

JUNE 2010 CXC MATHEMATICS PAPER 2

Section I

1. a. (i) **Required to calculate:** $1\frac{1}{2} - \frac{2}{5}$
 $4\frac{2}{5} \times \frac{3}{4}$

Calculation:

Working with the numerator to get

$$1\frac{1}{2} - \frac{2}{5}$$

$$= 1\frac{5-4}{10}$$

$$= 1\frac{1}{10} \text{ or } \frac{11}{10}$$

Now, working with the denominator to get

$$4\frac{2}{5} \times \frac{3}{4} = \frac{22}{5} \times \frac{3}{4}$$

$$= \frac{33}{10}$$

Hence, the calculation reduces to

$$\frac{1\frac{1}{2} - \frac{2}{5}}{4\frac{2}{5} \times \frac{3}{4}} = \frac{\frac{11}{10}}{\frac{33}{10}}$$

$$= \frac{11}{10} \times \frac{10}{33}$$

$$= \frac{1}{3} \text{ (exact value)}$$

(ii) **Required to calculate:** $2.5^2 - \frac{2.89}{17}$

Calculation:

The arithmetic is not too difficult but to save valuable time we use the calculator which is allowable.

$$2.5^2 - \frac{2.89}{17} = 6.25 - 0.17$$

$$= 6.08$$

$$= 6.1 \text{ to 2 significant figures}$$

b. **Data:** 150 T-shirts cost \$1 920

(i) **Required to calculate:** The cost of 1 T-shirt

Calculation:

$$\begin{aligned}\text{The cost of 1 T-shirt} &= \$ \frac{1920}{150} \\ &= \$12.80\end{aligned}$$

(ii) **Required to Calculate:** The amount for 150 T-shirts @ \$ 19.99 each

Calculation:

$$\begin{aligned}\text{Cost of 150 T-shirts @ \$ 19.99 each} &= 150 \times \$19.99 \\ &= \$2998.50\end{aligned}$$

(iii) **Required To calculate:** The profit

Calculation:

The sales price exceeds the cost price so there is a profit made.

$$\begin{aligned}\text{The profit} &= \text{Total received from sales} - \text{Total cost price} \\ &= \$2998.50 - \$1920.00 \\ &= \$1078.50\end{aligned}$$

(iv) **Required to calculate:** The percentage profit

Calculation:

$$\begin{aligned}\text{The profit, as a percentage of the cost price} &= \frac{1078.50}{1920.00} \times 100\% \\ &= 56.2\% \\ &= 56\% \text{ (to the nearest whole no.)}\end{aligned}$$

2. a. **Data:** $a = -1$, $b = 2$ and $c = -3$

(i) **Required to calculate:** $a + b + c$

Calculation:

We substitute for the value of a, b and c from the data to get

$$\begin{aligned}a + b + c &= (-1) + 2 + (-3) \\ &= -1 + 2 - 3 \\ &= 2 - 4 \\ &= -2\end{aligned}$$

(ii) **Required to calculate:** $b^2 - c^2$

Calculation:

We substitute for b and c to get

$$\begin{aligned}b^2 - c^2 &= (2)^2 - (-3)^2 \\ &= 4 - 9 \\ &= -5\end{aligned}$$

- b. (i) **Required to express:** Statement given as an algebraic expression.

Solution:

Seven times the sum of x and y .

The sum of x and $y = x + y$

$$\begin{aligned} \text{Hence, seven times the sum of } x \text{ and } y &= 7 \times (x + y) \\ &= 7(x + y) \end{aligned}$$

- (ii) **Required to express:** Statement given as an algebraic expression.

Solution:

The product of 2 consecutive (whole) numbers when the smaller is y .

If the smaller number is y , then the next or larger number is $y + 1$.

$$\begin{aligned} \text{Hence, the product} &= y \times (y + 1) \\ &= y(y + 1) \end{aligned}$$

- c. **Data:** $2x + y = 7$ and $x - 2y = 1$

Required to calculate: x and y

Calculation:

Let

$$2x + y = 7 \quad \dots(1)$$

$$x - 2y = 1 \quad \dots(2)$$

Using the method of substitution

From (1) we obtain y in terms of x

$$y = 7 - 2x$$

Substituting this expression into equation (2) to obtain an equation in x only

$$x - 2(7 - 2x) = 1$$

$$x - 14 + 4x = 1$$

$$5x = 15$$

$$x = 3$$

Substituting $x = 3$

$$\begin{aligned} y &= 7 - 2(3) \\ &= 1 \end{aligned}$$

$$\therefore x = 3 \text{ and } y = 1$$

OR

Using the method of elimination

Let

$$2x + y = 7 \quad \dots(1)$$

$$x - 2y = 1 \quad \dots(2)$$

Equation (1) \times 2

$$4x + 2y = 14 \quad \dots(3)$$

Equation (2) + equation (3)

$$\text{Gives } 4x + 2y + x - 2y = 14 + 1$$

This eliminates y and reduces to $5x = 15$ and $x = 3$

Substituting $x = 3$ into (1)

$$2(3) + y = 7$$

$$y = 7 - 6$$

$$y = 1$$

$$\therefore x = 3 \text{ and } y = 1$$

OR

Using the matrix method

$$2x + y = 7 \quad \dots(1)$$

$$x - 2y = 1 \quad \dots(2)$$

Using the coefficients of x and of y in the given equations we get

$$\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \end{pmatrix} \quad \text{Matrix equation}$$

Let $A = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$, the 2×2 matrix from the matrix equation.

