FAS-PASS Maths

CSEC MATHEMATICS MAY- JUNE 2011
PAPER 2
Section I

1. (a) (i) **Required to calculate:** $\dfrac{2\frac{1}{4}+1\frac{1}{8}}{4\frac{1}{2}}$.

Calculation:

$$\dfrac{2\frac{1}{4}+1\frac{1}{8}}{4\frac{1}{2}}$$

We first work the numerator

$$2\frac{1}{4}+1\frac{1}{8}$$

$$3\ \frac{2(1)+1(1)}{8}=3\frac{3}{8}$$

Hence,

$$\frac{Numerator}{Denominator}=3\tfrac{3}{8}\ \div 4\tfrac{1}{2}$$

Converting both numerator and denominator into improper fractions. Then

inverting the denominator and multiplying to get:

$$=\frac{27}{8}\div\frac{9}{2}$$

$$=\frac{27}{8}\times\frac{2}{9}$$

$$=\frac{3}{4}\text{ (in exact form)}$$

(ii) **Required to calculate:** $3.96\times 0.25-\sqrt{0.0256}$

Calculation:

The arithmetic is a bit cumbersome, especially with finding the square root of 0.0256. So, we use the calculator to find 3.96 x 0.25, then √0.0256, to get

$$3.96\times 0.25-\sqrt{0.0256}=0.99-0.16$$

$$=0.83\text{ (in exact form)}$$

(b) **Required to calculate:** *W*,*X*,*Y* and *Z*.

Calculation:

The total cost, W, of 6 ½ kg of rice @ \$2.40 per kg $= 6\frac{1}{2} \times \$2.40$

$= \$\ 15.60$

Hence the value of $W = \$15.60$

The cost of 4 bags of potatoes = \$52.80

Hence the unit price of potatoes, $X = \frac{\$52.80}{4}$

$= \$\ 13.20$

Hence the value of $X = \$13.20$

The cost of Y cartons of milk @ \$2.35 each = \$14.10

$$\text{The value of } Y = \frac{\text{Total cost of milk}}{\text{Cost of 1 carton}}$$

$$= \frac{\$14.10}{\$2.35}$$

$$= 6$$

Hence, $Y = 6$

Z% VAT = \$9.90

Sub-total = \$82.50

$$Z = \frac{\text{V.A.T.}}{\text{Sub-Total}} \times 100$$

$$= \frac{\$9.90}{\$82.50} \times 100$$

$$= 12$$

The value of $Z = 12$

2. (a) **Required To Write:** $\frac{x-2}{3} + \frac{x+1}{4}$ as a fraction in its lowest terms.

Solution:

The LCM of 4 and 3 is 12. So,

$$\frac{x-2}{3} + \frac{x+1}{4}$$

$$\frac{4(x-2)+3(x+1)}{12} = \frac{(4x-8)+(3x+3)}{12}$$

$$= \frac{(7x-5)}{12} \text{ (as a fraction in its lowest terms)}$$

(b) **Data:** $a * b = (a+b)^2 - 2ab$

Required to calculate: $3 * 4$

Calculation:
In the binary operation of $3*4$, the value of
$a=3,\ b=4$

$\therefore 3*4=((3)+(4))^2-2(3)(4)$
$=(7)^2-24$
$=49-24$
$=25$

(c) (i) **Required to factorise:** xy^3+x^2y

Solution:
We separate the common terms from each of the two terms. These are shown underlined.

$xy^3+x^2y=\underline{xy}.y^2+x.\underline{xy}$
$=xy\left(y^2+x\right)$

(ii) **Required to factorise:** $2mh-2nh-3mk+3nk$

Solution:
The four terms are grouped into two terms of two each. We factor out the common factor in each of the two groups to get:

$2mh-2nh-3mk+3nk=2h\left(\underline{m-n}\right)-3k\left(\underline{m-n}\right)$
$=\left(m-n\right)\left(2h-3k\right)$

(d) **Data:** Table of values of the variables *x* and *y*.
Required to find: the value of *a* and of *b*
Solution:

x	***y***
2	12
5	*a*
b	48

y varies directly as *x*.
This may be expressed as
$y\propto x$
$\therefore y=kx$
(where *k* is the constant of proportion or constant of proportionality)

When $x=2$ and $y=12$
(data from the table of values)
$12=k(2)$
$12=2k$
$\therefore k=6$

Hence the equation can now be expressed as $y=6x$

When $x=5$, $y=a$

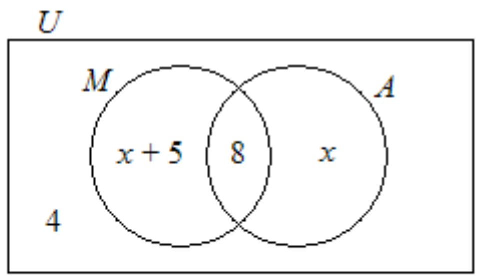
$\therefore a=k(5)$
$a=(6)(5)$
$=30$

When $x=b$, $y=48$

$\therefore 48=k(b)$
$48=6(b)$
$48=6b$
$\therefore b=8$

Hence, the value of $a=30$ and the value of $b=8$.

3. (a) **Data:** A given Venn diagram, $U=\{\text{Students in a class of 35}\}$



(i) **Required to find:** The number of students who do not study either Art or Music.
Solution:

The number of students who do not belong to the set M or the set A is 4 as shown in the complement region of $M\cup A$ and is the number of students who do not study either Art or Music.

