JUNE 2009 CXC MATHEMATICS GENERAL PROFICIENCY (PAPER 2)

Section I

1. a. (i) **Required To Calculate:** $\dfrac{2\frac{2}{3}+1\frac{1}{5}}{6\frac{2}{5}}$

Calculation:
Numerator:

$$2\frac{2}{3}+1\frac{1}{5}$$

$$3\frac{5(2)+3(1)}{15}=3\frac{13}{15}$$

$$\frac{\text{Numerator}}{\text{Denominator}}=\frac{3\frac{13}{15}}{6\frac{2}{5}}$$

$$=\frac{\frac{58}{15}}{\frac{32}{5}}$$

$$=\frac{58}{15}\times\frac{5}{32}$$

$$=\frac{58}{96}$$

$$=\frac{29}{48}(exact)$$

(ii) **Required To Calculate:** $\sqrt{\left(\dfrac{0.0256}{25}\right)}$

Calculation:

$$\sqrt{\left(\frac{0.0256}{25}\right)}=0.032$$

$$=3.2\times10^{-2} \text{ in standard from}$$

b. **Data:** Truck driver earns \$560 for a 40 hour work week.

(i) **Required To Calculate:** His hourly rate.

Calculation:

$$\text{Hourly rate of the driver} = \frac{\$560}{40}$$
$$= \$14 \text{ per hour}$$

(ii) **Data:** Overtime is paid at one and a half times the basic rate.

Required To Calculate: Overtime wage for 10 hours of overtime.

Calculation:

$$\text{Overtime rate} = 1\frac{1}{2} \times \$14 \text{ per hour}$$
$$= \$21 \text{ per hour}$$

$$\text{Overtime wage for 10 hours of overtime} = 21 \times 10$$
$$= \$210$$

(iii) **Required To Calculate:** Total wages earned by driver for a 55 hour week.

Calculation:

For a 55 hour work week, total wages

= \$560 for the first 40 hours + $(\$21 \times 15)$ for the next 15 hours of overtime

$$= \$560 + (\$21 \times 15)$$
$$= \$560 + \$315$$
$$= \$875$$

2. a. **Required To Factorise:** (i) $2ax + 3ay - 2bx - 3by$, $(ii)\,5x^2 - 20$ and $(iii)\,3x^2 + 4x - 15$.

Solution:

(i) $2ax + 3ay - 2bx - 3by = a(2x + 3y) - b(2x + 3y)$

$= (2x + 3y)(a - b)$

(ii) $5x^2 - 20 = 5(x^2 - 4)$

$= 5\{(x)^2 - (2)^2\}$

(This is a difference of 2 squares)

$= 5(x - 2)(x + 2)$

(iii) $3x^2 + 4x - 15 = (3x - 5)(x + 3)$

b. **Data:** Cost of biscuits at $\$x$ per pack and ice cream at $\$y$ per cup.

(i) **Required To Find:** Pair of simultaneous equations in x and y to represent the information given.

Solution:

1 pack of biscuits and 2 cups of ice cream cost $\$\{(1 \times x) + (2 \times y)\}$

Hence, $x + 2y = 8$ (data)

Similarly, 3 packs of biscuits and 1 cup of ice cream cost $\$\{(3 \times x) + (1 \times y)\}$.

Hence, $3x + y = 9$

(ii) **Required To Solve:** Equations to find x and y.

Solution:

Let

$x + 2y = 8 \quad \ldots(1)$

$3x + y = 9 \quad \ldots(2)$

From equation (1)

$x = 8 - 2y$

Substitute in (2)

$$3(8 - 2y) + y = 9$$
$$24 - 6y + y = 9$$
$$24 - 5y = 9$$
$$24 - 9 = 5y$$
$$5y = 15$$
$$y = 3$$

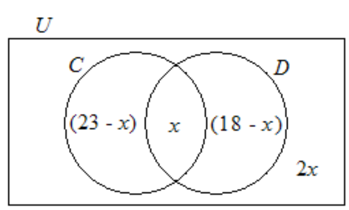
Substitute $y = 3$ $\quad x = 8 - 2(3)$
$\quad = 2$

$\therefore$ 1 pack of biscuits costs \$2 and 1 cup of ice cream costs \$3.

3. a. **Data:** Survey results of 50 students who owned cellular phones and digital cameras.

(i) **Required To Complete:** Venn diagram to represent the information given.

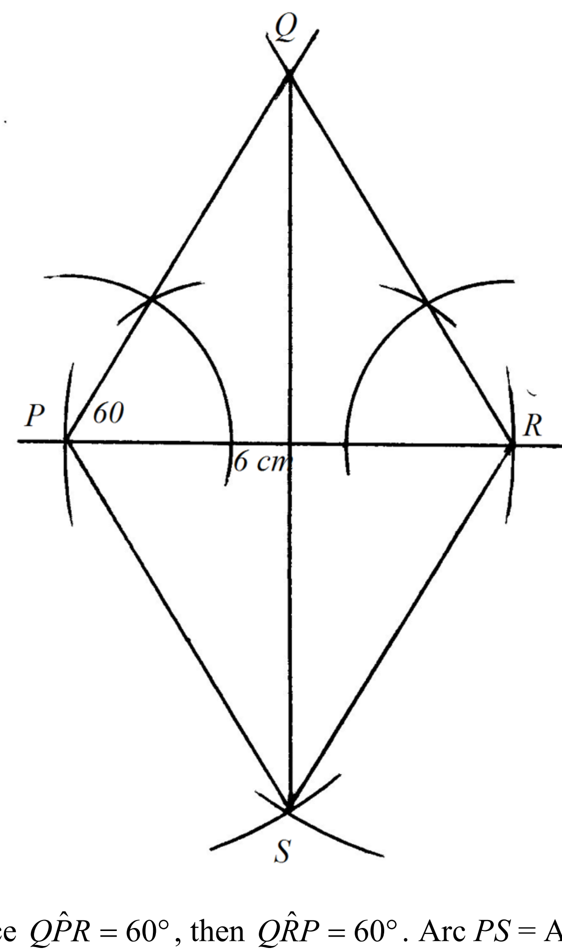
Solution:



(ii) **Required To Find:** Expression in terms of x for the total number of students in the survey.
Solution:
Total number of students in the survey
$= (23 - x) + x + (18 - x) + 2x$
$= 41 + x$

(iii) **Required To Calculate:** x
Calculation:
$41 + x = 50$ (data)
$x = 50 - 41$
$= 9$

b. (i) **Required To Construct:** Rhombus *PQRS*, with diagonal *PR* = 6 cm and $R\hat{P}Q = 60°$.
Solution:



Since $Q\hat{P}R = 60°$, then $Q\hat{R}P = 60°$. Arc *PS* = Arc RS = Arc *PQ*.

(ii) **Required To Find:** Length of *QS*.
Solution:
PS = 10.4 cm (by measurement).

4. a. **Data:** Table showing the readings of an aircraft flight record.

