CSEC JANUARY 2011 MATHEMATIC - PAPER 2

Section I

1. (a) (i) **Required to calculate:** $\left(5.8^2 + 1.02\right) \times 2.5$

Calculation:
The arithmetic is simple but would take a bit of time. So we choose the calculator.

$$\left\{(5.8)^2 + 1.02\right\} \times 2.5 = (33.64 + 1.02) \times 2.5$$
$$= 34.66 \times 2.5$$
$$= 86.65 \text{ (in exact form)}$$

(ii) **Required to calculate:** $\dfrac{2\frac{4}{9}}{4\frac{2}{3}} - \dfrac{3}{7}$

Calculation:
We convert the numerator and denominator of the first term to improper fractions. Then we simplify by inverting the denominator and multiplying by the numerator

$$\frac{2\frac{4}{9}}{4\frac{2}{3}} = \frac{\frac{22}{9}}{\frac{14}{3}}$$
$$= \frac{22}{9} \times \frac{3}{14}$$
$$= \frac{22}{42}$$
$$= \frac{11}{21}$$

And now we subtract the second term from our reduced first term

$$\frac{11}{21} - \frac{3}{7}$$
$$\frac{11 - 3(3)}{21} = \frac{2}{21}$$

Therefore, $\dfrac{2\frac{4}{9}}{4\frac{2}{3}} - \dfrac{3}{7} = \dfrac{2}{21}$ (in exact form).

(b) **Data:** Employee is paid a basic wage of \$9.50 per hour for a 40 hour week and one and a half times this rate for overtime.

(i) **Required to calculate:** Basic weekly wage for one employee.

Calculation:
Basic weekly wage of one employee for a 40-hour work week
= hourly rate × number of hours in a basic work week
$= \$9.50 \times 40$
$= \$380.00$

(ii) **Required to calculate:** Overtime wage for an employee who works 6 hours overtime.

Calculation:

The overtime rate $= \$9.20 \times 1\frac{1}{2}$ per hour
$= \$14.25$ per hour

Hence overtime wage for 6 hours overtime $= \$14.25 \times 6$
$= \$85.50$

(iii) **Required to calculate:** Total amount paid in overtime wages for for 30 employees.

Calculation:
Basic wage of 30 employees in one 40 hour week
=Number of employees × basic work week wage
$= 30 \times \$380.00$
$= \$11400.00$

Total wages paid = \$12 084.00 (data)

$\therefore$ Total paid in overtime wages $= \$12\,084.00 - \$11\,400.00$
$= \$684.00$

(iv) **Required to calculate:** The number of overtime hours worked.

Calculation:
$\therefore$ Total number of overtime hours worked
= Overtime wages ÷ overtime rate
$= \dfrac{\$684.00}{\$14.25 \text{ per hour}}$
$= 48$ hours

2. (a) **Required to simplify:** $\frac{2x}{5}-\frac{x}{3}$

Solution:
The LCM of 3 and 5 is 15. So,
simplifying to get

$$\frac{2x}{5}-\frac{x}{3}$$

$$\frac{3(2x)-5(x)}{15}=\frac{6x-5x}{15}$$

$$=\frac{x}{15} \text{ (as a single fraction)}$$

(b) **Required to factorise:** a^2b+2ab

Solution:
Factorising
We re-write the terms to see the common terms than can be factored out.

$$a^2b+2ab=a.ab+2.ab$$

$$=ab(a+2)$$

(c) **Data:** $q=\frac{p^2-r}{t}$

Required to express: p as the subject of the formula

Solution:
We cross multiply so as to obtain a linear equation.
Then, we place all terms involving p on one side of the equation and simplify.

$$\frac{q}{1}=\frac{(p^2-r)}{t}$$

$$q\times t=(p^2-r)\times 1$$

$$qt=p^2-r$$

$$\therefore qt+r=p^2$$

$$p=\sqrt{qt+r}$$

(d) **Data:** Table showing the number of donuts in two types of boxes.

(i) **Required to calculate:** The total number of donuts sold.

Calculation:

Number of donuts in 8 small boxes with x each $= (8 \times x) = 8x$

Number of donuts in 5 large boxes with $(2x+3)$ each $= 5(2x+3)$

Total number of donuts $= (8 \times x) + 5(2x+3)$
$= 8x + 10x + 15$
$= 18x + 15$

(ii) **Data:** Total number of donuts sold = 195.

Required to calculate: The number of donuts in the various boxes

Calculation:
We equate
$18x + 15 = 195$ (data)
$18x = 195 - 15$
$= 180$
$x = \frac{180}{18}$
$= 10$

a) Hence, the number of donuts in a small box = 10

b) Hence, the number of donuts in a large box $= 2(10) + 3$
$= 20 + 3$
$= 23$

3. (a) **Required to simplify:** $7p^5q^3 \times 2p^2q$

Solution:
Simplifying and grouping to easily see the law of indices being applied
$7p^5q^3 \times 2p^2q = 7 \times 2 \times p^{5+2}q^{3+1}$
$= 14p^7q^4$

(b) **Data:** One carton of milk measures 6 cm by 4 cm by 10 cm.

(i) **Required to calculate:** Volume of each carton.

Calculation:
Volume= length by width by height $= 6 \times 4 \times 10$
$= 240 \text{ cm}^3$

(ii) **Data:** An ice cream recipe requires 3 litres of milk.

Required to calculate: The number of cartons of milk to be bought

Calculation:
Recipe for ice cream requires 3 litres of milk $= 3 \times 1000\,\text{cm}^3$

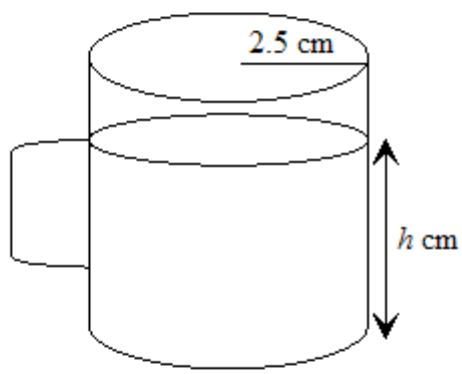
Hence, the number of cartons required $= \dfrac{3 \times 1000}{240}$
$= 12.5$ cartons

It is unlikely that cartons are sold in 'halves' and so it is expected the purchaser would have to buy 13 cartons.

(iii) **Data:** Carton of milk is poured into a cylindrical cup of internal diameter 5 cm.

Required to calculate: The height, *h* cm, of the milk inside the cup

Calculation:



Let the height of the milk in the cup be *h* cm.
The volume of milk in the cup should be the volume of milk in 1 carton.

$$\therefore \pi r^2 h = \text{Volume}$$
$$3.14 \times (2.5)^2 \times h = 240$$
$$h = \frac{240}{3.14 \times (2.5)^2}$$
$$= 12.22$$
$$= 12.2 \text{ cm to 3 significant figures}$$

4. (a) **Data:** $U = \{1, 2, 3, ..., 12\}$, $H = \{\text{Odd numbers between 4 and 12}\}$ and $J = \{\text{Prime numbers from 1 - 12}\}$

(i) **Required to list:** The member of set *H*.