We wish to find the inverse of A and multiply the matrix equation by the inverse of A , denoted by A^{-1}

$$\begin{aligned} \det A &= 2(-2) - 1(1) \\ &= -4 - 1 \\ &= -5 \end{aligned}$$

$$A^{-1} = -\frac{1}{5} \begin{pmatrix} -2 & -1 \\ -1 & 2 \end{pmatrix}$$

$$-\frac{1}{5} \begin{pmatrix} -2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{5} \begin{pmatrix} -2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 7 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{5} \begin{pmatrix} -15 \\ -5 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

Both sides are now 2×1 matrices and are equal

So, equating corresponding entries we obtain

$$\therefore x = 3 \text{ and } y = 1$$

d. **Required to factorise:** $4y^2 - z^2$, $2ax - 2ay - bx + by$ and $3x^2 + 10x - 8$

Solution:

$$(i) \quad 4y^2 - z^2 = (2y)^2 - (z)^2$$

This is re-written in the form of a difference of two squares and which is a standard form for factorising

$$\text{And so } 4y^2 - z^2 = (2y - z)(2y + z)$$

- (ii) The four terms are grouped into two groups of two and factoring out the common term of each group

$$2ax - 2ay - bx + by = 2a(x - y) - b(x - y)$$

Further factoring will give

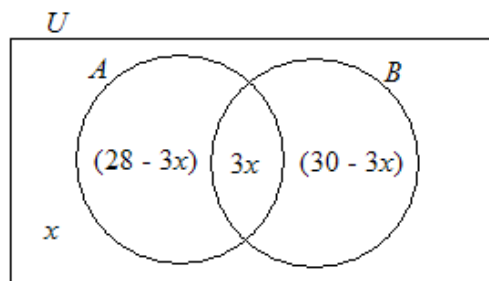
$$= (x - y)(2a - b)$$

(iii) $3x^2 + 10x - 8 = (3x - 2)(x + 4)$

3. a. **Data:** Survey on 40 tourists regarding who visited Antigua and/or Barbados.

- (i) **Required to complete:** Venn diagram given to represent the information given.

Solution:



- (ii) **Required to find:** An expression for the total number of tourists.

Solution:

The total number of tourists is the sum of the numbers of each of the subsets of the Universal set.

$$= (28 - 3x) + 3x + (30 - 3x) + x$$

$$= 28 - 3x + 3x + 30 - 3x + x$$

$$= 58 - 2x$$

- (iii) **Required to calculate:** x

Calculation:

$$58 - 2x = 40 \text{ (data)}$$

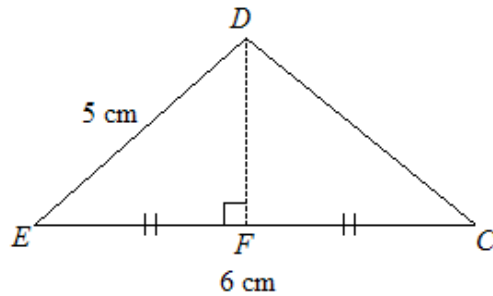
$$2x = 18$$

$$x = 9$$

- b. **Data:** Diagram of a prism.

- (i) **Required to calculate:** The length of EF

Calculation:



F is the midpoint of EC (data)

$$\begin{aligned}\therefore EF &= \frac{6}{2} \\ &= 3 \text{ cm}\end{aligned}$$

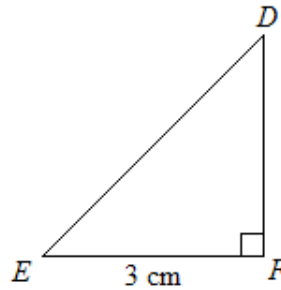
(ii) **Required to calculate:** Length of DF .

Calculation:

The straight line drawn from D to the midpoint of EC is perpendicular to EC . $\hat{D}EF$ is a right angle.

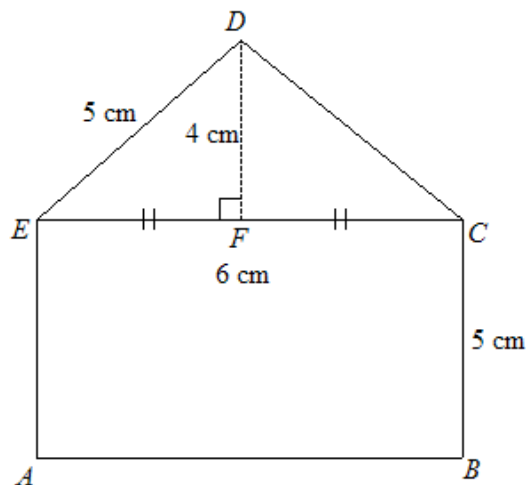
Applying Pythagoras' theorem to triangle DEF ,

$$\begin{aligned}EF^2 + DF^2 &= DE^2 \\ \therefore DF &= \sqrt{DE^2 - EF^2} \\ &= \sqrt{(5)^2 - (3)^2} \\ &= 4 \text{ cm}\end{aligned}$$



(iii) **Required to calculate:** Area of the face $ABCDE$.

Calculation:



The area of $\triangle DEC = \frac{6 \times 4}{2} = 12 \text{ cm}^2$

The area of rectangle $ABCE = 6 \times 5 = 30 \text{ cm}^2$

Hence, the area of the entire face $ABCDE = 12 + 30 = 42 \text{ cm}^2$

4. a. **Data:** $y = kx^2$ and $y = 50$ when $x = 10$.

(i) **Required to calculate:** k

Calculation:

$y = 50$ when $x = 10$ (data)

Substituting gives

$\therefore 50 = k(10)^2$

$$k = \frac{50}{100}$$

$$= \frac{1}{2}$$

(ii) **Required to calculate:** y when $x = 30$.

Calculation:

$$y = \frac{1}{2}x^2$$

When

$x = 30$

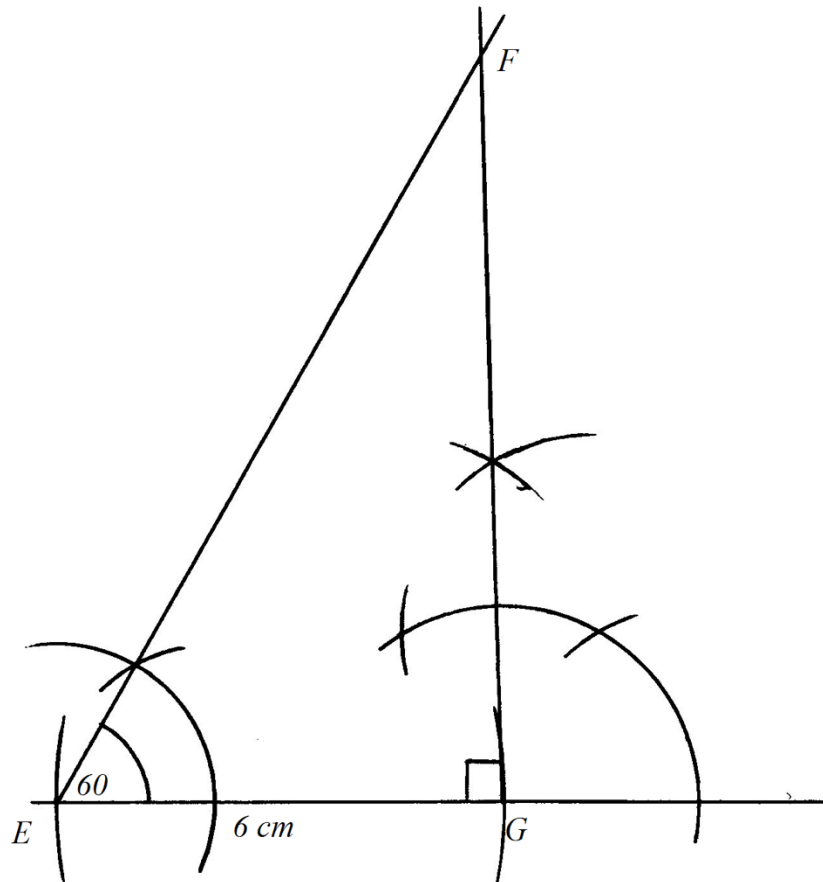
$$y = \frac{1}{2}(30)^2$$

$$= 450$$

b. (i) **Required to construct:** Triangle EFG , in which, $EG = 6 \text{ cm}$,

$\hat{FEG} = 60^\circ$ and $\hat{EGF} = 90^\circ$

Solution: We cut off $EG = 6 \text{ cm}$ from a line drawn longer than 6 cm
At E we construct an angle of 60° and at G we construct an angle of 90° . These construction lines are produced, if necessary, meet at F .



(ii) (a) **Required to find:** Length of EF .

Solution:

$EF = 12.0$ cm (by measurement using a ruler)

(b) **Required to find:** \hat{EFG}

Solution:

$\hat{EFG} = 30^\circ$ (by measurement using a protractor)

5. a. **Data:** $f(x) = 2x - 5$ and $g(x) = x^2 + 3$

(i) (a) **Required to calculate:** $f(4)$

Calculation:

Substitute $x = 4$ to get

$$f(4) = 2(4) - 5$$

$$= 8 - 5$$

$$= 3$$

(b) **Required to calculate:** $gf(4)$

Calculation:

Since $f(4) = 3$, from (i) (a) then

$$\begin{aligned} gf(4) &= g(3) \\ &= (3)^2 + 3 \\ &= 9 + 3 \\ &= 12 \end{aligned}$$

OR

We could have found the composite function $gf(x)$ and $gf(4)$ to give the same result.

(ii) **Required to calculate:** $f^{-1}(x)$

Calculation:

Let

$$y = 2x - 5$$

$$y + 5 = 2x$$

$$\frac{y + 5}{2} = x$$

Interchanging x and y to obtain

$$f^{-1}(x) = \frac{x + 5}{2}$$

b. **Data:** Diagram of the quadratic $x^2 + 2x - 3$ drawn on the $x - y$ plane.

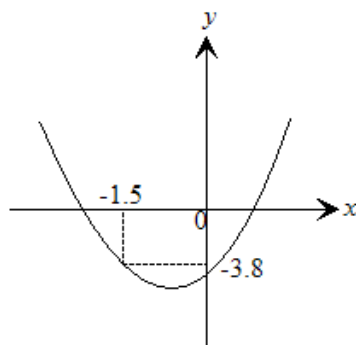
(i) **Required to find:** Scale used on the $x -$ axis.

Solution:

On the $x -$ axis, the scale used is $2 \text{ cm} \equiv 1 \text{ unit}$ or $1 \text{ cm} \equiv 0.5 \text{ unit}$

(ii) **Required to find:** y when $x = -1.5$

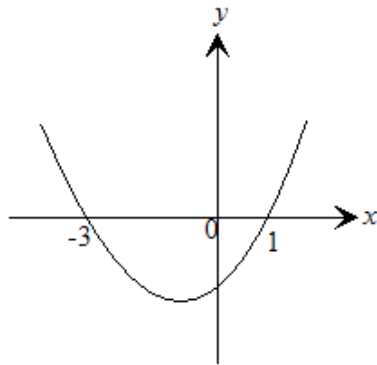
Solution:



When $x = -1.5$, $y = -3.8$ (obtained by a read off)

(iii) **Required to find:** x when $y = 0$

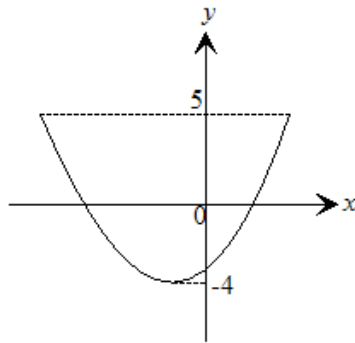
Solution:



When $y = 0$, $x = -3$ or 1 the points where the curve cuts the horizontal axis (read off)

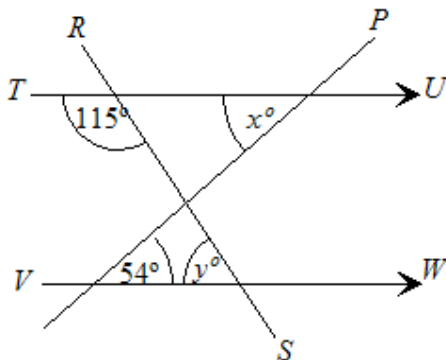
(iv) **Required to find:** The range of y for which $-4 \leq x \leq 2$.

Solution:



For $-4 \leq x \leq 2$, $-4 \leq y \leq 5$ is of the form $a \leq y \leq b$, where $a = -4 \in \mathfrak{R}$ and $b = 5 \in \mathfrak{R}$. This is illustrated on the diagram shown above.

6. a. **Data:** Diagram as shown below.



