JANUARY 2010 CXC MATHEMATICS - PAPER 2

Section I

1. a. **Required to calculate:** $\frac{2.76}{0.8}+8.7^2$

Calculation:
The arithmetic is not difficult in this question, but we could save valuable time by using the calculator and which is allowable. So,

$$\frac{2.76}{0.8}+8.7^2=3.45+75.69 \text{ (Using the calculator)}$$
$$=79.14 \text{ (exact value)}$$

b. **Data:** Salesman salary is \$3 140 per month together with 2% commission on sales.

(i) **Required to calculate:** Fixed salary for the year.
Calculation:
The total fixed salary for the year of 2009 = monthly salary x 12
$$=\$3140\times 12$$
$$=\$37\,680$$

(ii) **Required to calculate:** Commission for the year.
Calculation:
The commission for the year of 2009 $=2\% \text{ of } \$720\,000$
$$=\frac{2}{100}\times\$720\,000$$
$$=\$14\,400$$

(iii) **Required to calculate:** Total income for the year.
Calculation:
The total income for the year of 2009 = The total fixed salary for 2009 + The total earned in commission for 2009
$$=\$37\,680+\$14\,400$$
$$=\$52\,080$$

c. **Data:** $1\frac{1}{3}$ cups of milk and 2 cups of pancake mix produces 8 pancakes.

(i) **Required to calculate:** Number of cups of pancake mix to make 12 pancakes.
Calculation:
From the given instructions:
8 pancakes are produced from using 2 cups of mix.

Hence, 1 pancake is produced from $\frac{2}{8}$ cups of mix.

And, 12 pancakes will be produced from $\frac{2}{8} \times 12$ cups of mix.

$= 3$ cups of pancake mix.

(ii) **Required to calculate:** Number of pancakes made from 5 cups of milk.
Calculation:

$1\frac{1}{3}$ cups of milk produces 8 pancakes.

So, 1 cup of milk will produce $\frac{8}{1\frac{1}{3}}$ pancakes.

Hence, 5 cups of milk will produce $\frac{8}{1\frac{1}{3}} \times 5$ pancakes.

$= \left(\frac{8}{1} \times \frac{3}{4} \times 5\right)$ pancakes.

$= 30$ pancakes.

2\. a. **Data:** $a = 6,\ b = -4$ and $c = 8$.

Required to calculate: $\frac{a^2 + b}{c - b}$

Calculation:
Substituting the values given to the variables to obtain

$$\frac{a^2 + b}{c - b} = \frac{(6)^2 + (-4)}{8 - (-4)}$$
$$= \frac{36 - 4}{8 + 4}$$
$$= \frac{32}{12}$$
$$= 2\frac{8}{12}$$
$$= 2\frac{2}{3} \text{ (in lowest terms)}$$

b. (i) **Required to simplify:** $3(x - y) + 4(x + 2y)$
Solution:

Expanding by using the distributive law to get,

$$3(x-y)+4(x+2y) = 3x-3y+4x+8y$$
$$= 3x+4x+8y-3y$$
$$= 7x+5y$$

(ii) **Required to simplify:** $\dfrac{4x^2 \times 3x^4}{6x^3}$

Solution:
Separating the coefficients from the variables to simplify, we use the laws of indices to get

$$\frac{4x^2 \times 3x^4}{6x^3} = \frac{4\times 3}{6} \times \frac{x^2 \times x^4}{x^3}$$
$$= 2x^{2+4-3}$$
$$= 2x^3$$

c. **Data:** $x-3 < 3x-7$.

(i) **Required to find:** x

Solution:
Using the same principles used to solve equations, we obtain

$$x-3 < 3x-7$$
$$-3+7 < 3x-x$$
$$4 < 2x$$
$$\therefore 2x > 4$$
$$\div 2$$
$$x > 2$$

(ii) **Required to find:** Smallest integer value of x that satisfies the inequality.

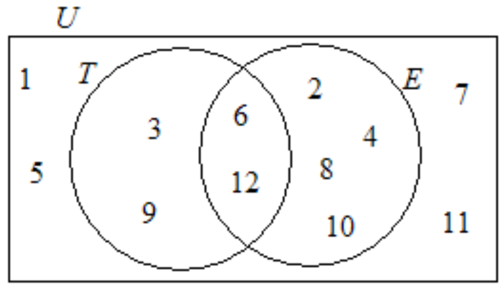
Solution:
$x > 2$ *and*
$x \in Z \qquad Z = \{\ldots,-3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ldots\}$
$\therefore x_{\min} = 3$

The smallest integer that satisfies the inequality is the next integer that is greater than 2 and which is 3.

3. a. **Data:** $U = \{1, 2,\ 3,\ldots,12\}$, T = {multiples of 3} and E = {even numbers}

(i) **Required to draw:** Venn diagram to represent the information given.

Solution:
$T = \{3,\ 6,\ 9,\ 12\}$
$E = \{2,\ 4,\ 6,\ 8,\ 10,\ 12\}$
Hence,



(ii) (a) **Required to list:** Members of $T \cap E$
Solution:

The elements that are common to both sets T and E
$T \cap E = \{6, 12\}$
as illustrated on the Venn diagram.

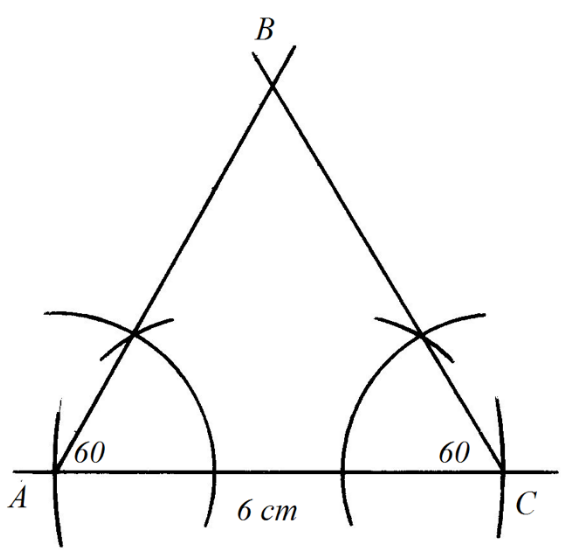
(b) **Required to list:** Members of $(T \cup E)'$
Solution:
First we list and then deduce the complement
$T \cup E = \{2, 3, 4, 6, 8, 9, 10, 12\}$
$(T \cup E)' = \{1, 5, 7, 11\}$
as illustrated on the Venn diagram.

b. (i) **Required to construct:** Triangle ABC where $AC = 6$ cm, $A\hat{C}B = 60°$ and $C\hat{A}B = 60°$.
Solution

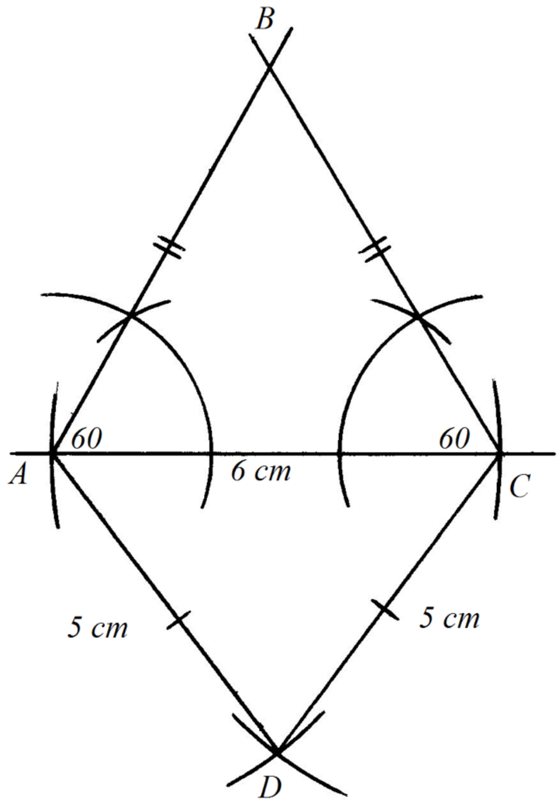
Draw a straight line longer than 6 cm and cut off $AC = 6$ cm.