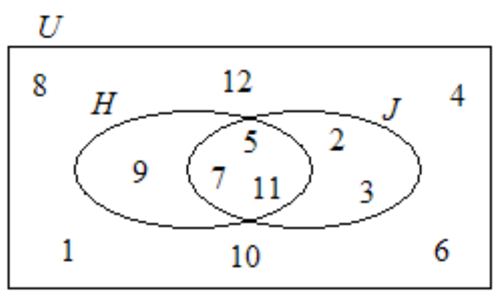
Solution:
Odd numbers are NOT divisible by 2. So,
$H = \{5, 7, 9, 11\}$

(ii) **Required to list:** The members of set *J*.

Solution:
Prime numbers have only two factors, itself and 1. Remember 1 is NOT prime. So,
$J = \{2, 3, 5, 7, 11\}$

(iii) **Required to draw:** A Venn diagram showing sets *U*, *H* and *J*.

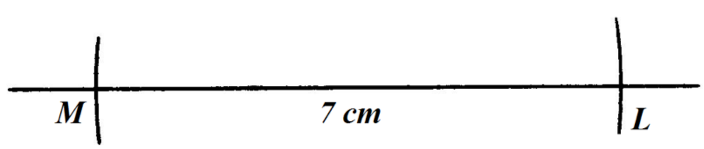
Solution:



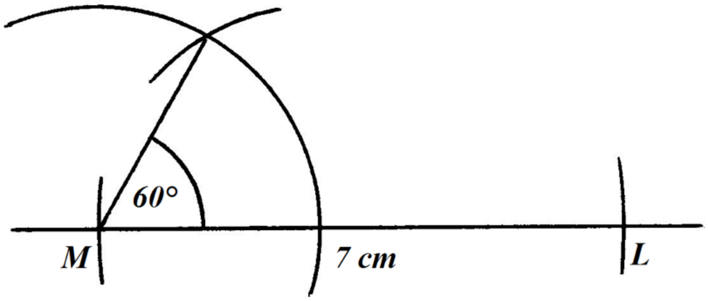
(b) (i) **Required to construct:** Triangle *LMN* with $L\hat{M}N = 60°$, *MN* = 9 cm and *LM* = 7 cm.

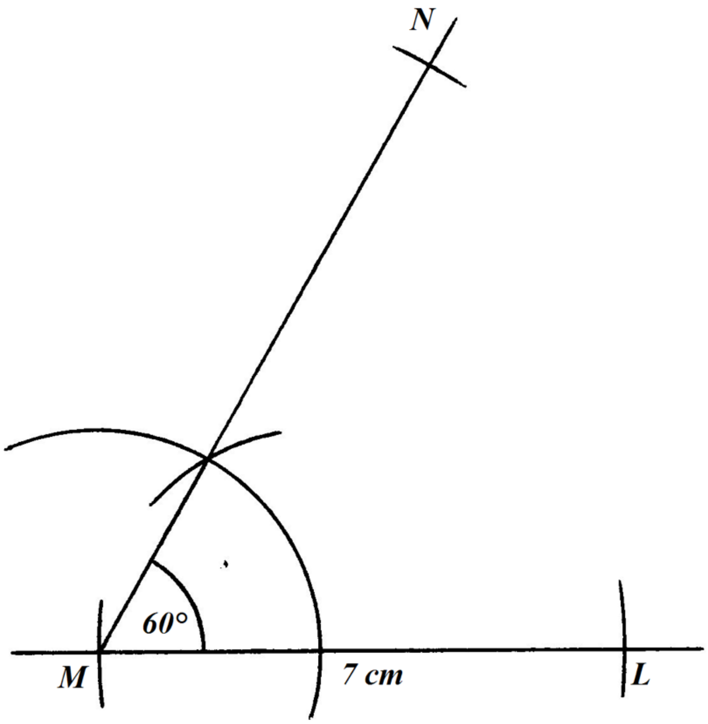
Solution:
(The construction is shown in stages to assist with candidates learning, though it is expected that it be done on one diagram.)

From a straight line longer than 7 cm, a length of 7 cm is cut off.

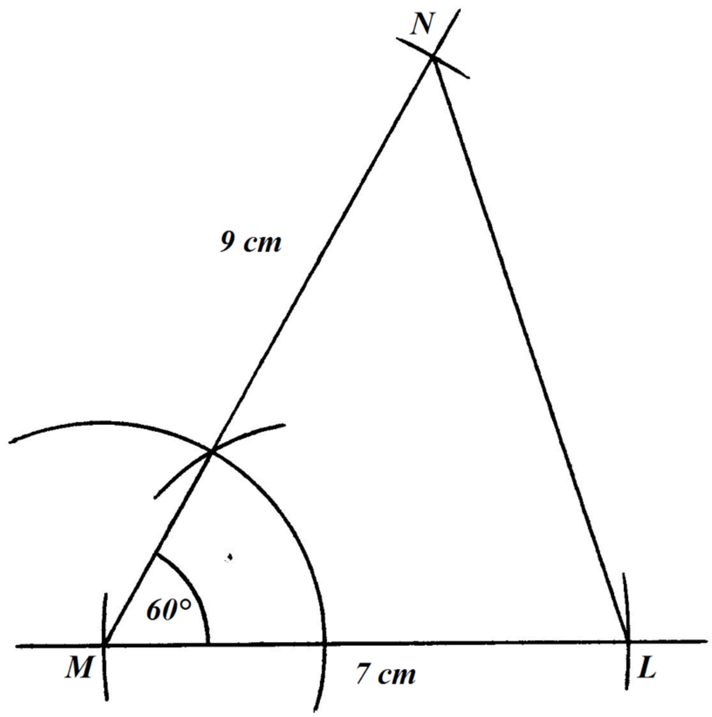


At *M*, we construct an angle of 60^0





We cut off *MN* = 9 cm to locate *N*. Join *N* to *L* to complete the triangle.



