FAS-PASS Maths

JANUARY 2009 CXC MATHEMATICS GENERAL PROFICIENCY (PAPER 2)

Section I

1. a. **Required To Calculate:** $\dfrac{3\frac{3}{4}}{2\frac{1}{3}-\frac{5}{6}}$

Calculation:
Denominator:

$2\frac{1}{3}-\frac{5}{6}$

$2\frac{2-5}{6}=1\frac{8-5}{6}$

$=1\frac{1}{2}$

and

$\dfrac{3\frac{3}{4}}{1\frac{1}{2}}=\dfrac{\frac{15}{4}}{\frac{3}{2}}$

$=\frac{15}{4}\times\frac{2}{3}$

$=\frac{5}{2}$

$=2\frac{1}{2}$

b. (i) **Required To Calculate:** Value of one BDS$ in EC$
Calculation:

$BDS\$2\,000\equiv EC\$2\,700$

$BDS\$1\equiv EC\$\frac{2\,700}{2\,000}$

$=EC\$1.35$

(ii) **Required To Calculate:** Amount of BDS$ received for exchanging EC$432.00

Calculation:

$$\text{EC\$}1.35 \equiv \text{BDS\$}1.00$$

$$\text{EC\$}1.00 \equiv \text{BDS\$}\frac{1.00}{1.35}$$

$$\text{EC\$}432 \equiv \text{BDS\$}\frac{1.00}{1.35} \times 432$$

$$= \text{BDS\$}320$$

c. **Data:** Principal $24 000 receives 8% per annum compound interest.
Required To Calculate: Amount of money after 2 years.
Calculation:

$$\text{Interest at the end of } 1^{\text{st}} \text{ year} = \frac{8}{100} \times 24\,000$$
$$= \$1920$$

$$\text{Principal at start of second year} = \$24\,000 + \$1920$$
$$= \$25\,920$$

$$\text{Interest at the end of } 2^{\text{nd}} \text{ year} = \frac{8}{100} \times 25\,920$$
$$= \$2\,073.60$$

Hence, total amount of money in the account at the end of 2 years

$$= \$25\,920 + \$2\,073.60$$
$$= \$27\,993.60$$

OR

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$A = 24\,000\left(1 + \frac{8}{100}\right)^2$$

$$= 24\,000(1.08)^2$$

$$= \$27\,993.60$$

2. a. **Required To Simplify:** $\frac{2m}{n}-\frac{5m}{3n}$

Solution:

$$\frac{2m}{n}-\frac{5m}{3n}$$

$$\frac{3(2m)-5m}{3n}=\frac{6m-5m}{3n}$$

$$=\frac{m}{3n}\text{ as a single fraction}$$

b. **Data:** $a*b=a^2-b$

Required To Calculate: $5*2$

Calculation:

$$5*2=(5)^2-2$$

$$=25-2$$

$$=23$$

c. **Required To Factorise:** $3x-6y+x^2-2xy$

Solution:

$$3x-6y+x^2-2xy=3(x-2y)+x(x-2y)$$

$$=(x-2y)(3+x)$$

d. **Data:** A 21 cm drinking straw cut into 3 pieces of varying length.

(i) **Required To Find:** Length of each piece in terms of x.

Solution:

Length of 1st piece $=x$ cm (data)

Length of 2nd piece $=(x-3)$cm (data)

Length of 3rd piece $=2\times x$

$=2x$ cm (data)

(ii) **Required To Find:** Expression in terms of x to represent the sum of the lengths of all three pieces.

Solution:

Sum of the lengths of all three pieces of straw

$$=x+(x-3)+2x$$

$$=x+x+2x-3$$

$$=(4x-3)\text{ cm}$$

(iii) **Required To Calculate:** x

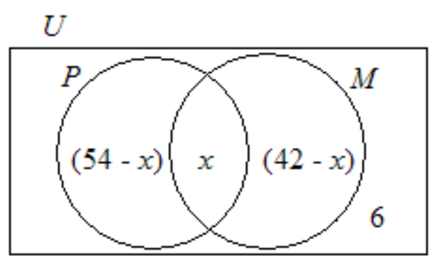
Calculation:

$$4x-3=21$$
$$4x=21+3$$
$$4x=24$$
$$x=\frac{24}{4}$$
$$=6$$

3. a. **Data:** School of 90 form 5 students where 54 study P.E., 42 Music, x both and 6 neither.

(i)-(ii) **Required To Complete:** Venn diagram to represent the information given.

Solution:

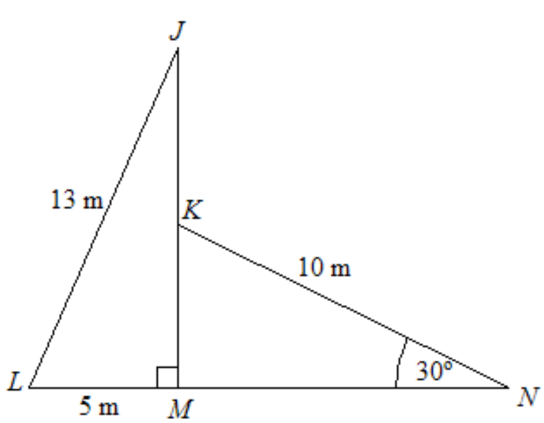


(iii) **Required To Calculate:** x

Calculation:

$$(54-x)+x+(42-x)=90$$
$$102-x=90$$
$$x=102-90$$
$$=12$$

b. **Data:** Diagram as shown below.