Construct a 60^0 angle at A.
Construct a 60^0 angle at C.

The line from A meets the line from C at B.

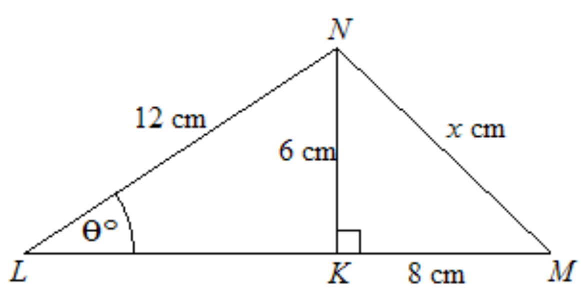


(ii) **Required to complete:** Diagram to show the kite *ABCD*, in which $AD = 5$ cm.
Solution:
Arcs of length 5 cm drawn with centre *A* and centre *C* to meet at *D* on the opposite side of *B,* to form the kite.



(iii) **Required to find:** $D\hat{A}C$
Solution:
$D\hat{A}C = 53°$ (when measured by the protractor).

4. a. **Data:** Diagram as shown below.



(i) **Required to calculate:** *x*.
Calculation:

Applying Pythagoras' theorem to the right-angled triangle *NKM*

$x^2 = (6)^2 + (8)^2$ (Pythagoras' Theorem)

$$x = \sqrt{36+64}$$
$$= \sqrt{100}$$
$$= 10 \text{ cm}$$

(ii) **Required to calculate:** θ

Calculation:

Triangle *NLK* is a right–angled triangle. So,

$$\sin\theta° = \frac{6 \text{ cm}}{12 \text{ cm}}$$
$$\therefore \theta° = \sin^{-1}\left(\frac{6}{12}\right)$$
$$\therefore \theta° = 30°$$
$$\theta = 30$$

b. **Data:** Map with scale 1 : 1 250

(i) **Required to find:** Distance from *S* to *F* on the map.

Solution:

SF measures 7.8 cm (to 1 decimal place by the ruler).

(ii) **Required to calculate:** Actual distance *SF*.

Calculation:

The actual distance *SF* $= (7.8 \times 1250)$cm according to the scale.

(100 cm = 1 m)

$$= \frac{7.8 \times 1250}{100} \text{ m}$$
$$= 97.5 \text{ m to 1 decimal place}$$

(iii) **Data:** Time taken to run from *S* to *F* = 9.72 seconds.

(a) **Required to calculate:** Average speed in ms^{-1}

Calculation:

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$
$$= \frac{97.5\text{m}}{9.72\text{s}}$$
$$= 10.03 \text{ ms}^{-1}$$
$$= 10 \text{ ms}^{-1} (\text{to 1 decimal place})$$

(b) **Required to calculate:** Average speed in kmh^{-1}

Calculation:

1 km =1 000 m

$1\text{ h} = 3\ 600\text{ s}$

So

$$\text{Average speed} = \frac{\frac{97.5}{1000}\text{ km}}{\frac{9.72}{3600}\text{ h}}$$

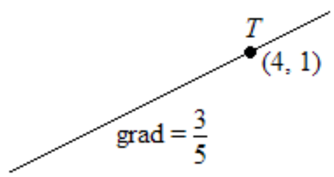
$$= 36.1\underline{1}\ \text{kmh}^{-1}$$

$$= 36.1\ \text{kmh}^{-1}\ \text{(to 3 significant figures)}$$

5. a. **Data:** Line passes through $T(4, 1)$ and has a gradient of $\frac{3}{5}$.

Required to find: Equation of the line.

Solution:



To find the equation of the line we require the coordinates of 1 point on the line and the gradient of the line. These we have and applying the formula we get

$$\frac{y-1}{x-4} = \frac{3}{5}$$

$$5(y-1) = 3(x-4)$$

$$5y - 5 = 3x - 12$$

$$5y = 3x - 7$$

OR any other equivalent form.

b. (i) **Required to draw:** Triangle ABC, from the coordinates of the vertices given.

Solution:

This is shown on the diagram below

(ii) **Required to draw:** The line $y = 2$.

Solution:

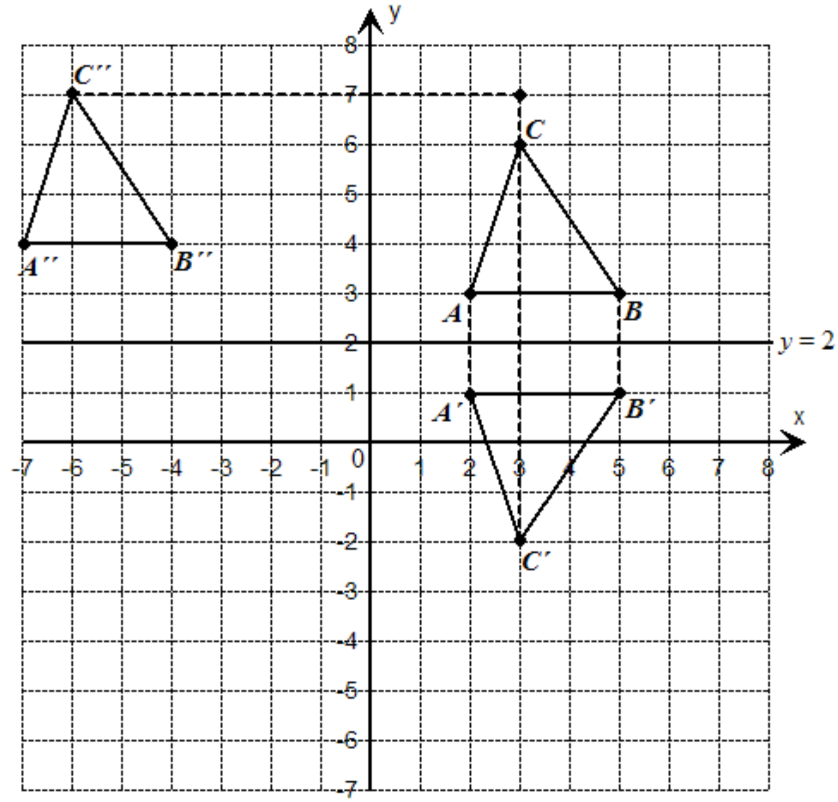
This is shown on the diagram below

(iii) **Required to draw:** Image of triangle ABC under a reflection in the line $y = 2$ and label it $A'B'C'$.

Solution:

This is shown on the diagram below

(iv) **Required to draw:** $A''B''C''$ using the coordinates for the vertices given.
Solution:
This is shown on diagram below.



