FAS-PASS Maths

# CSEC MATHEMATICS JANUARY 2012

## Section I

1. (a) (i) **Required to calculate:** $\left(1\frac{3}{4}\right)^2 \div 3\frac{1}{2}$

**Calculation:**
The arithmetic is relatively simple so there is no need for the calculator. All working can be shown.
We convert the mixed fractions to improper fractions.

$$\left(1\frac{3}{4}\right)^2 \div 3\frac{1}{2} = \left(\frac{7}{4}\right)^2 \div \frac{7}{2}$$
$$= \frac{49}{16} \div \frac{7}{2}$$
$$= \frac{49}{16} \times \frac{2}{7}$$
$$= \frac{7}{8}$$
(as a fraction in exact form)

(ii) **Required to calculate:** $\sqrt{0.0529} + 0.216$

**Calculation:**
To find the root by arithmetic is cumbersome, so we use the calculator.

$$\sqrt{0.0529} + 0.216 = 0.23 + 0.216$$
$$= 0.446$$
$$= 4.46 \times 10^{-1}$$

(This is the exact value and it is expressed in standard form)

(b) **Data:** Basic wage of typist = $22.50 per hour for a 40-hour work week

Overtime rate = $1\frac{1}{2}$ the basic hourly rate

**Required to calculate:**
(i) Typist's basic weekly wage
(ii) Over time wage for one hour of overtime work
(iii) Wage earned for overtime if she worked for a total of 52 hours
(iv) Number of overtime hours worked to obtain a total wage of $1 440.00

**Solution:**
(i) The typist's basic weekly wage = The basic hourly Rate $\times$Number of hours in a basic work week, (no overtime).

$$= \$22.50 \times 40$$
$$= \$900.00$$

(ii) The overtime wage for one hour of overtime work

$= 1\frac{1}{2} \times$ basic hourly rate

$= 1\frac{1}{2} \times \$22.50$

$= \$33.75$

(iii) The overtime wage obtained for a total of 52 hours (which includes the hours in a basic working week) worked
= Number of overtime hours $\times$ Overtime rate
Number of overtime hours = (52 - 40) = 12 hours

Hence, overtime wage earned = 12 x \$ 33.75
$= \$405.00$

(iv) The number of overtime hours that were worked
= The overtime wage $\div$ The overtime rate
= (Total wage – Basic wage) $\div$ Overtime rate

$= (\$1440 - \$900) \div \$33.75$

$= \$540 \div \$33.75$

$= 16$ hours

2. (a) **Data:** $3x + 2y = 13 \ldots (1)$

$x - 2y = -1 \ \ldots \ (2)$

**Required to calculate:** The value of $x$ and of $y$

**Solution:**

Using the method of substitution

From eq. (2) we express $x$ in terms of $y$

$x - 2y = -1$

$x = 2y - 1 \ldots (3)$

Substituting equation (3) into equation (1) to arrive and one equation in one unknown and which is solvable.

$$3x + 2y = 13$$
$$3(2y - 1) + 2y = 13$$
$$6y - 3 + 2y = 13$$
$$8y = 16$$
$$y = \frac{16}{8}$$
$$y = 2$$

Now, substituting $y = 2$ into equation (1)

$$x - 2y = -1$$
$$x - 2(2) = -1$$
$$x - 4 = -1$$
$$x = -1 + 4$$
$$x = 3$$
$$\therefore x = 3,\ y = 2$$

(We could also have used the methods of elimination, graphical or matrix to obtain the same result)

(b) **Required to factorise:**

(i) $x^2 - 16$

(ii) $2x^2 - 3x + 8x - 12$

**Solution:**

(i) $x^2 - 16$

We re-arrange the terms of the expression to look like:

$= (x)^2 - (4)^2$ This is in the form of a difference of two squares

$= (x-4)(x+4)$

(ii) $2x^2 - 3x + 8x = 12$

$2x^2 + 8x - 3x - 12$

$2x(x+4) - 3(x+4)$

$(x+4)(2x-3)$

(c) **Data:** Adult ticket costs \$30.00 each
Children's ticket costs \$15.00 each
A company bought 28 tickets

(i) **Required to find:** an expression, in terms of $x$, if $x$ tickets were for adults

a) the number of tickets for children
b) the amount spent on tickets for adults
c) the amount spent on tickets for children

**Solution:**

a) $x$ tickets were for adults.
$\therefore$The number of tickets for children
= Total number of tickets – Number of tickets for adults
= 28 - $x$

b) The amount of money spent on tickets for adults
= Cost of ticket for one adult $\times$ The number of tickets for adults
$= \$30 \times x$
$= \$30x$

c) The amount of money spent on tickets for children
= The cost of ticket for one child $\times$ the number of tickets for children
$= \$15 \times (28 - x)$
$= \$15(28 - x)$

(ii) **Required to show:** The amount spent on 28 tickets is $\$(15x + 420)$
**Solution:**
The total amount spent on all 28 tickets
= The amount spent on adult tickets + the amount spent on children tickets
$= 30x + 15(28 - x)$
$= 30x + 420 - 15x$
$= \$(15x + 420)$ Q.E.D.

