmatrix} \begin{pmatrix} 61 \\ 89 \end{pmatrix}$$

$$= \begin{pmatrix} \left(-\frac{13}{55} \times 61\right) + \left(\frac{12}{55} \times 89\right) \\ \left(\frac{10}{55} \times 61\right) + \left(-\frac{5}{55} \times 89\right) \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{793}{55} + \frac{1068}{55} \\ \frac{610}{55} + \frac{-445}{55} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{275}{55} \\ \frac{165}{55} \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

Both matrices are of order 2 x 1 and are equal matrices
Equating corresponding entries we conclude that
$x = 5$ and $y = 3$
And so,
a) Mass of 1 lollipop = 5 grams
b) Mass of 1 toffee = 3 grams.

OR
We could use the graphical method to solve for *x* and for *y*.
We treat both equations as straight lines and draw their graphs on the same axes. Their point of intersection will be the solution.

$5x + 12y = 61$ …(1)
$10x + 13y = 89$…(2)

Let 'line 1' be $5x + 12y = 61$

When $x = 0,\ y = 5\frac{1}{12}$, When $y = 0,\ x = 12\frac{1}{5}$

We plot the points $\left(0,\ 5\frac{1}{12}\right)$ and $\left(12\frac{1}{5},\ 0\right)$ for the line $5x + 12y = 61$.

Similarly we let 'line 2' be $10x + 13y = 89$

When $x = 0,\ y = 6\frac{11}{13}$, When $y = 0,\ x = 8\frac{9}{10}$

We plot the points $\left(0,\ 6\frac{11}{13}\right)$ and $\left(8\frac{9}{10},\ 0\right)$ for the line $10x + 13y = 89$.

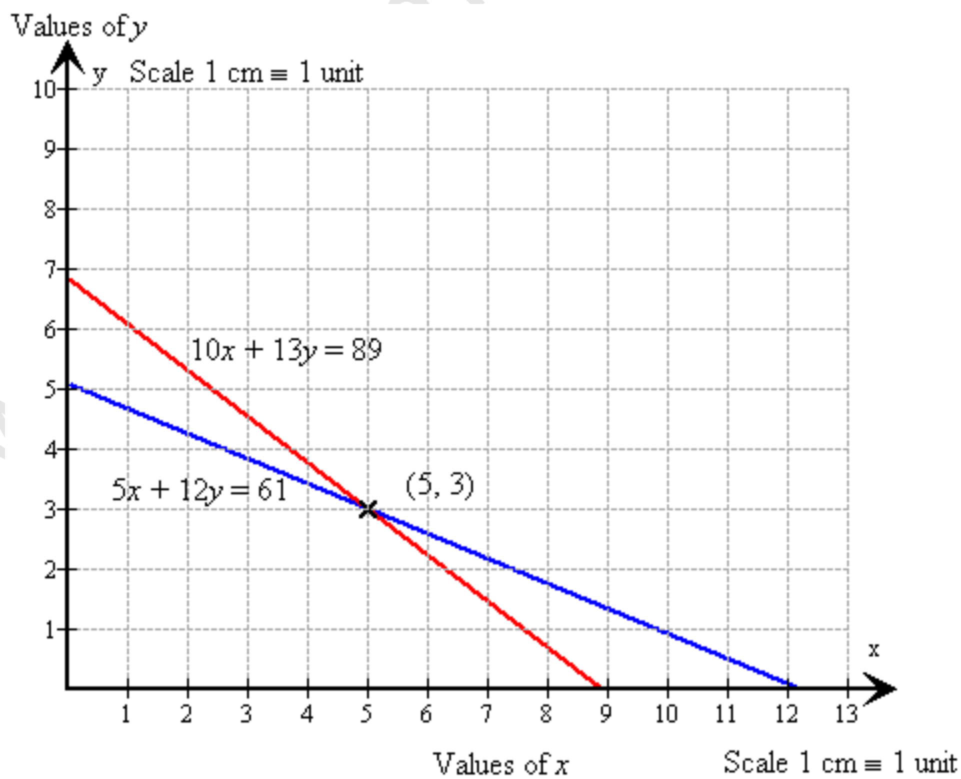
The lines are drawn on the same axes and we observe that they intersect at the point with coordinates, $(5,\ 3)$.

$\therefore x = 5$ and $y = 3$.

So, again we have,

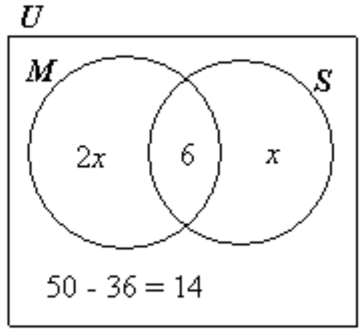
a) Mass of 1 lollipop = 5 grams

b) Mass of 1 toffee = 3 grams.



3. (a) **Data:** The numbers of students in a class who received awards for Mathematics and/or Science.

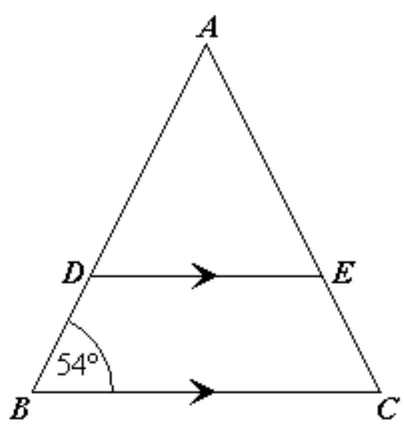
(i) **Required to represent:** The information on the incomplete Venn diagram that was given
Solution:



(ii) **Required to calculate:** the value of x
Calculation:
The sum of the number of elements in all the subsets of the universal set must total 50. This is the total number of students in the class.

$$\therefore 2x+6+x+14=50$$
$$3x+20=50$$
$$3x=50-20$$
$$3x=30$$
$$x=\frac{30}{3}$$
$$=10$$

(b) **Data:**



Above is an isosceles triangle, ABC, with $AB = AC$ and DE parallel to BC.