(v) **Required to describe:** Single transformation that maps ΔABC onto $\Delta A''B''C''$.
Solution:
The image is congruent to the object and is neither flipped nor re-oriented. The transformation is a translation. We can deduce the translation vector by observing one object-image set of points.

$$C \xrightarrow{T=\begin{pmatrix}-9\\1\end{pmatrix}} C''$$

The point C is mapped onto C'' by a horizontal shift of 9 units to the left and a vertical shift of 1 unit upwards. This may be represented by the translation, T, where $T = \begin{pmatrix}-9\\1\end{pmatrix}$.

Hence, ΔABC is mapped onto $\Delta A''B''C''$ by a translation, T, where we define $T = \begin{pmatrix}-9\\1\end{pmatrix}$.

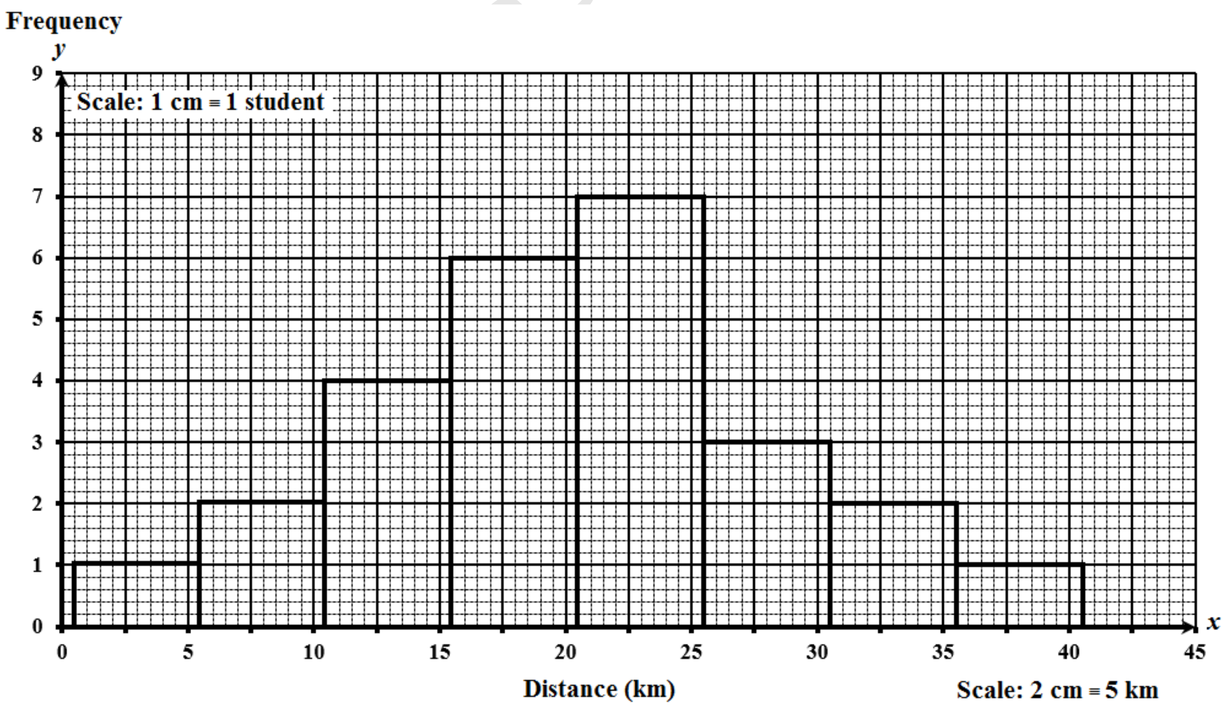
6. **Data:** Records of the distances travelled to school by 26 students of a class.

a. **Required to complete:** Frequency table to represent the data given.
Solution:
The data is that of a continuous variable.
UCL-Upper class limit
LCL-Lower class limit
LCB-Lower class boundary
UCB-Upper class boundary

Distance in km L.C.L-U.C.L	X L.C.B-U.C.B	Frequency
1 – 5	$0.5 \le x < 5.5$	1
6 – 10	$5.5 \le x < 10.5$	2
11 – 15	$10.5 \le x < 15.5$	4
16 – 20	$15.5 \le x < 20.5$	6
21 – 25	$20.5 \le x < 25.5$	7
26 – 30	$25.5 \le x < 30.5$	3
31 – 35	$30.5 \le x < 35.5$	2
36 – 40	$35.5 \le x < 40.5$	1

$\sum f = 26$

b. **Required to qraw:** Histogram to represent the data given.
Solution:



c. **Required to calculate:** Probability that a randomly chosen student travelled 26 km or more to school.

Calculation:

$$P(\text{student travelled} \geq 26\text{km}) = \frac{\text{No. of student travelling³26km}}{\text{Total no. of students}}$$

$$= \frac{3+2+1}{\sum f = 26}$$

$$= \frac{6}{26}$$

$$= \frac{3}{13}$$

d. **Required to explain:** Whether the mean, mode or median is most appropriate for estimating the cost of transportation service for the school.

Solution:

The mean is most appropriate for estimating the cost of transportation. This is because the mean takes into consideration each student and the actual distance that is covered by each student. If the mean is required, we may use the formula:

$\bar{x} = \dfrac{\sum fx}{\sum f}$, where

$\bar{x} =$ mean

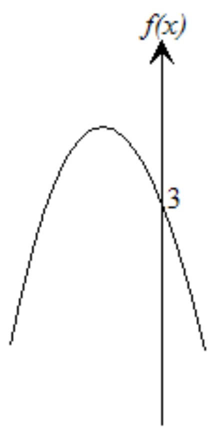
$x =$ mid-class interval or the mid interval value of the classes

$f =$ frequency

7. **Data:** Graph representing $f(x) = ax^2 + bx + c$.

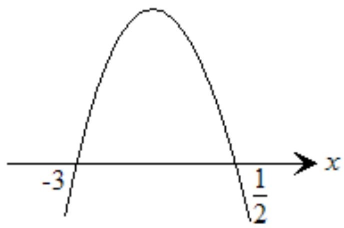
(i) **Required to find:** $f(x)$ when $x = 0$

Solution:



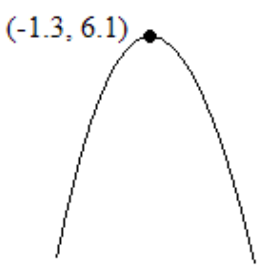
When $x = 0$, $f(x) = 3$ or $f(0) = 3$

(ii) **Required to find:** x when $f(x)=0$
Solution:



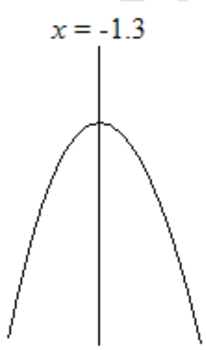
When $f(x)=0$, $x=\frac{1}{2}$ and $x=3$. These are the points where the curve cuts the horizontal axis.

(iii) **Required to find:** Coordinates of the maximum point.
Solution:



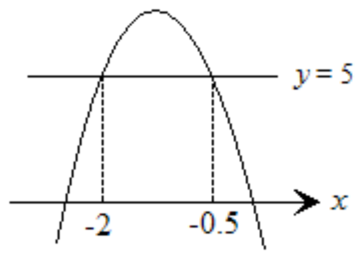
The coordinates of the maximum point are (-1.3, 6.1).