- (i) **Required to calculate:** x
Calculation:
 $x = 54^\circ$
 (Alternate angles to angle PVW)

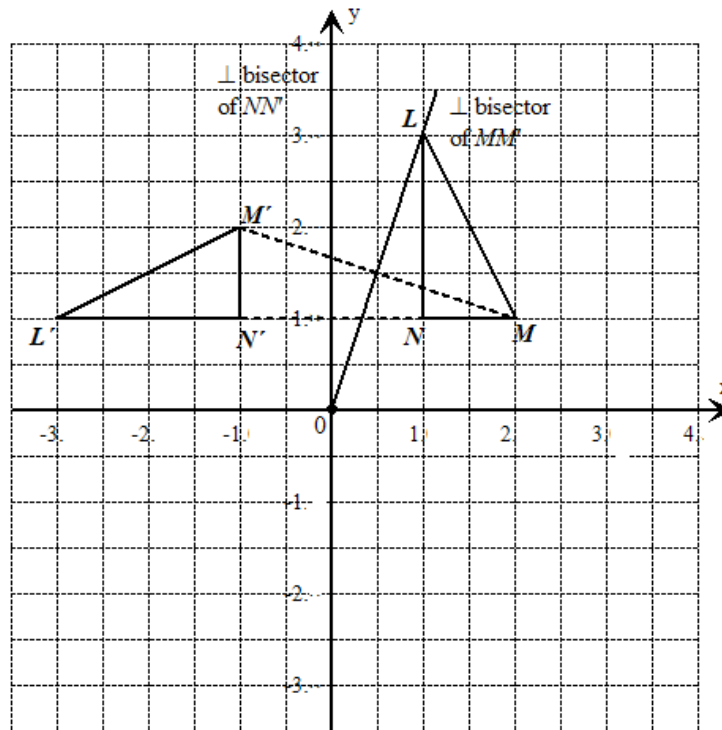
- (ii) **Required to calculate:** y
Calculation:
 $y + 115^\circ = 180^\circ$
 $\therefore y = 65^\circ$
 (When parallel lines are cut by a transversal, co-interior angles are supplementary).

b. **Data:** Diagram showing, $\triangle LMN$ and its image $\triangle L'M'N'$ after undergoing a rotation.

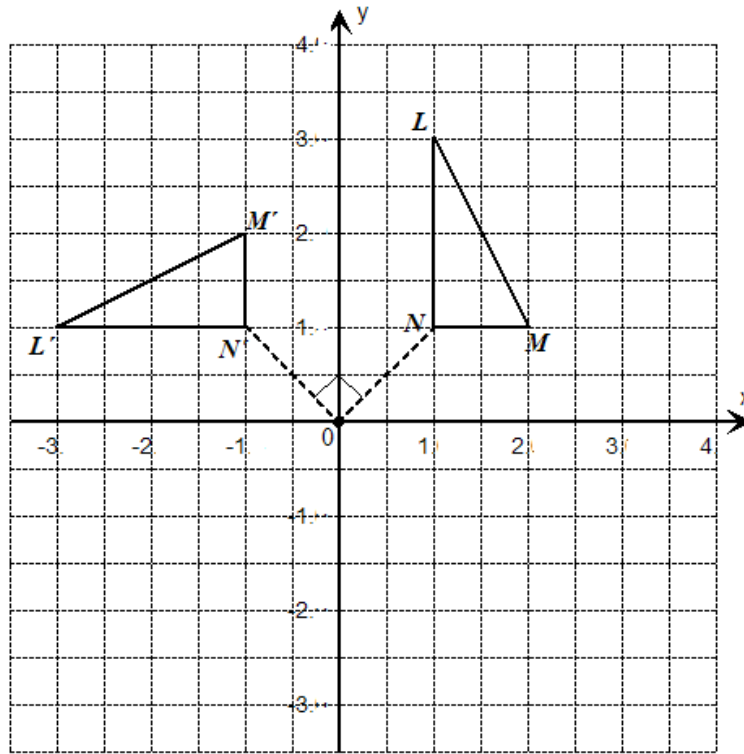
- (i) **Required to describe:** The transformation fully by stating the center, and direction of rotation.

Solution:

To find the center of rotation, choose two pairs of object-image points, say, M and M' and N and N' . Construct the perpendicular bisectors of MM' and NN' . Extend both bisectors so as to meet. Since both meet at O , O is the center of rotation. It is unnecessary to have repeated the process with a third bisector as all these perpendicular bisectors are concurrent.



To find the angle of rotation, choose any pair of object and image points, say, N and N' . Join each point to the centre of rotation to form the angle NON' . This is the angle of rotation.



By measurement, $\widehat{NON'} = 90^\circ$

Therefore, the angle of rotation is 90° .

The rotation from ON to ON' is anti-clockwise.

$\triangle LMN \longrightarrow \triangle L'M'N'$ by a rotation of 90° about O in an anti-clockwise direction.

- (ii) **Required to state:** Two geometrical relationships between $\triangle LMN$ and $\triangle L'M'N'$.

Solution:

$\triangle LMN \longrightarrow \triangle L'M'N'$ by a rotation which is a congruent transformation.

$\triangle LMN \equiv \triangle L'M'N'$ (all corresponding sides **AND** all corresponding angles of the object are the same as that of the image).

- (iii) **Required to calculate:** Coordinates of L after it undergoes a translation of

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Calculation:

$$L \xrightarrow{T = \begin{pmatrix} 1 \\ -2 \end{pmatrix}} L'$$

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \xrightarrow{T = \begin{pmatrix} 1 \\ -2 \end{pmatrix}} \begin{pmatrix} 1+1 \\ 3+(-2) \end{pmatrix}$$

The image is $\begin{pmatrix} 1+1 \\ 3+(-2) \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Therefore, the coordinates of the image of L is $(2, 1)$.

7. **Data:** Records of the distance that a ball was thrown by 24 students.

a. **Required To Complete:** Frequency table for the data given.

Solution:

This is a continuous variable

LCL-Lower class limit

UCL-Upper class limit

LCB-lower class boundary

UCB-upper class boundary

Distance in m L.C.L- U.C.L	L.C.B.	U.C.B.	Frequency, f
20 – 29	$19.5 \leq x < 29.5$		3
30 – 39	$29.5 \leq x < 39.5$		5
40 – 49	$39.5 \leq x < 49.5$		8
50 – 59	$49.5 \leq x < 59.5$		6
60 – 69	$59.5 \leq x < 69.5$		2

$$\sum f = 24$$

b. **Required to state:** Lower boundary for the class interval 20 – 29.