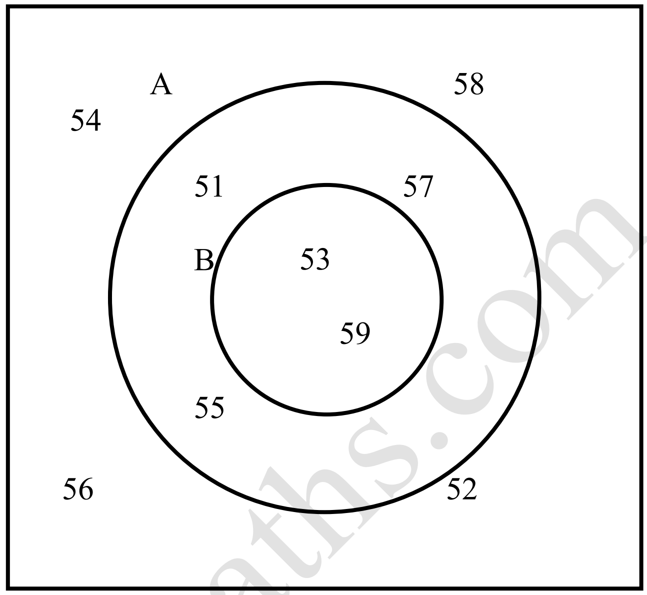
(iii) **Data:** Cost of 28 tickets = \$660
**Required to calculate:** The number of adult tickets bought
**Solution:**
The total cost of tickets = \$660
$\therefore 660 = 15x + 420$
$660 - 420 = 15x$
$240 = 15x$
$x = \dfrac{240}{15}$
$x = 16$
$\therefore$Since $x$ represents the number of adult tickets, then the number of adult tickets bought is 16.

3. (a) **Data:** $U = \{51, 52, 53, 54, 55, 56, 57, 58, 59\}$
$A = \{\text{Odd numbers}\}$
$B = \{\text{Prime numbers}\}$
**Required To:**
(i) list the members of set $A$
(ii) list the members of set $B$
(iii) Draw a Venn diagram to represent the sets $A$, $B$ and $U$.

**Solution:**

(i) $A = \{51, 53, 55, 57, 59\}$ (ii) $B = \{53, 59\}$

(iii) U



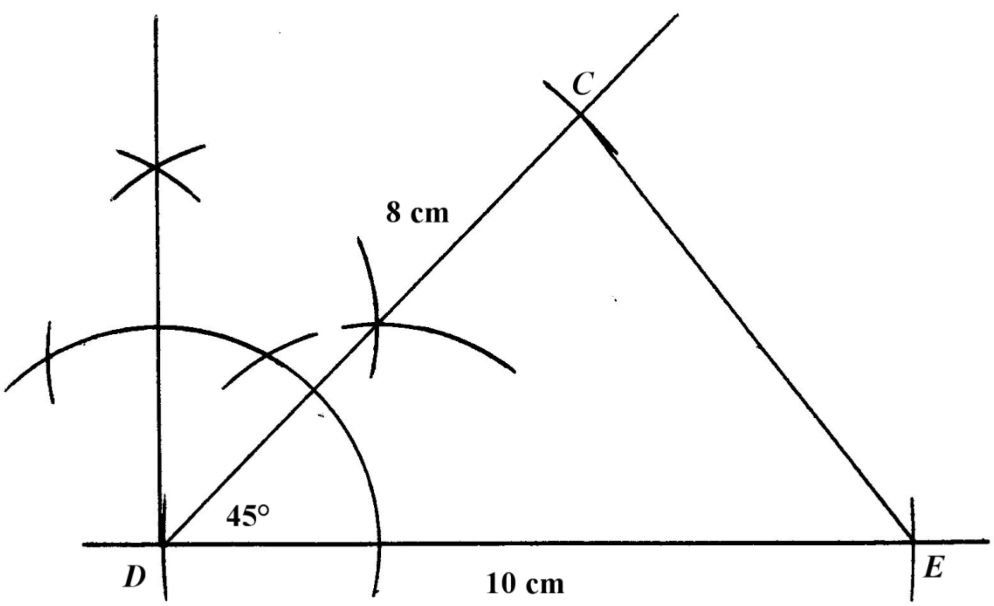

(b) (i) **Required to construct:**

a) triangle, *CDE* in which *DE* = 10cm, *DC* = 8cm and angle *CDE* = 45°.

b) line, *CF*, perpendicular to *DE* such that *F* lies on *DE*.