(iv) **Required to find:** Equation of the axis of symmetry.
Solution:

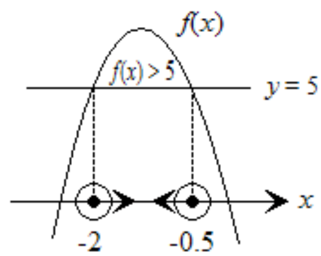


Equation is $x=-1.3$, the vertical which passes through the maximum point.

(v) **Required to find:** x when $f(x)=5$.
Solution:
The horizontal line $f(x)$ or $y = 5$ cuts the curve at $x = -2$ and at $x = -0.5$



(vi) **Required to find:** $f(x) > 5$.
Solution:
We are seeking the range of values of x for which $f(x)$ lies above the line $y = 5$



$\{x : -2 < x < -0.5\}$

8. **Data:** Table as shown.

No. *n*, of hexagons in the pattern	1	2	3	4	5	20	n
No. of sticks, *S* used for the pattern	6	11	16	x	y	z	S

a. (i) **Required to find:** x
Solution:
Notice that the value of S = 6, 11, 16, …which are multiples of 5 and exceeded by 1 because we notice that:-

$6 = 1(5)+1$
$11 = 2(5)+1$
$16 = 3(5)+1$

Hence, the pattern or relation that connects S and n is
$S = n(5)+1$

(i) $S = 5n+1$
When $n = 4$, $S = 5(4)+1 \quad = 21 \qquad \therefore x = 21$

(ii) **Required to find:** y

Solution:

When $n = 5$ $S = 5(5) + 1 \quad = 26$ $\therefore y = 26$

(iii) **Required to find:** z

Solution:

When $n = 20$ $S = 5(20) + 1 \quad = 101$

$\therefore z = 101$

b. **Required to find:** Expression in terms of *n* for *S*.

Solution:

$S = 5n + 1$ (as found and shown above)

c. **Data:** 76 sticks were used to make a pattern of *h* hexagons.

Required to calculate: h

Calculation:

When

$S = 76$

Hence, (data)

$76 = 5(n) + 1$

$5n = 75$

$n = 15$

The number of hexagons = 15 and $h = 15$.

Section II

9. a. **Data:** $E = \frac{1}{2}mv^2$

(i) **Required to express**: v in terms of E and m.
Solution:

We cross multiply to make the equation in a linear and more

manageable form.

$$E = \frac{1}{2}mv^2$$

$$\frac{E}{1} = \frac{mv^2}{2}$$

$$2 \times E = mv^2 \times 1$$

$$mv^2 = 2E$$

$$v^2 = \frac{2E}{m}$$

$$v = \sqrt{\frac{2E}{m}}$$

(ii) **Required to calculate:** v when $E = 45$ and $m = 13$.
Calculation:
When we substitute $E = 45$ and $m = 13$ we obtain

$$v = \sqrt{\frac{2(45)}{13}}$$

$$= \sqrt{\frac{90}{13}}$$

$$= 2.63\underline{1}$$

$$= 2.63 \text{ (to 2 decimal places)}$$

b. **Data:** $g(x) = 3x^2 - 8x + 2$

(i) **Required to express:** $g(x)$ in the form $a(x+b)^2 + c$.
Solution:

$$3x^2 - 8x + 2 = 3\left(x^2 - \frac{8}{3}x\right) + 2$$

Half the coefficient of x is $\frac{1}{2}\left(-\frac{8}{3}\right) = -\frac{4}{3}$

$$3x^2 - 8x + 2 = 3\left(x - \frac{4}{3}\right)^2 + *$$

And where the value of * is to be determined.

$$3\left(x-\frac{4}{3}\right)^2 = 3\left(x^2-\frac{8}{3}x+\frac{16}{9}\right)$$

$$= 3x^2 - 8x + 5\frac{1}{3}$$

$$\begin{array}{r} -3\frac{1}{3} = * \\ \hline 2 \\ \hline \end{array}$$

Hence, $3x^2 - 8x + 2 \equiv 3\left(x-\frac{4}{3}\right)^2 - 3\frac{1}{3}$ and which is of the form $a(x+b)^2 + c$, where

$a = 3 \in \Re$, $b = -\frac{4}{3} \in \Re$ and $c = -3\frac{1}{3} \in \Re$.

OR

We could expand the desired form and equate the coefficients with the given form

$$3x^2 - 8x + 2 = a(x+b)^2 + c$$
$$= a\left(x^2 + 2bx + b^2\right) + c$$
$$= ax^2 + 2abx + ab^2 + c$$

Equating the coefficient of x^2 we get

$a = 3 \in \Re$

Equating the coefficient of x we get

$$2(3)b = -8$$
$$b = -\frac{8}{2(3)}$$
$$= -\frac{4}{3} \in \Re$$

Equating the constant we get

$$3\left(-\frac{4}{3}\right)^2 + c = 2$$
$$5\frac{1}{3} + c = 2$$
$$c = -3\frac{1}{3} \in \Re$$

$\therefore 3x^2 - 8x + 2 \equiv 3\left(x-\frac{4}{3}\right)^2 - 3\frac{1}{3}$ which is the required form.

(ii) **Required to solve:** $g(x) = 0$

Solution:

Using the quadratic equation formula

$$g(x) = 3x^2 - 8x + 2 = 0$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{8 \pm \sqrt{64 - 24}}{6}$$

$$= \frac{8 \pm \sqrt{40}}{6}$$

$$= 0.27\underline{9} \text{ or } 2.38\underline{7}$$

$$= 0.28 \text{ or } 2.39 \text{ to 2 decimal places}$$

OR

Using the form obtained when we completed the square

$$g(x) = 3x^2 - 8x + 2 = 0$$

$$\therefore 3\left(x - \frac{4}{3}\right)^2 - 3\frac{1}{3} = 0$$

$$3\left(x - \frac{4}{3}\right)^2 = 3\frac{1}{3}$$

$$\left(x - \frac{4}{3}\right)^2 = \frac{3\frac{1}{3}}{3}$$

$$\left(x - \frac{4}{3}\right)^2 = \frac{10}{9}$$

Taking the square root

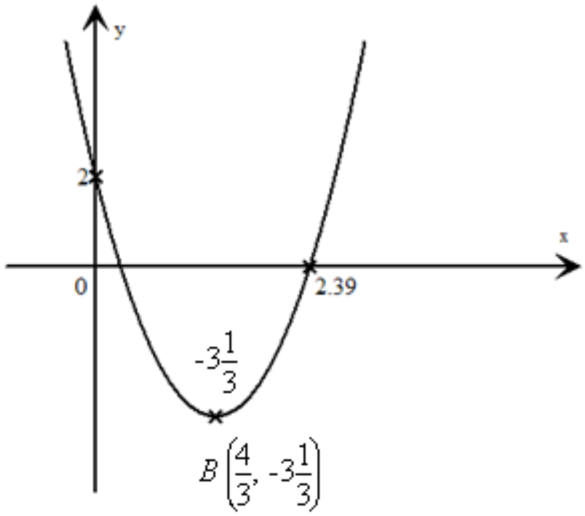
$$\left(x - \frac{4}{3}\right) = \pm\frac{\sqrt{10}}{3}$$

$$x = \frac{4 \pm \sqrt{10}}{3}$$

$$= 0.27\underline{9} \text{ or } 2.38\underline{7}$$

$$= 0.28 \text{ or } 2.39$$

(iii) **Required to sketch:** The graph of $g(x)$.
Solution:



(a) **Required to find:** The y – coordinate of A.
Solution:
$g(x) = 3x^2 - 8x + 2$
At A, $x = 0$ $\qquad$ $g(0) = 3(0)^2 - 8(0) + 2$
$= 2$
Hence, $A = (0, 2)$ and the y – coordinate of A is 2.