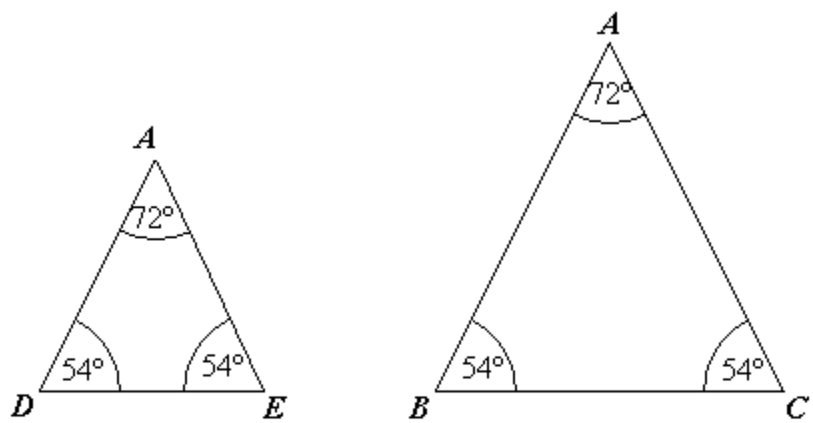
(i) a) **Required to calculate:** $\angle BAC$
Calculation:

$A\hat{C}B = 54°$ (The base angles of an isosceles triangle are equal)

$B\hat{A}C = 180° - (54° + 54°)$
$= 72°$
(The sum of the three interior angles of a triangle = 180°)

b) **Required to calculate:** $\angle AED$
Calculation:
Angle AED = Angle ACB $= 54^0$
(Corresponding angles are equal when parallel lines are cut by a transversal)

(ii) **Required to explain:** Why triangle *ABC* and *ADE* are similar but not congruent.
Solution:



$A\hat{D}E = A\hat{B}C$

$A\hat{E}D = A\hat{C}B$

$D\hat{A}E = B\hat{A}C$ (These are the same angle)

The three angles of ΔABC are equal to the three angles of ΔADE. The two triangles are equiangular or similar. That is, they have the same shape but not the same size.

However, none of the four required conditions are satisfied for two triangles to be congruent. These are:

i. Three sides equal
ii. Two sides and the included angle
iii. Two angles and the corresponding side
iv. Right angle, hypotenuse and one side

In fact, none of their corresponding sides are equal.
So, the triangles are therefore similar, but not congruent.

4. (a) (i) **Required to make:** r the subject in $r - h = rh$

Solution:

All terms involving r are kept on one side of the equation.
We simplify and make r the subject.

$$r - h = rh$$
$$r - rh = h$$
$$r(1-h) = h$$
$$r = \frac{h}{1-h}$$

(ii) **Required to make:** r the subject in $V = \pi r^2 h$

Solution:

$$V = \pi r^2 h$$
$$\pi r^2 h = V$$
$$r^2 = \frac{V}{\pi h}$$
$$r = \sqrt{\frac{V}{\pi h}}$$

(b) **Data:** $f(x) = 2x + 5,\ g(x) = \frac{x-3}{2}$

(i) **Required to calculate:** $f^{-1}(19)$

Calculation:

We first find f^{-1} and then $f^{-1}(19)$

Let $y = f(x)$

$$\therefore y = 2x + 5$$
$$y - 5 = 2x$$
$$x = \frac{y-5}{2}$$

Replacing y by x we get:

$$f^{-1}(x) = \frac{x-5}{2}$$
$$\therefore f^{-1}(19) = \frac{(19)-5}{2}$$
$$= \frac{14}{2}$$
$$= 7$$

(ii) **Required to find:** $gf(3)$

Calculation:

We will find g*f(x)* and then *gf* of this value

$$g(x)=\frac{x-3}{2}$$

$$\therefore gf(x)=\frac{(2x+5)-3}{2}$$

$$=\frac{2x+2}{2}$$

$$=x+1$$

$$gf(3)=3+1$$

$$=4$$

OR

We will find *f(*3) and then *g* of this value

$$f(3)=2(3)+5$$

$$=11$$

$$gf(3)\equiv g(11)$$

$$=\frac{11-3}{2}$$

$$=\frac{8}{2}$$

$$=4$$

(c) **Data:** Equation of the line, *GH* is, $3x+2y=15$.

(i) **Required to determine:** The gradient of the line *GH*.

Solution:

We express the equation in the form *y* = *mx* + *c* and where *m* is the gradient.

$$3x+2y=15$$

$$2y=-3x+15$$

$\div 2$

$$y=-\frac{3}{2}x+\frac{15}{2}$$

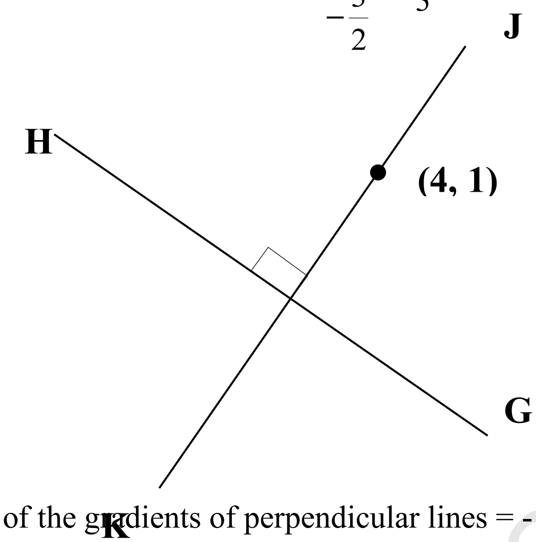
Which is of the form, $y=mx+c$, where $m=-\frac{3}{2}$ is the gradient.

(ii) **Data:** *JK* passes through $(4, 1)$ and is perpendicular to *GH*.

Required to find: The equation of *JK*

Solution:
JK and HG are perpendicular

The gradient of $JK = \frac{-1}{-\frac{3}{2}} = \frac{2}{3}$



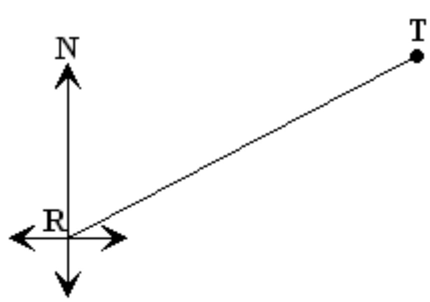
(since the product of the gradients of perpendicular lines = - 1).