(i) **Required To Calculate:** Distance travelled in km.
Calculation:
The distance travelled in km = Final reading – Previous reading

$$= 1083 - 957$$

$$= 126 \text{ km}$$

(ii) **Required To Calculate:** Average speed of the aircraft in kmh^{-1}
Calculation:

$$\text{The time of flight } = \begin{array}{r} 09:07 \\ -\ \underline{08:55} \\ \underline{\quad 12} \end{array} \text{ minutes}$$

$$= \frac{12}{60} \text{ hours}$$

$$\text{Hence, average speed } = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{126}{\frac{12}{60}}$$

$$= 630 \text{ kmh}^{-1}$$

b. **Data:** Map with a scale of 1 : 50 000 showing two points *L* and *M*.

(i) **Required To Measure:** Distance on the map from *L* to *M*.
Solution:
Distance *LM* on the map = 7.0 cm (by measurement)

(ii) **Required To Calculate:** Actual distance from *L* to *M*.
Calculation:

$$\text{Actual distance from L to M } = \frac{7 \times 50\,000}{1000 \times 100} \text{ km}$$

$$= 3.5 \text{ km}$$

(iii) **Data:** Actual distance on the map between 2 points is 4.5 km.
Required To Calculate: Distance on the map between the 2 points.

Calculation:

Actual distance between 2 point = 4.5 km $= 4.5 \times 1000 \times 100$ cm

Distance between 2 points on map $= (4.5 \times 1000 \times 100) \div 50\,000$
$= 9$ cm

5. a. **Data:** $f(x) = 2x - 5$ and $g(x) = x^2 - 31$

(i) **Required To Calculate:** $f(-2)$.

Calculation:

$$\begin{aligned} f(-2) &= 2(-2) - 5 \\ &= -4 - 5 \\ &= -9 \end{aligned}$$

(ii) **Required To Calculate:** $gf(1)$.

Calculation:

$$\begin{aligned} f(1) &= 2(1) - 5 \\ &= 2 - 5 \\ &= -3 \end{aligned}$$

$$\begin{aligned} gf(1) &= g(-3) \\ &= (-3)^2 - 31 \\ &= 9 - 31 \\ &= -22 \end{aligned}$$

(iii) **Required To Calculate:** $f^{-1}(3)$.

Calculation:

Let

$$y = 2x - 5$$
$$y + 5 = 2x$$
$$x = \frac{y+5}{2}$$

Replace y by x

$f^{-1}(x) = \frac{x+5}{2}$ Hence, $f^{-1}(3) = (3 + 5) / 2 = 4$

b. **Data:** $y = x^2 + 2x - 3$ and an incomplete table of x and y values.

(i) **Required To Complete:** Table of x and y values.

Solution:

x	-4	-3	-2	-1	0	1	2
y	5	(0)	-3	-4	-3	(0)	5

When $x = -3$ $\quad y = (-3)^2 + 2(-3) - 3$

$= 9 - 6 - 3$

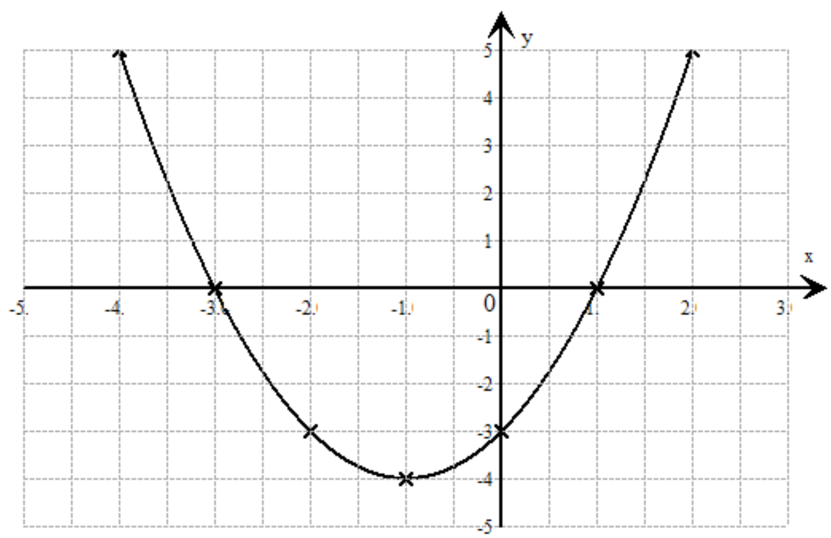
$= 0$

When $x = 1$ $\quad y = (1)^2 + 2(1) - 3$

$= 0$

(ii) **Required To Draw:** Graph of $y = x^2 + 2x - 3$ for $-4 \le x \le 2$.

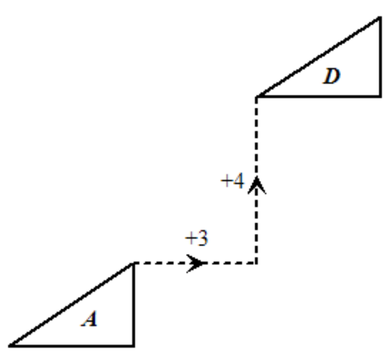
Solution:



6. **Data:** Diagram showing triangles *A*, *B* and *D*.

a. (i) **Required To Find:** Transformation which maps triangle *A* onto triangle *D*.

Solution:

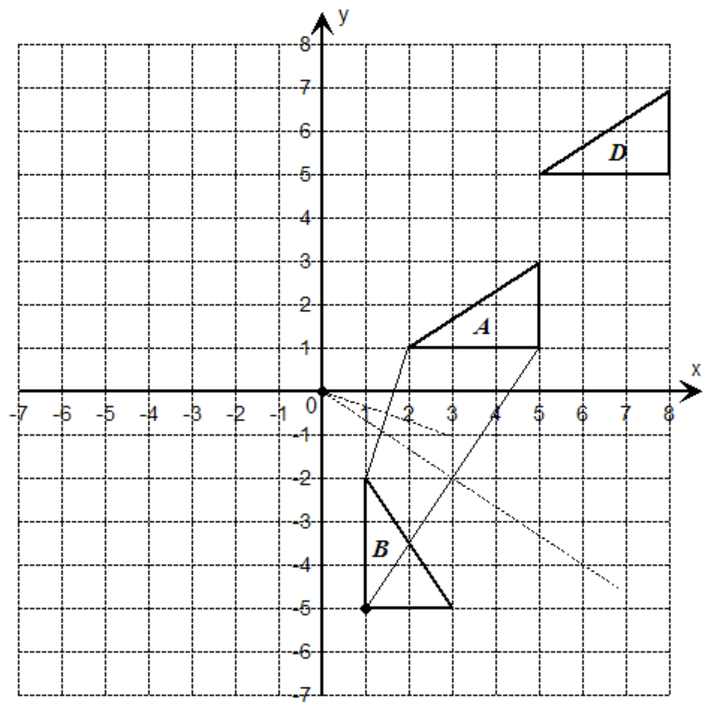


Triangle A is mapped onto triangle D by a horizontal shift of 3 units to the right and 4 units vertically upwards. This may be represented by the translation, T, where $T = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

(ii) **Required To Find:** Transformation which maps triangle A onto triangle B.