**Solution:**

a) We draw a line longer than 10 cm and cut off *DE* 10 cm.
At *D* we construct an angle of $90^0$ and bisect it to obtain angle *CDE* = $45^0$.
We cut off 8 cm with the compass to find *C*.
Join *C* to *E* to complete the triangle.



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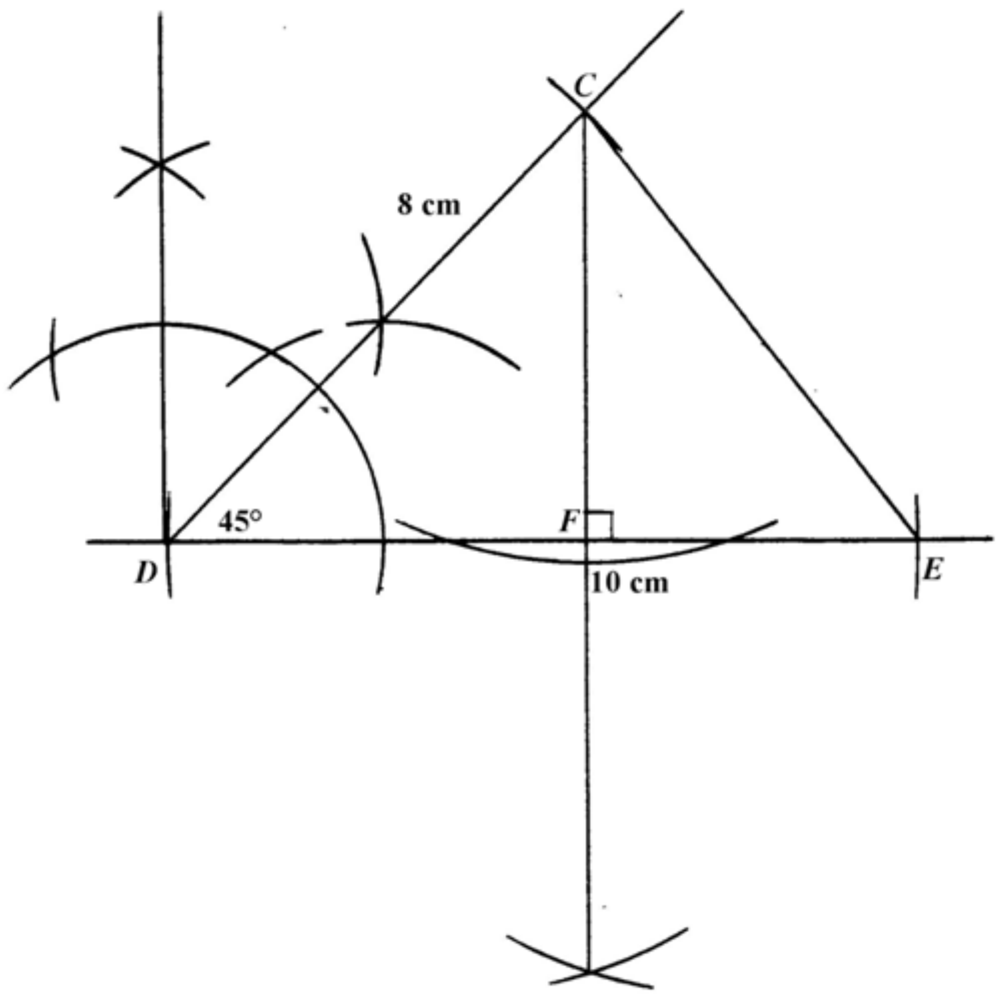
b) The perpendicular from $C$ to $DE$ is constructed meeting $DE$ at $F$




(ii) **Required to measure:** Size of $D\hat{C}E$
**Solution:** Using a protractor we get
Angle $DCE$ = 82° (by measurement)

4. (a) **Data:** Table showing part of a bus schedule.
**Required to calculate:**
(i) time spent at Chagville
(ii) time taken to travel from Belleview to Chagville
(iii) the distance, in km, between Belleview and Chagville, if the bus travelled at an average speed of 54 $kmh^{-1}$.
**Solution:**

| Town | Arrive | Depart |
|---|---|---|
| **Belleview** | | 6:40 am |
| **Chagville** | 7:35 am | 7:45 am |
| **St. Andrews** | 8:00 am | |

(i) The time spent at Chagville
= The departure time from Chagville – The arrival time at Chagville
$= 7:45 \text{ am} - 7:35 \text{ am}$
$= 10 \text{ minutes}$

(ii) The time taken to travel from Belleview to Chagville
= The arrival time at Chagville – The departure time from Belleview
$= 7:35 \text{ am} - 6:40 \text{ am}$
$= 55 \text{ minutes}$