(i) **Required To Calculate:** Length of *MK*.
Calculation:

$$\frac{MK}{10} = \sin 30°$$

$$MK = 10\sin 30°$$

$$= 5 \text{ m}$$

(ii) **Required To Calculate:** Length of *JK*.
Calculation:

$$JM = \sqrt{(13)^2 - (5)^2} \qquad \text{(Pythagoras' Theorem)}$$

$$= \sqrt{144}$$

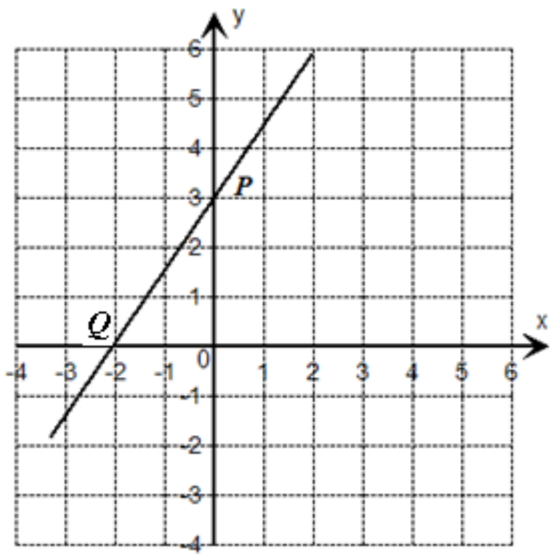
$$= 12 \text{ m}$$

Length of *JK* = Length of *JM* – Length of *KM*

$$= 12 - 5$$

$$= 7 \text{ m}$$

4. **Data:** Diagram with straight line cutting the axes at *P* and *Q*.



a. **Required To State:** Coordinates of *P* and *Q*.
Solution:
Line cuts the y – axis at 3, $\therefore P = (0,\ 3)$.
Line cuts the x – axis at – 2, $\therefore Q = (-2,\ 0)$.

b. (i) **Required To Find:** Gradient of *PQ*.
Solution:

$$\text{Gradient of } PQ = \frac{3-0}{0-(-2)}$$
$$= \frac{3}{2}$$

(ii) **Required To Find:** Equation of *PQ*.
Solution:
Equation of *PQ* is

$$\frac{y-3}{x-0} = \frac{3}{2} \qquad \text{Using point at } P$$
$$2y - 6 = 3x$$
$$2y = 3x + 6$$
$$= y = \frac{3}{2}x + 3$$

OR

$$\frac{y-0}{x-(-2)} = \frac{3}{2} \qquad \text{Using point at } Q$$
$$2y = 3x + 6$$

OR

$y = mx + c$ is the general equation of a straight line where gradient, $m = \frac{3}{2}$ and intercept on the vertical axis is 3.

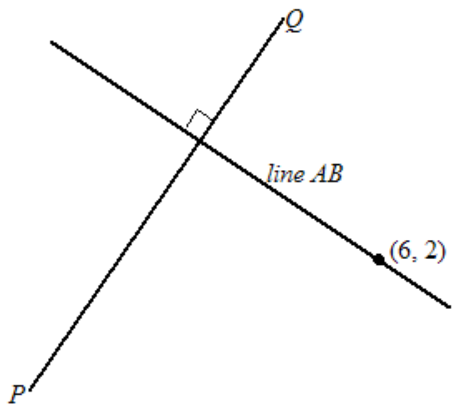
$$\therefore y = \frac{3}{2}x + 3$$

c. **Data:** Point (-8, *t*) lies on *PQ*.
Required To Calculate: *t*
Calculation:

Equation of *PQ* is $y = \frac{3}{2}x + 3$.

$$t = \frac{3}{2}(-8) + 3$$
$$t = -12 = 3$$
$$= -9$$

(d) **Data:** *AB* is perpendicular to *PQ* and passes through (6, 2).
Required To Find: Equation of *AB*.
Solution:

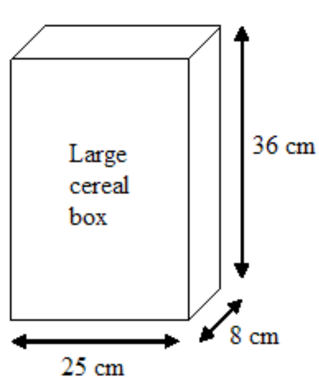


Gradient of $PQ = \frac{3}{2}$, then gradient of $AB = -\frac{2}{3}$.
(Product of the gradients of perpendicular lines = -1).
Equation of *AB* is

$$\frac{y-2}{x-6} = -\frac{2}{3}$$
$$3(y-2) = -2(x-6)$$
$$3y - 6 = -2x + 2$$
$$3y = -2x + 12$$
$$3y = -2x + 18$$
$$\div 3$$
$$y = -\frac{2}{3}x + 6$$

is of the form $y = mx + c$, where $m = -\frac{2}{3}$ and $c = 6$.