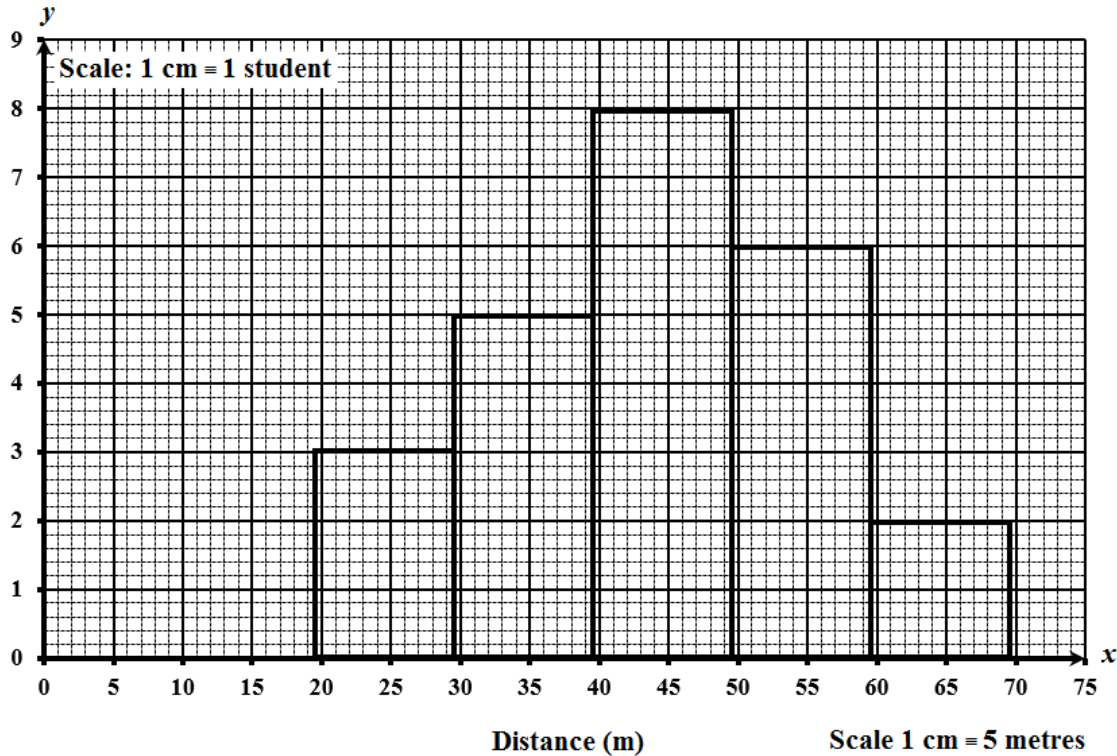
Solution:

In the class interval 20 – 29, the lower class limit is 20 but the lower class boundary is 19.5 as illustrated on the modified table

c. **Required to draw:** Histogram to illustrate the data given.

Solution:

No. of students



- d. (i) **Required to find:** The number of students who threw the ball 50 metres or more.

Solution:

The number of students who threw the ball 50 m or more = $6 + 2 = 8$

- (ii) **Required to find:** The probability that a randomly chosen student threw the ball 50 metres or more.

Solution:

$$P(\text{Student threw the ball } \geq 50 \text{ m}) = \frac{\text{No. of students who threw the ball } \geq 50 \text{ m}}{\text{Total no. of students}}$$

$$= \frac{2+6}{24}$$

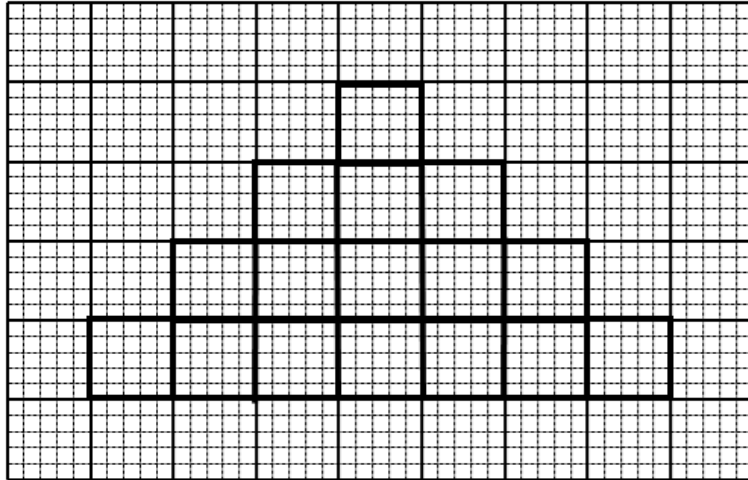
$$= \frac{8}{24}$$

$$= \frac{1}{3}$$

8. **Data:** Sequence of diagrams made up of squares of side 1 cm.

- a. **Required to draw:** The 4th figure in the sequence.

Solution:



b. **Required to complete:** Table given.

Solution:

The area of the figure is the square of the number of the figure

The perimeter of the figure is obtained by calculating the figure number times six then subtracting 2.

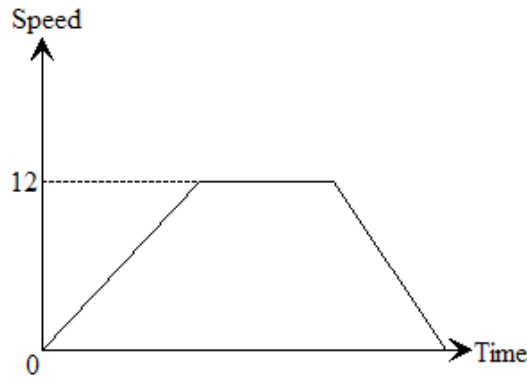
Figure	Area of figure, cm^2	Perimeter of figure, cm
1	1	$1 \times 6 - 2 = 4$
2	4	$2 \times 6 - 2 = 10$
3	9	$3 \times 6 - 2 = 16$
4	$4^2 = 16$	$4 \times 6 - 2 = 22$
5	$5^2 = 25$	$5 \times 6 - 2 = 28$
15	$15^2 = 225$	$15 \times 6 - 2 = 88$
n	n^2	$n \times 6 - 2 = 6n - 2$

Section II

9. a. **Data:** Speed – time graph of an athlete during a race.