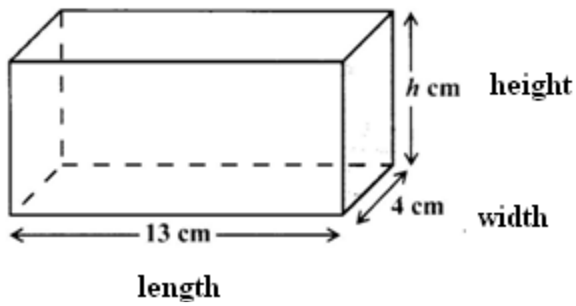
(iii) The distance between Belleview and Chagville
= The time in hours to travel from Belleview to Chagville $\times$ The average Speed during the journey
$= \frac{55}{60} \times 54$
$= 49\frac{1}{2} \text{ km}$

(b) **Data:** The base area of a cylindrical bucket = 300 $cm^2$
4.8 litres of water was poured into bucket
**Required to calculate:** Height of water in the bucket
**Solution:**
The volume of water in the cylindrical bucket $= 300 \times h$
(where we take $h$ as the height of water in the bucket)
Volume of water in the bucket = 4.8 litres
Recall 1 litre = 1000 $cm^3$
$4.8 \text{ litres} = 4.8 \times 1000 = 4800 \text{ cm}^3$
Therefore,
$300 \times h = 4800$
$h = \frac{4800}{300}$
$= 16 \text{ cm}$

(c) **Data:** Length of cuboid = 13 cm
Width of cuboid = 4 cm
Height of cuboid = $h$ cm

**Required to:**
(i) find an expression for the area of the shaded face
(ii) write an expression for the volume of the cuboid, in terms of $h$
(iii) calculate $h$, if the volume of the cuboid is 286 $cm^2$

**Solution:**




(i) The area of the shaded face in cm² = $h \times w$
$= 4 \times h$
$= 4h$

(ii) The volume of the cuboid in cm³ = length x width x height
$= 13 \times 4 \times h$
$= 52h$

(iii) The volume of the cuboid in cm³ = 286 cm³
$\therefore 286 = 52h$
$h = \dfrac{286}{52}$
$h = 5.5$

5. (a) **Data:** Two triangles *JKL* and *MLP*.
*JK* is parallel to *ML*, *LM* = *MP* and *KLP* is a straight line.
Angle *JLM* = 22°, angle *LMP* = 36°
**Required to find:**
(i) $M\hat{L}P$
(ii) $L\hat{J}K$
(iii) $J\hat{K}L$
(iv) $K\hat{L}J$
**Solution:**
(i) $M\hat{L}P = 180° - 36°$ (sum of angles in a triangle is $180^0$)
$M\hat{L}P = 144°$
$= \dfrac{144°}{2}$
$= 72°$ (The base angles of an isosceles triangle are equal)
(ii) $L\hat{J}K = 22°$ (Alternate to $J\hat{L}M$ angles are equal, when parallel lines are cut by a transversal)
(iii) $J\hat{K}L = 72°$ (Corresponding angles with $M\hat{L}P$ when parallel lines are cut by a transversal, corresponding angles are equal)

(iv) $K\hat{L}J = 180° - (22° + 72°) \; = 86°$
OR
Sum of angles in a triangle is $180^0$

(b) **Data:** Diagram showing $PQR$ and its image $P'Q'R'$.
**Required to:**
(i) state the coordinates of $P$ and of $Q$
(ii) describe fully the transformation that maps triangle $PQR$ onto triangle $P'Q'R'$
(iii) Write the coordinates of images $P$ and $Q$ under the translation $\begin{pmatrix} 3 \\ -6 \end{pmatrix}$.

**Solution:**
(i) By a read off from the diagram, $P = (2,\ 1)$ and $Q = (4,\ 3)$.

(ii) Triangle $PQR$ is mapped onto triangle $P'Q'R'$ by a reflection in the $x$ – axis.
(The perpendicular bisector of the line joining any set of object-image points, for example, $PP'$ or $QQ'$ or $RR'$ is seen as the $x$ –axis).

(iii) Writing $P\ (2, 1)$ as a matrix $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, we obtain,

$$P\begin{pmatrix} 2 \\ 1 \end{pmatrix} \xrightarrow{\begin{pmatrix} 3 \\ -6 \end{pmatrix}} P''$$

$$P'' = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -6 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -5 \end{pmatrix}$$

$$\therefore P'' = (5, -5)$$

Writing $Q\ (4, 3)$ as a matrix $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$, we obtain,

$$Q\begin{pmatrix} 4 \\ 3 \end{pmatrix} \xrightarrow{\begin{pmatrix} 3 \\ -6 \end{pmatrix}} Q''$$

$$Q'' = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ -6 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ -3 \end{pmatrix}$$

$$\therefore Q'' = (7, -3)$$

6. **Data:** An incomplete table with corresponding values of *x* and *y* for the function $y = x^2 - 2x - 3$ for integer values from -2 to 4

**Required to:**
(a) copy and complete the table
(b) plot the graph of $y = x^2 - 2x - 3$ for $-2 \le x \le 4$
(c) use graph to estimate the value of *y* when $x = 3.5$

**Solution:**
(a) We substitute the values of $x = -1$ and $x = 2$ in the equation to find their corresponding values of *y*.