(b) **Required to find:** The x – coordinate of C.
Solution:
The curve $g(x)$ cuts the x – axis at $\frac{4 \pm \sqrt{10}}{3}$, that is at the points $\left(\frac{4-\sqrt{10}}{3}, 0\right)$ and $\left(\frac{4+\sqrt{10}}{3}, 0\right)$ on the horizontal axis. This approximates to (0.28, 0) and (2.39, 0).
C is the point with the greater x value. Therefore, the x – coordinate of C is $\frac{4+\sqrt{10}}{3} \approx 2.39$.

(c) **Required to find:** The x and y coordinates of B.
Solution:
$g(x) = 3x^2 - 8x + 2$
(Assuming that B is the minimum point of $g(x)$.)
The axis of symmetry of $g(x)$ is $x = \frac{-(-8)}{2(3)}$

The vertical at $x = \frac{4}{3}$

The axis of symmetry passes through the minimum point of $g(x)$.

Therefore, the x – coordinate of the minimum point is $x = \frac{4}{3}$.

To find the corresponding the y coordinate we get

$$\begin{aligned} g\left(\frac{4}{3}\right) &= 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + 2 \\ &= \frac{16}{3} - \frac{32}{3} + 2 \\ &= -3\frac{1}{3} \end{aligned}$$

The y – coordinate of B is $-3\frac{1}{3}$ and so $B = \left(1\frac{1}{3}, -3\frac{1}{3}\right)$.

OR

$$\begin{aligned} g(x) &= 3x^2 - 8x + 2 \\ &= 3\left(x - \frac{4}{3}\right)^2 - 3\frac{1}{3} \end{aligned}$$

$$3\left(x - \frac{4}{3}\right)^2 \geq 0 \quad \forall x$$

$$\begin{aligned} \therefore g(x)_{\min} &= 0 - 3\frac{1}{3} \\ &= -3\frac{1}{3} \end{aligned}$$

at

$$3\left(x - \frac{4}{3}\right)^2 = 0$$

$$\left(x - \frac{4}{3}\right)^2 = 0$$

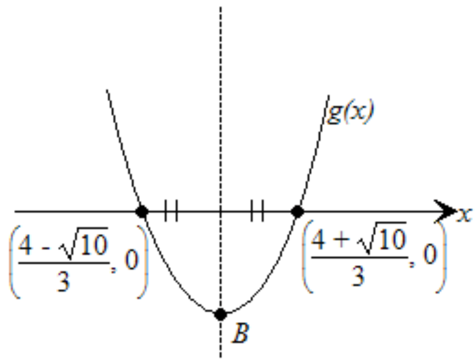
$$x - \frac{4}{3} = 0$$

$$\begin{aligned} x &= \frac{4}{3} \\ &= 1\frac{1}{3} \end{aligned}$$

The x – coordinate of $B = 1\frac{1}{3}$ and the y – coordinate of $B = -3\frac{1}{3}$

and B has coordinates $\left(1\frac{1}{3}, -3\frac{1}{3}\right)$.

OR



Since $g(x)$ is a quadratic graph and its shape is a parabola, then the x – coordinate of B is halfway between $\frac{4-\sqrt{10}}{3}$ and $\frac{4+\sqrt{10}}{3}$, that is

$$x = \frac{\frac{4-\sqrt{10}}{3} + \frac{4+\sqrt{10}}{3}}{2}$$

$$= \frac{\frac{4}{3} - \frac{\sqrt{10}}{3} + \frac{4}{3} + \frac{\sqrt{10}}{3}}{2}$$

$$= \frac{4}{3}$$

And

$$g\left(\frac{4}{3}\right) = -3\frac{1}{3}$$

Therefore, x coordinate of B is $\frac{4}{3}$ and y coordinate of B is $-3\frac{1}{3}$ and the coordinates of B is $\left(1\frac{1}{3}, -3\frac{1}{3}\right)$.

10. **Data:** Manager wishes to make x small pizzas and y large pizzas and the oven can hold no more than 20 pizzas.

a. (i) **Required to find:** Inequality to represent the information given.

Solution:

Number of small pizzas = x

Number of large pizzas = y

Total must not be more than 20,

Hence

$x + y \qquad \leq 20$

$\therefore x + y \leq 20 \qquad \ldots(1)$

(ii) **Data:** Ingredient for each small pizza cost \$15 and ingredients for each large pizza cost \$30. Manager plans to spend no more than \$450 on ingredients.

Required to find: Inequality to represent the information given

Solution:

Cost of x small pizzas at \$15 each $= \$(15 \times x)$

Cost of y large pizzas at \$30 each $= \$(30 \times y)$

Total must be no more than \$450.

So $\$(15x + 30y)$ must be $\leq \$450$

We $\div 15$ to simplify the inequation to obtain

$x + 2y \leq 30 \qquad \ldots(2)$

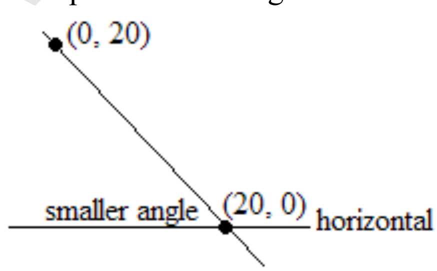
b. (i)-(ii) **Required to draw:** Graphs of the lines associated with the inequalities and shade the feasible region.

Solution:

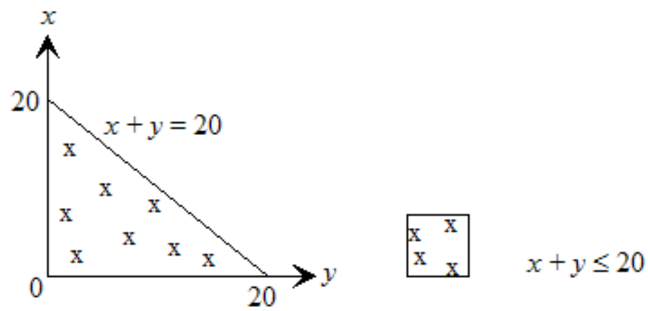
We draw the line by obtaining 2 points on the line $x + y = 20$.

x	y
0	20
20	0

We draw a horizontal to intersect the line. The side with the smaller (acute) angle will represent the $\leq$ region.