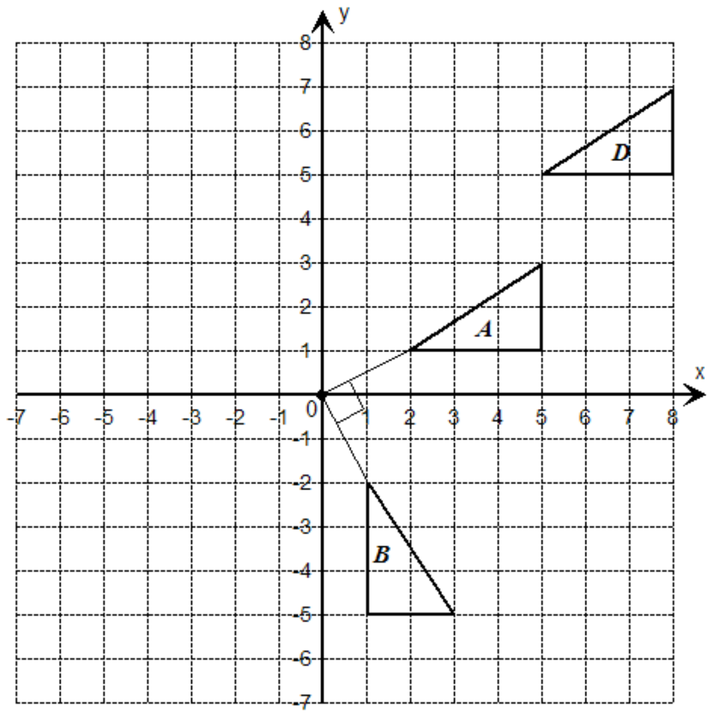
Solution:

Since orientation changes, B is a rotation of A.



The above diagram shows how to obtain the centre of rotation.

Join two pairs of corresponding image and object points.
Then construct the perpendicular bisector of each line.
The point at which the two bisectors meet is the center of rotation.
The origin is therefore the center of rotation.



To obtain the angle of rotation measure the angle *AOB*.

Triangle *A* is mapped onto triangle *B* by a rotation of 90° clockwise about *O*.

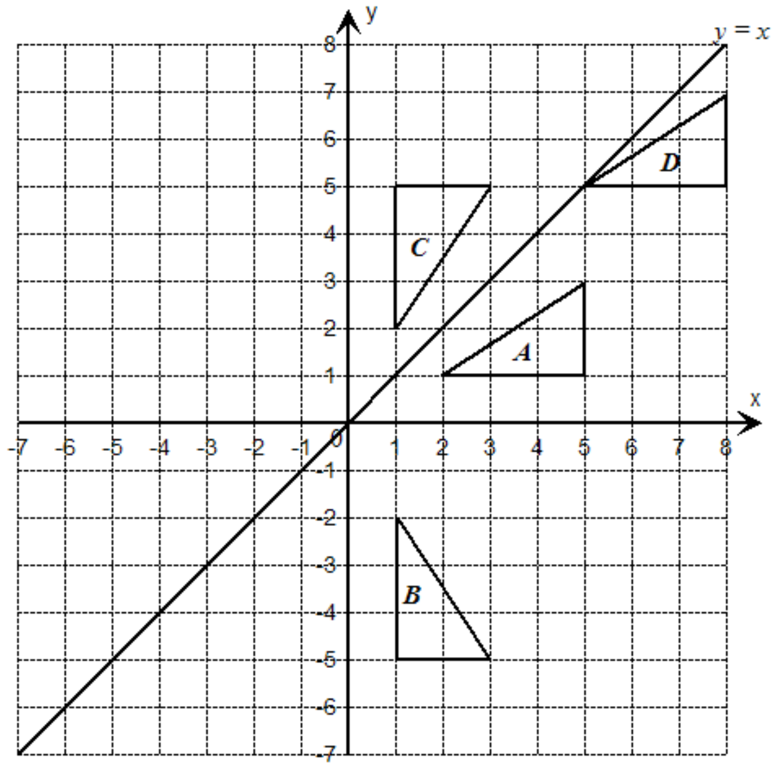
$$A \xrightarrow{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} B$$

(iii) **Required To Find:** Coordinates of the vertices of triangle *C*.
Solution:
Triangle *A* is mapped onto triangle *C* after a reflection in the line $y = x$.

$$A \xrightarrow{\text{Reflection in } y = x} C$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 5 & 5 \\ 1 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 5 \end{pmatrix}$$

The vertices of *C* are (1, 2), (1, 5) and (3, 5).



7. **Data:** Table showing the waiting time of students at a school canteen.

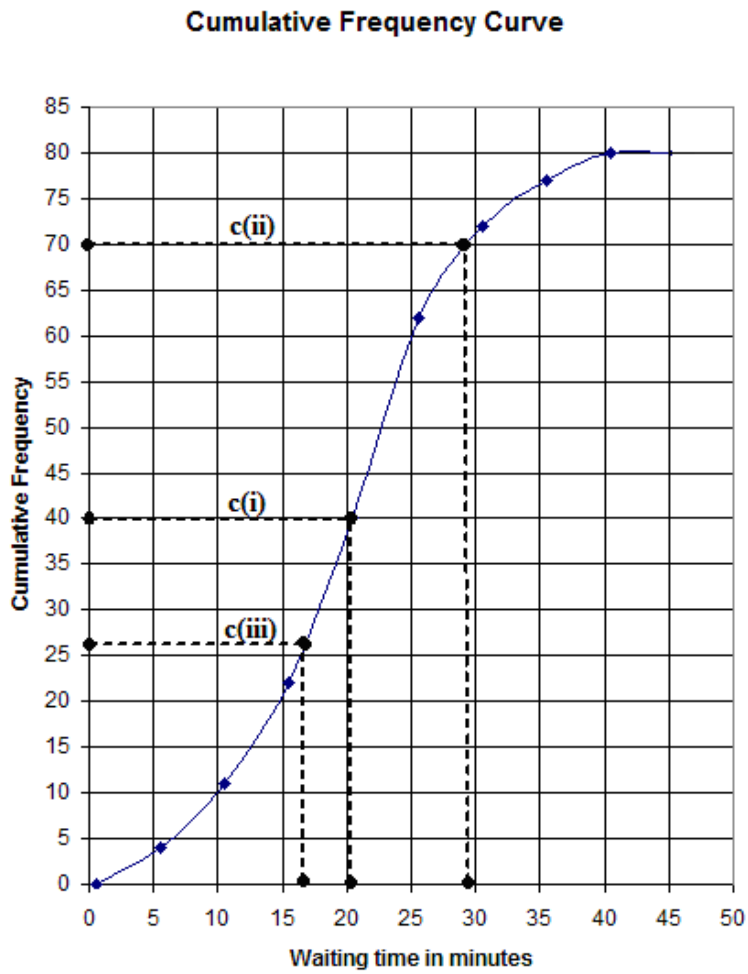
a. **Required To Complete:** Table showing cumulative frequency.
Solution:

The data is that of a continuous variable.