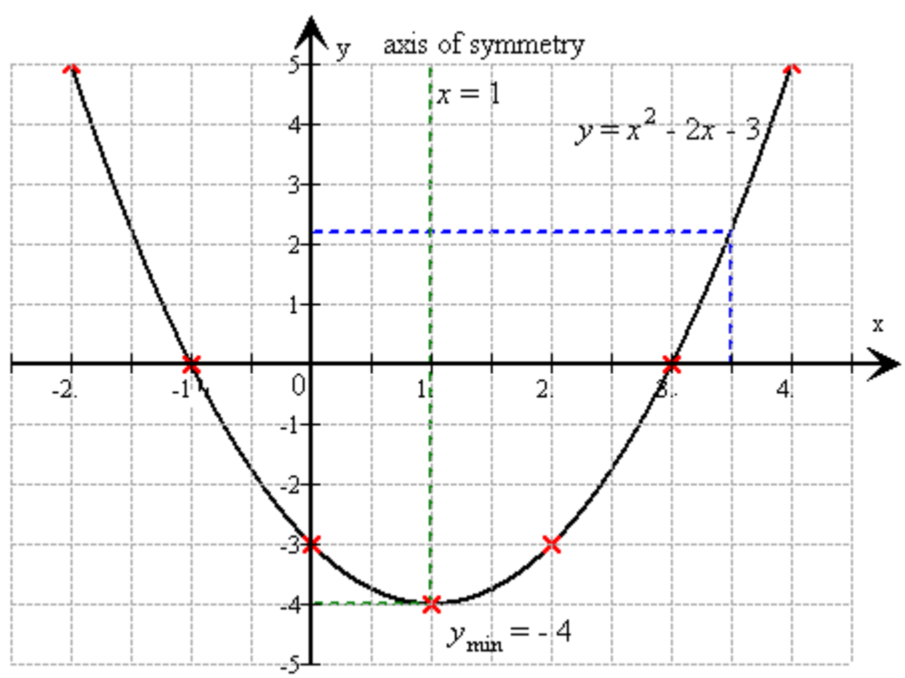
When $x = -1$
$$y = x^2 - 2x - 3$$
$$= (-1)^2 - 2(-1) - 3$$
$$= 1 + 2 - 3$$
$$= 0$$

When $x = 2$
$$y = x^2 - 2x - 3$$
$$= (2)^2 - 2(2) - 3$$
$$= 4 - 4 - 3$$
$$= -3$$

The completed table is shown below

| ***x*** | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| ***y*** | 5 | **0** | -3 | -4 | **-3** | 0 | 5 |

(b)




(c) The vertical from $x = 3.5$ is drawn to meet the curve. From the point of meeting, a horizontal is drawn to meet the vertical axis at $y = 2.2$
When $x = 3.5$, $y = 2.2$ (obtained by a read-off)



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(d) **Required To:**

(i) write the equation of the axis of symmetry

(ii) estimate the minimum value of the function $y$

(iii) state the solutions of the equation $x^2 - 2x - 3 = 0$

**Solution:**

(i) The equation of the axis of symmetry is $x = 1$ (the vertical drawn from the minimum point on the curve)

(ii) Minimum value of function is $y = -4$. (The horizontal is drawn from the minimum point at $x = 1$, to meet the vertical axis.

(iii) Values of the solutions of the equation $x^2 - 2x - 3 = 0$ occur at the points where the graph cuts the $x$ – axis. These are seen at the points where $x = -1$ and at $x = 3$.

7. (a) **Data:** Histogram showing distribution of heights of seedlings in a sample.