5. Data:



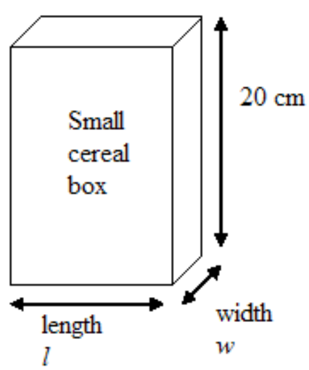
a. **Required To Calculate:** Volume of large cereal box.
Calculation:
$$\text{Volume} = (25 \times 8 \times 36)$$
$$= 7\,200\ \text{cm}^3$$

b. **Required To Calculate:** Total surface area of large cereal box.
Calculation:
Area of front and back faces $= 2(25 \times 36)\ \text{cm}^2$
Area of left and right faces $= 2(8 \times 36)\ \text{cm}^2$
Area of base and top faces $= 2(25 \times 8)\ \text{cm}^2$

Total area of large cereal box
$$= 2(25 \times 36) + 2(8 \times 36) + 2(25 \times 8)$$
$$= 1800 + 576 + 400$$
$$= 2\,776\ \text{cm}^2$$

c. **Data:** One large box can fill six small boxes each of equal volume.
(i) **Required To Calculate:** Volume of one small cereal box.
Calculation:
Volume of large cereal box = 6 × Volume of small cereal box
$$\text{Volume of small cereal box} = \frac{7\,200}{6}$$
$$= 1\,200\ \text{cm}^3$$

(ii) **Required To List:** Two different pairs of values which the company can use for the height and width of a small box.
Solution:



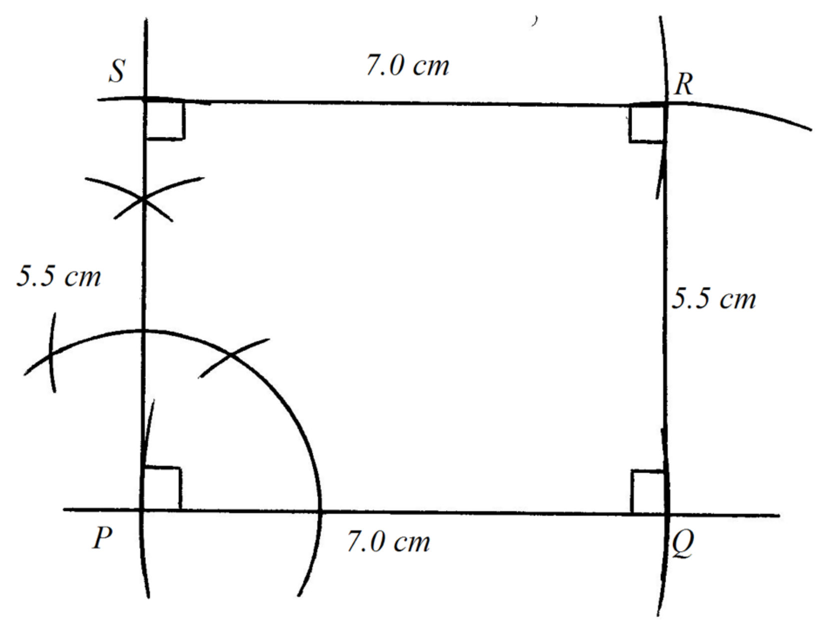
In a small cereal box, let
length = l cm and width = w cm.

$l \times w \times 20 = 1200$

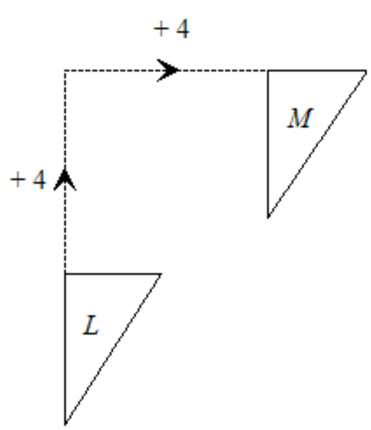
$l \times w = 60$

There are an infinite number of possible choices for values of l and w such that $l \times w = 60$.
For simplicity, we may choose two integral values, such as
Let $l = 10$ and $w = 6$ and $l = 12$ and $w = 5$ as the two pairs of values.

6. a. **Required To Construct:** Rectangle *PQRS* with *PQ* = 7.0 cm and *QR* = 5.5 cm.
Solution:



b. **Data:** $\Delta L \longrightarrow \Delta M$
(i) **Required To Calculate:** Translation vector that maps ΔL onto ΔM.
Calculation:



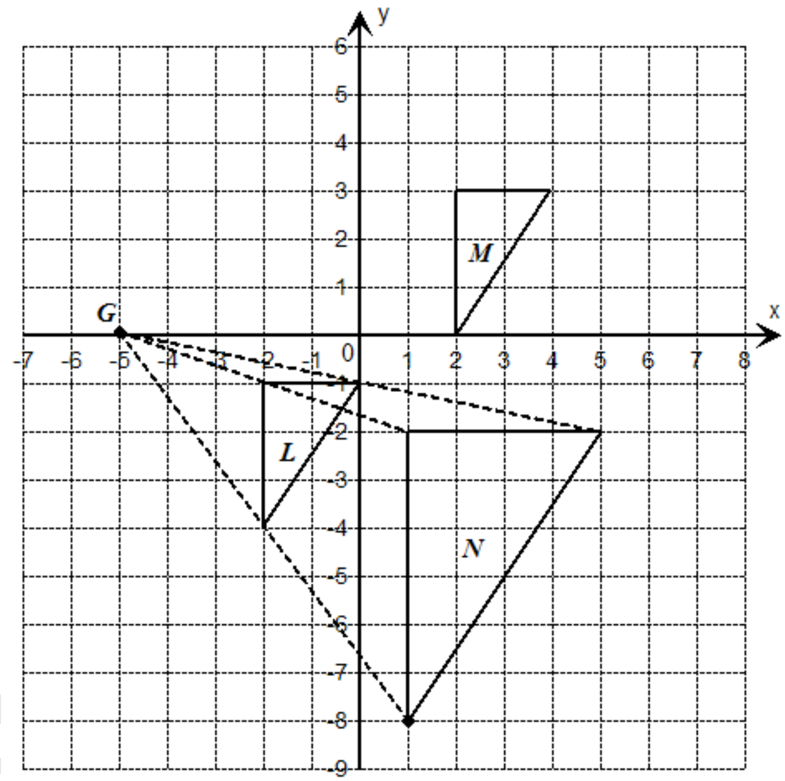
L is mapped onto *M* by a horizontal shift of 4 units to the right and 4 units vertically upwards. This may be represented by the translation vector, *T*, where

$T = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$.

(ii) **Data:** $\Delta L \longrightarrow \Delta N$, by an enlargement, say *E*.

(a) **Required To Find:** Centre of enlargement, *G* on diagram.

Solution:



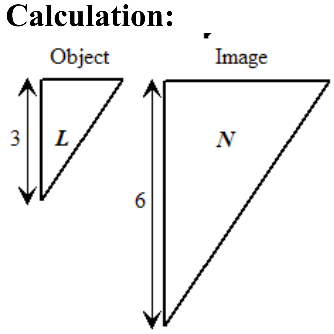
(b) **Required To State:** Coordinates of *G*.

Solution:

G is (-5, 0), as seen on the diagram.

(c) **Required To Calculate:** Scale factor of the enlargement.

Calculation:



Measuring the image length and its corresponding object length.

$$\frac{\text{Image Length}}{\text{Object Length}} = \frac{6}{3}$$
$$= 2$$

Therefore, scale factor of the enlargement is 2.

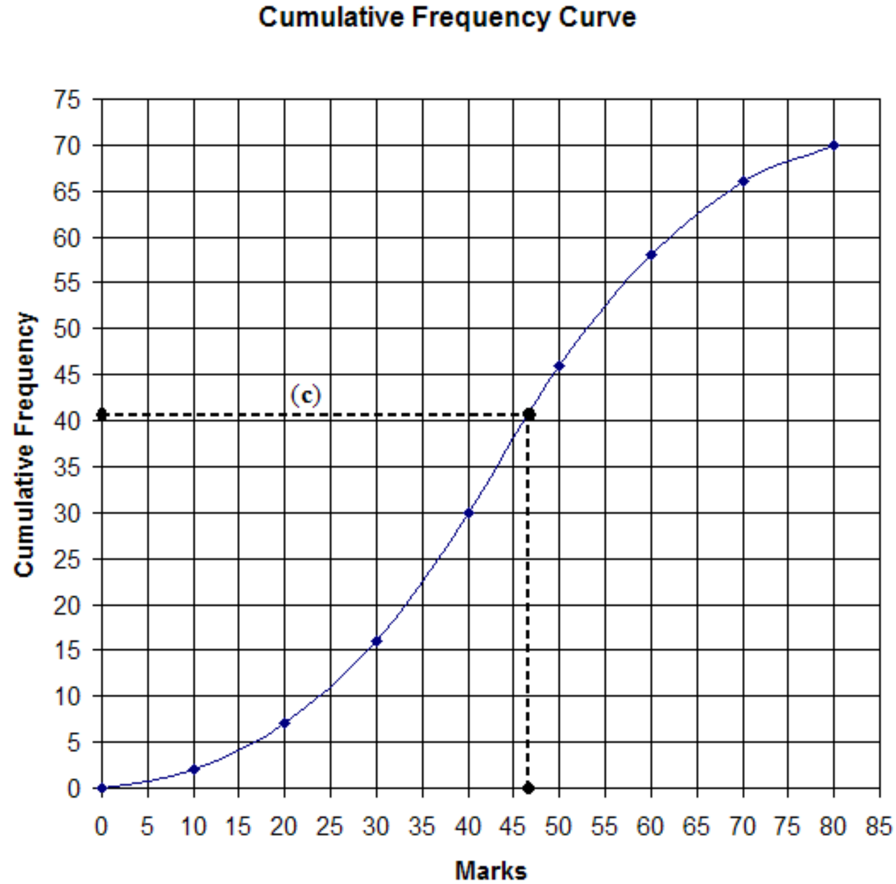
7. **Data:** Given table of values for marks in a test obtained by 70 students.
   a. **Required To Complete:** Table to show cumulative frequency distribution.
   Solution:
   Modifying the table.
   Distribution – discrete variable.

Marks L.C.B U.C.B.	**Frequency. *f***	**Cumulative frequency (C.F.)**	**Points to be plotted (U.C.B, C.F.)**
			(0, 0)
1 – 10	2	2	(10, 2)
11 – 20	5	7	(20, 7)
21 – 30	9	16	(30, 16)
31 – 40	14	$14+16=30$	(40, 30)
41 – 50	16	$16+30=46$	(50, 46)
51 – 60	12	$12+46=58$	(60, 58)
61 – 70	8	$8+58=66$	(70, 66)
71 – 80	4	$4+66=70$	(80, 70)

A cumulative frequency curve must start from the horizontal axis. By checking the values of U.C.B., we find that an initial point with U.C.B = 0 and C.F. = 0.

b. (i) **Required To Draw:** Cumulative frequency curve for the information given.