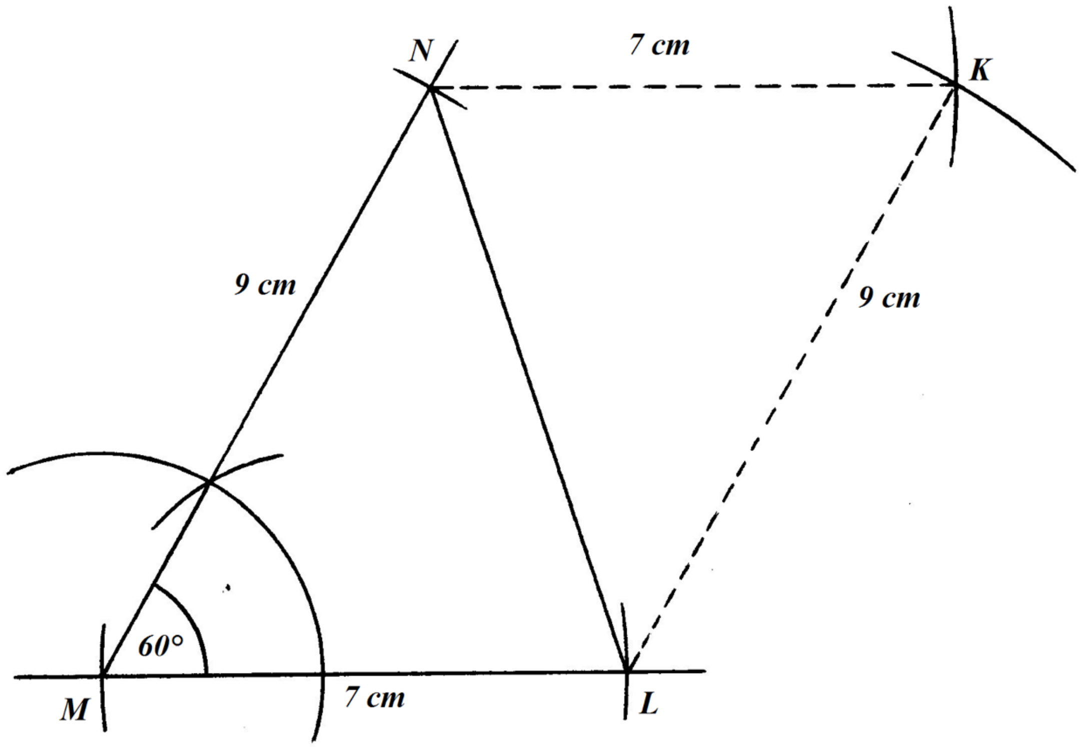
(ii) **Required to state:** The size of $M\hat{N}L$.

Solution:
$M\hat{N}L = 48°$ (by measurement, using the protractor)

(iii) **Required to show:** The point *K*, such that *KLMN* is a parallelogram.

The opposite sides of a parallelogram are equal in length.
With center *L*, an arc of radius 9 cm is drawn.
With center *N*, an arc of radius 7 cm is drawn.
The arcs intersect at *K*.

Solution:



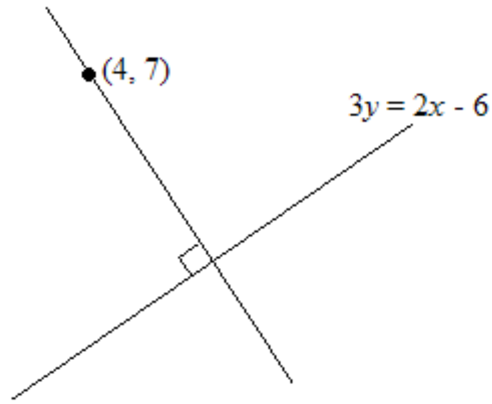
5. (a) **Data:** Equation of a straight line $3y = 2x - 6$

(i) **Required to find:** Gradient of the line.

Solution:
$3y = 2x - 6$
$\div 3$
$y = \frac{2}{3}x - 2$ is now reduced to the form of $y = mx + c$, where
$m = \frac{2}{3}$ is the gradient is read off.

(ii) **Required to find:** Equation of a perpendicular to the given line passing through $(4,\ 7)$.

Solution:



To find the equation of a line, we require the coordinates of (i)1 point on the line and (ii) the gradient of the line.

The gradient of any line perpendicular to the given line $= \dfrac{-1}{\frac{2}{3}}$

$= -\dfrac{3}{2}$

(The product of the gradients of perpendicular lines = -1)

$\therefore$ Equation of line perpendicular to the given line, passing through the point $(4,\ 7)$ is therefore

$\dfrac{y-7}{x-4} = -\dfrac{3}{2}$

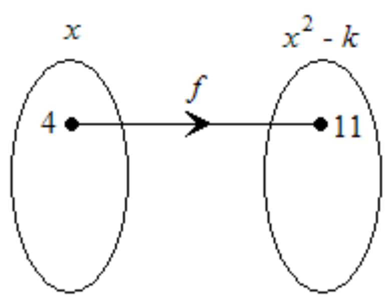
$2y - 14 = -3x + 12$

$2y = -3x + 26$ or expressed in any other equivalent form.

(b) **Data:** Diagram illustrating $f : x \to x^2 - k$ where $x \in \{3,\ 4,\ 5, ...10\}$.

(i) **Required to calculate:** The value of *k*.

Calculation:



$x=4$ is mapped onto 11 as shown on the mapping diagram.
Substituting we get,

$$\therefore (4)^2 - k = 11$$
$$16 - k = 11$$
$$k = 16 - 11$$
$$= 5$$

(ii) **Required to calculate:** $f(3)$

Calculation:
Substituting $x = 3$ in $f(x)$

$$\therefore f : x \rightarrow x^2 - 5$$
$$f(3) = (3)^2 - 5$$
$$= 9 - 5$$
$$= 4$$

(iii) **Required to calculate:** The value of x when $f(x) = 95$.

Calculation:

$$f(x) = 95$$
$$\therefore x^2 - 5 = 95$$
$$x^2 = 100$$
$$x = \sqrt{100}$$
$$= 10 \text{ or } -10$$

Since x is positive (data) we take $x = 10$ only.

6. **Data:** Graph showing the monthly sales at a school cafeteria.

(i) **Required to complete:** The table showing the sales for each month.

Solution:

Month	Jan	Feb	Mar	Apr	May
Sales in \$Thousands	38	35	27	15	10

(ii) **Required to find:** The two months in between which there was the greatest decrease in sales.

Solution:
The difference in sales between January to February = \$38 000 - \$35 000
= \$3 000
The difference in sales between February to March = \$35 000 - \$27 000
= \$8 000
The difference in sales between March to April = \$27 000 - \$15 000
= \$12 000
The difference in sales between April to May = \$15 000 - \$ 10 000
= \$5 000

$\therefore$ The greatest decrease in sales occurred between the months of March and April. **This may also be deduced from the graph by the 'branch' or line segment with the steepest gradient. This occurs between March and April.**

(iii) **Required to calculate:** The mean monthly sales.

Calculation:
The mean monthly sales from Jan to May

$$= \frac{\text{Total sales from Jan to May}}{\text{Total number of months from Jan to May}}$$

$$= \frac{\$(38\,000 + 35\,000 + 27\,000 + 15\,000 + 10\,000)}{5}$$

$$= \frac{\$125\,000}{5}$$

$$= \$25\,000$$

(iv) **Data:** The total sales from January to June was \$150 000.

Required to calculate: Sales for the month of June.

Calculation:
Sales in June = Total sales from Jan to June – Total sales from Jan to May

$$= \$150\,000 - \$125\,000$$

$$= \$25\,000$$

(v) **Required To Compare:** The sales in June with the sales in the five previous months.

Solution:
The sales in the month of June was the same as the average or mean sales for the first five months.
It showed a significant increase from the last two months, but still fell below that of the first three months.

It is also the only month that showed an increase in sales since January.

7. **Data:** Diagram showing the image $R'S'T'$ of RST under a transformation.

(i) **Required to state:** The coordinates of R and R'.