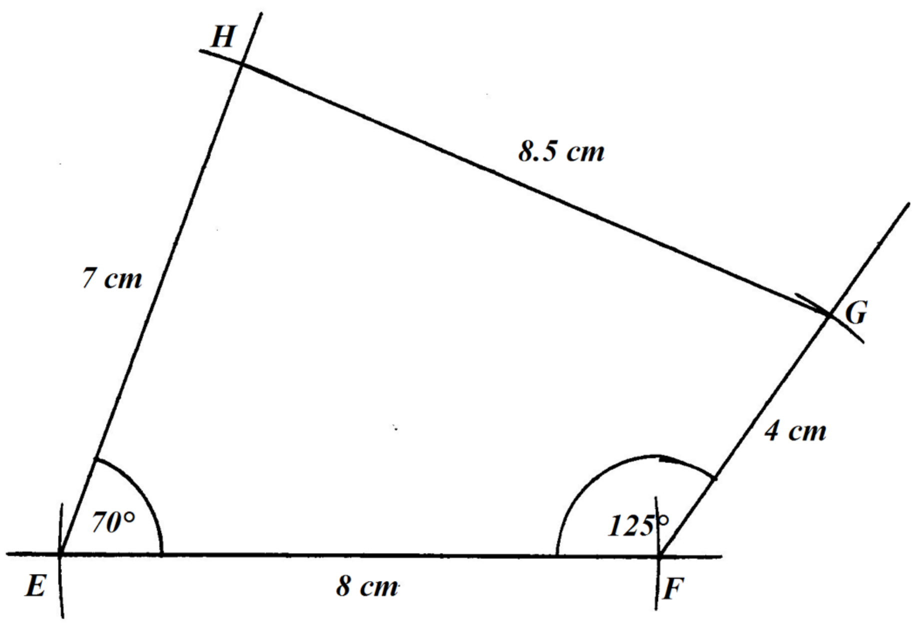
(This number is seen on the diagram in $(M\cup A)'$.

(ii) **Required to find:** The value of x.
Solution:
The sum of all the numbers of all the subsets of the Universal set in the diagram.
$=(x+5)+8+(x)+4$
$=2x+17$

The total number of students $=35$ (data)
Hence, we may equate
$\therefore 2x+17=35$
$2x=35-17$
$x=9$

(iii) **Required to find:** The number of students studying Music only.
Solution:
The value of $x = 9$
n (M only) = (9 + 5) as shown on the diagram
$= 14$ students

(b) (i) **Data:** $EF = 8$ cm, angle EFG = 125°, FG = 4 cm, angle HEF = 70°, EH = 7cm
Required: To draw the quadrilateral with the above measurements.
Solution:
Draw a straight line longer than 8 cm and cut off EF 8 cm.
At E we draw an angle of 70^0 and at F we draw an angle of 125^0
Cut off $FG = 4$ cm
Cut off $GH = 8.5$ cm



(ii) **Required to find:** The length of GH.
Solution:
$GH = 8.5$ cm (by measurement using the ruler)

4. (a) (i) **Data:** $5 - 2x < 9$

Required to find: x

Solution:
The procedure is much the same as solving an equation

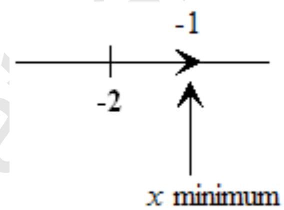
$$5-2x<9$$
$$5<9+2x$$
$$5-9<2x$$
$$-4<2x$$
$$\therefore \frac{-4}{2}<x$$
$$-2<x$$
$$x>-2$$

(ii) **Data:** $x \in Z$

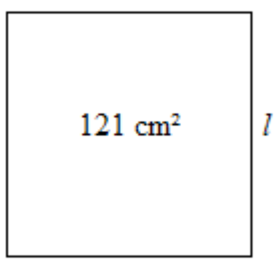
Required to find: The smallest value of x

Solution:

$$x \in Z$$
$$\therefore x_{\min} = -1$$



(b) (i) **Data:**



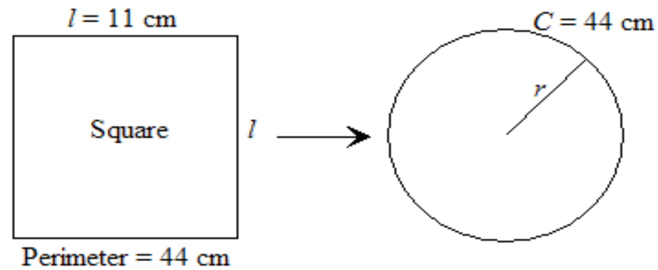
a) **Required to calculate:** l, length of side.
Calculation:
The area of a square is side × side. So,

$$l \times l = 121 \text{ cm}^2$$
$$\therefore l = \sqrt{121} \text{ cm}$$
$$= 11 \text{ cm}$$

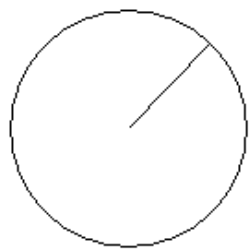
b) **Required to find:** Perimeter of square.
Calculation:
Perimeter $= l \times 4$
The side, l, was found to be 11 cm
So, perimeter is

$$= 11 \times 4$$
$$= 44 \text{ cm}$$

(ii) a) **Data:**



Required to find: The radius of the circle
Solution:
The circumference of the circle is the same as the perimeter of the Square (data)