Solutio



(ii) **Required To State:** Assumption made when drawing the curve through the point (0, 0).
Solution:
In drawing the curve with a starting point at (0, 0), we are assuming that no student obtained a mark of 0. This also seems clear from the table of values that is given.

c. **Required To Find:** Number of students who passed the test.
Solution:
If the pass mark is 47 (which corresponds to a cumulative frequency value of 41), then the number of students who passed the test is $70 - 41 = 29$.

d. **Required To Calculate:** Probability randomly chosen student had a mark less than or equal to 30.

Calculation:

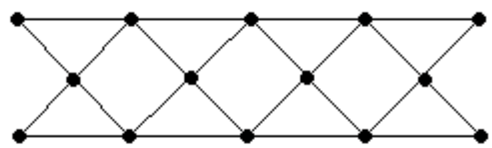
$$P(\text{Student obtained a mark} \leq 30) = \frac{\text{No. of students who obtained a mark} \leq 30}{\text{Total no. of students} = \sum f}$$

$$= \frac{16}{70}$$

$$= \frac{8}{35}$$

8. **Data:** Pattern made up of lines and dots.

a. **Required To Draw:** Fourth diagram in the sequence.
Solution:
The fourth diagram in the sequence is



b. **Required To Complete:** Table given.
Solution:

No. of dots, *d*	Pattern connecting *l* and *d*	No. of line segments, *l*
5	$2\times5-4$	6
8	$2\times8-4$	12
11	$2\times11-4$	18
⋮	⋮	⋮
(i) 62	$2\times62-4$	120
⋮	⋮	⋮
(ii) 92	$2\times92-4$	180

c. (i) **Required To Find:** No. of dots in the 6th diagram of the sequence.
Solution:
The sequence of dots *d* is

$d =$
5 — 1^{st}
$5+3=8$ — 2^{nd}
$8+3=11$ — 3^{rd}
$11+3=14$ — 4^{th}
$14+3=17$ — 5^{th}
$17+3=20$ — 6^{th}

$\therefore$ In the 6^{th} diagram of the sequence, the number of dots, $d = 20$.

(ii) **Required To Find:** Number of line segments in the 7^{th} diagram of the sequence.

Solution:

In the 7^{th} diagram

$d =$

20 6^{th}

$3 + 20 = 23$ 7^{th}

$$\therefore \text{No. of line segments } = 2 \times 23 - 4$$
$$= 46 - 4$$
$$= 42$$

(iii) **Required To Find:** The rule which relates l to d.

Solution:

By observing the pattern values of d and l

	2 is constant $\times$value of d – constant = 4
5	$2 \times 5 - 4$
8	$2 \times 8 - 4$
11	$2 \times 11 - 4$
62	$2 \times 62 - 4$
92	$2 \times 92 - 4$

$$l = 2 \times d - 4$$
$$l = 2d - 4$$

9. a. **Data:** $\frac{p}{2} = \sqrt{\frac{t + r}{g}}$

Required To Make: t the subject of the formula.

Solution:

$$\frac{p}{2} = \sqrt{\frac{t + r}{g}}$$

Squaring to remove $\sqrt{\ }$

$$\left(\frac{p}{2}\right)^2 = \left(\sqrt{\frac{t + r}{g}}\right)^2$$

$$\frac{p^2}{4} = \frac{t + r}{g}$$

$$g \times p^2 = 4(t + r)$$

$$gp^2 = 4t + 4r$$
$$gp^2 - 4r = 4t$$
$$4t = gp^2 - 4r$$
$$t = \frac{gp^2 - 4r}{4} \text{ or } \frac{gp^2}{4} - r$$

b. **Data:** $f(x) = 2x^2 - 4x - 13$

(i) **Required To Express:** $f(x)$ in the form $f(x) = a(x+h)^2 + k$.

Solution:

$$f(x) = 2x^2 - 4x - 13$$
$$= 2(x^2 - 2x) - 13$$

Half the coefficient of $-2x = \frac{1}{2}(-2)$
$$= -1$$

$$f(x) = 2(x-1)^2 + *$$
$$2(x^2 - 2x + 1) = 2x^2 - 4x + 2$$
$$\underline{-15} = *$$
$$\underline{-13}$$

Hence, $f(x) = 2x^2 - 4x - 13 \equiv 2(x-1)^2 - 15$ is of the form $a(x+h)^2 + k$, where $a = 2,\ h = -1$ and $k = -15$.

OR

$$2x^2 - 4x - 13 = a(x+h)^2 + k$$
$$= a(x^2 + 2hx + h^2) + k$$
$$= ax^2 + 2ahx + ah^2 + k$$

Equating coefficient of x^2.
$a = 2$
Equating coefficient of x.
$-4 = 2(2)h$
$h = -1$
Equating constants.
$2(-1)^2 + k = -13$
$k = -15$

$\therefore 2x^2 - 4x - 13 \equiv 2(x-1)^2 - 15$, where a, h and k are already given.