Waiting time in minutes, t	L.C.B U.C.B.	No. of students, f	Cumulative frequency	Points to be plotted (U.C.B., C.F.)
				(0.5, 0)
1 – 5	$0.5 \le t < 5.5$	4	4	(5.5, 4)
6 – 10	$5.5 \le t < 10.5$	7	11	(10.5, 11)
11 – 15	$10.5 \le t < 15.5$	11	22	(15.5, 22)
16 – 20	$15.5 \le t < 20.5$	18	40	(20.5, 40)
21 – 25	$20.5 \le t < 25.5$	22	62	(25.5, 62)
26 – 30	$25.5 \le t < 30.5$	10	72	(30.5, 72)
31 – 35	$30.5 \le t < 35.5$	5	77	(35.5, 77)
36 – 40	$35.5 \le t < 40.5$	3	80	(40.5, 80)

The point (0.5, 0) is obtained by extrapolation so that the curve starts from the horizontal axis.

b. **Required To Complete:** Cumulative frequency curve.
Solution:



c. (i) **Required To Find:** Median for the data:
Solution:
From the graph, or the table of values, the median for the data = 20.5 minutes.

(ii) **Required To Find:** Number of students who waited for no more than 29 minutes.
Solution:
From the graph, 70 students waited no longer than 29 minutes.

(iii) **Required To Calculate:** Probability a randomly chosen student waited no more than 17 minutes.
Solution:

$$P(\text{Student waited} \leq 17 \text{ minutes}) = \frac{\text{No. of students who waited} \leq 17\text{mins}}{\text{Total no. of students}}$$

$$= \frac{27}{80}$$

8. **Data:** Sequence of diagrams.
Required To Draw: Next diagram in the sequence.
Solution:
Answer Sheet for Question 8

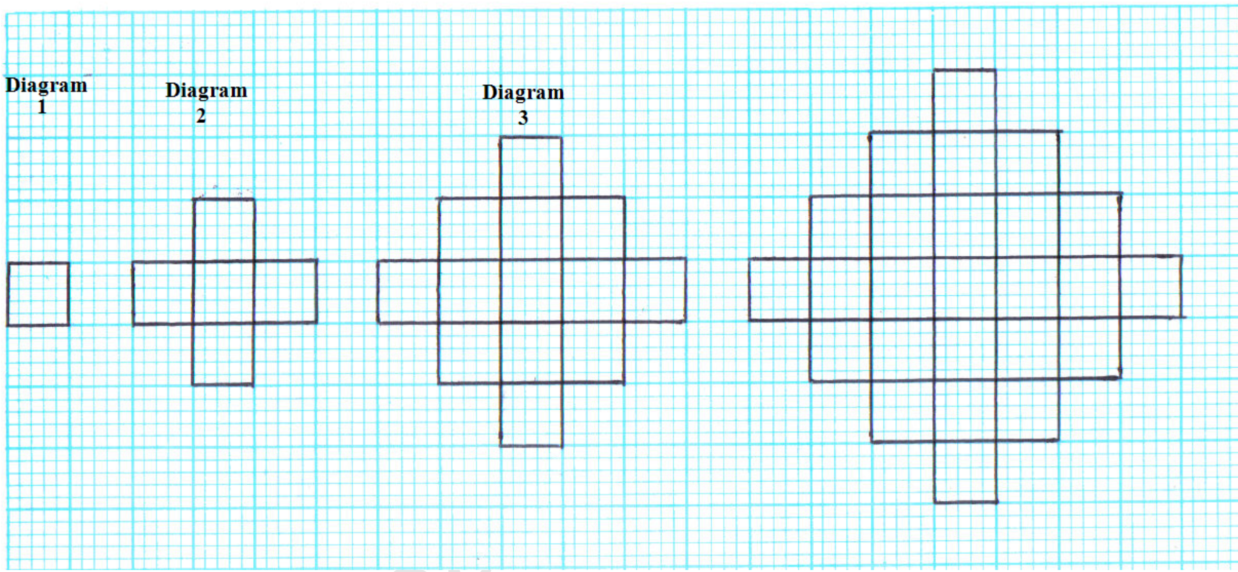


Diagram Number	Number of Unit Squares	Pattern for Calculating Number of Unit Squares
1	1	$1^2 + 0^2$
2	5	$2^2 + 1^2$
3	13	$3^2 + 2^2$
4	25	$4^2 + 3^2$
10	181	$10^2 + 9^2$
15	421	$15^2 + 14^2$

We notice that the expression in column 3 is the square of the number in column 1 added to the square of the number in column 1 of the previous row.
Column 2 is the result of column 3.

(i) **Required To Complete:** The table given for diagram 4.

Solution:
Column 3 should be $4^2 + 3^2$

Column 2 $= 4^2 + 3^2$
$= 16 + 9$
$= 25$

(ii) **Required To Complete:** The table given for diagram 10.

Solution:
Column 3 should be $10^2 + 9^2$
Column 2 $= 10^2 + 9^2$
$= 100 + 81$
$= 181$

(iii) **Required To Complete:** The table given for diagram with 421 unit squares.

Solution:
Let column 1 be *n*.
Therefore, column 3 will be $n^2 + (n-1)^2$.

$$n^2 + (n-1)^2 = 421$$
$$n^2 + n^2 - 2n + 1 = 421$$
$$2n^2 - 2n - 420 = 0$$
$$n^2 - n - 210 = 0$$

$$(n-15)(n+14) = 0$$
$$n = 15 \text{ or } -14$$
$$n \neq -\text{ve}$$
$$\therefore n = 15$$

Hence, in (iii), column 1 is 15, and column 3 is $15^2 + 14^2$.

Section II

9. a. **Data:** $y = 4 - 2x$ and $y = 2x^2 - 3x + 1$.

Required To Calculate: x and y.

Calculation:

Let

$y = 4 - 2x$ …(1)

and $y = 2x^2 - 3x + 1$ …(2)

Equating (1) and (2)

$$4 - 2x = 2x^2 - 3x + 1$$

$$2x^2 - 3x + 1 - 4 + 2x = 0$$

$$2x^2 - x - 3 = 0$$

$$(2x - 3)(x + 1) = 0$$

$$x = -1 \text{ or } 1\frac{1}{2}$$

When $x = -1$ $\quad y = 4 - 2(-1)$
$= 6$

When $x = 1\frac{1}{2}$ $\quad y = 4 - 2\left(1\frac{1}{2}\right)$
$= 1$

Hence, $x = -1$ and $y = 6$ **OR** $x = 1\frac{1}{2}$ and $y = 1$.

b. **Required To Express:** $2x^2 - 3x + 1$ in the form $a(x + h)^2 + k$.

Solution:

$$a(x + h)^2 + k = a(x^2 + 2hx + h^2) + k$$
$$= ax^2 + 2ahx + ah^2 + k$$

Equating coefficient of x^2.