Hence, the equation of *JK* is

$$\frac{y-1}{x-4}=\frac{2}{3}$$
$$3(y-1)=2(x-4)$$
$$3y-3=2x-8$$
$$3y=2x-8+3$$
$$3y=2x-5$$
$$y=\frac{2}{3}x-\frac{5}{3}$$

(Or expressed in any other equivalent form)

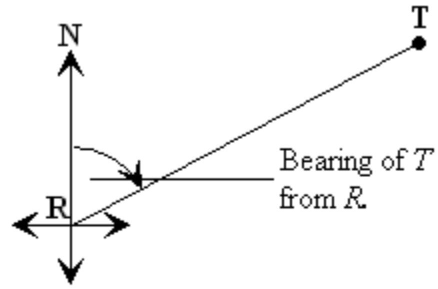
5. (a) **Data:** Diagram showing a scale drawing of the line *RT* and the North direction. Scale 1 cm ≡ 30 m.



(i) **Required to find:** The length of *RT*.
Solution:

By measurement, the length of $RT = 5.8$ cm (using a ruler)

(ii) **Required to find:** The bearing of T from R.
Solution:



By measurement with the protractor, the angle $NRT = 65°$.
$\therefore$ The bearing of T from R is 065°
(A bearing always being expressed in three digits)

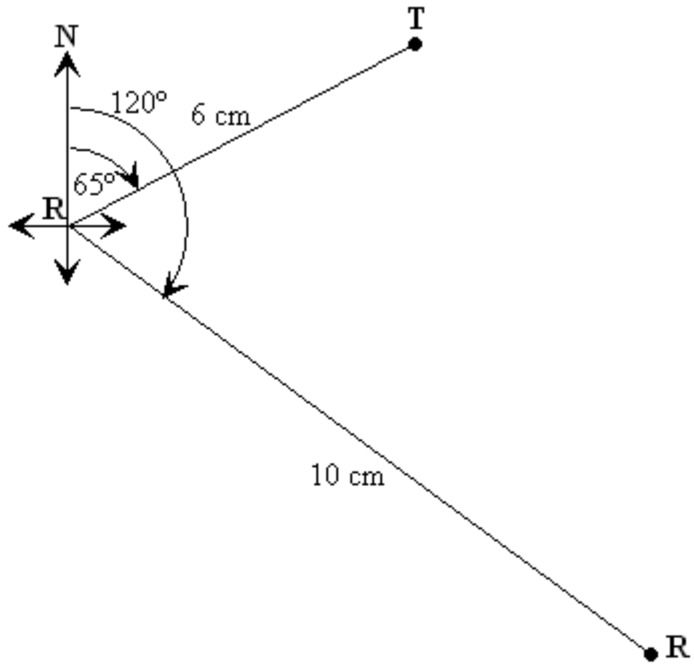
(iii) **Required to calculate:** The actual distance RT.
Calculation:
Scale is 1 cm ≡ 30 m (the scale on the diagram given)

So 1cm = 30 m in actuality
Scale 1 : 30 × 100 = 1 : 3 000

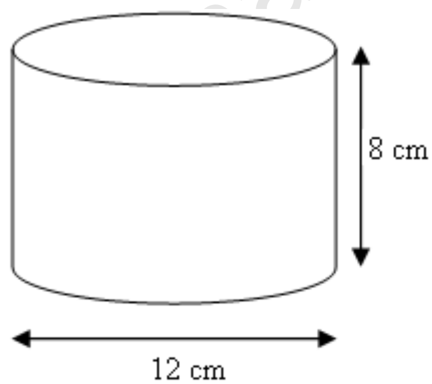
Hence, the actual distance $RT = 5.8 \times 30$ m
$= 5.8 \times 30$ m
$= 174$ m

(b) (i) **Data:** M is 300 m from R.
Required to find: The distance RM on the scale diagram
Solution:
M is 300 m from R.
1 m = 100 cm
$\therefore$ The actual length of RM is $300\text{ m} = 300 \times 100$ cm.
On the scale drawing, RM should therefore be drawn to a length of
$\frac{300 \times 100}{3000} = 10$ cm.

(ii) and (iii)
RM is to be drawn 10 cm long and angle NRM should be 120°. This is drawn and shown, on the diagram.



6. (a) **Data:**



The cylinder with dimensions as shown

(i) **Required to calculate:** The radius of the cylinder.
Calculation:
Diameter = 12 cm

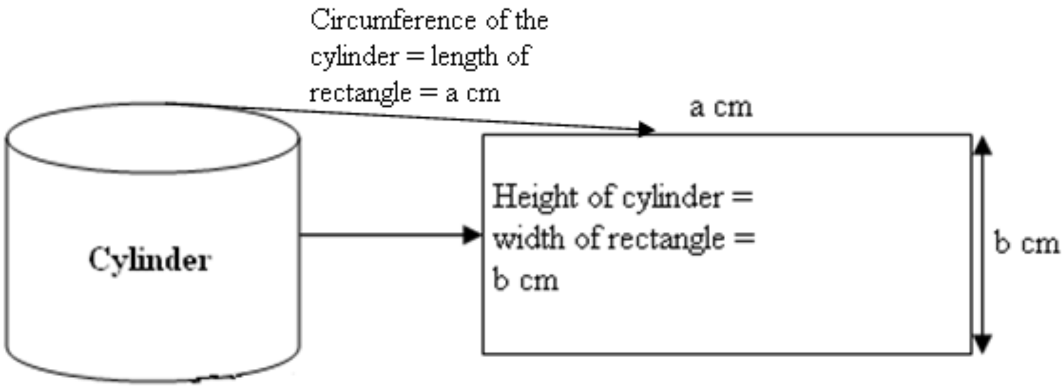
$\therefore \text{Radius} = \dfrac{12 \text{ cm}}{2}$ (radius = ½ diameter)

$= 6 \text{ cm}$

(ii) **Required to calculate:** The circumference of the cross-section.
Calculation:

$$\begin{aligned}\text{Circumference of the cross-section} &= 2\pi \times \text{Radius} \\ &= 2 \times 3.14 \times 6 \\ &= 37.68 \text{ cm}\end{aligned}$$

(b) (i) **Data:** A rectangle showing the net of the cylinder.
Required to calculate: The value of *a* and of *b*.
Solution:



The circumference of the cylinder when outstretched forms the length of the rectangle. The height of the cylinder is the width of the cylinder.