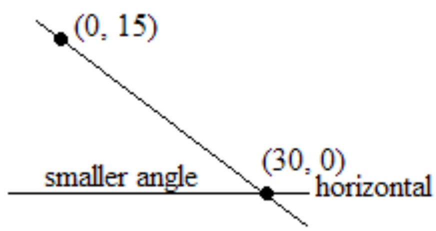
The region which satisfies $x + y \leq 20$ is



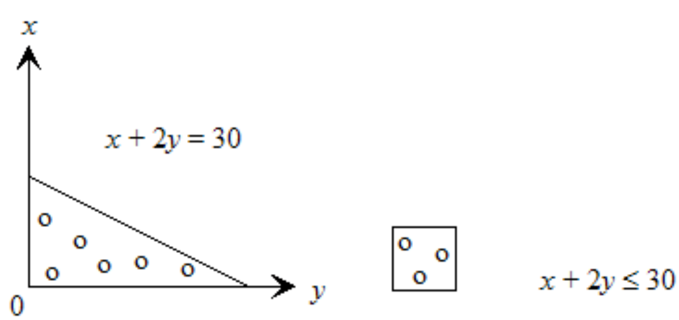
Obtaining 2 points so as to sketch the line $x+2y=30$.

x	y
0	15
30	0

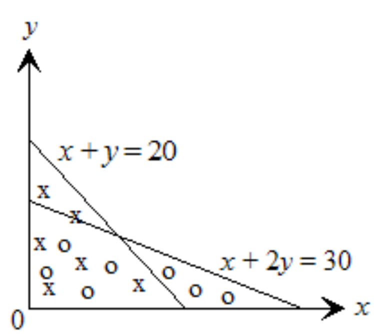
A horizontal is drawn through the line. The side with the smaller angle will represent the $\le$region.

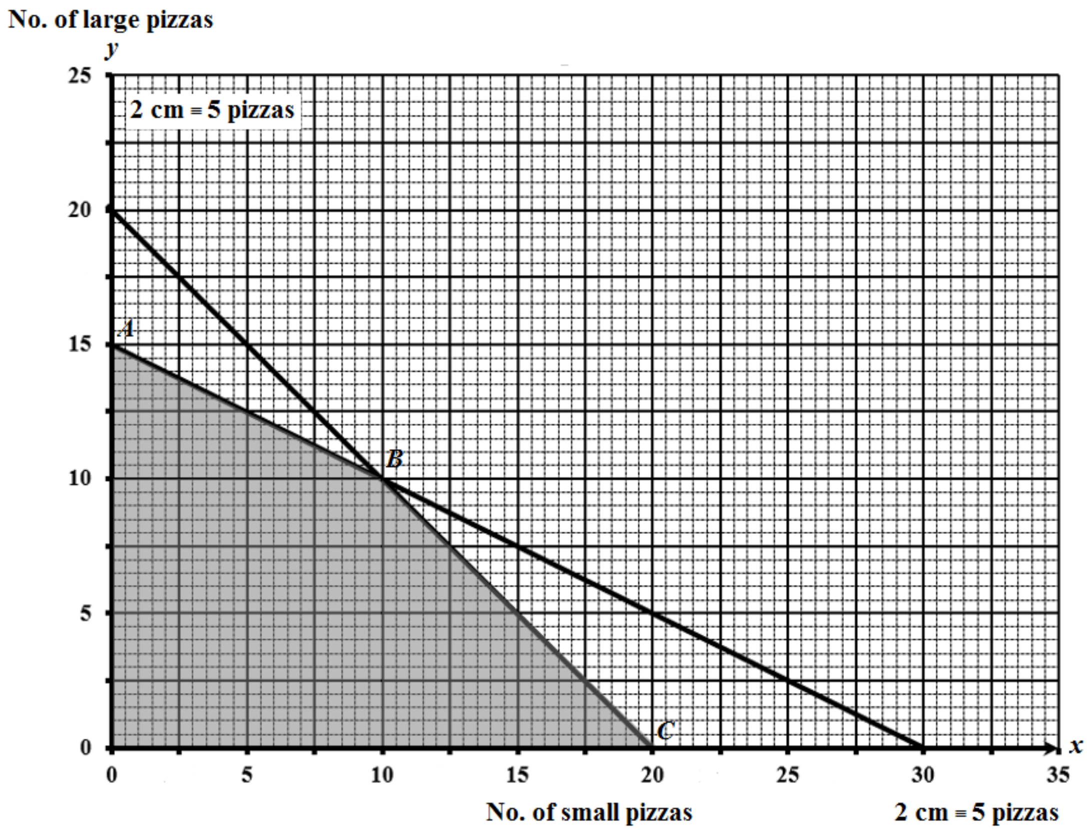


The region which satisfies $x+2y\le 30$ is



The feasible region is the area in which both shaded regions overlap.





(iii) **Required to state:** Coordinates of the shaded region.
Solution:
The shaded region $OABC$ represents the feasible region where
$O=(0,\ 0)$, $A=(0, 15)$, $B=(10,\ 10)$ and $C=(20,\ 0)$.

c. (i) **Required to find:** Expression in terms of x and y for the total profit made on the sale of pizzas.
Solution:
Let the total profit be P.
The profit on x small pizzas at \$8 each $=\$(8\times x)$
The profit on y large pizzas at \$20 each $=\$(20\times y)$
$P=\$8x+\$20y$
$P=\$(8x+20y)$

(ii) **Required to calculate:** Maximum profit made.
Calculation:
The vertices used for identifying the maximum profit are A (0, 15), B (10, 10) and C (20, 0). It makes no sense to test $x = 0$ and $y = 0$.

When $x=0$ and $y=15$

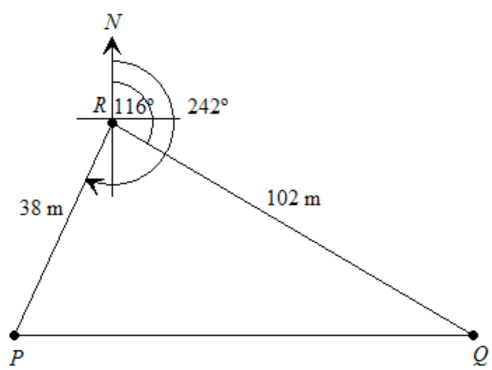
$P = 8(0) + 20(15)$
$= \$300$

When $x = 10$ and $y = 10$
$P = 8(10) + 20(10)$
$= \$280$

When $x = 20$ and $y = 0$
$P = 8(20) + 20(0)$
$= \$160$

Hence, the maximum profit, P, is \$300 as illustrated above and occurs when the shop sells 15 large pizzas.

11. a. **Data:** Diagram as shown below.



(i) **Required to prove:** $P\hat{R}Q = 126°$

Proof:

$$P\hat{R}Q + 116° = 242°$$
$$P\hat{R}Q = 242° - 116°$$
$$= 126°$$

Q.E.D.

(ii) **Required to calculate:** Distance PQ.

Calculation:

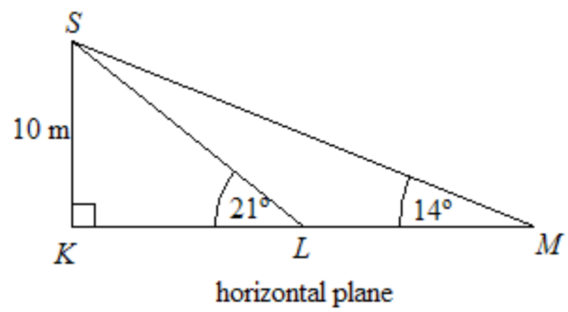
In triangle PQR, we have two sides and an included angle**.**
So we can apply the cosine law

$$PQ^2 = (38)^2 + (102)^2 - 2(38)(102)\cos 126° \text{ (cos } law\text{)}$$
$$= 16\,404.5$$
$$PQ = 128.0\text{ m}$$
$$= 128\text{ m to the nearest m.}$$

b. **Data:** Vertical pole, 10 m standing on a horizontal plane with points, *K*, *L* and *M*.