**Required to:** copy and complete the table

**Solution:**

We modify the table to look like:
L.C.L-lower class limit
U.C.L-upper class limit
L.C.B-lower class boundary
U.C.B-Upper class boundary

| **Height in cm, $x$ L.C.L –U.C.L** | **L.C.B $\leq x \leq$ U.C.B.** | **Midpoint OR Mid-class interval** | **Frequency, $f$** |
|---|---|---|---|
| 1 – 10 | $0.5 \leq x < 10.5$ | 5.5 | 18 |
| 11 – 20 | $10.5 \leq x < 20.5$ | 15.5 | 25 |
| 21 – 30 | $20.5 \leq x < 30.5$ | 25.5 | 23 |
| 31 – 40 | $30.5 \leq x < 40.5$ | 35.5 | 20 |
| 41 – 50 | $40.5 \leq x < 50.5$ | 45.5 | 14 |
| $\sum fx = 2420$ | | | $\sum f = 100$ |

(b) **Required to determine:**

(i) the modal class interval

(ii) the number of seedlings in the sample

(iii) the mean height of the seedlings

(iv) the probability that a seedling chosen at random has a height that is greater than 30 cm

**Solution:**

(i) Modal class interval = 11 – 20 (since the most amount of seedlings occurs in this class)

(ii) The number of seedlings = 18 + 25 + 23 + 20 + 14
= 100

(iii) Mean, $\bar{x} = \dfrac{\sum fx}{\sum f}$, where $f$ = frequency and $x$ = midpoint or mid class interval

and $\sum fx = (5.5\times18)+(15.5\times25)+(25.5\times23)+(35.5\times20)+(45.5\times14)$
$= 2420$

So, the mean height of seedlings $= \dfrac{\sum fx}{\sum f}$

$= \dfrac{2420}{100}$

$= 24.2$ cm

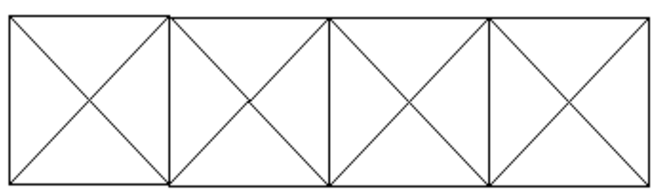
(iv) $P(\text{Seedling is greater than 30 cm}) = \dfrac{\text{No. of seedlings greater than 30 cm}}{\text{Total no. of seedlings}}$

$= \dfrac{34}{100}$

$= \dfrac{17}{50}$

The probability may also be expressed as the exact fraction of 0.34 or as the percentage of 34%.

8. (a) **Data:** Table of values and diagrams showing a sequence of shapes.
**Required to draw:** the 4th shape in the pattern
**Solution:**



(b) **Required to:** copy and complete the table for:
(i) Figure 4
(ii) Figure 10
**Solution:**
The completed table is shown below.

| | Total Number of Straws | |
|---|---|---|
| Figure | Formula | Number |
| 1 | $1(6)-0$ | 6 |
| 2 | $2(6)-1$ | 11 |
| 3 | $3(6)-2$ | 16 |
| 4 | $4(6)-3$ | 21 |
| ⋮ | ⋮ | ⋮ |
| 10 | $10(6)-9$ | 51 |

(c) **Required to find:** The figure in the sequence which uses 106 straws.
**Solution:**
In general,
We notice, the total no. of straws
= {Figure number × (6)}– {(Figure number – 1)}

Let's say the figure number be $n$
Then 106 $= (n \text{ x } 6) - (n - 1)$
$106 = 5n + 1$

$$n = \frac{106-1}{5}$$
$$= \frac{105}{5}$$
$$= 21$$

$\therefore$ Figure 21 has 106 straws.

(d) **Required to find:** an expression, in $n$, for the number of straws in the $n^{\text{th}}$ pattern
**Solution:**
Given that $n$ is the figure number,
The total no. of straws used in the $n^{\text{th}}$ pattern $= n(6)-(n-1)$

$= 5n + 1$
(This was found from before)

**Section II**

9. (a) **Data:** $y=\frac{2x+3}{x-4}$

**Required to:**

(i) make $x$ the subject of the formula

(ii) determine the inverse of $f(x)=\frac{2x+3}{x-4}$, $x\neq 4$

(iii) find the value of $x$ for which $f(x)=0$

**Solution:**

(i) $y=\frac{2x+3}{x-4}$

Cross multiply to obtain a linear form and then to make $x$ the subject

$$(x-4)y=2x+3$$
$$xy-4y=2x+3$$
$$xy-2x=4y+3$$
$$(y-2)x=4y+3$$
$$x=\frac{4y+3}{y-2}$$

(ii) $f(x)=\frac{2x+3}{x-4}$

Let $y=f(x)$

Making $x$ the subject of the formula was completed in previous part. We interchange $x$ and $y$ in the expression to obtain the inverse. The expression for $y$ will now be $f^{-1}$

$$y=\frac{4x+3}{x-2}$$
$$\therefore f^{-1}(x)=\frac{4x+3}{x-2}$$

(iii) Let

$$\frac{2x+3}{x-4}=0$$
$$2x+3=0$$
$$2x=-3$$
$$x=-\frac{3}{2}$$

So, $f(x)=0$ when $x=-\frac{3}{2}$



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(b) **Data:** Diagrams showing the graphs of lines $x = 6$, $x + y = 40$ and $3y = x$.
**Required to:**
(i) state the other two inequalities which define the shaded region
(ii) identify the three pairs of values for which $p$ has a maximum or minimum value
(iii) identify the pair of values which makes $p$ and a maximum
**Solution:**
(i) The two other inequalities which define the shaded region are $x \geq 6$ and $x + y \leq 40$.