Therefore $a = 37.8$ cm and $b = 8$ cm

(ii) **Required to calculate**: The area of the curved surface of the cylinder.
Calculation:
Area of the curved surface of the circular cylinder

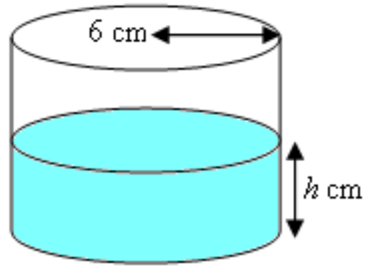
$$= 2\pi rh$$
$$= 2 \times 3.14 \times 6 \times 8$$
$$= 301.44\ cm^2$$

OR

Since the curved surface of the cylinder, when outstretched is in actuality the rectangle, then they both have the same area.

The area of the rectangle
$$= a \times b$$
$$= 37.68 \times 8$$
$$= 301.44 \text{ cm}^2$$

(c) **Data:** 0.5 litres of water is poured into the cylinder.
Required to calculate: The height of water in the cylinder.
Calculation:



The volume of water in the cylinder = 0.5 litres

$= 0.5 \times 1000 \qquad (1 \text{ litre} = 1000 \text{ cm}^3)$

$= 500 \text{ cm}^3$

Let the height of water in the cylinder be *h* cm.

$\therefore \pi(6)^2 \times h = 500$

$$h = \frac{500}{3.14 \times 6 \times 6}$$

$$h = 4.4\underline{\underline{2}}$$

$$= 4.4 \text{ cm to one decimal place}$$

7. **Data:** Table showing the scores obtained from 100 children in a competition.

(a) (i) **Required to state:** The modal class interval.

Solution:

The modal class interval is 20 – 29 since this class corresponds to the one with the highest frequency.

(ii) **Required to state:** The class interval in which 19.4 would lie.

Solution:

To obtain a score of 19.4 clearly indicates that the data is continuous and NOT discrete.

To answer (ii) and subsequent parts of the question it is helpful to improve on the given table, as shown below.

LCL-Lower class limit

UCL-Upper class limit

LCB-Lower class boundary

UCB-Upper class boundary

Continuous Data				
Score, x L.C.L. U.C.L. (Lower and upper class limit)	**L.C.B. U.C.B.** (Lower and upper class boundary**)**	**Class Midpoint** $\frac{\text{L.C.B.+U.C.B.}}{2}$	**Frequency, f**	$f \times x$ **Frequency × Class Midpoint**
$0-9$	$0 \le x < 9.5$	4.5	8	36
$10-19$	$9.5 \le x < 19.5$	14.5	13	188.5
$20-29$	$19.5 \le x < 29.5$	$\frac{19.5+29.5}{2}$ $=24.5$	25	25×24.5 $=612.5$
$30-39$	$29.5 \le x < 39.5$	$\frac{29.5+39.5}{2}$ $=34.5$	22	22×34.5 $=759$
$40-49$	$39.5 \le x < 49.5$	$\frac{39.5+49.5}{2}$ $=44.5$	20	20×44.5 $=890$
$50-59$	$49.5 \le x < 59.5$	$\frac{49.5+59.5}{2}$ $=54.5$	12	12×54.5 $=654$
			$\sum f = 100$	

The score 19.4 would therefore lie in the class interval, 10 – 19 or $9.5 \le x < 19.5$.

(b) **Required to calculate:** The mean score
Calculation:
For grouped data we use the formula

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$\bar{x} = \text{mean}$, $\sum = \text{sum of}$, $f = \text{frequency}$, $x = \text{midpoint of class interval}$

$$\therefore \bar{x} = \frac{36+188.5+612.5+759+890+654}{100}$$
$$= \frac{3140}{100}$$
$$= 31.4$$

(c) **Required to explain:** Why the value of the mean is an estimate of the true value.
Solution:
To obtain the true or correct mean we would need the actual scores from the raw data. However, the data is, ‘grouped data’ and so we used the class midpoint or mid-class interval. The actual values of the distribution may vary much, above or below, from this midpoint value, which we are using. Hence, the mean obtained by using the midpoint of a class will only give an estimate of the mean.

(d) **Data:** Student must score at least 40 points in order to qualify for the next round.
Required to find: The probability that a student qualifies for the next round
Solution:
To qualify for Round 2, students must score at least 40 points, that is, their score must be ≥ 40.
So,
P(Student scores 40 or more and hence qualifies for Round 2)

$$= \frac{\text{No. of students scoring 40 or more points}}{\text{Total no. of students}}$$

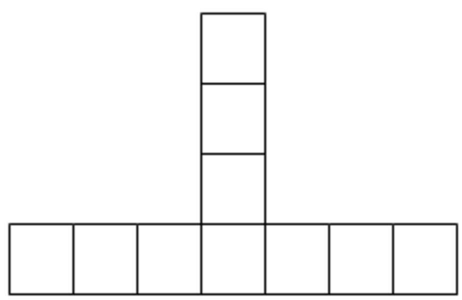
$$= \frac{20+12}{100}$$

$$= \frac{32}{100}$$

$$= \frac{8}{25}$$

8. **Data:** The first three diagrams in a sequence.

(a) **Required to draw:** The fourth diagram in the sequence.
Solution:
By observing the sequence, the fourth diagram of the sequence should have 3 squares drawn to the left, three squares drawn to the right and three squares drawn upwards from a center square. The completed fourth diagram is shown below.



$n = 4$

(b) **Data:** Table showing a column of the diagram, *n,* and a column for the number of squares.

(i) **Required to calculate:** The value of *a*
Calculation:

Diagram *n*	**Number of squares**
1	1
2	4
3	7
4	10

We look for a pattern between the number of squares, S, and the diagram number, n. The number of squares in each pattern increases by 3 when n increases by 1. We can see this in

1, 1+3 = 4, 4 + 3 = 7, 7 + 3 = 10 and so on
$\therefore$ The number of squares is obtained from n, by first multiplying n by 3 to give $3n$
In each case, the number of squares is now 2 less than the value of $3n$.
We find that the number of squares, which we refer to as S to be $=3n-2$.
We can test to confirm the formula that we have created.
When $n=1$, number of squares $=3(1)-2=1$

When $n=2$, number of squares $=3(2)-2=4$

When $n=3$, number of squares $=3(3)-2=7$

When $n=4$, number of squares $=3(4)-2=10$

$\therefore a=10$

(ii) **Required to calculate:** b
Calculation:
When $n=10$, we substitute to get
No. of squares $=3(10)-2=28$
$\therefore b=28$

(iii) **Required to calculate:** c
Calculation:
No. of squares = 40, we substitute to get c

$$\therefore 3n-2=40$$
$$3n=42$$
$$n=\frac{42}{3}$$
$$=14$$

Hence, $c=14$.