Since the perimeter of the square is 44 cm, then,

$$2\pi r = 44$$

$$2 \times \frac{22}{7} \times r = 44$$

$$\therefore r = 7 \text{ cm}$$

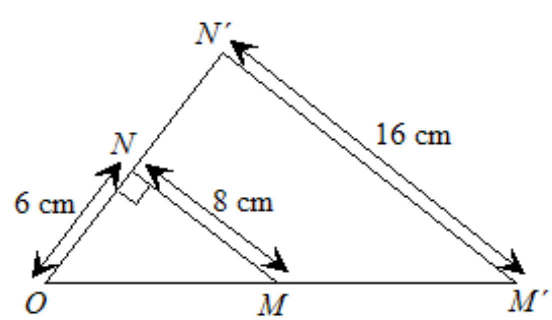
b) **Required to find:** Area of the circle.
Solution:

$$A = \pi r^2$$

$$= \left(\frac{22}{7}\right)(7)^2$$

$$= 154 \text{ cm}^2$$

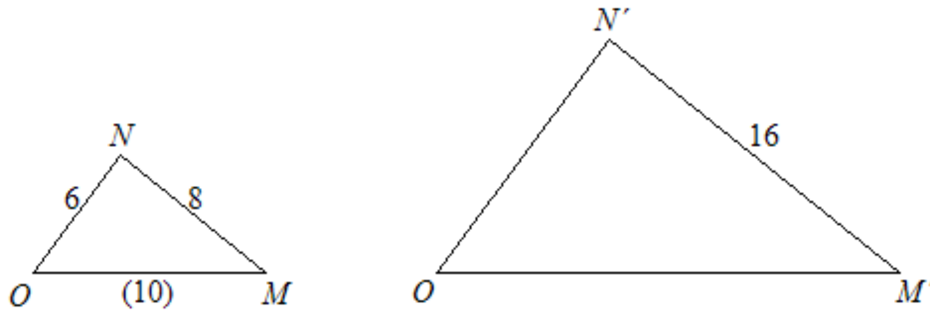
5. (a) **Data:**



Triangle $OM'N'$ is the image of triangle OMN after undergoing an enlargement, about the center O.

(i) **Required to Find:** Value of k, the scale factor.
Solution:
$OMN \xrightarrow{Enlargement} OM'N'$,

The center of enlargement is O.
Separating the 2 triangles so we may better view them.



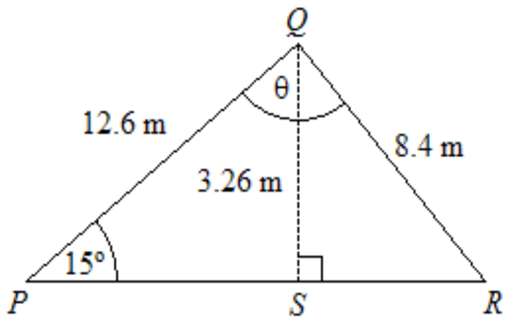
$$\text{Scale Factor} = \frac{\text{Image Length}}{\text{Object Length}}$$

$$\text{Choosing } \frac{N'M'}{NM}$$
$$= \frac{16}{8}$$
$$= 2$$

(ii) **Required to calculate:** The length of OM.
Calculation:
$OM^2 = (6)^2 + (8)^2$ (by using Pythagoras' Theorem)
$= 36 + 64$
$OM = \sqrt{100}$
$= 10$ cm

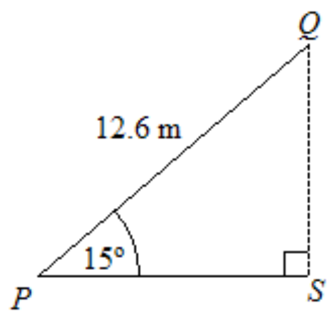
(iii) **Required to calculate:** The length of OM'.
Calculation:
The scale factor of the enlargement is 2
$$\frac{OM'}{OM} = \frac{2}{1}$$
$$\frac{OM'}{10} = \frac{2}{1}$$
$$OM' = 20 \text{ cm}$$

(b) **Data:**



$PQ = 12.6\,\text{cm},\ QR = 8.4\,\text{cm}$ and $Q\hat{P}R = 15°$

(i) **Required to find:** Length of *QS*, to 3 significant figures.
Solution:
Looking separately at the right-angled triangle *PQS*.



$\sin 15^{o} = \dfrac{QS}{12.6}$ (definition)

$QS = 12.6 \times (\sin 15^{o})$

$= 3.26\ \text{m}$ to 3 s.f.

(ii) **Required to find:** $R\hat{Q}S$ to 3 significant figures.
Solution:
Considering the right-angled triangle *RQS*

$$\cos R\hat{Q}S = \frac{3.261}{8.4}$$

$$R\hat{Q}S = \cos^{-1}\left(\frac{3.261}{8.4}\right)$$

$$= 67.15°$$

$$= 67.2° \text{ (to 3 s.f.)}$$

(iii) **Required to find:** The area of ΔPQR
Solution:

$$P\hat{Q}S = 180° - (90° + 15°)$$

$$= 75°$$

(The sum of angles in a triangle = 180°)

$$\therefore P\hat{Q}R = 67.2° + 75°$$

$$= 142.21°$$

In triangle PRQ we have two sides and the included angle. So,

$$\text{Area of } \Delta PQR = \frac{1}{2}(QP \times QR)\sin P\hat{Q}R$$
$$= \frac{1}{2}(12.6 \times 8.4)\sin P\hat{Q}R$$
$$= \frac{1}{2}(12.6 \times 8.4)\sin 142.21°$$
$$= 32.4\underline{\underline{2}}$$
$$= 32.4 \text{ cm}^2 \text{(to 3 s.f.)}$$

6. (a) **Data:** $f(x) = 6x + 8$ and $g(x) = \frac{x-2}{3}$.