(i) **Required to complete:** Diagram showing the information given.

Solution:



(ii) (a) **Required To Calculate:** Length of *KL*.

Calculation:

Consider the right-angled triangle *SKL*

$$\tan 21° = \frac{10}{KL}$$

$$KL = \frac{10}{\tan 21°}$$
$$= 26.05$$
$$= 26.1 \text{ m} (\text{to one decimal place}).$$

(b) **Required to calculate:** Length of *LM*.

Calculation:

Consider the right-angled triangle *SKM*

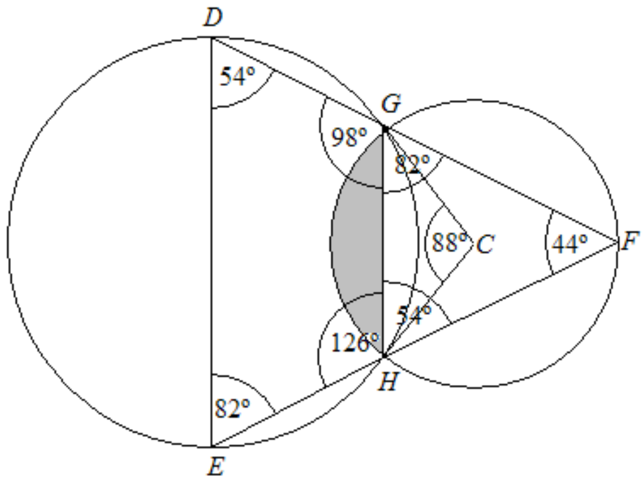
$$\tan 14° = \frac{10}{KM}$$

$$\therefore KM = \frac{10}{\tan 14°}$$
$$= 40.11 \text{m}$$

The length of *LM* = The length of *KM* – The length of *KL*

$$= 40.11 - 26.05$$
$$= 14.06$$
$$= 14.1 \text{ m (to one decimal place)}$$

12. a. **Data:** Diagram shown below with $G\hat{C}H = 88°$ and $G\hat{H}E = 126°$.



(i) **Required to calculate:** $G\hat{F}H$
Calculation:

$$G\hat{F}H = \frac{1}{2}(88°)$$
$$= 44°$$

(The angle subtended by a chord (*GH*) at the center of a circle, (angle *GCH*) is twice the angle it subtends at the circumference, (angle *GFH*) standing on the same arc).

(ii) **Required to calculate:** $G\hat{D}E$
Calculation:

$$G\hat{D}E = 180° - 126°$$
$$= 54°$$

(The opposite angles of a cyclic quadrilateral (*DEHG*) are supplementary.)

(iii) **Required to calculate:** $D\hat{E}F$
Calculation:

$$G\hat{H}F = 180° - 126°$$
$$= 54°$$

(Angles at a point on a straight line total 180^0).

$$H\hat{G}F = 180° - (54° + 44°)$$
$$= 82°$$

(The sum of the angles in a triangle = 180°).

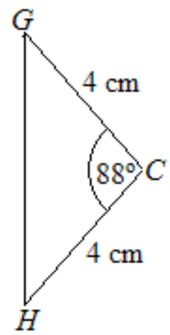
$$D\hat{G}H = 180° - 82°$$
$$= 98°$$

(Angles at a point on a straight line totals 180^0).

$D\hat{E}F = 180° - 98°$
$= 82°$
(The opposite angles of a cyclic quadrilateral are supplementary).

b. **Data:** $GC = 4$ cm

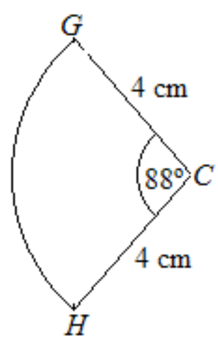
(i) **Required to calculate:** Area of ΔGCH
Calculation:



We know two sides and the included angle.

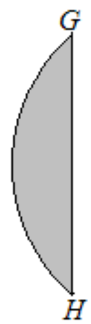
So, area of $\Delta GCH = \frac{1}{2}(4)(4)\sin 88°$
$= 7.995\text{ cm}^2$
$= 8.00\text{ cm}^2$

(ii) **Required to calculate:** Area of the minor sector *GCH*.
Calculation:



The area of the minor sector $GCA = \frac{88°}{360°} \times 3.14(4)^2$
$= 12.28\underline{0}\text{ cm}^2$
$= 12.28\text{ cm}^2$

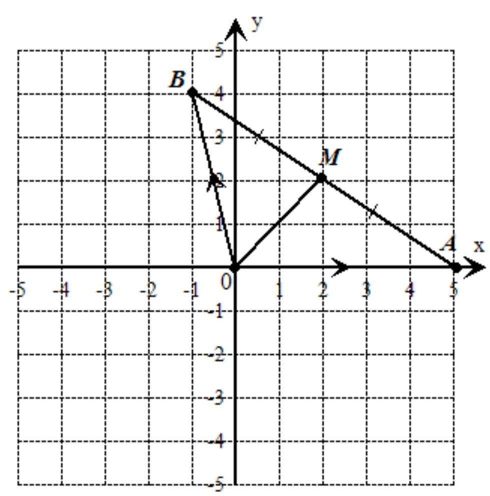
(iii) **Required to calculate:** Area of the shaded segment.
Calculation:



The area of the shaded segment = The area of sector GCH – The area of ΔGCH

$$= 12.280 - 7.995$$
$$= 4.28\underline{5}\ \text{cm}^2$$
$$= 4.29\ \text{cm}^2$$

13. a. **Data:** Vector diagram as shown below.



(i) (a) **Required to express:** $\overrightarrow{OB}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

Solution:
If,
$B = (-1,\ 4)$

$$\overrightarrow{OB} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

This is of the form $\begin{pmatrix} a \\ b \end{pmatrix}$, where $a = -1$ and $b = 4$.

(b) **Required to express:** $\overrightarrow{OA} + \overrightarrow{OB}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

Solution:

Applying the vector triangle law

$$OA + OB = \begin{pmatrix} 5 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$
$$= \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

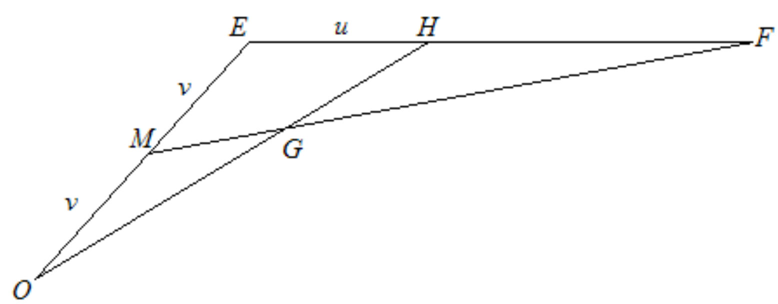
which is of the form $\begin{pmatrix} a \\ b \end{pmatrix}$, where $a = 4$ and $b = 4$.