(ii) **Required To Find:** Values of x at which $f(x)$ cuts the $x-$axis

Solution:

Assuming the graph is $f(x)=2x^2-4x-13$, $f(x)$ cuts the $x-$axis at $f(x)=0$.

Let

$2x^2-4x-13=0$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(-13)}}{2(2)}$$

$$=\frac{4\pm\sqrt{16+104}}{4}$$

$$=\frac{4\pm\sqrt{120}}{4}$$

$$=\frac{4\pm\sqrt{4\times 30}}{4}$$

$$=\frac{4\pm 2\sqrt{30}}{4}$$

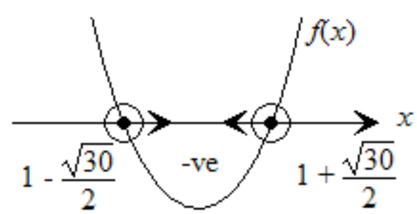
$$=1\pm\frac{\sqrt{30}}{2}$$

$\therefore f(x)$ cuts the $x-$axis at $x=1+\frac{\sqrt{30}}{2}$ and $1-\frac{\sqrt{30}}{2}$.

(iii) **Required To Find:** Interval for which $f(x)\le 0$.

Solution:

$f(x)=2x^2-4x-13$. Coefficient of $x^2>0$, therefore $f(x)$ is a parabola in shape with a minimum point.



$f(x)\le 0$ for $\left\{x:1-\frac{\sqrt{30}}{2}\le x\le 1+\frac{\sqrt{30}}{2}\right\}$

(iv) **Required To Find:** Minimum value of $f(x)$.

Solution:

$$f(x) = 2x^2 - 4x - 13$$
$$= 2(x-1)^2 - 15$$
$$2(x-1)^2 \geq 0 \qquad \forall x$$
$$\therefore f(x)_{\min} = 0 - 15$$
$$= -15$$

OR

$$f(x) = 2x^2 - 4x - 13$$

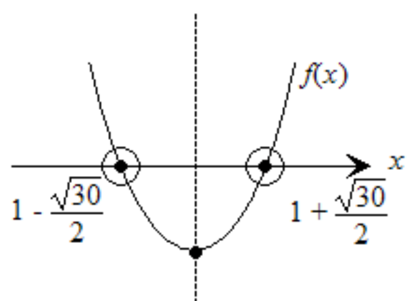
The axis of symmetry of $f(x)$ is $x = \dfrac{-(-4)}{2(2)}$

$$= 1$$

The axis of symmetry passes through the minimum point and hence $x = 1$ at the minimum point.

$$f(1) = 2(1)^2 - 4(1) - 13$$
$$= -15$$
$$\therefore f(x)_{\min} = -15$$

OR



The x coordinates of the minimum point is half way between the x value and which $f(x)$ cuts the x – axis $= \dfrac{1 + \dfrac{\sqrt{30}}{2} + 1 - \dfrac{\sqrt{30}}{2}}{2}$

$$= \frac{2}{2}$$
$$= 1$$
$$f(1) = -15$$
$$\therefore f(x)_{\min} = -15$$

(v) **Required To Find:** x at which $f(x)$ is minimum.

Solution:

$$f(x) = 2(x-1)^2 - 15$$

$f(x)_{\min} = -15$ at $2(x-1)^2 = 0$

That is, $(x-1)^2 = 0$ and $x = 1$.

10. a. **Data:** $f : x \to x - 3$ and $g : x \to x^2 - 1$.

(i) **Required To Calculate:** $f(6)$.

Calculation:

$$f(6) = 6 - 3$$
$$= 3$$

(ii) **Required To Calculate:** $f^{-1}(x)$.

Calculation:

Let

$$y = x - 3$$
$$y + 3 = x$$
$$x = y + 3$$

Replace y by x

$$f^{-1} : x \to x + 3$$

(iii) **Required To Prove:** $fg(2) = fg(-2) = 0$.

Proof:

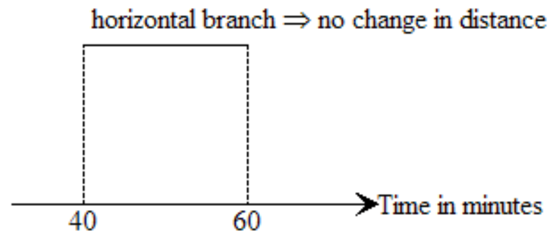
$$g(2) = (2)^2 - 1$$
$$= 4 - 1$$
$$= 3$$
$$fg(2) \equiv f(3)$$
$$= 3 - 3$$
$$= 0$$
$$g(-2) = (-2)^2 - 1$$
$$= 4 - 1$$
$$= 3$$
$$fg(-2) = f(3)$$
$$= 3 - 3$$
$$= 0$$
$$fg(2) = fg(-2) = 0$$

Q.E.D.

b. **Data:** A distance time graph for a train travelling between stations A and B.

(i) **Required To Find:** Time during with train was a rest.

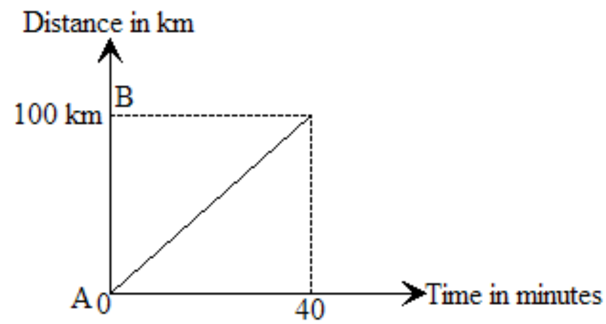
Solution:



Train was at rest at B for 60 – 40 = 20 minutes.