Solution:
Coordinates of $R = (2,\ 4)$ (a read-off from the diagram)

Coordinates of $R' = (2,\ 0)$ (a read-off from the diagram)

(ii) **Required to describe:** The transformation which maps triangle RST onto triangle $R'S'T'$.

Solution:
Reason:
- By observing the vertices of both triangles we would realise that they are the same perpendicular distance on opposite sides of the line $y = 2$.
- Both figures are congruent and the image is laterally inverted.

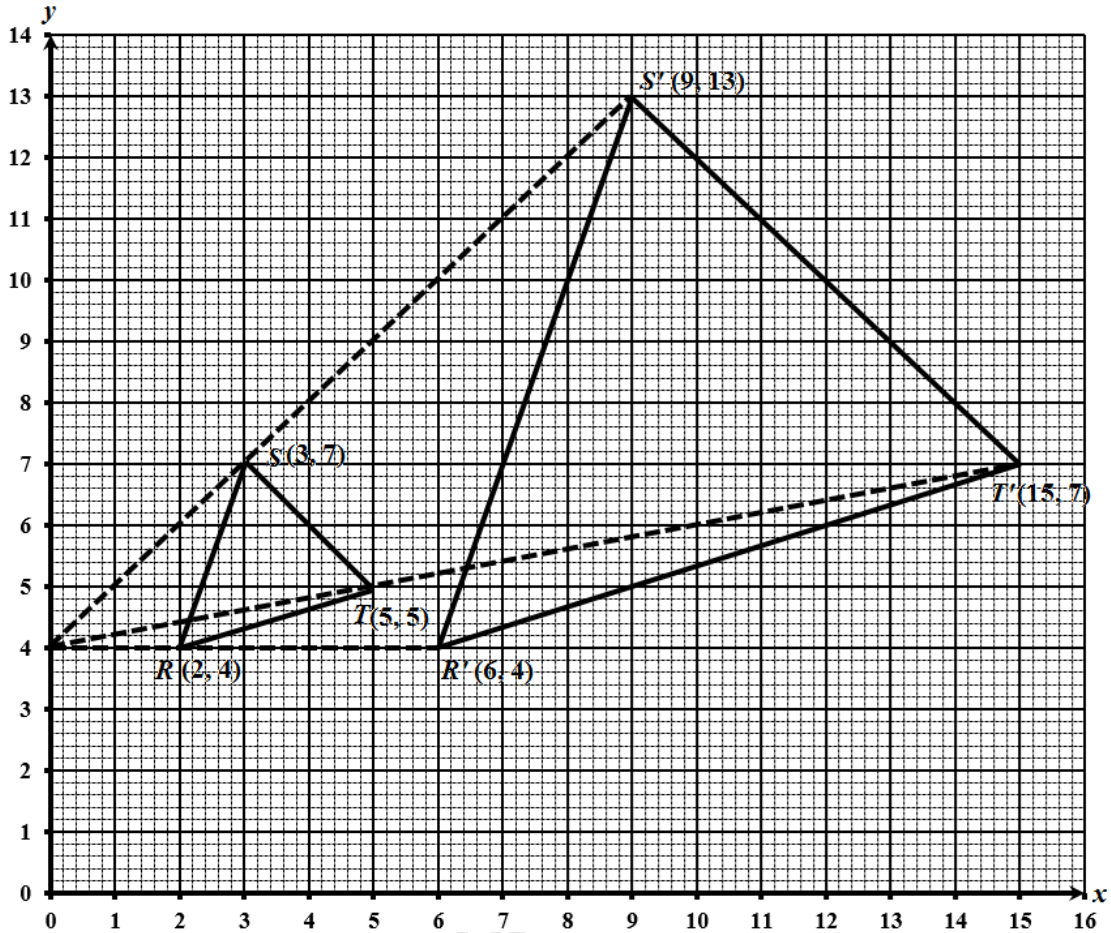
Hence, triangle RST is mapped onto triangle $R'S'T'$ by a reflection in the horizontal line $y = 2$.

(iii) **Data:** RST undergoes and enlargement of scale factor 3 and center $(0,\ 4)$.

a) **Required to draw:** $R''S''T''$

Solution:
Let center be C, $(0,\ 4)$.

$$CR'' = 3 \times CR \quad\quad CS'' = 3 \times CS \quad\quad CT'' = 3 \times CT$$
$$= (3 \times 2,\ 3 \times 0 + 4) \quad\quad = (3 \times 3,\ 3(7 - 4) + 4) \quad\quad = (3 \times 5,\ 3(1) + 4)$$
$$R'' = (6,\ 4) \quad\quad S'' = (9,\ 13) \quad\quad T'' = (15,\ 7)$$



OR

We may translate the vertices of triangle *RST* and most importantly the center of enlargement under the translation vector $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$, so as to make the center of rotation *O*.

$$R = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \qquad S = \begin{pmatrix} 3 \\ 7 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \qquad T = \begin{pmatrix} 5 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

Because the centre of rotation is now *O* and the scale factor is 3, we can multiply *R*, *S* and *T*, by the matrix for enlargement, centre *O* and scale factor 3, and which is $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$, to obtain the vertices of the enlarged triangle.

$$R\text{ becomes} = \begin{pmatrix} 3 & 0 \\ 3 & 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

$$S\text{ becomes} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \times \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 9 \end{pmatrix}$$

$$T\text{ becomes} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \times \begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 15 \\ 3 \end{pmatrix}$$

Finally, since we initially translated triangle *RST* under the translation $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$, we need to reverse the process and translate these vertices, using the negative of that translation vector and which is now $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$, to get triangle $R''S''T''$.

And so,

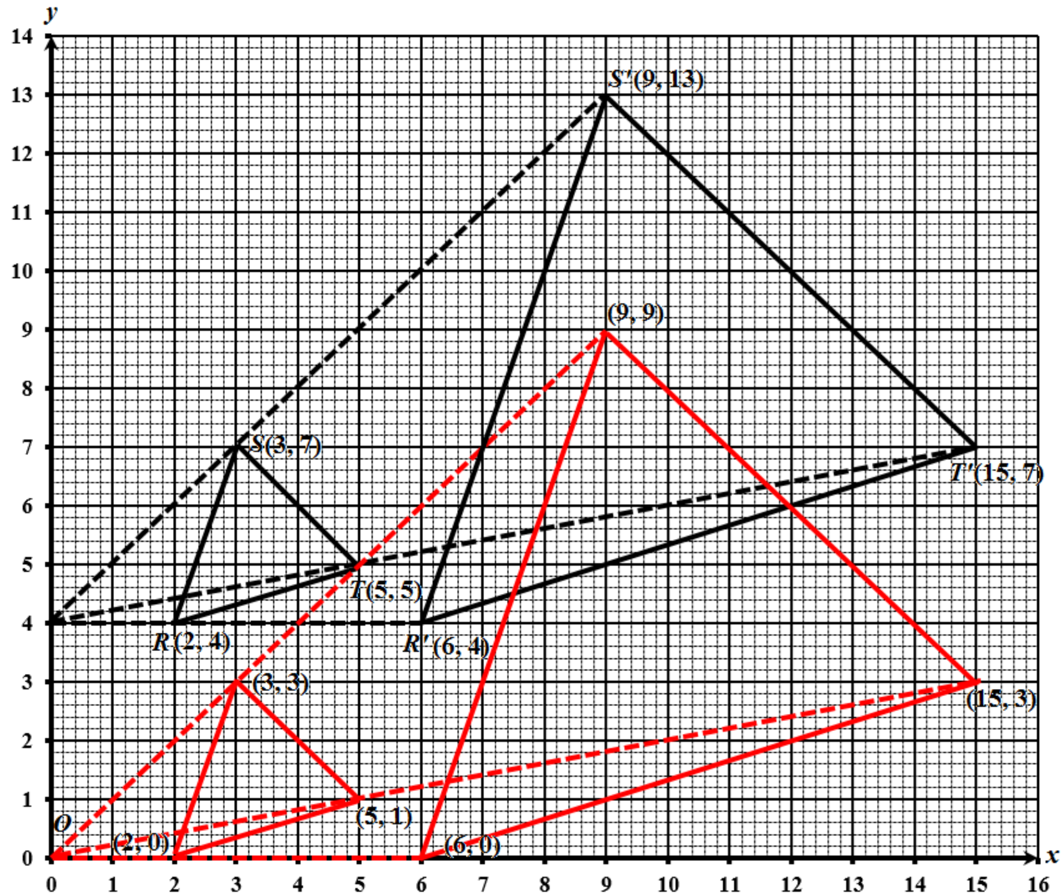
$$R'' = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