(i) **Required to calculate:** $g\left(\frac{1}{2}\right)$.

Calculation:
We substitute $x = ½$ in $g(x)$ to get

$$g\left(\frac{1}{2}\right) = \frac{\left(\frac{1}{2}\right)-2}{3} = -\frac{1}{2}$$

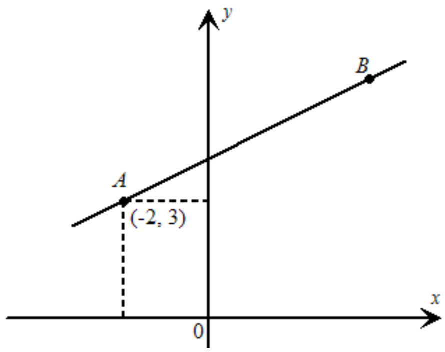
(ii) **Required to calculate:** An expression for $gf(x)$.
Calculation:
We could find $gf(x)$ by replacing x in $g(x)$ by $f(x)$.

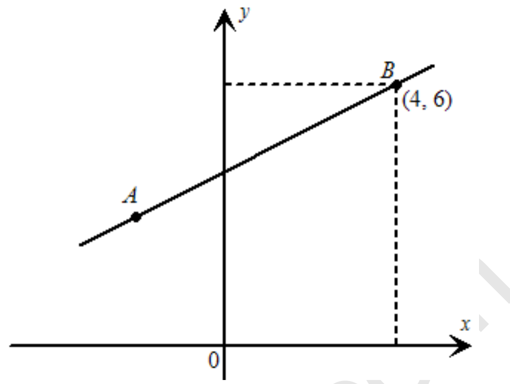
$$gf(x) = g(f(x))$$
$$= g(6x + 8)$$
$$= \frac{(6x+8)-2}{3} = \frac{6x+6}{3}$$
$$= 2x + 2$$

(iii) **Required to calculate:** $f^{-1}(x)$
Calculation:
Let $f(x)$ be y.
$y = 6x + 8$
Interchange x and y.

$$x = 6y + 8$$
$$x - 8 = 6y$$
$$y = \frac{x-8}{6}$$
$$\therefore f^{-1}(x) = \frac{x-8}{6}$$

(b) (i) **Required to find:** The coordinates of A and of B.
Solution:





Read off from the graph
$A=(-2,\ 3)$

Read off from the graph
$B=(4,\ 6)$

(ii) **Required to find:** The gradient of AB.
Solution: Using the formula for gradient, we get

$$\text{Gradient } = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-3}{4-(-2)} = \frac{1}{2}$$

Using A as $(x_1,\ y_1)$ and B as $(x_2,\ y_2)$ OR vice –versa

(iii) **Required to find:** The equation of the line passing through A and B.
Solution:
Choosing $A=(-2,\ 3)$
Then using $\frac{y-y_1}{x-x_1}=m$, where $(x_1,\ y_1)$ is a point on the line and m is the gradient. We get

$$\frac{y-3}{x-(-2)}=\frac{1}{2}$$
$$2(y-3)=x+2$$
$$2y-6=x+2$$
$$2y=x+2+6$$
$$2y=x+8$$
$$y=\frac{1}{2}x+4$$

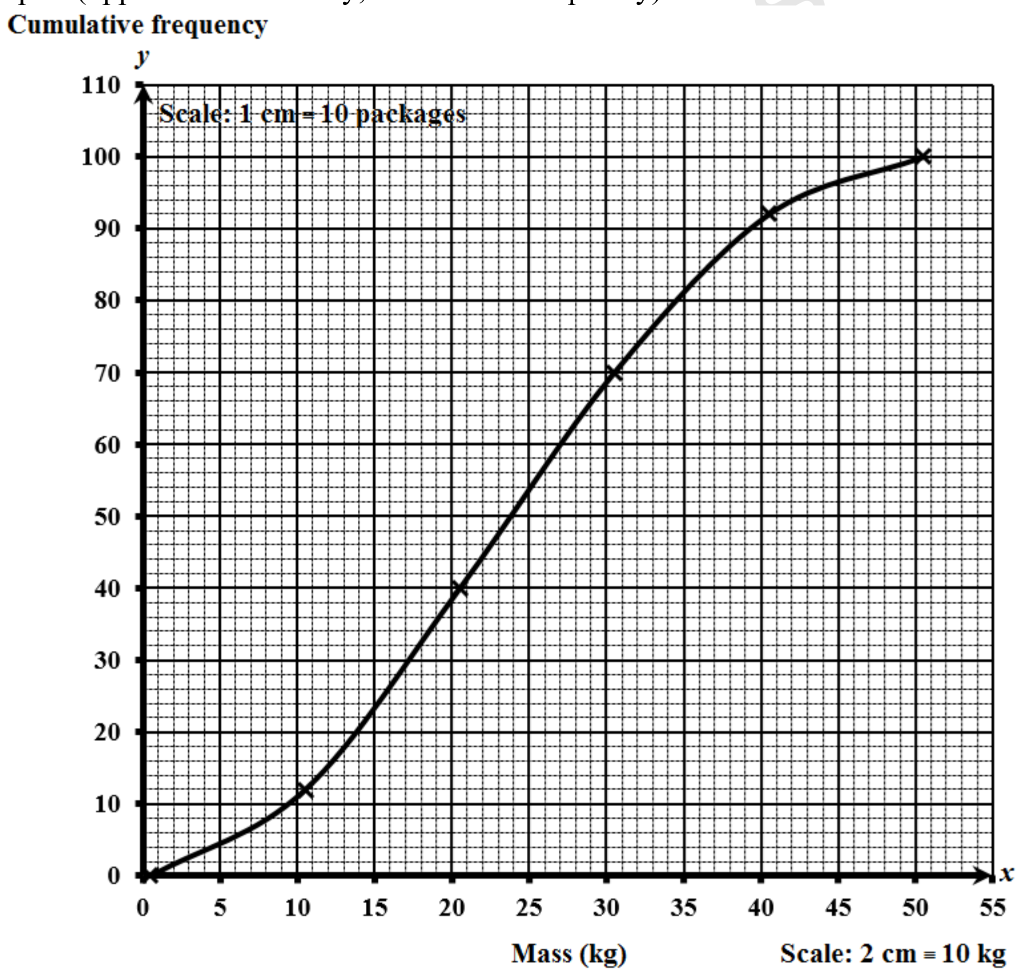
If, instead, we had chosen B as the point, the equation would have been the same.