(ii) **Required To Find:** Average speed of the train from A to B.

Solution:



$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{100 \text{ km}}{\left(\frac{40-0}{60}\right)\text{hr}}$$

$$= \frac{100}{\frac{2}{3}} \text{kmh}^{-1}$$

$$= 150 \text{ kmh}^{-1}$$

(iii) **Required To Find:** Time taken for the train to travel from B to C.

Solution:

$$\text{Time taken from B to C} = \frac{\text{Distance from B to C}}{\text{Average speed from B to C}}$$

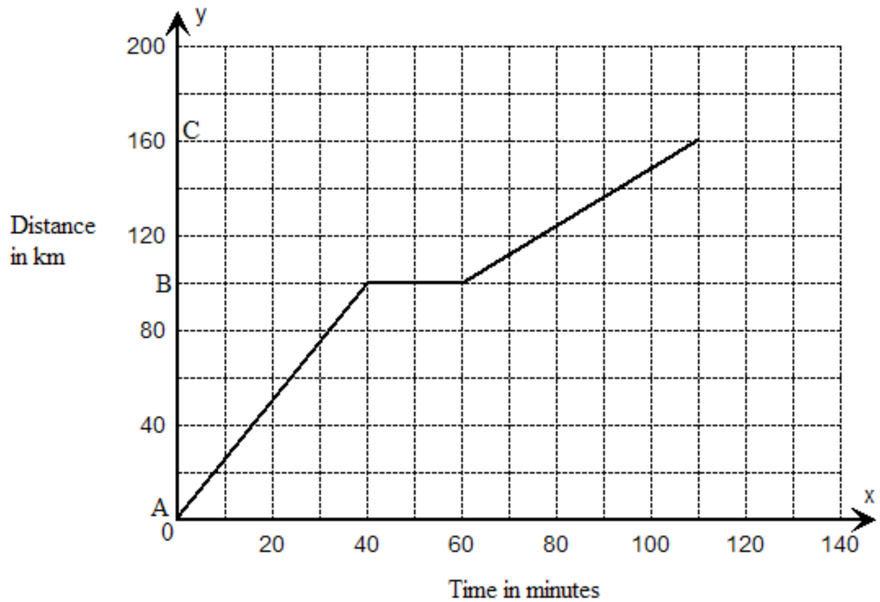
$$= \frac{50 \text{ km}}{60 \text{kmh}^{-1}}$$

$$= \frac{5}{6} \text{ hour}$$

$$= 50 \text{ minutes}$$

(iv) **Required To Draw:** Line segment which describes the journey from B to C.

Solution:



11. a. **Data:** Table of values for $y = \frac{1}{2}\tan x$.

(i) **Required To Complete:** Table of values given.

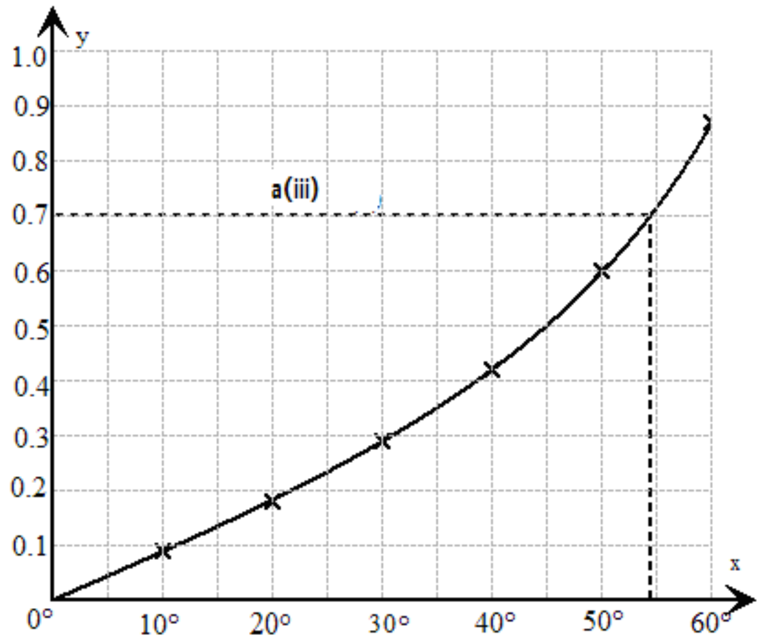
Solution:

x	10°	20°	30°	40°	50°	60°°
y	0.09	0.18	0.29	0.42	0.60	0.87

Note: When $x = 10°$, $y = 0.09$ which was erroneously given as 0.13 in the exam paper. This value has been altered.

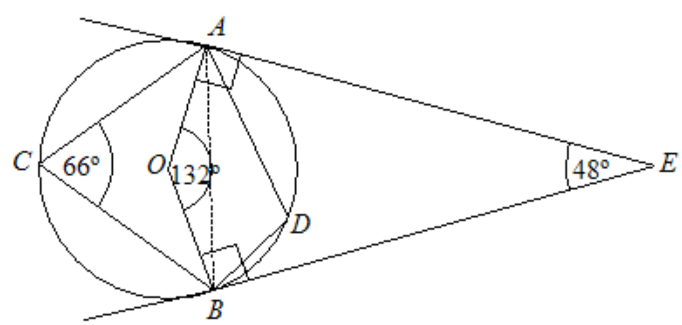
(ii) **Required To Draw:** Graph of $y = \frac{1}{2}\tan x$.