(c) **Required to write:** The number of squares in the n^{th} diagram of the sequence.
Solution:
The number of squares in the n^{th}diagram$=3n-2$. This formula that was created before.

Section II

9. (a) **Data:** The velocity, y ms^{-1}, of a particle after x seconds is given by $y = \frac{3}{x}$, $x \neq 0$.

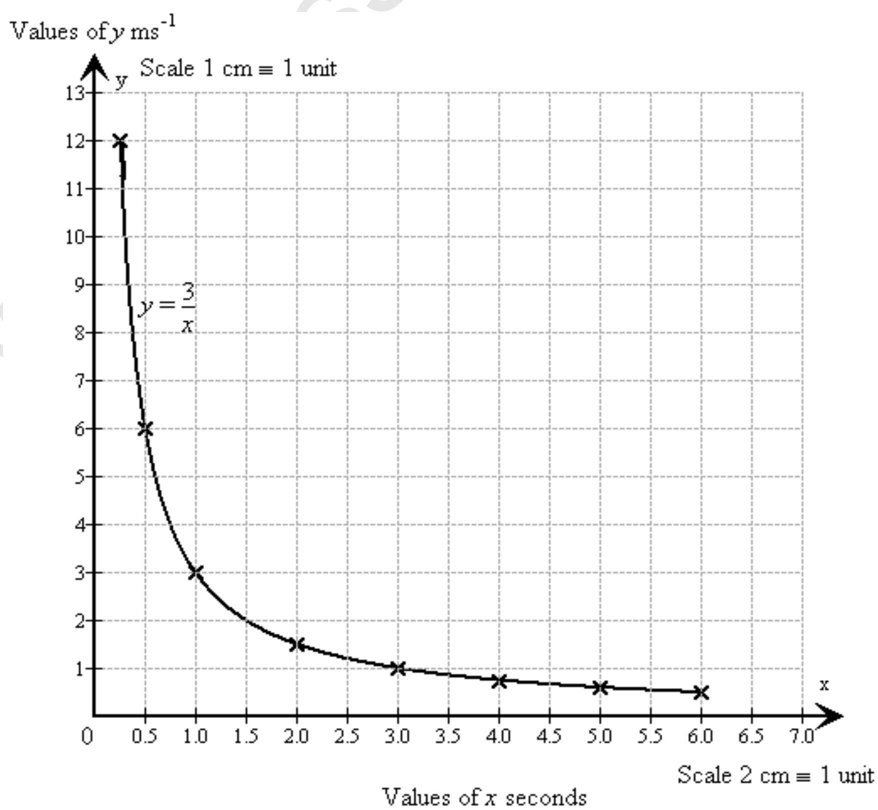
(i) **Required to complete:** The table given.
Solution:
When $x = 0.5$, $y = \frac{3}{0.5} = 6$
When $x = 3$, $y = \frac{3}{3} = 1$
When $x = 5$, $y = \frac{3}{05} = 0.6$

The completed table now looks like:

x seconds	0.25	0.5	1	2	3	4	5	6
y ms^{-1}	12	6	3	1.5	1	0.75	0.6	0.5

(ii) **Required to draw:** The graph of $y = \frac{3}{x}$, $x \neq 0$ using the points obtained.

Solution:
The graph is drawn as shown below, for the values on the table and using the scale given in the question.



(b) (i) **Required to write:** $f(x)=3x^2-5x+1$ in the form $a(x-h)^2+k$.

Solution:

We expand the required form and then equate coefficients with the given equation**.**

$$a(x-h)^2+k=a(x-h)(x-h)+k$$
$$=a(x^2-2hx+h^2)+k$$
$$=ax^2-2ahx+ah^2+k$$

$$\therefore 3x^2-5x+1=ax^2-2ahx+ah^2+k$$

Equating coefficients in x^2

$3=a$

Equating the coefficient of x and recalling $a=3$

$$-2ah=-5$$
$$\therefore -2(3)h=-5$$
$$h=\frac{-5}{-6}$$
$$=\frac{5}{6}$$

Equating the constant terms and recalling $a=3$ and $h=\frac{5}{6}$

$$1=ah^2+k$$
$$\therefore 1=3\left(\frac{5}{6}\right)^2+k$$
$$1=3\left(\frac{25}{36}\right)+k$$
$$k=1-\frac{25}{12}$$
$$=1-2\frac{1}{12}$$
$$k=-1\frac{1}{12}$$

Hence, $a=3$, $h=\frac{5}{6}$ and $k=-1\frac{1}{12}$ and a, h and k are constants.

And $3x^2-5x+1=3\left(x-\frac{5}{6}\right)^2-1\frac{1}{12}$

OR

$$3x^2 - 5x + 1 = 3\left(x^2 - \frac{5}{3}x\right) + 1$$

One half the coefficient of the x term is $\frac{1}{2}\left(-\frac{5}{3}\right) = -\frac{5}{6}$

This is what we use to help us to construct the terms within the brackets and which has to be squared.

$$\therefore 3x^2 - 5x + 1 = 3\left(x - \frac{5}{6}\right)^2 + *$$

$$3\left(x - \frac{5}{6}\right)\left(x - \frac{5}{6}\right)$$

$$3\left(x^2 - \frac{5}{3}x + \frac{25}{36}\right)$$

$$3x^2 - 5x + 2\frac{1}{12}$$

Since we require the expression to be equal to $3x^2 - 5x + 1$, then the value of $* = -1\frac{1}{12}$.

$\therefore 3x^2 - 5x + 1 = 3\left(x - \frac{5}{6}\right)^2 - 1\frac{1}{12}$, which is of the form $a(x-h)^2 + k$,

where $a = 3$, $h = \frac{5}{6}$ and $k = -1\frac{1}{12}$ and a, h and k are constants.

(ii) **Required to determine:** The minimum value of $f(x)$ and the value of x for which this minimum value occurs.