$a = 2 \in \Re$

Equating coefficient of x.

$-3 = 2(2)h$

$h = -\frac{3}{4} \in \Re$

Equating constants.

$$2\left(-\frac{3}{4}\right)^2 + k = 1$$

$$\frac{9}{8} + k = 1$$

$$k = -\frac{1}{8} \in \Re$$

And $2x^2 - 3x + 1 \equiv 2\left(x - \frac{3}{4}\right)^2 - \frac{1}{8}$.

OR

$$2x^2 - 3x + 1 = 2\left(x^2 - \frac{3}{2}x\right) + 1$$

Half the coefficient of x is $\frac{1}{2}\left(-\frac{3}{2}\right) = -\frac{3}{4}$.

$$= 2\left(x - \frac{3}{4}\right)^2 + *$$

$$2\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) = 2x^2 - 3x + 1\frac{1}{8}$$

$$\begin{array}{r} -\ \frac{1}{8} = * \\ \hline 1 \\ \hline \end{array}$$

Hence, $2x^2 - 3x + 1 \equiv 2\left(x - \frac{3}{4}\right)^2 - \frac{1}{8}$ is of the form $a(x+h)^2 + k$, where $a = 2 \in \Re$, $h = -\frac{3}{4} \in \Re$ and $k = -\frac{1}{8} \in \Re$.

c. (i) **Required To Find:** The minimum value of $2x^2 - 3x + 1$.

Solution:

Let

$$y = 2x^2 - 3x + 1$$

$$= 2\left(x - \frac{3}{4}\right)^2 - \frac{1}{8}$$

$$2\left(x - \frac{3}{4}\right)^2 \geq 0 \qquad \forall x$$

$$\therefore y_{\min} = 0 - \frac{1}{8}$$

$$= -\frac{1}{8}$$

(ii) **Required To Find:** x at which minimum value occurs.

Solution:

At

$$2\left(x-\frac{3}{4}\right)^2=0$$

$$\left(x-\frac{3}{4}\right)^2=0$$

$$x=\frac{3}{4}$$

Minimum value of $2x^2-3x+1$ is $-\frac{1}{8}$ at $x=\frac{3}{4}$.

OR

The minimum value of $2x^2-3x+1$ occurs at

$$x=\frac{-(-3)}{2(2)}$$

$$=\frac{3}{4}$$

When $x=\frac{3}{4}$ the minimum value is

$$2\left(\frac{3}{4}\right)^2-3\left(\frac{3}{4}\right)+1=2\left(\frac{9}{16}\right)-2\frac{1}{4}+1$$

$$=1\frac{1}{8}-2\frac{1}{4}+1$$

$$=2\frac{1}{8}-2\frac{1}{4}$$

$$=-\frac{1}{8}$$

d. **Required To Sketch:** The graph of $2x^2-3x+1$ showing coordinates of the minimum point, the y intercept and the values of x where the graph cuts the x – axis.

Solution:

$$y=2x^2-3x+1$$

$$=2\left(x-\frac{3}{4}\right)^2-\frac{1}{8}$$

Let

$$2\left(x-\frac{3}{4}\right)^2-\frac{1}{8}=0$$

$$2\left(x-\frac{3}{4}\right)^2=\frac{1}{8}$$

$$\left(x-\frac{3}{4}\right)^2=\frac{1}{16}$$

$$x-\frac{3}{4}=\pm\frac{1}{4}$$

$$x=\frac{3}{4}\pm\frac{1}{4}$$

$$=1 \text{ or } \frac{1}{2}$$

OR

$$2x^2-3x+1=0$$

$$(2x-1)(x-1)=0$$

$$x=\frac{1}{2} \text{ or } 1$$

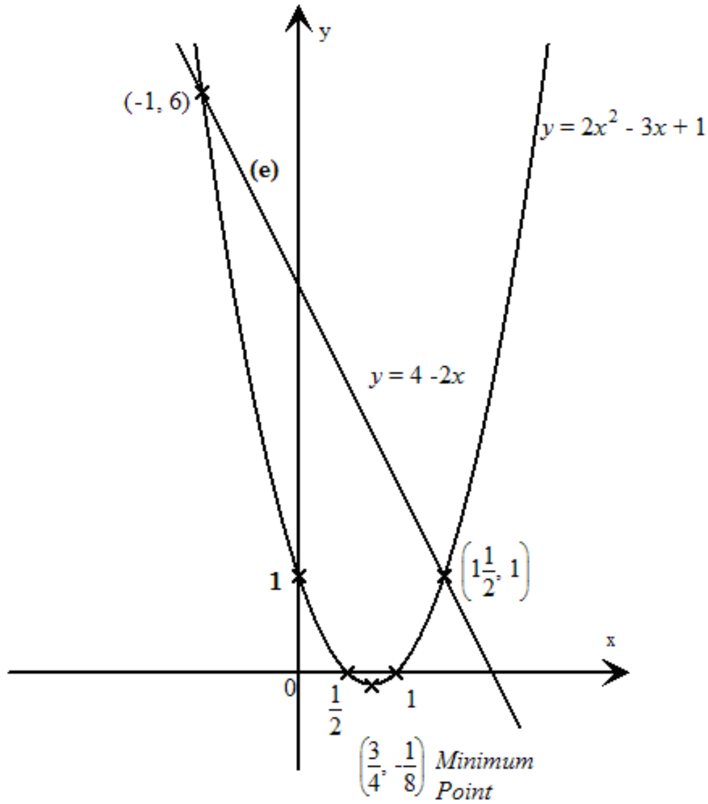
and the curve cuts the $x-$axis at 1 and $\frac{1}{2}$.

When $x=0$ $\quad y=2(0)^2-3(0)+1$
$\quad =1$

y cuts the $y-$axis at 1.

$y_{\min}=-\frac{1}{8}$ at $x=\frac{3}{4}$.

$\therefore \left(\frac{3}{4},-\frac{1}{8}\right)$ is the minimum point of $y=2x^2-3x+1$.



10. **Data:** Conditions involving a shop's demands when buying x guitars and y violins.

a. (i) **Required To Find:** The inequality to represent the information given.
Solution:
No. of violins must be less or equal to the no. of guitars.
$y \le x$ …(1)

(ii) **Data:** Cost of one guitar is \$150 and the cost of one violin is \$300.
Required To Find: The inequality to represent the information given.
Solution:
The cost of x guitars at \$150 each and y violins at \$300 each is
$(\$150 \times x) + (\$300 \times y)$.
Maximum amount to spend is \$4 500.
$\therefore 150x + 300y \le 4\,500$
$\div 150$
$$x + 2y \le 30 \quad ...(2)$$

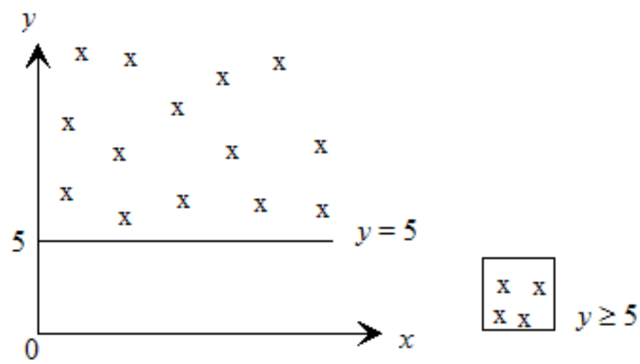
(iii) **Data:** Owner of the shop must buy at least 5 violins.
Required To Find: The inequality to represent the information given.
Solution:
The owner must buy at least 5 violins.
$\therefore y \ge 5$ …(3)

b. (i)-(ii) **Required To Draw:** The graphs of the lines associated with the inequalities.