(i) (a) **Required to find:** Maximum speed.

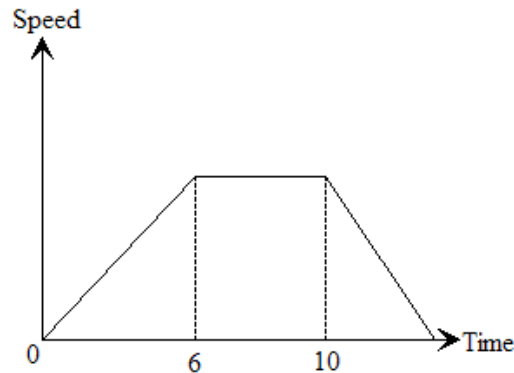
Solution:



The maximum speed is 12 ms^{-1} (obtained by a read off).

(b) **Required to find:** Number of seconds for which the speed is constant.

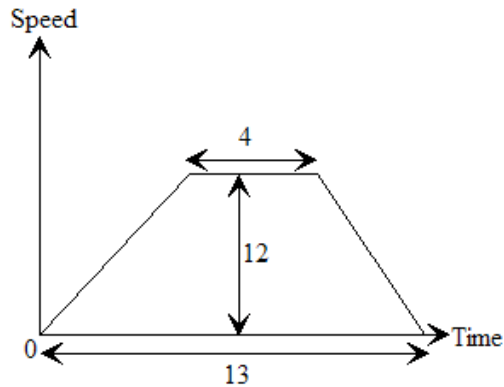
Solution:



The speed was constant for $(10 - 6) = 4$ seconds. This is shown by the branch which is a horizontal line (acceleration is the gradient = 0)

(c) **Required to find:** Total distance covered during the race.

Solution:



The total distance covered by the athlete is found by calculating the area under the graph. This region is a trapezium

$$\begin{aligned} \text{Therefore the total distance covered} &= \frac{1}{2}(4 + 13)12 \\ &= \frac{17 \times 12}{2} \\ &= 102 \text{ m} \end{aligned}$$

- (ii) (a) **Required to find:** Time period during which the speed of the athlete is increasing.
Solution:
 The athlete's speed was increasing from $t = 0$ to $t = 6$, i.e a period of 6 seconds, as illustrated by the 'branch' of the graph with the positive gradient.
- (b) **Required to find:** Time period during which the speed of the athlete is decreasing.
Solution:
 The athlete's speed was decreasing from the period $t = 10$ to $t = 13$, a period of 3 seconds, as shown by the 'branch' of the graph with the negative gradient.
- (c) **Required to find:** Time period during which the athlete's acceleration is 0.
Solution:
 Acceleration = 0 means that velocity is horizontal.
 Acceleration = 0 from $t = 6$ to $t = 10$, a period of 4 seconds as illustrated on the graph by the 'branch' that is horizontal.

b. **Data:** Farmer supplies x pumpkins and y melons to neighbours.

- (i) **Required to find:** Inequality to represent the third condition.

Solution:

The total number of pumpkins and melons must NOT exceed 12.
‘Not greater’ means ‘less than or equal to’.

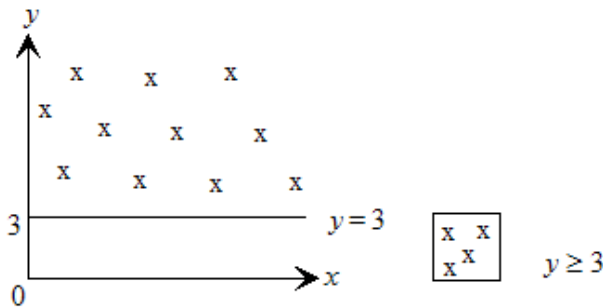
Hence,
 $x + y \leq 12$

- (ii) **Required to draw:** The graphs of the line associated with the three inequalities.

Solution:

The line $y = 3$ is a horizontal line.

The region which satisfies $y \geq 3$ is the region above the line

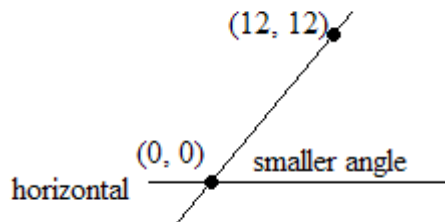


Obtaining 2 points on the line $y = x$.

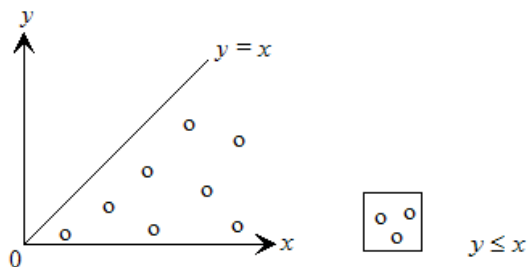
x	y
0	0
12	12

A horizontal is drawn through the line.

The side with the smaller or acute angle represents the \leq region.



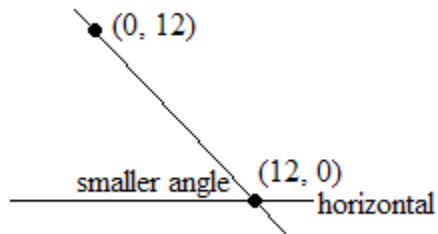
The region which satisfies $y \leq x$ is shown shaded