Solution:

$$3x^2 - 5x + 1 = 3\left(x - \frac{5}{6}\right)^2 - 1\frac{1}{12}$$

Any term to be squared, such as $\left(x - \frac{5}{6}\right)^2 \geq 0 \quad \forall x$

$$\therefore 3\left(x - \frac{5}{6}\right)^2 \geq 0$$

Hence, the minimum value of $f(x) = 0 - 1\frac{1}{12}$

$$= -1\frac{1}{12}$$

$f(x)$ will have a minimum value when $3\left(x-\frac{5}{6}\right)^2=0$

$$\therefore \left(x-\frac{5}{6}\right)^2=0$$

$$x=\frac{5}{6}$$

OR

When $f(x)=ax^2+bx+c$, the minimum or maximum value occurs at $x=\frac{-b}{2a}$, since this is the equation of the axis of symmetry of the quadratic curve and it passes through the maximum or the minimum point. So, the *x*-coordinate of the maximum or the minimum point of a quadratic is always $x=\frac{-b}{2a}$

Hence, the minimum value of $f(x)$ occurs when

$$x=\frac{-(-5)}{2(3)}$$
$$=\frac{5}{6}$$

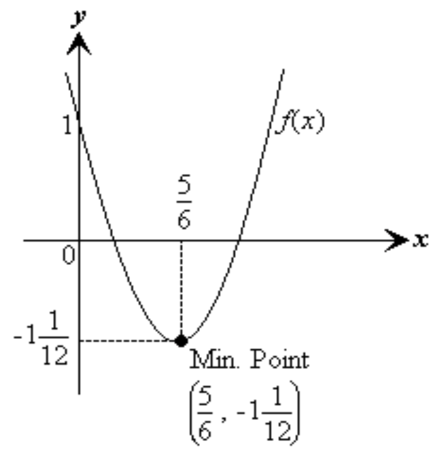
When $x=\frac{5}{6}$

$$f(x)=3\left(\frac{5}{6}\right)^2-5\left(\frac{5}{6}\right)+1$$
$$=-1\frac{1}{12}$$

Hence, the minimum value of $f(x)=-1\frac{1}{12}$ at $x=\frac{5}{6}$.

OR

We could also have drawn the graph of $f(x)$ and read off the minimum point for the solution and the *x*-value at which it occurs. This is not the best procedure (unless it was specified) as inexact answers often prove difficult to read off.



This is a longer method and since the values have to be read off by eye, highly accurate answers are not expected, due to these limitations. The x-value of $\frac{5}{6}$ would be difficult to read off as well as the minimum value of $-1\frac{1}{12}$

(Also, since any method is acceptable as none was specified, students who are more mathematically aware could have used differential calculus.)

(iii) **Required to solve:** The equation $3x^2-5x+1=0$, giving the answer correct to 2 decimal places.
Solution:

If $3x^2-5x+1=0$, then using the form obtained by completing the square we get:

$$3\left(x-\frac{5}{6}\right)^2-1\frac{1}{12}=0$$

$$3\left(x-\frac{5}{6}\right)^2=\frac{13}{12}$$

$$\left(x-\frac{5}{6}\right)^2=\frac{13}{36}$$

$$x-\frac{5}{6}=\frac{\pm\sqrt{13}}{6}$$

$$x=\frac{5}{6}\pm\frac{\sqrt{13}}{6}$$

$$=\frac{5\pm\sqrt{13}}{6}$$

$$=1.43\underline{\underline{4}} \text{ or } 0.23\underline{\underline{2}}$$

$$=1.43 \text{ or } 0.23 \text{ to 2 decimal places}$$

OR

Using the quadratic equation formula

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)}$$

$$= \frac{5 \pm \sqrt{25-12}}{6}$$

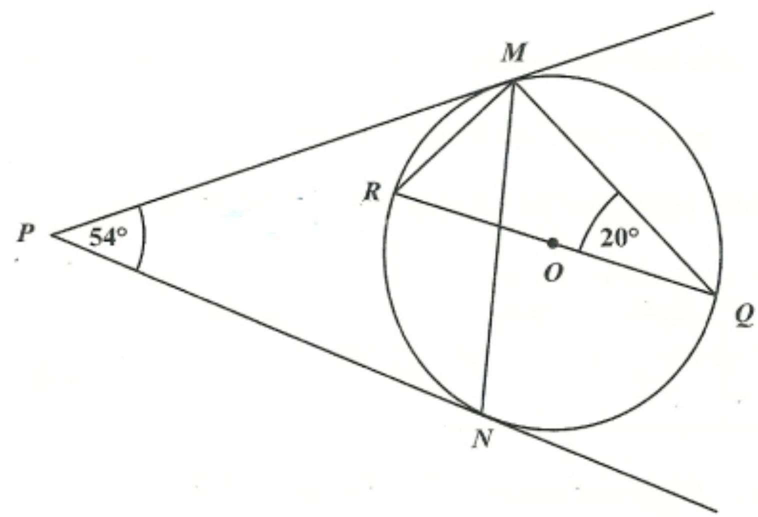
$$= \frac{5 \pm \sqrt{13}}{6}$$

$$= \frac{5+\sqrt{13}}{6} \text{ or } \frac{5-\sqrt{13}}{6}$$

$$= 1.43\underline{\underline{4}} \text{ or } 0.23\underline{\underline{2}}$$

$$= 1.43 \text{ or } 0.23 \text{ to 2 decimal places}$$

10. (a) **Data:**



In the above diagram PM and PN are tangents, PQ is a diameter of the circle, center, O, $R\hat{Q}M = 20°$ and $M\hat{P}N = 54°$.