(ii) The coordinates of the vertices of the triangle are $(6,\ 2)$, $(6,\ 34)$ and $(30,\ 10)$ and to identify the three pairs of $(x,\ y)$ for which $p$ has a maximum or a minimum value.

(iii) We substitute the corresponding values of $x$ and $y$ to obtain the value of $p$

When $x = 6$ and $y = 2$

$$\begin{aligned} p &= 4x + 3y \\ &= 4(6) + 3(2) \\ &= 30 \end{aligned}$$

When $x = 6$ and $y = 34$

$$\begin{aligned} p &= 4x + 3y \\ &= 4(6) + 3(34) \\ &= 126 \end{aligned}$$

When $x = 30$ and $y = 10$

$$\begin{aligned} p &= 4x + 3y \\ &= 4(30) + 3(10) \\ &= 150 \end{aligned}$$

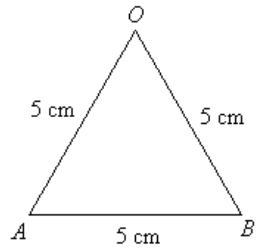
$\therefore$ The pair $(30,\ 10)$ makes $p$ a maximum and which is 150 as shown.

10. (a) **Data:** Diagram showing a regular hexagon with center $O$ and $AO = 8$ cm.
**Required To:**
(i) determine the size of angle $AOB$.
(ii) calculate, to the nearest whole number, the area of the hexagon
**Solution:**
(i) A regular hexagon is made up of six identical or congruent equilateral triangles as shown.
In an equilateral triangle, each interior angle is 60°.
Therefore, angle $AOB = 60°$.

(ii) Let us consider the triangle $AOB$

Let $S$ be ½ of the perimeter of triangle $AOB$

$$S = \frac{5+5+5}{2}$$
$$= 7.5$$



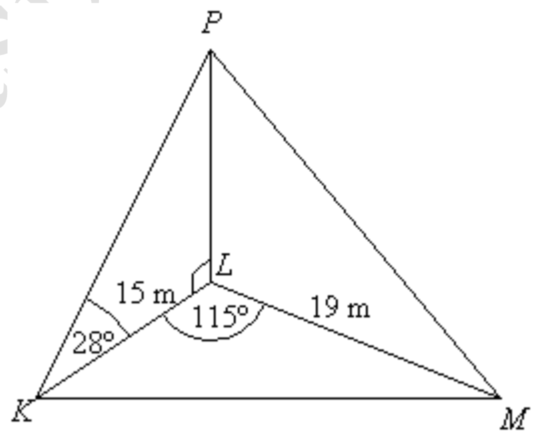

Since the lengths of all three sides of the triangle are known, we may use Heron's formula to obtain the area.

$$\therefore \text{Area} = \sqrt{7.5(7.5-5)(7.5-5)(7.5-5)}$$
$$= \sqrt{7.5 \times 2.5 \times 2.5 \times 2.5}$$
$$= \sqrt{117.1875}$$
$$= 6 \times \sqrt{117.1875}$$
$$= 64.9$$
$$\approx 65 \text{ cm}^2$$

(b) **Data:** Diagram showing a vertical pole *PL* standing on a horizontal plane *KLM*, where the angle of elevation of *P* from *K* is 28°, *KL* = 15 m, *LM* = 19 m and $K\hat{L}M = 115°$.

(i) **Required to copy:** the diagram, showing the angle of elevation and one right angle.

**Solution:**




$K\hat{L}P$ is a right angle because it was stated that *PL* is vertical and *KLM* is a horizontal plane. A horizontal plane and a vertical line will meet at a right angles. Also angle *PLW* will be a right angle and for the same reason.

(ii) **Required to calculate:**

a) *PL*

b) *KM*

c) the angle of elevation of *P* from *M*

**Solution:**

a) $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\tan 28^\circ = \dfrac{PL}{15}$

$PL = 15\tan 28^\circ$

$PL = 7.9\underline{\underline{7}}\text{ m}$

$= 8.0\text{ m (to 2 significant figures)}$

b) In the triangle KLM, we have two sides and the included angle. So we can apply the 'Cosine rule' to the triangle $KLM$:

$KM^2 = LM^2 + KL^2 - 2(LM)(KL)\cos K\hat{L}M$

$= (19)^2 + (15)^2 - 2(19)(15)\cos 115^\circ$

$= 586 + 240.89$

$= 826.89$

$KL = \sqrt{826.89}$

$= 28.\underline{\underline{7}}\text{ m}$

$\approx 29\text{ m (to 2 significant figures)}$

c) Angle of elevation of $P$ from $M$ is shown as $P\hat{M}L$.