7. (a), (b) **Data:** Number of packages = 100

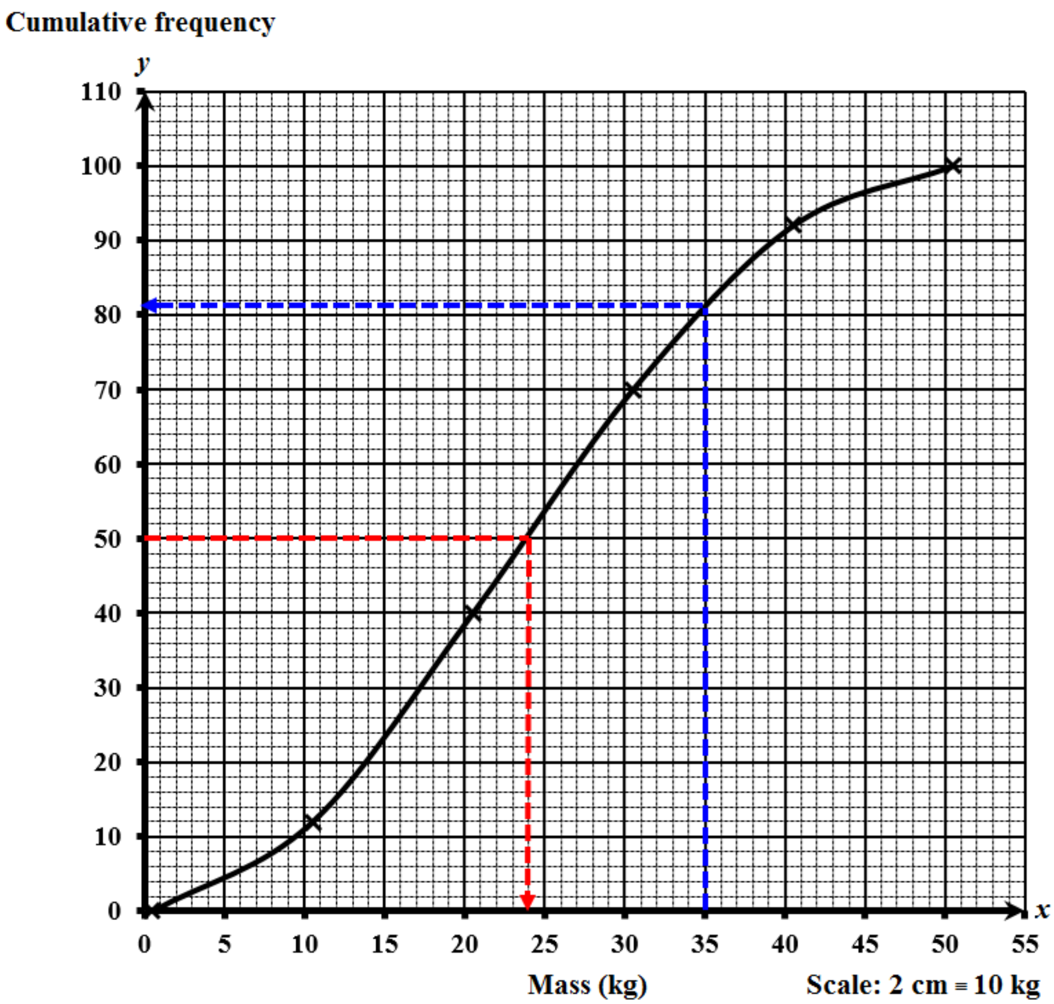
Mass is a continuous variable
L.C.B-lower class boundary
U.C.B-upper class boundary

Mass , *m* (kg)	**L.C.B U.C.B.**	**No. of Packages (Frequency)**	**Cumulative Frequency**	**Points to Plot (U.C.B., C.F.)**
-	-	0	0	(0.5, 0)
1 – 10	$0.5 \leq m < 10.5$	12	0 + 12 = 12	(10.5, 12)
11 – 20	$10.5 \leq m < 20.5$	28	12 + 28 = 40	(20.5, 40)
21 – 30	$20.5 \leq m < 30.5$	30	40 + 30 = 70	(30.5, 70)
31 – 40	$30.5 \leq m < 40.5$	22	70 + 22 = 92	(40.5, 92)
41 – 50	$40.5 \leq m < 50.5$	8	92 + 8 = 100	(50.5, 100)

We plot (upper class boundary, cumulative frequency)



(c) (i) **Required to find:** Median mass
Solution:



½ of CF is 50

The horizontal at 50 is drawn to meet the curve. At the point of meeting, a vertical is dropped to obtain the read off that is the median

Median mass = 24 kg (as shown on the diagram)

(ii) **Required To Find:** Probability that the mass of a package is less than 35 kg

Solution:

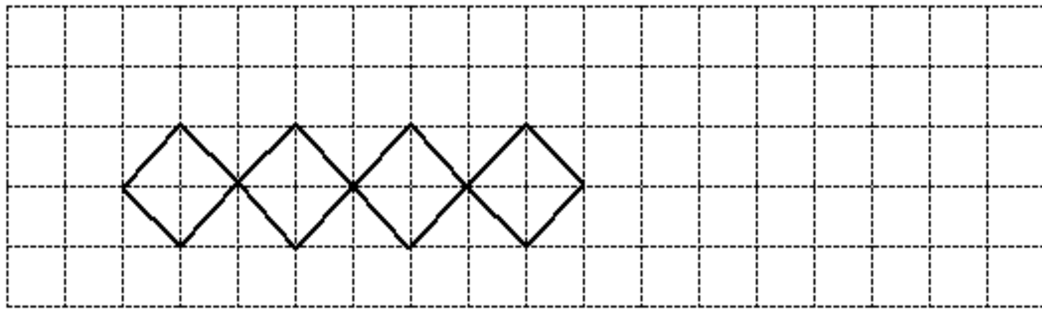
The vertical at 35 is drawn to meet the curve. At the point of meeting, a horizontal is drawn to obtain the read off that is the number of packages that is < 35 kg and which is 81.

$$P(\text{Mass} < 35 \text{ kg}) = \frac{\text{No. of packages} < 35 \text{ kg}}{\text{Total no. of packages}}$$

$$= \frac{81}{100}$$

8. **Answer Sheet for Question 8**

(a)



(b) (i) Number of the diagram x 4

$= 6 \times 4 = 24$ sticks

(ii) The number of the diagram is 7

The number of sticks = 7 x 4 = 7(4)

The rule connecting *t* and *s* gives

$1 + \left(\frac{3}{4} \times 7(4)\right) = 22$

Hence, the number of thumb tacks in the seventh diagram is 22

(c)

	No. of sticks ***s***	**Rule Connecting** ***t*** **and** ***s***	**No. of Thumb Tacks** ***t***
	4	$1 + \left(\frac{3}{4} \times 4\right)$	4
	8	$1 + \left(\frac{3}{4} \times 8\right)$	7
	12	$1 + \left(\frac{3}{4} \times 12\right)$	10
(i)	52	$1 + \left(\frac{3}{4} \times 52\right)$	40
(ii)	72	$1 + \left(\frac{3}{4} \times x\right) = 55$ $\frac{3}{4}x = 54$ $x = \frac{54 \times 4}{3}$ $= 72$	55

(d) $t = 1 + \left(\frac{3}{4} \times s\right)$

Section II

9. (a) **Data:** $y = x^2 - x + 3$ …(1) and $y = 6 - 3x$ …(2)

Required to solve: The value of x and of y

Solution:

Solving simultaneously

Equating the equations (1) and (2) since both are equal to y.

We obtain a quadratic in x.

$$x^2 - x + 3 = 6 - 3x$$
$$x^2 - x + 3 - 6 + 3x = 0$$
$$x^2 + 2x - 3 = 0$$
$$(x+3)(x-1) = 0$$
$$x = 1 \text{ or } -3$$

When $x = 1$
$$y = 6 - 3(1)$$
$$= 6 - 3$$
$$= 3$$

When $x = -3$
$$y = 6 - 3(-3)$$
$$= 6 + 9$$
$$= 15$$

Hence, $x = 1$, $y = 3$ **OR** $x = -3$, $y = 1$

(b) (i) **Data:** $4x^2 - 8x - 2 \equiv a(x+h)^2 + k$, a, h and k are constants.

Required to express: $4x^2 - 8x - 2$ in the form $a(x+h)^2 + k$.

Solution:

$$4x^2 - 8x - 2 = 4(x^2 - 2x) - 2$$
$$= 4[(x-1)^2 - 1] - 2$$
$$= 4(x-1)^2 - 4 - 2$$
$$= 4(x-1)^2 - 6$$

$\frac{1}{2}$ coefficient of $-2x = \frac{1}{2}(-2)$

$= -1$

Therefore $4(x-1)^2 - 6$ is of the form $a(x+h)^2 + k$ where $a = 4$, $h = -1$ and $k = -6$.

(Note that we could have also expanded the right hand side and equated coefficients to obtain the same solution)

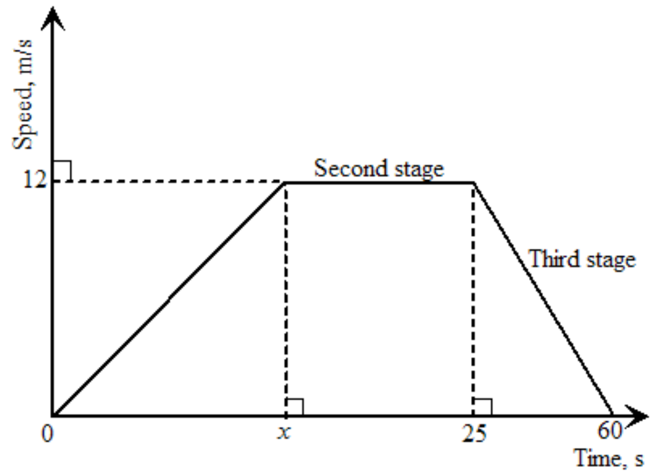
(ii) **Required to find:** The x – coordinate of the minimum point on $f(x)$