Solution:

The line $y = 5$ is a straight horizontal line.

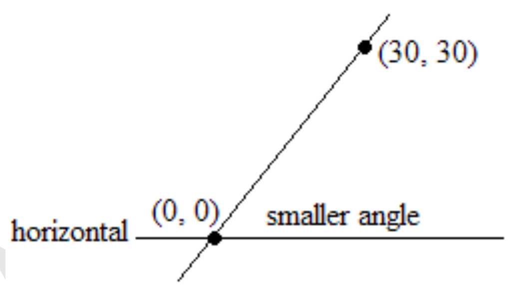
The region which satisfies $y \geq 5$ is



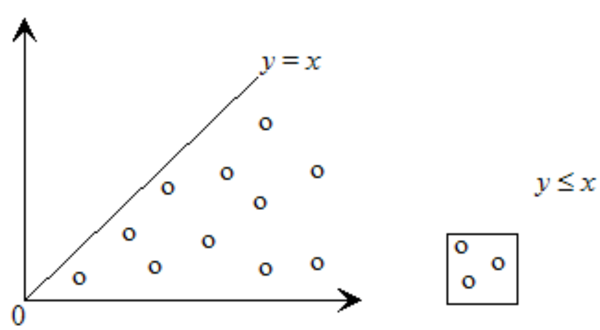
Obtaining two points on the line $y = x$.

x	y
0	0
30	30

The region with the smaller angle represents the $\leq$ region.



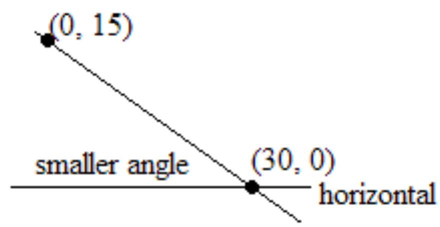
The region which satisfies $y \leq x$ is



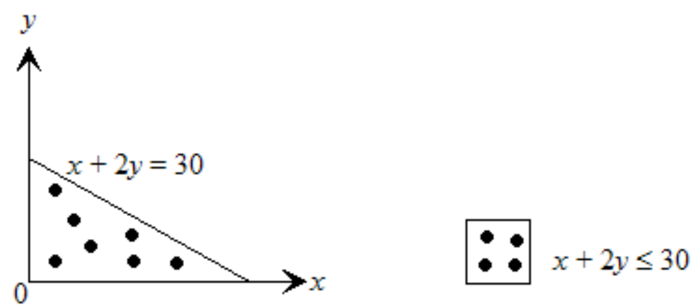
Obtaining 2 points on the line $x + 2y = 30$.

x	y
0	15
30	0

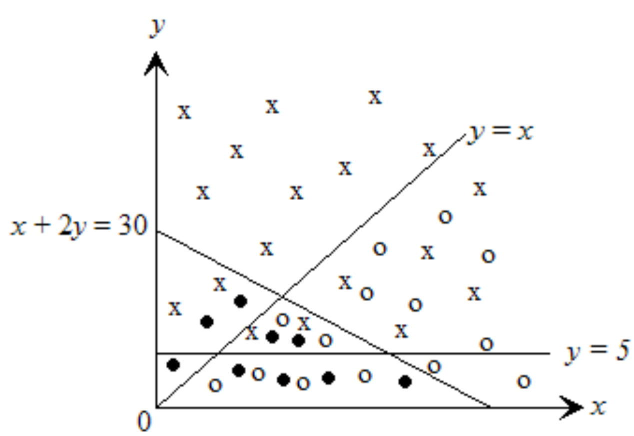
The region with the smaller angle represents the $\leq$ region.

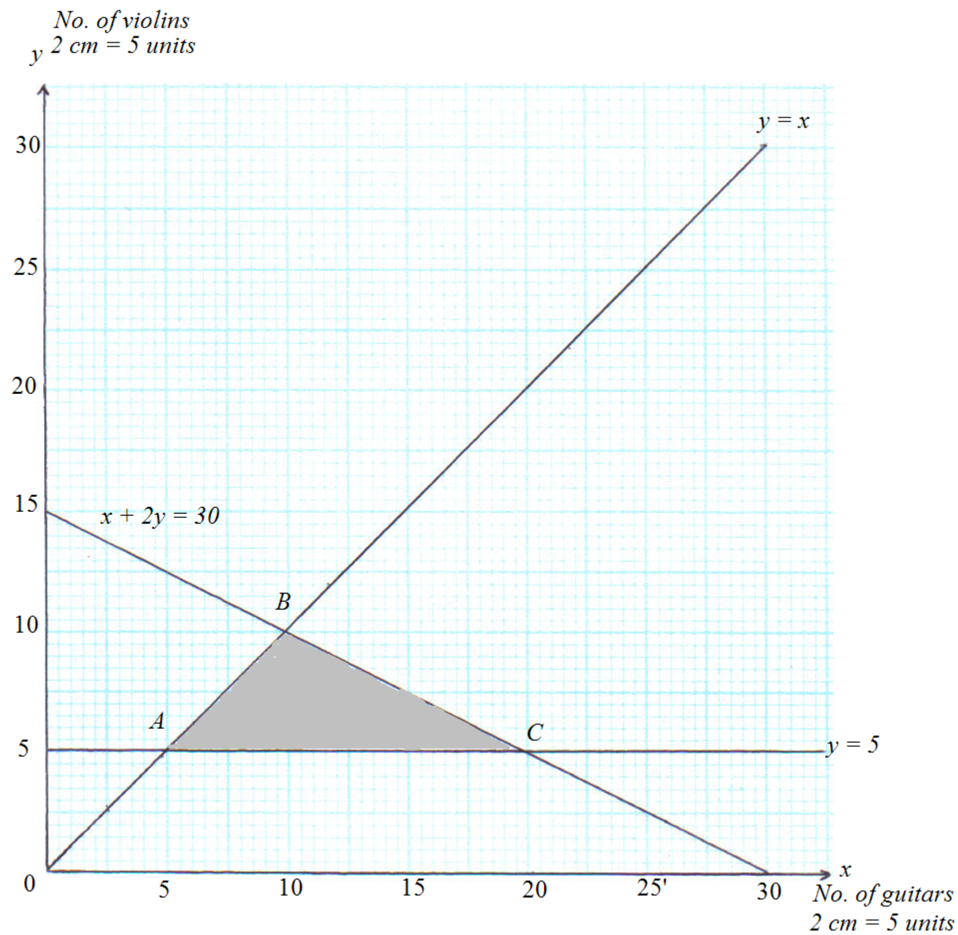


The region which satisfies $x + 2y \leq 30$ is



The feasible region is the area in which all three previously shaded regions overlap.