$\tan P\hat{M}L = \dfrac{PL}{LM}$

$= \dfrac{8}{19}$

$P\hat{M}L = \tan^{-1}\left(\dfrac{8}{19}\right)$

$\approx 22.\underline{\underline{7}}^\circ$

$\approx 23^\circ \text{ (to 2 significant figures)}$

11. (a) **Data:** Diagram showing position vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

(i) **Required to find:** in the form $\begin{pmatrix} x \\ y \end{pmatrix}$

a) $\overrightarrow{OA}$

b) $\overrightarrow{OB}$

c) $\overrightarrow{BA}$





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**Solution:**

Since $A$ has coordinates (-1,3), then we may express the vectors, measured from a fixed point $O$ as

a) $\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ where $x = -1$ and $y = 3$.

Since B has coordinates (5,1) then similarly

b) $\overrightarrow{OB} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x = 5$ and $y = 1$.

c) $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA}$ (by the vector triangle law)

$$= \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -5-1 \\ -1+3 \end{pmatrix}$$

$= \begin{pmatrix} -6 \\ 2 \end{pmatrix}$ is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x$ = -6 and $y$ = 2

(ii) **Data:** $G$ is the midpoint of the line $AB$.

**Required to find:** in the form $\begin{pmatrix} x \\ y \end{pmatrix}$,

a) $\overrightarrow{BG}$

b) $\overrightarrow{OG}$

**Solution:**

a) Since $G$ is the midpoint of the line $AB$,

$\overrightarrow{BG} = \frac{1}{2}\overrightarrow{BA}$ (data)

$$\frac{1}{2}\overrightarrow{BA} = \frac{1}{2}\begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

$= \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x$ = -3 and $y$ = 1

b) $\overrightarrow{OG} = \overrightarrow{OB} + \overrightarrow{BG}$

$$= \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$= \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x = 2$ and $y = 2$.

(b) **Data:** $L = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $M = \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}$

**Required to evaluate:**

(i) $L + 2M$

(ii) $LM$

**Solution:**

(i) $L + 2M = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} + 2\begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}$

We now simplify and add the corresponding entries to obtain

$$= \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} + \begin{pmatrix} -2 & 6 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 8 \\ 1 & 8 \end{pmatrix}$$

(ii) $LM = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$

We check for the conformability of the matrices $L$ and $M$ under the operation of multiplication. We will obtain a matrix a 2 x 2 matrix. Each entry of the result is computed to give:

$$e_{11} = (3 \times -1) + (2 \times 0)$$
$$= -3$$

$$e_{12} = (3 \times 3) + (2 \times 2)$$
$$= 13$$

$$e_{21} = (1 \times -1) + (4 \times 0)$$
$$= -1$$

$$e_{22} = (1 \times 3) + (4 \times 2)$$
$$= 11$$

$$\therefore LM = \begin{pmatrix} -3 & 13 \\ -1 & 11 \end{pmatrix}$$

(c) **Data:** $Q = \begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix}$

**Required to find:**

(i) $Q^{-1}$

(ii) the value of $x$ and of $y$ in the equation $\begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$

**Solution:**

(i)

When $Q$ is of the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$Q^{-1} = \frac{1}{|Q|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\therefore Q^{-1} = \frac{1}{ad - bc}\begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}$$

$$= \frac{1}{(1)(4) - (-2)(-1)}\begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & -1 \\ -\frac{1}{2} & 2 \end{pmatrix}$$

(ii) $\begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$

Recall: $Q = \begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix}$ and $Q^{-1} = \begin{pmatrix} \frac{1}{2} & -1 \\ -\frac{1}{2} & 2 \end{pmatrix}$

Multiply both sides by $Q^{-1}$

A matrix multiplied by its inverse gives the identity matrix and the identity matrix multiplied by any matrix gives the same matrix. Hence,

$$\begin{pmatrix} \frac{1}{2} & -1 \\ -\frac{1}{2} & 2 \end{pmatrix} \begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -1 \\ -\frac{1}{2} & 2 \end{pmatrix} \begin{pmatrix} 8 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e_{11} \\ e_{12} \end{pmatrix}$$

$$e_{11} = \left(\frac{1}{2} \times 8\right) + (-1 \times 3)$$
$$= 1$$

$$e_{12} = \left(-\frac{1}{2} \times 8\right) + (2 \times 3)$$
$$= 2$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Both are equal 2 x 1 matrices, so equating corresponding entries we will obtain

$x = 1$ and $y = 2$