Solution:

$$4(x-1)^2 - 6$$
$$4(x-1)^2 \geq 0, \ \forall x$$
$$\therefore \text{minimum } f(x) = 4(0) - 6 = -6$$

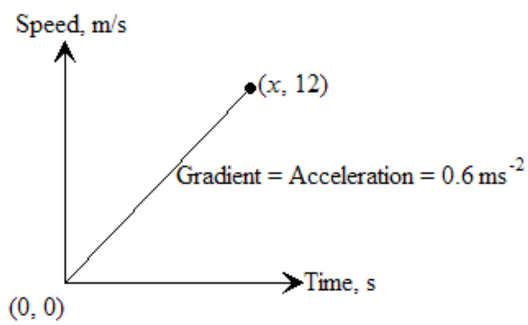
This occurs when $4(x-1)^2 = 0$ and $x = 1$

(c) **Data:**



During the first stage the car's speed increases from 0 ms^{-1} to 12 ms^{-1}.
Acceleration = 0.6 ms^{-2} is constant and shown by a straight line branch.

(i) **Required to find:** Value of x.
Solution: The gradient of the line on a velocity-time graph gives the acceleration

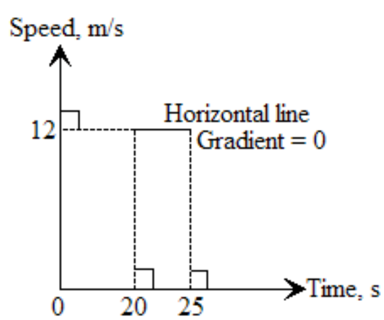


$$\frac{12-0}{x-0}=0.6$$

$$x=\frac{12}{0.6}$$

$$=20 \text{ s}$$

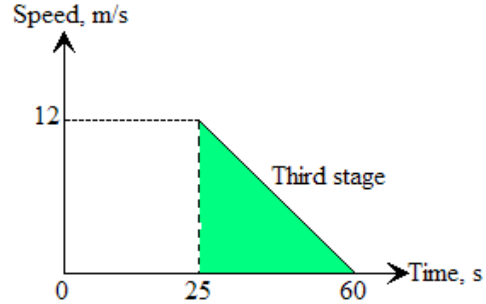
(ii) **Required to explain:** What takes place during the second stage.
Solution:



In the second stage of the journey, from 20 to 25 seconds, the velocity is constant as the gradient is 0. There is no acceleration or the acceleration is 0 ms^{-2}. The branch is a horizontal line and the car is moving at the constant speed of 12 ms^{-1}.

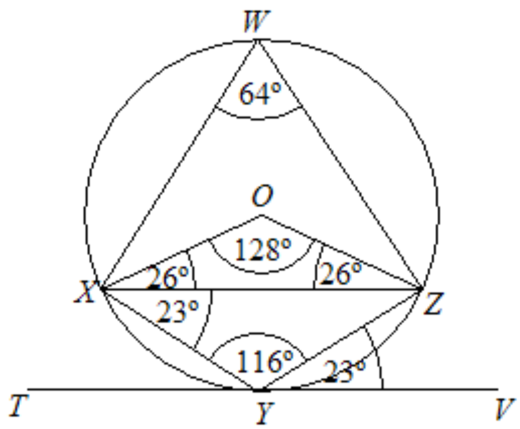
(iii) **Required to find:** The distance travelled during the third stage.
Solution:

The area under a speed – time graph gives distance covered. The third stage is shown as the shaded triangle



$$\text{Area} = \frac{(60-25)\times 12}{2}$$
$$= 210\,\text{m}$$

10. (a) **Data:**



(i) **Required to calculate:** $X\hat{O}Z$
Calculation:

$X\hat{Y}Z = 180° - 64^0$
$= 116°$

(The opposite angles of the cyclic quadrilateral *WXYZ* are supplementary).

(ii) **Required to calculate:** $Y\hat{X}Z$
Calculation:

$Y\hat{X}Z = 23°$

(The angle made by the tangent to a circle and a chord, angle *VYZ* at the point of contact = angle in the alternate segment, angle *YXZ*.)

(iii) **Required to calculate:** $O\hat{X}Z$
Calculation:

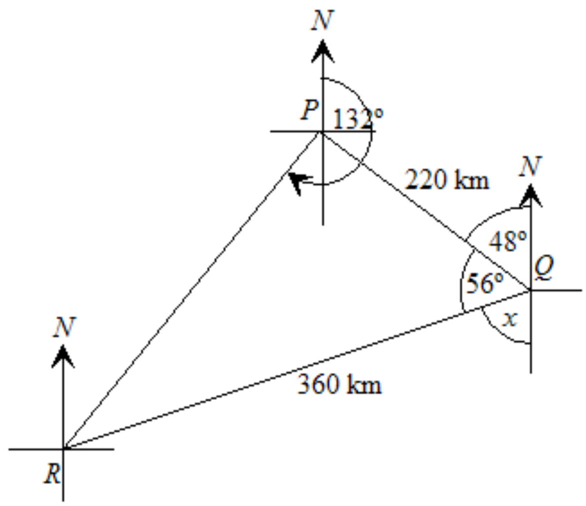
OX = *OZ* (radii of the same circle)
Triangle *OXZ* is isosceles (two sides are equal) and the base angles are therefore equal

$$O\hat{X}Z = \frac{(180° - 128°)}{2}$$ (The sum of angles in a triangle = 180º).