$$S'' = \begin{pmatrix} 9 \\ 9 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 9 \\ 13 \end{pmatrix}$$

$$T'' = \begin{pmatrix} 15 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 15 \\ 7 \end{pmatrix}$$

The following diagram illustrates the process.

The initial transformation (enlargement with center O and scale factor 3 is shown in red. The final image, after the opposite translation, is shown in black.



Required to calculate: The area of triangle $R''S''T''$.

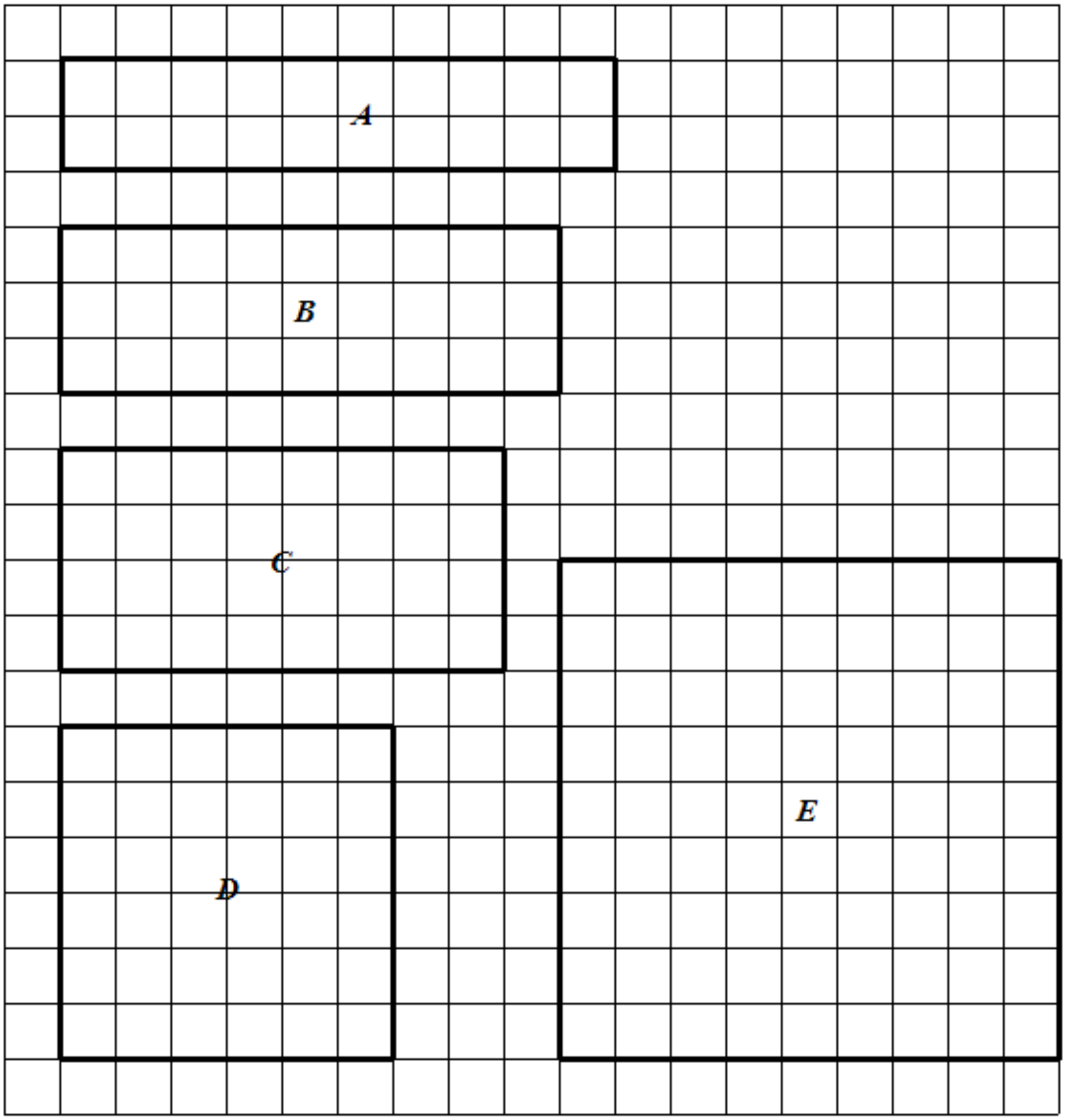
Calculation:
Area of triangle $RST = 4$ square units.
Under an enlargement, the image increases by the square of the scale factor.
Hence, the area of triangle $R''S''T'' = (3)^2 \times 4 \;\; = 36$ square units

b) **Required to state:** Two geometrical relationships between triangles RST and $R''S''T''$.

Solution:

- Triangle RST is similar to triangle $R''S''T''$, that is $\hat{R} = \hat{R}''$, $\hat{S} = \hat{S}''$ and $\hat{T} = \hat{T}''$. The image is an enlargement of the object.
- The ratios of the corresponding sides of image and object are all the same. For example, $\frac{R''S''}{RS} = \frac{S''T''}{ST} = \frac{R''T''}{RT} = 3$ which is the scale factor.

Answer Sheet for Question 8



Rectangle	Length	Width	Area (square units)	Perimeter (units)
A	10	2	20	24
B	9	3	27	24
C	8	4	32	24
D	6	6	36	24
E	9	9	81	36

8. (a) (i) a) **Required to draw:** Rectangle *B*

Solution:
Area = 27 square units
Perimeter = 24 units

Let $lw = 27$ …(1)
and $2l + 2w = 24$ …(2)
Solving simultaneously to find *l* and *w*
Equation (2) $\div 2$
$l + w = 12$
$w = 12 - l$

Substitute this expression in equation (1)
$l(12 - l) = 27$
$l^2 - 12l + 27 = 0$
$(l - 9)(l - 3) = 0$
$\therefore l = 9$ or $l = 3$

Hence, the value of $l = 9$ and $w = 3$ for the rectangle *B*

b) **Required to draw:** Rectangle *C*.