(i) **Required to calculate:** The size of $M\hat{R}Q$

Calculation:

Considering the triangle RMQ

$R\hat{M}Q = 90°$ (The angle in a semicircle is a right angle)

Considering the triangle RMQ

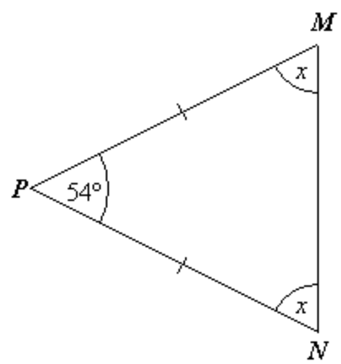
$$\therefore M\hat{R}Q = 180° - (90° + 20°)$$

$$= 70°$$

(The sum of the interior angles in a triangle = 180°)

(ii) **Required to calculate:** The size of $P\hat{M}R$
Calculation:
$P\hat{M}R = 20°$
(The angle made a tangent (*PM*) to a circle and a chord (*MR*), at the point of contact(*M*) is equal to the angle in the alternate segment, (angle *MQR*)

(iii) **Required to calculate:** $P\hat{N}M$
Calculation:



$PM = PN$
(The two tangents that can be drawn to a circle from any point outside the circle are equal in length.)
Hence, triangle *PMN* is isosceles (two sides equal) and the two base angles *PMN* and *PNM* are equal.
The sum of the interior angles in a triangle = 180°
So,

$$P\hat{M}N = P\hat{N}M$$
$$= \frac{180° - 54°}{2}$$
$$= \frac{126°}{2}$$
$$= 63°$$

(b) **Data:**

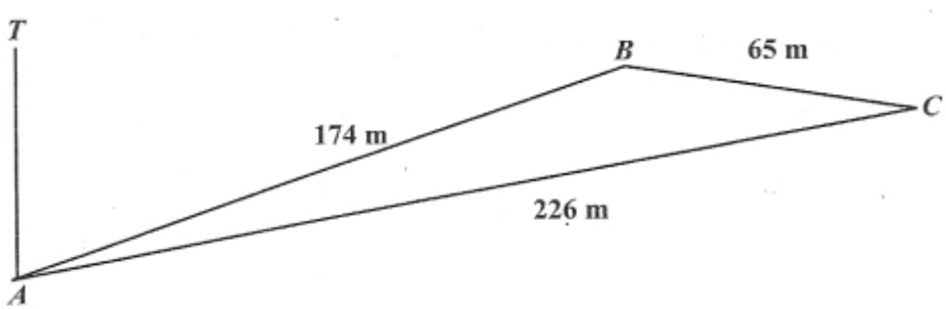


Diagram showing three points *A*, *B* and *C* lying on a horizontal plane

(i) a) **Required to calculate:** The size of $A\hat{B}C$

Calculation:

We now apply the cosine rule to triangle *ABC*.

$b = 226$, $a = 65$ and $c = 174$

$$b^2 = a^2 + c^2 - 2ac\cos\hat{B}$$

$$(226)^2 = (65)^2 + (174)^2 - 2(65)(174)\cos\hat{B}$$

$$51076 = 4225 + 30276 - 22620\cos\hat{B}$$

$$\therefore 22620\cos\hat{B} = 4225 + 30276 - 51076$$
$$= -16575$$

$$\cos\hat{B} = \frac{-16575}{22620}$$
$$= -0.73276$$

Since $\cos\hat{B} =$ a negative value, then $\hat{B}$ is obtuse.

Recall:

$$\cos(180° - x) = -\cos x$$

$$\therefore \hat{B} = 180° - \cos^{-1}(0.73276)$$
$$= 180° - 42.881°$$
$$= 137.119°$$
$$= 137.1° \text{ (to the nearest } 0.1°\text{)}$$

b) **Required to calculate:** The area of ΔABC.

Calculation:

In the triangle ABC, we know two sides and the included angle**;** *a*, *c* and angle *B*

$$\text{Area of } \Delta ABC = \frac{1}{2}(\text{side})(\text{side}) \times \text{sine of the included angle}$$
$$= \frac{1}{2}(174)(65)\sin 137.119°$$
$$= 3848.1\underline{\underline{4}} \text{ m}^2$$
$$= 3848.1 \text{ (to 1 decimal place)}$$

OR

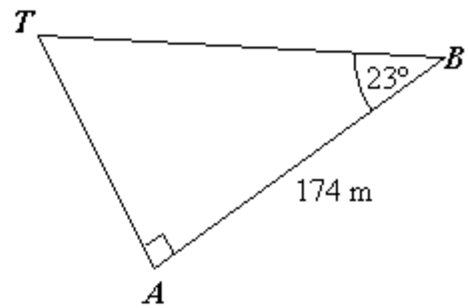
Since we know all three sides of the triangle then we may use Heron's Formula

One half of the perimeter of the triangle, $S = \frac{1}{2}(174 + 65 + 226) \quad = 232.5$

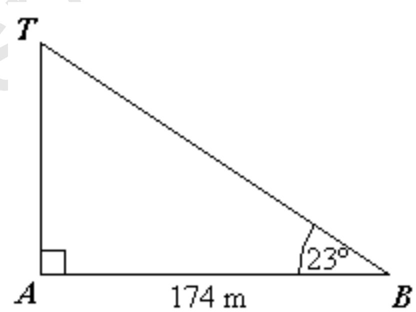
Area $= \sqrt{(232.5)(232.5 - 226)(232.5 - 174)(232.5 - 65)}$
$= \sqrt{232.5 \times 6.5 \times 58.5 \times 167.5}$
$= 3848.1 \text{ m}^2$ (to 1 decimal place)

(ii) **Data:** The angle of elevation of T from B is 23°.

a) **Required to draw:** Triangle TAB
Solution:



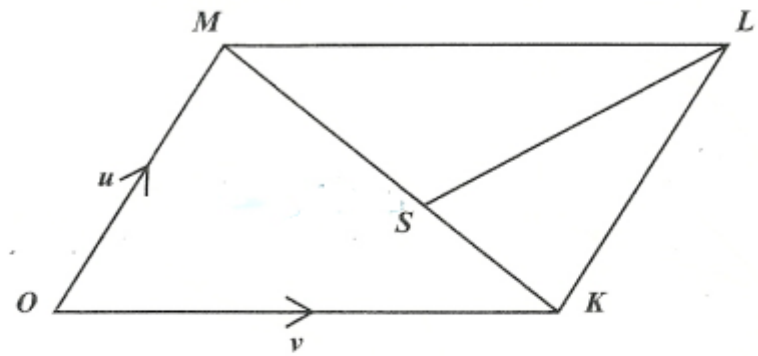
This is how the triangle appears on the diagram since it is drawn as a 3-d figure.
This can be shown better when the diagram is drawn plane as:



b) **Required to calculate:** The height of the lighthouse
Calculation:
Since the triangle is right-angled, then from the data we can use the ratio of tangent to obtain the length of TA.

$$\frac{TA}{174} = \tan 23°$$
$$\therefore TA = 174 \tan 23°$$
$$= 73.858 \text{ m}$$
$$= 73.86 \text{ m to 2 decimal places}$$

11. (a) **Data:**



OKLM is a parallelogram. $OK = v$, $OM = u$ and $MS = 2\ SK$.