$$= \frac{52°}{2}$$

$$= 26°$$

(b) **Data:**



(i) **Required to calculate:** The value of x.
Calculation:

$$x° = 180° - (48° + 56°)$$
$$= 76°$$ (The angles at a point in a straight line = 180º)

(ii) **Required to calculate:** The length of RP.
Calculation:
Applying the cosine rule to triangle PQR since we have two sides and the included angle

$$(RP)^2 = (220)^2 + (360)^2 - 2(220)(360)\cos 56°$$
$$= 48400 + 129600 - (158\,400)(\cos 56°)$$
$$RP = \sqrt{178000 - (158\,400)(\cos 56°)}$$
$$= 299.\underline{\underline{0}}$$
$$= 299 \text{ km}$$

(iii) **Required to calculate:** The bearing of R from P.
Calculation:
Using the sine rule on triangle PQR

$$\frac{RQ}{\sin R\hat{P}Q} = \frac{RP}{\sin 56°}$$

$$\frac{360}{\sin R\hat{P}Q} = \frac{299}{\sin 56°}$$

$$\sin R\hat{P}Q = \frac{360 \times \sin 56°}{299}$$

$$R\hat{P}Q = \sin^{-1}\left(\frac{360 \times \sin 56°}{299}\right)$$

$$= 86.5°$$

Bearing is direction measured from North in a clockwise direction

$\therefore$ Bearing of R from P: $132^0 + 86.5^0$

$= 218.5^0$

11. (a) **Data:** Matrix, $M = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$

Required to calculate: Inverse of the matrix

Calculation:

We first find the determinant of M

$$\det M = (4)(3) - (5)(2)$$
$$= 12 - 10$$
$$= 2$$

Now we find the inverse of M.

$$M^{-1} = \frac{1}{2}\begin{pmatrix} 4 & -(5) \\ -(2) & 3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{4}{2} & -\frac{5}{2} \\ -\frac{2}{2} & \frac{3}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -2\frac{1}{2} \\ -1 & 1\frac{1}{2} \end{pmatrix}$$

(b) **Data:** $R\ (7,\ 2) \xrightarrow{M = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}} R'\ (2,\ -7)$

$T\ (-5,\ 4) \xrightarrow{M = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}} T'\ (4,\ 5)$

(i) **Required to calculate:** The value of a and of b.

Calculation:

$$\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}\begin{pmatrix} 7 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

$$2\times2 \quad 2\times1 = \begin{pmatrix} e_{11} \\ e_{12} \end{pmatrix}$$

$$\begin{pmatrix} 2a \\ 7b \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

Both are 2x1 matrices and are equal. So equating corresponding entries we obtain

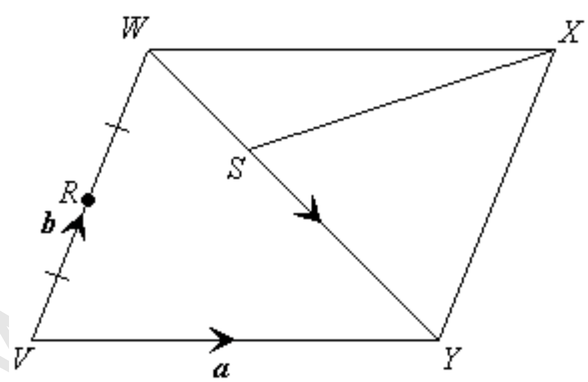
$2a = 2$ $\qquad$ $7b = -7$

$a = 1$ $\qquad$ $b = -1$

$$\therefore M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(ii) **Required to describe:** The transformation that M represents.
Solution:
$M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is the matrix that represents a 90^0 clockwise rotation about the origin.

(c) **Data:** *WXYV* is a parallelogram.



$VY = \boldsymbol{a}$

$VW = \boldsymbol{b}$

$WS : SY = 1 : 2$

(i) a) **Required To Write:** An expression in terms of a and b for $\overrightarrow{WY}$.
Solution:

Applying the vector triangle law

$WY = WV + VY$
$= \boldsymbol{a} + (-\boldsymbol{b})$

b) **Required to write:** An expression in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ for $\overrightarrow{WS}$.
Solution:

$$WS = \frac{1}{3}WY$$
$$= \frac{1}{3}(\boldsymbol{a} - \boldsymbol{b})$$
$$= \frac{\boldsymbol{a}-\boldsymbol{b}}{3} \text{ or } \frac{1}{3}\boldsymbol{a} - \frac{1}{3}\boldsymbol{b}$$

c) **Required To Write:** An expression in terms of a and b for $\overrightarrow{SX}$.
Solution:
Applying the vector triangle law

$$SX = SW + WX \qquad (SW = -WX)$$
$$= -\frac{1}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b} + \boldsymbol{a}$$
$$= \frac{2}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$$

(ii) **Required To Prove:** R, S and X are collinear.
Solution:
Applying the vector triangle law

$$RX = RW + WX \qquad \overrightarrow{SW} = -\overrightarrow{WS}$$
$$= \frac{1}{2}\boldsymbol{b} + \boldsymbol{a}$$
$$= \boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$$

$$SX = \frac{2}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$$
$$= \frac{2}{3}\left(\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}\right)$$
$$= \frac{2}{3}RX$$
$$\downarrow$$
Scalar Multiple

$\overrightarrow{SX}$ is a scalar multiple of $\overrightarrow{RX}$ and therefore these vectors are parallel.
But X is a common point on both vectors. $\therefore$ R, S and X are collinear.