(ii) **Required to calculate:** Midpoint of AB.
Calculation:
Using the midpoint formula

$$M = \left(\frac{-1+5}{2}, \frac{4+0}{2} \right)$$
$$= (2, 2)$$

is of the form (x, y), where $x = 2$ and $y = 2$.

b. **Data:** Diagram with OE, EF and MF are straight lines. $EF = 3EH$, $MF = 5MG$ and M is the midpoint of OE. $\overrightarrow{OM} = v$ and $\overrightarrow{EH} = u$.



(i) (a) **Required to express:** $\overrightarrow{HF}$ in terms of u and v.
Solution:

$$\overrightarrow{EF} = 3\overrightarrow{EH}$$
$$\overrightarrow{EF} = 3u$$
$$\overrightarrow{HF} = 3u - u$$
$$= 2u$$

(b) **Required to express:** $\overrightarrow{MF}$ in terms of u and v.
Solution:
(Since M is the midpoint of OE, then $\overrightarrow{ME} = \overrightarrow{OM} = v$).
Using the vector triangle law, we get

$\overrightarrow{MF} = \overrightarrow{ME} + \overrightarrow{EF}$
$= v + 3u$

(c) **Required to express:** $\overrightarrow{OH}$ in terms of u and v.
Solution:
$\overrightarrow{OE} = v + v$
$= 2v$
$\overrightarrow{OH} = \overrightarrow{OE} + \overrightarrow{EH}$
$= 2v + u$

(ii) **Required to prove:** $\overrightarrow{OG} = \frac{3}{5}(2v + u)$

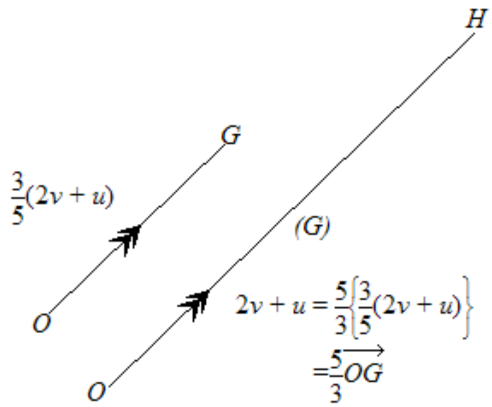
Proof:
Using the vector triangle law
$\overrightarrow{OG} = \overrightarrow{OM} + \overrightarrow{MG}$
$= v + \overrightarrow{MG}$

If $\overrightarrow{MF} = 5\overrightarrow{MG}$, then $\overrightarrow{MG} = \frac{1}{5}\overrightarrow{MF}$
$= \frac{1}{5}(v + 3u)$

$\therefore \overrightarrow{OG} = v + \frac{1}{5}(v + 3u)$
$= v + \frac{1}{5}v + \frac{3}{5}u$
$= \frac{3}{5}u + \frac{6}{5}v$
$= \frac{3}{5}(u + 2v)$

Q.E.D.

(iii) **Required to prove:** O, G and H lie on a straight line.
Solution:



$\overrightarrow{OG}=\frac{3}{5}(2v+u)$

$\overrightarrow{OH}=2v+u$

$\quad=\frac{5}{3}\overrightarrow{OG}$

Since $\overrightarrow{OG}$ is a scalar multiple, which is $\frac{5}{3}$ of $\overrightarrow{OH}$, then $\overrightarrow{OG}$ and $\overrightarrow{OH}$ are parallel.

Since O is a common point on both lines then, G lies on $\overrightarrow{OH}$ (as shown on the diagram). Therefore, O, G and H all lie on the same straight line and are collinear.

Q.E.D.

14. a. **Data:** $L=\begin{pmatrix}3 & 2\\ 1 & 4\end{pmatrix}$ and $N=\begin{pmatrix}-1 & 3\\ 0 & 2\end{pmatrix}$

Required to calculate: $L-N^2$

Calculation:

We will multiply N by N to obtain N^2 and subtract the product from L

$$N^2 = \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$$

$$\begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix} = \begin{pmatrix} (-1\times -1)+(3\times 0) & (-1\times 3)+(3\times 2) \\ (0\times -1)+(2\times 0) & (0\times 3)+(2\times 2) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$$

$$L - N^2 = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 3-1 & 2-3 \\ 1-0 & 4-4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$$

b. **Data:** $M = \begin{pmatrix} x & 12 \\ 3 & x \end{pmatrix}$ is a singular matrix.

Required to calculate: x
Calculation:

$$M = \begin{pmatrix} x & 12 \\ 3 & x \end{pmatrix}$$

If M is singular, then the $\det M = 0$.
Hence,

$$(x \times x) - (3 \times 12) = 0$$
$$x^2 - 36 = 0$$
$$x^2 = 36$$
$$x = \pm 6$$

c. **Data:** (2, 1) is mapped onto (-1, 2) by $R = \begin{pmatrix} 0 & p \\ q & 0 \end{pmatrix}$.

Required to calculate: The value of p and of q.
Calculation:

$$\begin{pmatrix}2\\1\end{pmatrix} \xrightarrow{\begin{pmatrix}0 & p\\ q & 0\end{pmatrix}} \begin{pmatrix}-1\\2\end{pmatrix}$$

$$\begin{pmatrix}0 & p\\ q & 0\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}(0\times 2)+(p\times 1)\\ (q\times 2)+(0\times 1)\end{pmatrix}$$

$$= \begin{pmatrix}p\\2q\end{pmatrix}$$

$$\begin{pmatrix}p\\2q\end{pmatrix} = \begin{pmatrix}-1\\2\end{pmatrix}$$

Both are 2 x 1 matrices and are equal. So, equating corresponding entries, we obtain

$p=-1$ and $2q=2$

$q=1$

d. **Data:** (5, 3) is mapped onto (1,1) by $T=\begin{pmatrix}r\\s\end{pmatrix}$.

Required to calculate: The value of *r* and of *s*.

Calculation:

$$\begin{pmatrix}5\\3\end{pmatrix} \xrightarrow{T=\begin{pmatrix}r\\s\end{pmatrix}} \begin{pmatrix}1\\1\end{pmatrix}$$

$$\therefore \begin{pmatrix}5+r\\3+s\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$$

Both are 2 x 1 matrices and are equal. So, equating corresponding entries, we obtain

$5+r=1$

$r=-4$

and

$3+s=1$

$s=-2$

e. **Data:** The point (8, 5) undergoes the transformations R followed by *T*.

Required to calculate: The image of (8, 5).

Calculation:

Let

$$\begin{pmatrix}8\\5\end{pmatrix} \xrightarrow{R} \begin{pmatrix}x\\y\end{pmatrix}$$

Firstly we transform under *R*.

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 8 \\ 5 \end{pmatrix} = \begin{pmatrix} (0\times 8)+(-1\times 5) \\ (1\times 8)+(0\times 5) \end{pmatrix}$$

$$= \begin{pmatrix} -5 \\ 8 \end{pmatrix}$$

$$\therefore (x,\ y) = (-5,\ 8)$$

Secondly we transform under *T*

$$\begin{pmatrix} -5 \\ 8 \end{pmatrix} \xrightarrow{T=\begin{pmatrix} -4 \\ -2 \end{pmatrix}} \begin{pmatrix} -5+(-4) \\ 8+(-2) \end{pmatrix} = \begin{pmatrix} -9 \\ 6 \end{pmatrix}$$

Therefore, the image of (8, 5) after the combined transformations *R* followed by *T* is (-9, 6).