Solution:
So too,
$lw = 32$ …(1)
$2l + 2w = 24$ …(2)

And,
$l^2 - 12l + 32 = 0$
$(l - 4)(l - 8) = 0$
$\therefore l = 4$ or $l = 8$

Hence, $l = 8$ and $w = 4$ for the rectangle *C*

(ii) **Required to complete:** The table given.

Solution:
Answers are given above.

(b) **Required to find:** The length and width of rectangle *D*.

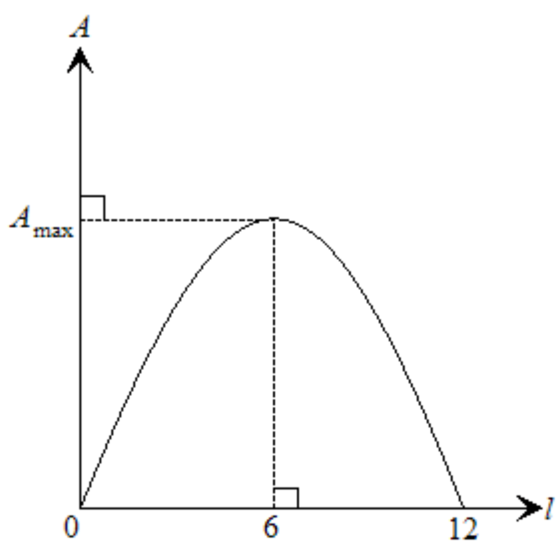
Solution:
Let P = perimeter and let A = area of the rectangle
$P = 2l + 2w$ and $A = lw$

$$P = 24$$
$$\therefore 2l + 2w = 24$$
$$l + w = 12$$
$$w = 12 - l$$

Therefore,
$$A = l(12 - l)$$
$$= 12l - l^2$$

When $A = 0$, $l = 12$ or 0.
In the expression of $A = 12l - l^2$. Note the coefficient of l^2 is negative.

Therefore, the graph of A vs l is a quadratic, has a maximum point, cuts the horizontal axis at 0 and 12 and is a more precisely described as a parabola, as expected for a quadratic.



The axis of symmetry occurs at:
$$l = \frac{-(-12)}{2(1)}$$
$$= 6$$
The maximum point occurs
When $l = 6$,
$$w = 12 - 6$$
$$= 6$$

$\therefore A_{\max}$ occurs at $w = 6$ and $l = 6$, that is, the rectangle is a square and
$$A_{\max} = 6 \times 6$$
$$= 36 \text{ square units}$$

Hence, for the rectangle D, $l = 6$, $w = 6$ and $A = 36$.

(c) **Required to find:** The length and width of rectangle *E*.

Solution:
So too, for *E* to have a maximum area, *E* must be a square.
So, when the perimeter = 36

$$\text{The length } = \frac{36}{4}$$
$$= 9 \text{ cm}$$

Therefore, for the rectangle *E*, $w = 9$ and $l = 9$.

$$\text{And the maximum area } = 9 \times 9$$
$$= 81 \text{ square units}$$

Section II

9. (a) **Data:** $f(x)=\dfrac{2x-7}{x}$ and $g(x)=\sqrt{x+3}$.

(i) **Required to calculate:** $f(5)$

Calculation:
We substitute $x = 5$ in $f(x)$ to get

$$f(5)=\frac{2(5)-7}{5}$$
$$=\frac{10-7}{5}$$
$$=\frac{3}{5}$$

(ii) a) **Required to find:** $f^{-1}(x)$

Solution:

$$f(x)=\frac{2x-7}{x}$$

Let

$$y=\frac{2x-7}{x}$$
$$xy=2x-7$$
$$xy-2x=-7$$
$$x(y-2)=-7$$
$$x=\frac{-7}{y-2}$$
$$=\frac{7}{2-y}$$

Now, replace y by x to obtain

$$\therefore f^{-1}(x)=\frac{7}{2-x},\ x\neq 2$$

b) **Required to find:** $gf(x)$

Solution:
Replace x in $f(x)$ by $g(x)$ to obtain

$$gf(x)=\sqrt{\frac{2x-7}{x}+3}$$
$$=\sqrt{\frac{2x-7}{x}+\frac{3x}{x}}$$
$$=\sqrt{\frac{2x-7+3x}{x}}$$
$$=\sqrt{\frac{5x-7}{x}}$$
$$=\sqrt{5-\frac{7}{x}} \quad ,x\neq 0$$

(b) (i) **Required to Express:** $1-6x-x^2$ in the form $k-a(x+h)^2$.

Solution:
$1-6x-x^2 \equiv k-a(x+h)^2$

Expanding the right hand side and then equate to the given quadratic equation.

$$k-a(x+h)^2 = k-a(x^2+2hx+h^2)$$
$$=-ax^2-2ahx+k-ah^2$$
$$=k-ah^2-2ahx-ax^2$$

Equating corresponding coefficients
For x^2
$-a=-1$
$\therefore a=1$

For x
$-2(1)h=-6$
$\therefore h=3$

Now, for the constant term
$1=k-1(3)^2$
$1=k-9$
$k=10$

OR

$1-6x-x^2 = 1-(x^2+6x),\ 1/2\, coeff.of\ x\ is\ 3$

$= ?-(x+3)^2$

$= ?-(x^2+6x+9)$

$= -x^2-6x-\ \ 9$

$+10$

1

$\therefore ? = 10$

Therefore, $1-6x-x^2 \equiv 10-(x+3)^2$ is of the form $k-a(x+h)^2$, where $k=10,\ a=1$ and $h=3$.

(ii) a) **Required to state:** The maximum value of $1-6x-x^2$.

Solution:
Recall
$1-6x-x^2 \equiv 10-(x+3)^2,$
$\geq 0, \forall x$

$\therefore$ The maximum value of $1-6x-x^2 = 10-0$
$= 10$

(This occurs at $(x+3)^2 = 0$ and when $x=-3$).

b) **Required to find:** The equation of the axis of symmetry.

Solution:
Let
$y = 1-6x-x^2$
$y = -x^2-6x+1$
is of the form ax^2+bx+c, where $a=-1,\ b=-6$ and $c=1$.

The equation of the axis of symmetry is $x = \dfrac{-b}{2a}$

$= \dfrac{-(-6)}{2(-1)}$

$= \dfrac{6}{-2}$

$= -3$

That is at the vertical with equation, $x=-3$.

(iii) **Required to find:** The roots of $1-6x-x^2$

Solution:

$1-6x-x^2=0$ or $-x^2-6x+1=0$ is of the form ax^2+bx+c, where $a=-1,\ b=-6$ and $c=1$.