(iii) **Required To State:** The coordinates of the vertices of the shaded region.
Solution:
The vertices of the region that satisfies all the inequalities in *ABC* (the feasible region).
A (5, 5) *B* (10, 10) *C* (20, 5)

c. **Data:** Profit of \$60 made on each guitar and a profit of \$100 made on each violin.

(i) **Required To Express:** The total profit in terms of *x* and *y*.
Solution:
Let the profit on *x* guitars at \$60 each and *y* violins at \$100 each be *P*.

$\therefore P = (60 \times x) + (100 \times y)$

$P = 60x + 100y$

(ii) **Required To Calculate:** Maximum profit.
Calculation:
Testing (10, 10)

$P = 60(10) + 100(10)$

$= \$1600$

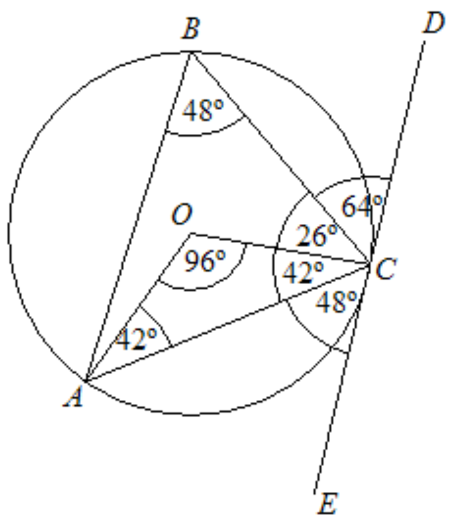
Testing (20, 5)

$P = 60(20) + 100(5)$

$= \$1700$

Maximum profit = \$1 700 when shop sells 20 guitars and 5 violins.

11. a. **Data:** Diagram showing a circle with centre *O*. Tangent *DCE* touches the circle at *C*. $A\hat{C}E = 48°$ and $O\hat{C}B = 26°$.



(i) **Required To Calculate:** $A\hat{B}C$

Calculation:

$A\hat{B}C = 48°$

(The angle made by a tangent to a circle and chord, at the point of contact is equal to the angle in the alternate segment).

(ii) **Required To Calculate:** $A\hat{O}C$

Calculation:

$A\hat{O}C = 2(48°)$

$= 96°$

(The angle subtended by a chord (*AC*) at the centre of a circle equals twice the angle it subtends at the circumference, standing on the same arc).

(iii) **Required To Calculate:** $B\hat{C}D$

Calculation:

$OC = OA$ (radii)

$O\hat{A}C = O\hat{C}A$

(The base angles of an isosceles triangle are equal).

$$O\hat{C}A = \frac{180° - 96°}{2}$$
$$= 42°$$

(The sum of the angles in a triangle = 180°).

$$B\hat{C}D = 180° - (48° + 42° + 26°)$$
$$= 64°$$

(Sum of angles in a triangle = 180°).

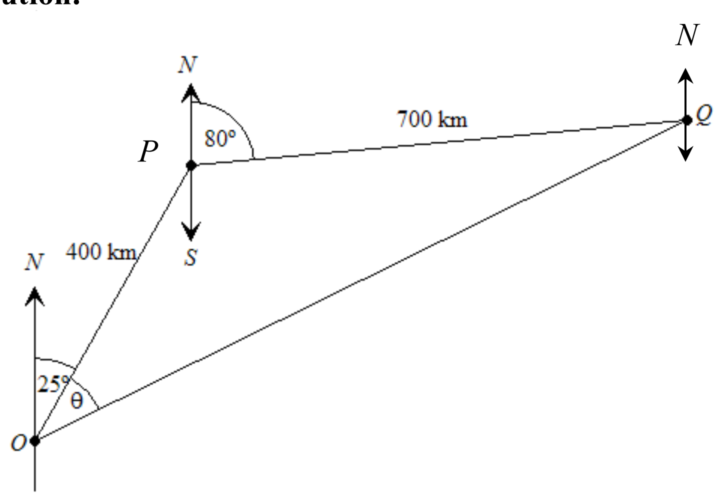
(iv) **Required To Calculate:** $B\hat{A}C$
Calculation:
$B\hat{A}C = 64°$
(The angle made by a tangent to a circle and a chord, at the point of contact equal the angle in the alternate segment).

(v) **Required To Calculate:** $O\hat{A}C$
Calculation:
$O\hat{A}C = 42°$ (from iii)

(iv) **Required To Calculate:** $O\hat{A}B$
Calculation:
$B\hat{A}C = 64°$
$O\hat{A}B = 64° - 42°$
$= 22°$

b. **Data:** Diagram showing two hurricane tracking stations relative to a fixed point.
(i) **Required To Complete:** Diagram labelling the bearings 025° and 080°.

Solution:



(ii) (a) **Required To Calculate:** $O\hat{P}Q$

Calculation:

$O\hat{P}S = 25°$ (Alternate angles)

$P\hat{Q}S = 180° - 80°$

$= 100°$ (Angles in a straight line)

$O\hat{P}Q = 100° + 25°$

$= 125°$

(b) **Required To Calculate:** Length of OQ.

Calculation:

$OQ^2 = (400)^2 + (700)^2 - 2(400)(700)\cos 125° \; (\cos law)$

$= 971202.8$

$OQ = 985.496$ km

$= 985$ km (to the nearest km)

(c) **Required To Calculate:** Bearing of Q from O.

Calculation:

Let $P\hat{O}Q = \theta°$

$\frac{700}{\sin\theta} = \frac{985.496}{\sin 125°} \; (\sin law)$

$\sin\theta = \frac{700 \sin 125°}{985.496}$

$= 0.5818$

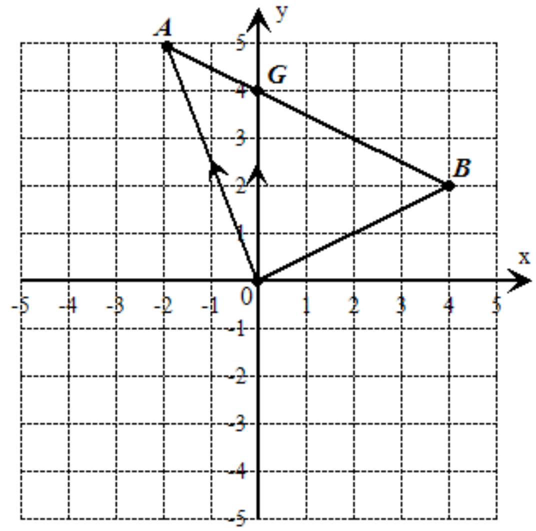
$\theta = 35.6°$

The bearing of Q from $O = 25° + 35.6°$

$= 060.6°$

12. NO SOLUTION HAS BEEN OFFERED FOR THIS QUESTION SINCE IT IS BASED ON EARTH GEOMETRY WHICH IS NO LONGER ON THE SYLLABUS

13. a. **Data:** $\overrightarrow{OA} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$.