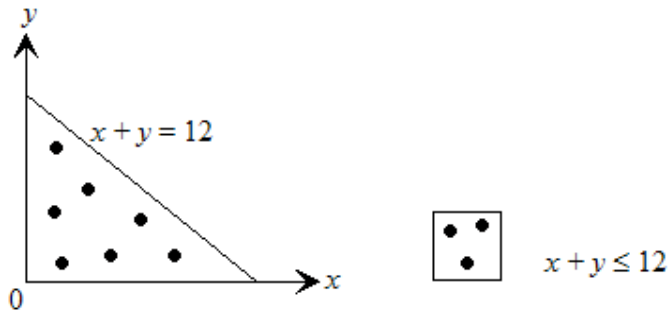
Obtaining 2 points on the line $x + y = 12$

x	y
0	12
12	0

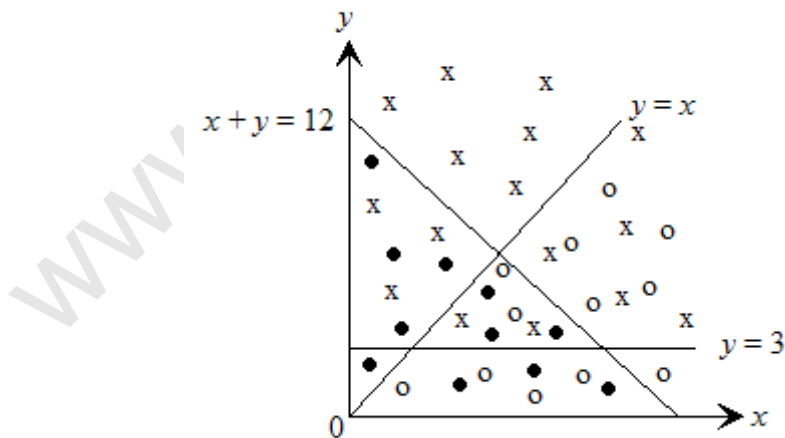
The side with the smaller or acute angle represents the \leq region.



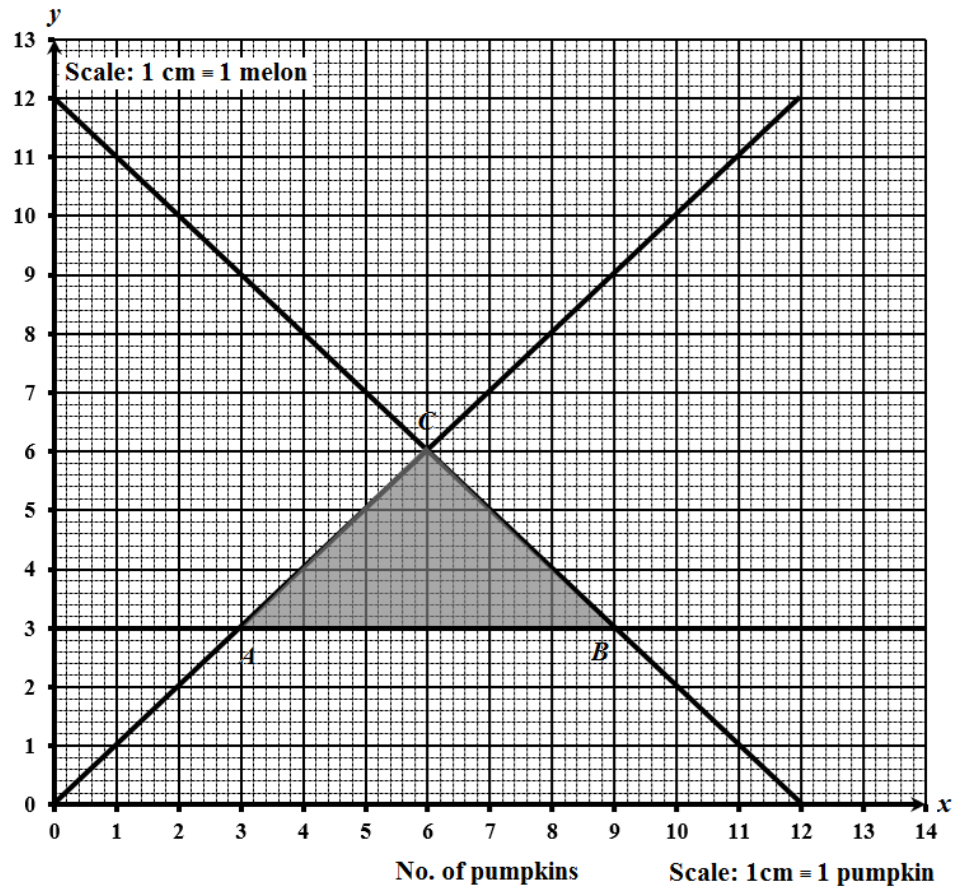
The region which satisfies $x + y \leq 12$ is shown shaded



The feasible region is represented by the area in which all three shaded regions overlap.



No. of melons

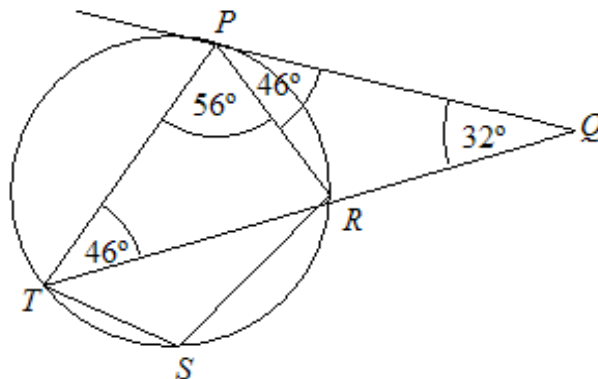


(iv) **Required To Find:** Minimum values of x and y .

Solution:

The region ABC as shown on the graph is the feasible region. Hence, the minimum x and y that satisfies the inequalities is at $x = 3$ and $y = 3$, at point A .

10. a. **Data:** Diagram as shown below.



(i) **Required to calculate:** $\hat{P}TR$

Calculation:

$$\hat{P}TR = 46^\circ$$

(The angle made by a tangent PQ to a circle and a chord, PR , at the point of contact, R , is equal to the angle in the alternate segment).

(ii) **Required to calculate:** $\hat{T}PR$

Calculation:

$$\begin{aligned}\hat{T}PR &= 180^\circ - (46^\circ + 46^\circ + 32^\circ) \\ &= 56^\circ\end{aligned}$$

(The sum of the angles in a triangle = 180°).

(iii) **Required to calculate:** $\hat{T}SR$

Calculation:

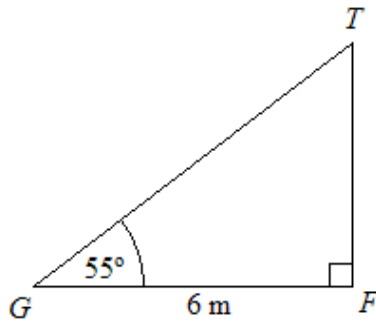
$$\begin{aligned}\hat{T}SR &= 180^\circ - 56^\circ \\ &= 124^\circ\end{aligned}$$

(The opposite angles in a cyclic quadrilateral $PRST$ are supplementary).

b. **Data:** Diagram of a flag pole on a horizontal plane.