Using the quadratic equation formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-6)\pm\sqrt{(-6)^2-4(-1)(1)}}{2(-1)}$$

$$=\frac{6\pm\sqrt{36+4}}{-2}$$

$$=\frac{6\pm\sqrt{40}}{-2}$$

$$=\frac{6+\sqrt{40}}{-2}\text{ or }\frac{6-\sqrt{40}}{-2}$$

$$=-6.16\underline{\underline{2}}\text{ or }0.16\underline{\underline{2}}$$

$$=-6.16\text{ or }0.16\text{ to 2 decimal places}$$

OR

$$1-6x-x^2=0$$

$$\therefore 10-(x+3)^2=0$$

$$\therefore (x+3)^2=10$$

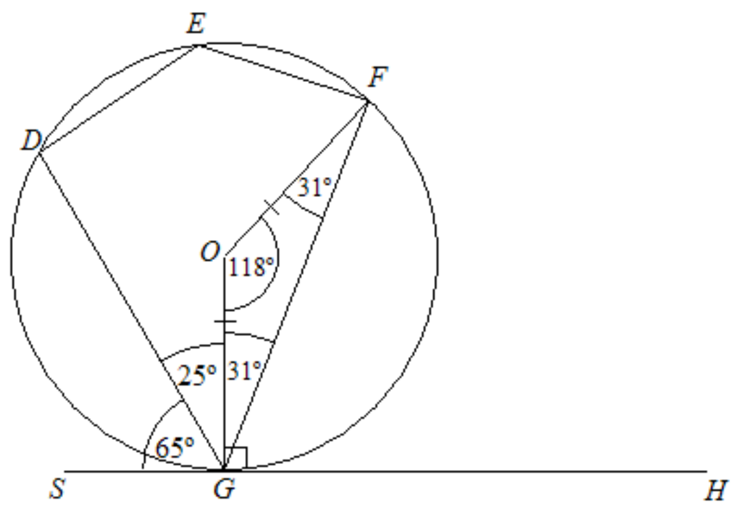
$$x+3=\pm\sqrt{10}$$

$$\therefore x=-3\pm\sqrt{10}$$

$$=-6.16\underline{\underline{2}}\text{ or }0.16\underline{\underline{2}}$$

$$=-6.16\text{ or }0.16\text{ to 2 decimal places}$$

10. (a) **Data:** Diagram as shown below.



(i) **Required to calculate:** $O\hat{G}F$

Calculation:
$OG = OF$ (radii of the same circle)

(The base angles of an isosceles triangle are equal)

$$O\hat{G}F = O\hat{F}G$$
$$= \frac{180° - 118°}{2}$$
$$= \frac{62°}{2}$$
$$= 31°$$

(The sum of the angles in a triangle =180º)

(ii) **Required to calculate:** $D\hat{E}F$
Calculation:
$O\hat{G}S = 90°$
(The angle made by the tangent *GS* to a circle and a radius, *OG*, at the point of contact = 90º)

$$\therefore O\hat{G}D = 90° - 65°$$
$$= 25°$$

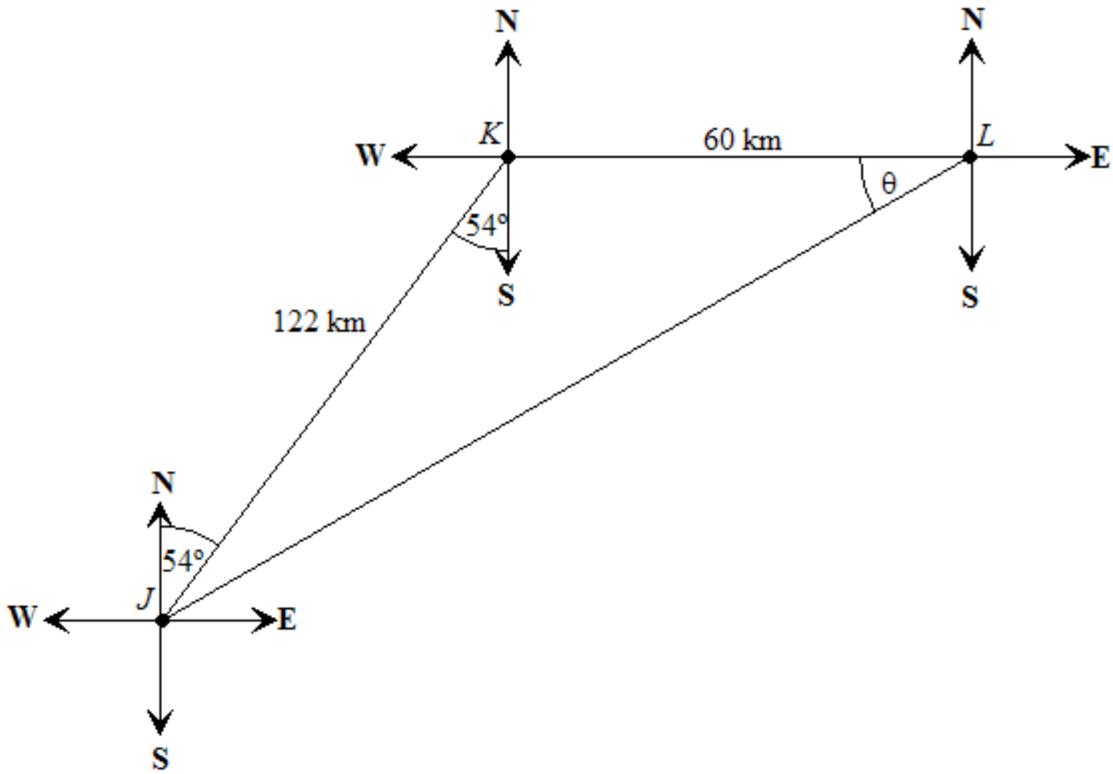
$$D\hat{G}F = 25° + 31°$$
$$= 56°$$

$$D\hat{E}F = 180° - 56°$$
$$= 124°$$

(The opposite angles of a cyclic quadrilateral, *DEFG*, are supplementary).

(b) (i) **Required to draw:** A diagram showing the information given.

Solution:



(ii) a) **Required to calculate:** $J\hat{K}L$

Calculation:

$J\hat{K}L = 90° + 54°$

$= 144°$

b) **Required to calculate:** JL

Calculation:

In triangle JKL we have two sides and the included angle. So, we can apply the cosine law to get

$JL^2 = (122)^2 + (60)^2 - 2(122)(60)\cos 144°$ (cos*ine law*)

$= 30\,328.0088$

$JL = 174.149$

$= 174.15$ km to 2 decimal places

c) **Required to calculate:** The bearing of J from L.

Calculation:

Let $K\hat{L}J$ be θ.

Applying the sine rule to triangle KLJ

$$\frac{122}{\sin\theta}=\frac{174.149}{\sin 144^\circ}$$

$$\therefore \sin\theta=\frac{122\times\sin 144^\circ}{174.149}$$

$$\therefore \theta=\sin^{-1}(0.4117)$$

$$\theta=24.31^\circ$$

The bearing of J from L $=270^\circ-24.31^\circ$

$=245.7^\circ$

$=245.7^\circ$ to the nearest 0.1°

11. (a) **Data:** Transformation matrix, $M=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $V\to V'$, $W\to W'$ under M.