(i) **Required To Express:** $\overrightarrow{AB}$ and $\overrightarrow{AG}$ in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$

$$= -\begin{pmatrix} -2 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -3 \end{pmatrix}$$

is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x = 6$ and y = - 3

$$\overrightarrow{AG} = \frac{1}{3}\overrightarrow{AB}$$

$$= \frac{1}{3}\begin{pmatrix} 6 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x = 2$ and $y = -1$.

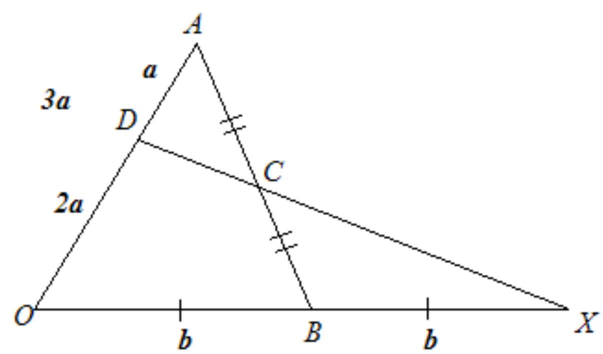
(ii) **Required To Express:** $\overrightarrow{OG}$ in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:

$$\overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{AG}$$
$$= \begin{pmatrix} -2 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$
$$= \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x = 0$ and $y = 4$.

b. **Data:** A vector diagram with B, the midpoint of OX, C the midpoint of AB and $OD = 2DA$, $\overrightarrow{OA} = 3a$ and $\overrightarrow{OB} = b$.



$$\overrightarrow{OB} = b$$
$$\overrightarrow{BX} = b$$
$$OD = 2DA$$
$$\overrightarrow{OD} = \tfrac{2}{3}\overrightarrow{OA}$$
$$= \tfrac{2}{3}(3a)$$
$$= 2a$$
$$\overrightarrow{DA} = a$$

(i) (a) **Required To Find:** $\overrightarrow{AB}$ in terms of ***a*** and ***b***.

Solution:

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= -(3a) + b$$
$$= -3a + b$$

(b) **Required To Find:** $\overrightarrow{AC}$ in terms of ***a*** and ***b***.

Solution:

$$\overrightarrow{AC} = \frac{1}{2}\overrightarrow{AB}$$
$$= \frac{1}{2}(-3a + b)$$
$$= -1\frac{1}{2}a + \frac{1}{2}b$$

(c) **Required To Find:** $\overrightarrow{DC}$ in terms of a and b.
Solution:

$$\overrightarrow{DC} = \overrightarrow{DA} + \overrightarrow{AC}$$
$$= a + (-1\tfrac{1}{2}a + \tfrac{1}{2}b)$$
$$= -\tfrac{1}{2}a + \tfrac{1}{2}b$$

(d) **Required To Find:** $\overrightarrow{DX}$ in terms of a and b.
Solution:

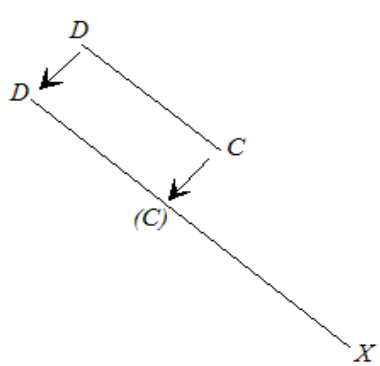
$$\overrightarrow{DX} = \overrightarrow{DO} + \overrightarrow{OX}$$
$$= -(2a) + 2b$$
$$= -2a + 2b$$

(ii) **Required To Find:** Two geometrical relationships between DX and DC.
Solution:

$$\overrightarrow{DX} = -2a + 2b$$
$$= 4(-\tfrac{1}{2}a + \tfrac{1}{2}b)$$
$$= 4\overrightarrow{DC}$$

$\therefore \left|\overrightarrow{DX}\right| = 4 \times \left|\overrightarrow{DC}\right|$ and $\overrightarrow{DX}$ is parallel to $\overrightarrow{DC}$.

(iii) **Required To Find:** One geometrical relationship between points D, C and X.
Solution:
Since $\overrightarrow{DC}$ and $\overrightarrow{DX}$ are parallel and D is a common point, then C must lie on DX. Therefore, D, C and X lie on the same straight line **OR** are collinear.



14. a. **Data:** $M = \begin{pmatrix} x & 4 \\ 3 & x \end{pmatrix}$ and $\det M = 13$.

Required To Calculate: x
Calculation:
$$\det M = (x \times x) - (4 \times 3)$$
$$= x^2 - 12$$
Hence,
$$x^2 - 12 = 13$$
$$\therefore x^2 = 25$$
$$x = \pm 5$$

b. **Data:** $R = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ and $S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.
R represents a reflection in the $y-$axis.
S represents a 90° clockwise rotation about O.

(i) **Required To Find:** Matrix to represent combined transformation S followed by R.
Solution:
The combined transformation S followed by R means S first and R second.
This is written as RS and is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

(ii) **Required To Calculate:** Image of the point (5, -2) under the combined transformation:
Calculation:
(5, -2) under RS is
$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} (0 \times 5) + (-1 \times -2) \\ (-1 \times 5) + (0 \times -2) \end{pmatrix}$$
$$= \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$
The image is (2, -5).

c. **Data:** $N = \begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix}$

(i) **Required To Calculate:** N^{-1}
Calculation:

$$\det N = (3\times 5) - (-1\times 2)$$
$$= 15 + 2$$
$$= 17$$

$$\therefore N^{-1} = \frac{1}{17}\begin{pmatrix} 5 & -(-1) \\ -(2) & 3 \end{pmatrix}$$
$$= \begin{pmatrix} \frac{5}{17} & \frac{1}{17} \\ -\frac{2}{17} & \frac{3}{17} \end{pmatrix}$$

(ii) **Required To Solve:** For x and y in $3x - y = 5$ and $2x + 5y = 9$.

Solution:

$3x - y = 5$

$2x + 5y = 9$

$$\begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \end{pmatrix}$$

$\times N^{-1}$

$$\begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix} \times N^{-1} \times \begin{pmatrix} x \\ y \end{pmatrix} = N^{-1}\begin{pmatrix} 5 \\ 9 \end{pmatrix}$$

$$I\begin{pmatrix} x \\ y \end{pmatrix} = N^{-1}\begin{pmatrix} 5 \\ 9 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{5}{17} & \frac{1}{17} \\ -\frac{2}{17} & \frac{3}{17} \end{pmatrix}\begin{pmatrix} 5 \\ 9 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{25}{17} + \frac{9}{17} \\ -\frac{10}{17} + \frac{27}{17} \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Equating corresponding entries.

$x = 2$ and $y = 1$