(i) **Required to calculate:** Height FT of the flag pole.

Calculation:



The triangle TFG is right-angled. So,

$$\tan 55^\circ = \frac{FT}{6}$$

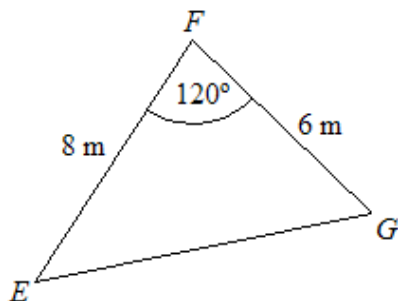
$$FT = 6 \tan 55^\circ$$

$$= 8.568$$

$$= 8.57 \text{ m (to 3 sig. fig)}$$

(ii) **Required to calculate:** The length of EG .

Calculation:



In triangle EFG , we have two sides and the included angle

Using the cosine rule:

$$EG^2 = (8)^2 + (6)^2 - 2(8)(6)\cos 120^\circ \text{ (cosine law)}$$

$$= 64 + 36 - 96\left(-\frac{1}{2}\right)$$

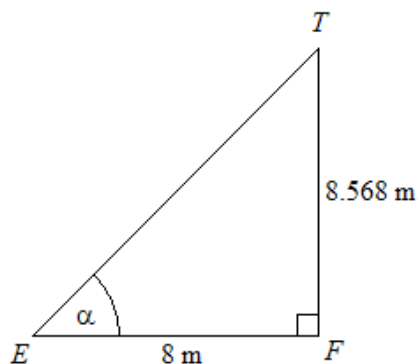
$$= 148$$

$$EG = 12.16$$

$$= 12.2 \text{ m (to 3 sig. fig)}$$

(iii) **Required to calculate:** The angle of elevation T from E .

Calculation:



Triangle EFT is a right angled triangle.

$$\text{Angle of elevation } T \text{ from } E \text{ is shown as } \alpha = \tan^{-1}\left(\frac{8.569}{8}\right)$$

$$= 46.96^\circ$$

$$= 47.0^\circ \text{ (to 3 sig. fig.)}$$

11. a. **Data:** $A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$.

(i) **Required to calculate:** AB .

Calculation:

We check for the conformability of the matrices under multiplication

$$\begin{aligned} AB &= \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix} \\ &= \begin{pmatrix} (1 \times 5) + (2 \times -2) & (1 \times -2) + (2 \times 1) \\ (2 \times 5) + (5 \times -2) & (2 \times -1) + (5 \times 1) \end{pmatrix} \\ &= \begin{pmatrix} 5 - 4 & -2 + 2 \\ 10 - 10 & -4 + 5 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

(ii) **Required to calculate:** B^{-1}

Calculation:

First we find the determinant of B

$$\begin{aligned} \det B &= (5 \times 1) - (-2 \times -2) \\ &= 5 - 4 \\ &= 1 \end{aligned}$$

$$\begin{aligned} \therefore B^{-1} &= \frac{1}{1} \begin{pmatrix} 1 & -(-2) \\ -(-2) & 5 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \end{aligned}$$

(iii) **Data:** $\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Required to express: $\begin{pmatrix} x \\ y \end{pmatrix}$ as a product of 2 matrices.

Solution:

Multiplying by B^{-1}

A matrix multiplied by its inverse is the identity matrix and the identity matrix multiplied by any matrix is the same matrix.

$$\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

(iv) **Required to calculate:** x and y

Calculation:

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} (1 \times 2) + (2 \times 3) \\ (2 \times 2) + (5 \times 3) \end{pmatrix} \\ &= \begin{pmatrix} 2 + 6 \\ 4 + 15 \end{pmatrix} \\ &= \begin{pmatrix} 8 \\ 19 \end{pmatrix} \end{aligned}$$

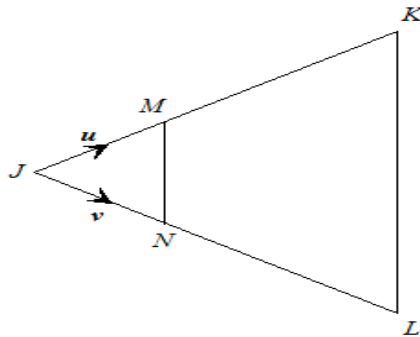
Both sides are 2×1 matrices and are equal. So, equating corresponding Entries we get

$\therefore x = 8$ and $y = 19$.

b. **Data:** Diagram showing $\triangle JKL$.

(i) **Required to draw:** Diagram given showing M and N .

Solution:



(ii) (a) **Required to express:** \overrightarrow{JK} in terms of u and v .

Solution:

$$\overrightarrow{JM} = u$$

$$\overrightarrow{MK} = 2u$$

$$\overrightarrow{JK} = 3u$$

Similarly

$$\overrightarrow{JN} = v$$

$$\overrightarrow{NL} = 2v$$

$$\overrightarrow{JL} = 3v$$

(b) **Required to express:** \overrightarrow{MN} in terms of u and v .

Solution:

Using the vector triangle law

$$\begin{aligned}\overrightarrow{MN} &= \overrightarrow{MJ} + \overrightarrow{JN} \\ &= -u + v\end{aligned}$$

(c) **Required To Express:** \overrightarrow{KL} in terms of u and v .

Solution:

Using the vector triangle law

$$\begin{aligned}\overrightarrow{KL} &= \overrightarrow{KJ} + \overrightarrow{JL} \\ &= -3u + 3v \\ &= 3(-u + v)\end{aligned}$$

(iii) **Required to find:** Two geometrical relationships between \overrightarrow{KL} and \overrightarrow{MN} .

Solution:

$\overrightarrow{KL} = 3 \times \overrightarrow{MN}$, that is a scalar multiple. Hence, \overrightarrow{KL} is parallel to \overrightarrow{MN} , so $|\overrightarrow{KL}| = 3|\overrightarrow{MN}|$, that is the length of \overrightarrow{KL} is 3 times the length of \overrightarrow{MN} .