(i) **Required To calculate:** The values of a, b, c and d.

Calculation:

$$M \times V = V'$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix}=\begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 3a+5b \\ 3c+5d \end{pmatrix}=\begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Both are 2×1 matrices and are equal

Equating corresponding entries to get

$$3a+5b=5 \quad \ldots(1)$$
$$3c+5d=-3 \quad \ldots(2)$$

So too,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 7 \\ 2 \end{pmatrix}=\begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 7a+2b \\ 7c+2d \end{pmatrix}=\begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

Equating corresponding entries

$7a+2b=2 \quad \ldots(3)$
$7c+2d=-7 \quad \ldots(4)$
Consider equations (1) and (3) and solving simultneously
$3a+5b=5 \quad \ldots(1)$
$7a+2b=2 \quad \ldots(3)$

Equation (1) $\times 7$
$21a+35b=35 \quad \ldots(5)$

Equation (3) $\times -3$
$-21a-6b=-6 \quad \ldots(6)$

Equation (5) + Equation (6)

$$\begin{array}{r} 21a+35b=35 \\ -21a-6b=-6 \\ \hline 29b=29 \\ \hline \end{array}$$

$\therefore b=1$

Substitute $b=1$ into equation (1)

$$3a+5(1)=5$$
$$3a=0$$
$$\therefore a=0$$

Consider equations (2) and (4) and solving simultneously
$3c+5d=-3 \quad \ldots(2)$
$7c+2d=-7 \quad \ldots(4)$

Equation (2) $\times 7$
$21c+35d=-21 \quad \ldots(7)$

Equation (4) $\times -3$
$-21c-6d=21 \quad \ldots(8)$
Equation (7) + Equation (8)

$$\begin{array}{r} 21c+35d=-21 \\ -21c-6d=21 \\ \hline 29d=0 \\ \hline \end{array}$$

$\therefore d=0$

Substitute $d=0$ into equation (2)

$$3c+5(0)=-3$$
$$3c=-3$$
$$\therefore c=-1$$

Therefore, $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(ii) **Required to state:** The coordinates of *Z*.

Solution:

$Z \xrightarrow{M} Z'$ (data)

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0x + 1y \\ -x + 0y \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} y \\ -x \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

Both are 2 x 1 matrices and are equal. So, equating corresponding entries we obtain

$y = 5$ and $-x = 1$

Hence, $x = -1$

$\therefore x = -1$ and $y = 5$ and so, $Z(x,\ y) = (-1,\ 5)$.

(iii) **Required to describe:** The transformation *M*.

Solution:

The transformation matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ represents a rotation of 90º clockwise about the origin *O*.

(b) **Data:** $P = (2,\ 7)$ and the vector $\overrightarrow{PR} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

(i) a) **Required to find:** $\overrightarrow{OP}$

Solution:

Since $P = (2,\ 7)$, then $\overrightarrow{OP} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ is of the form $\begin{pmatrix} a \\ b \end{pmatrix}$, where $a = 2$ and $b = 7$.

b) **Required to find:** $\overrightarrow{OR}$

Solution:

Let $R = (x, y)$

$\therefore \overrightarrow{OR} = \begin{pmatrix} x \\ y \end{pmatrix}$

Applying the vector triangle law

$$PR = PO + OR$$

$$\therefore \begin{pmatrix} 4 \\ -3 \end{pmatrix} = -\begin{pmatrix} 2 \\ 7 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} x-2 \\ y-7 \end{pmatrix}$$

Equating components

$x - 2 = 4$

$\therefore x = 6$

$y - 7 = -3$

$\therefore y = 4$

$\therefore \overrightarrow{OR} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$ is of the form $\begin{pmatrix} a \\ b \end{pmatrix}$, where $a = 6$ and $b = 4$.

(ii) **Data:** $S = (14, -2)$

a) **Required to find:** $\overrightarrow{RS}$

Solution:

$$S = (14, -2)$$

$$\therefore \overrightarrow{OS} = \begin{pmatrix} 14 \\ -2 \end{pmatrix}$$

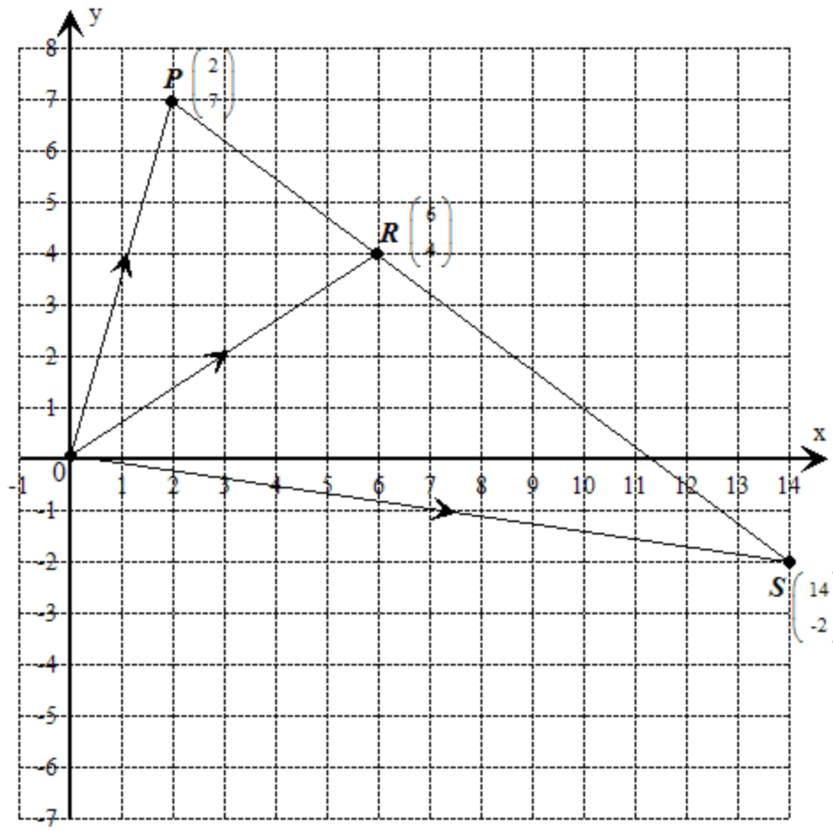
$$\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS}$$

$$= -\begin{pmatrix} 6 \\ 4 \end{pmatrix} + \begin{pmatrix} 14 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ -6 \end{pmatrix}$$

b) **Required to show:** P, R and S are collinear.

Solution:



$\overrightarrow{PR} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$\overrightarrow{RS} = \begin{pmatrix} 8 \\ -6 \end{pmatrix} = 2\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ (which is a scalar multiple of $\overrightarrow{PR}$, the scalar multiple being 2)

Therefore $\overrightarrow{RS}$ is parallel to $\overrightarrow{PR}$. *R* is a common point. So *P* and *R* must lie on the line with *PS*. Therefore, *P*, *R* and *S* are collinear.