(i) **Required to Express:** MK, in terms of *u* and *v*.
Solution:
Using the vector triangle law applied to triangle *MOK*

$$MK = MO + OK$$
$$= -(u) + v$$
$$= -u + v$$

(ii) **Required to Express:** SL, in terms of *u* and *v*.
Solution:
Using the vector triangle law applied to triangle *SLK*

$$SL = SK + KL$$

Since $MS = 2\ SK$, then

$$SK = \frac{1}{3}MK$$
$$= \frac{1}{3}(-u + v)$$

$\overrightarrow{KL} = u$ (The opposite sides of a parallelogram are parallel and equal. Hence, $KL = OM$ and are equivalent vectors)

$$\therefore SL = \frac{1}{3}(-u + v) + u$$
$$= -\frac{1}{3}u + \frac{1}{3}v + u$$
$$= \frac{1}{3}(2u + v)$$

(iii) **Required to express:** OS, in terms of *u* and *v*.
Solution:
Using the vector triangle law applied to triangle *OSK* though the triangle is shown incomplete in the diagram.

$$OS = OK + KS$$
$$= v + -\left(\frac{1}{3}(-u+v)\right)$$
$$= v + \frac{1}{3}u - \frac{1}{3}v$$
$$= \frac{1}{3}(u+2v)$$

(b) **Data:** $J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. The point *P* is mapped onto $(5,\ 4)$ under *J*.

Required to calculate: The coordinates of *P*.

Calculation:

Let *P* be $(x,\ y)$.

We multiply the coordinates of *P* by the matrix, *J*

$$P \xrightarrow{J} \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} (0 \times x) + (-1 \times y) \\ (1 \times x) + (0 \times y) \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} -y \\ x \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

Both matrices are 2 x 1 and are equal.
Equating corresponding entries we obtain $x = 4$ and $-y = 5$. Hence, $y = -5$

Hence, the coordinates of *P* are found to be $(4,\ -5)$.

OR

If an object point is mapped onto an image by a matrix, say *M*, then the inverse of *M* will map the image back to the object.

Let P' be $(5,\ 4)$.

$$P \xrightarrow{J} P'$$

$$\therefore P' \xrightarrow{J^{-1}} P$$

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

We now find the inverse of the matrix *J*, denoted by J^{-1}

Det J = $(0 \times 0) - (1 \times -1) = 1$

$$\therefore J^{-1}=\frac{1}{1}\begin{pmatrix} 0 & -(-1) \\ -(1) & 0 \end{pmatrix}$$

$$=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$J^{-1}\times P' = P$$

$$J^{-1}\times P' = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

$$=\begin{pmatrix} (0\times 5)+(1\times 4) \\ (-1\times 5)+(0\times 4) \end{pmatrix}$$

$$=\begin{pmatrix} 4 \\ -5 \end{pmatrix}=P$$

$$\therefore P=(4,\ -5)$$

(c) (i) **Required to write:** A matrix *H*, which represents an enlargement with scale factor 3, about the origin.

Solution:
The matrix representing an enlargement, center *O* and scale factor *k* is $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$.

$\therefore$ The matrix representing the enlargement, center *O* and scale factor 3 is $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

$$\therefore H=\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

(ii) **Required to find:** The coordinates of the image point when the object point $(5,\ -7)$ undergoes a combined transformation *H* followed by *J*.

Solution:
The transformation *H* followed by *J* is written *JH*.
Let $A=(5,\ -7)$
We shall first find the image of *A* under the transformation *H* and using this image as the new object, we find the image under the second transformation *J*.

$A \xrightarrow{JH}$ to give the final image

Firstly

$$A \xrightarrow{H} A'$$

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 5 \\ -7 \end{pmatrix} = \begin{pmatrix} (3\times5)+(0\times-7) \\ (0\times5)+(3\times-7) \end{pmatrix}$$

$$= \begin{pmatrix} 15 \\ -21 \end{pmatrix}$$

This is the image under H and now the object under J.
Secondly

$$A' \xrightarrow{J} A''$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 15 \\ -21 \end{pmatrix} = \begin{pmatrix} (0\times15)+(-1\times-21) \\ (1\times15)+(0\times-21) \end{pmatrix}$$

$$= \begin{pmatrix} 21 \\ 15 \end{pmatrix}$$

$\therefore \begin{pmatrix} 5 \\ -7 \end{pmatrix} \xrightarrow{JH} \begin{pmatrix} 21 \\ 15 \end{pmatrix}$ and the final image is $(21, 15)$.

(d) **Data:** The costs of three models of phones and the weekly sales of the models over a 2 - week period.

(i) **Required to find:** The 3×2 matrix which represents the sales over the 2 week period.

Solution: From the given table,

Week 1	Week 2
$2-\text{A}$	$0-\text{A}$
$5-\text{B}$	$6-\text{B}$
$3-\text{C}$	$10-\text{C}$

We consider the coefficients.

$\therefore$ The required 3×2 matrix is $\begin{pmatrix} 2 & 0 \\ 5 & 6 \\ 3 & 10 \end{pmatrix}_{3\times2}$.

(ii) **Required to write:** A 1×3 matrix representing the cost of the phones.

Solution:

Cost of model A = \$40 each
Cost of model B = \$55 each
Cost of model C = \$120 each

$\therefore$ The 1×3 matrix required is $\underset{1\times3}{\begin{pmatrix}40 & 55 & 120\end{pmatrix}}$.

(iii) **Required to find:** The multiplication of the matrices which give the sales over the two weeks.

Solution:

We check for the conformability to matrix multiplication.

$1\times3 \text{ X } 3\times2 = 1\times2$

$$\therefore \begin{pmatrix}40 & 55 & 120\end{pmatrix}\begin{pmatrix}2 & 0\\ 5 & 6\\ 3 & 10\end{pmatrix} = \begin{pmatrix}e_{11} & e_{12}\end{pmatrix}$$

If calculated, (though this has not been asked), then,

e_{11} = the sales for week 1 and e_{12} = the sales for week 2.