Solution:



(iii) **Required To Find:** x when $y = 0.7$.
Solution:
When $y = 0.7$, $x = 54.5°$ (Read off).

b. **Data:** Diagram as shown below.



(i) **Required To Calculate:** $O\hat{A}E$
Calculation:
$O\hat{A}E = 90°$
(The angle made by a tangent to a circle and a radius, at the point of contact = 90°). Similarly for $O\hat{B}E$.

(ii) **Required To Calculate:** $A\hat{O}B$
Calculation:
$$A\hat{O}B = 360° - (90° + 90° + 48°)$$
$$= 132°$$

(The sum of the angles in a quadrilateral = 360°).

(iii) **Required To Calculate:** $A\hat{C}B$
Calculation:
A chord AB subtends twice the angle at the centre of a circle, that it subtends at the circumference, standing on the same arc.

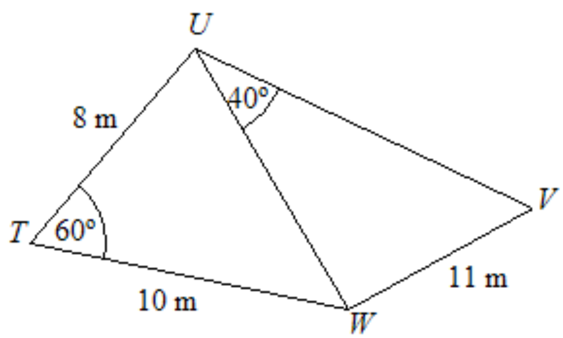
$$\therefore A\hat{C}B = \frac{1}{2}(132°)$$
$$= 66°$$

(iv) **Required To Calculate:** $A\hat{D}B$
Calculation:

$$A\hat{D}B = 180° - 66°$$
$$= 114°$$

(The opposite angles of cyclic quadrilateral are supplementary).

12. a. **Data:** Diagram as shown below.



(i) **Required To Calculate:** Length of UW.
Calculation:

$$UW^2 = (8)^2 + (10)^2 - 2(8)(10)\cos 60° \quad \text{(Cosine Rule)}$$
$$= 64 + 100 - 160 \times \frac{1}{2}$$
$$= 164 - 80$$
$$= 84$$
$$UW = \sqrt{84}$$
$$= 9.16\underline{5}$$
$$= 9.17\,\text{m (to 2 decimal places)}$$

(ii) **Required To Calculate:** $U\hat{V}W$
Calculation:

$$\frac{9.165}{\sin U\hat{V}W} = \frac{11}{\sin 40°}$$

$$\sin U\hat{V}W = \frac{9.165 \sin 40°}{11}$$

$$= 0.535$$

$$U\hat{V}W = \sin^{-1}(0.535)$$

$$= 32.34°$$

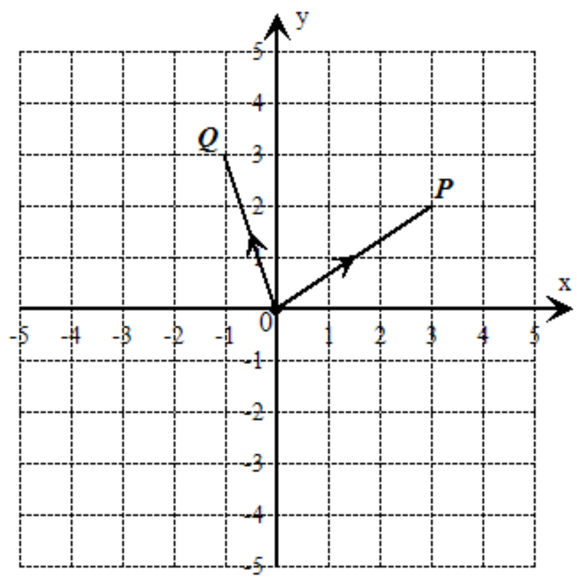
$$= 32.3° \text{ (to the nearest } 0.1°)$$

(iii) **Required To Calculate:** Area of ΔTUW
Calculation:

Area = ½ (8 x 10) sin 60^0
=34.64 cm^2

b.NO SOLUTION HAS BEEN OFFERED AS THIS PART OF THE QUESTION IS ON EARTH GEOMETRY WHICH HAS BEEN REMOVED FROM THE COURSE.

13. **Data:** Diagram showing vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$.



a. (i) **Required To Express:** $\overrightarrow{OP}$ in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:
P is the point (3, 2), as seen on the diagram.

$\overrightarrow{OP} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ where $x = 3$ and $y = 2$.

(ii) **Required To Express:** $\overrightarrow{OQ}$ in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:
Q is the point (-1, 3) as seen on the diagram.
$\overrightarrow{OQ} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ where $x = -1$ and $y = 3$.

b. **Data:** *R* has coordinates (8, 9).

(i) **Required To Express:** $\overrightarrow{QR}$ in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:
R is (8, 9).

$$\overrightarrow{OR} = \begin{pmatrix} 8 \\ 9 \end{pmatrix}$$

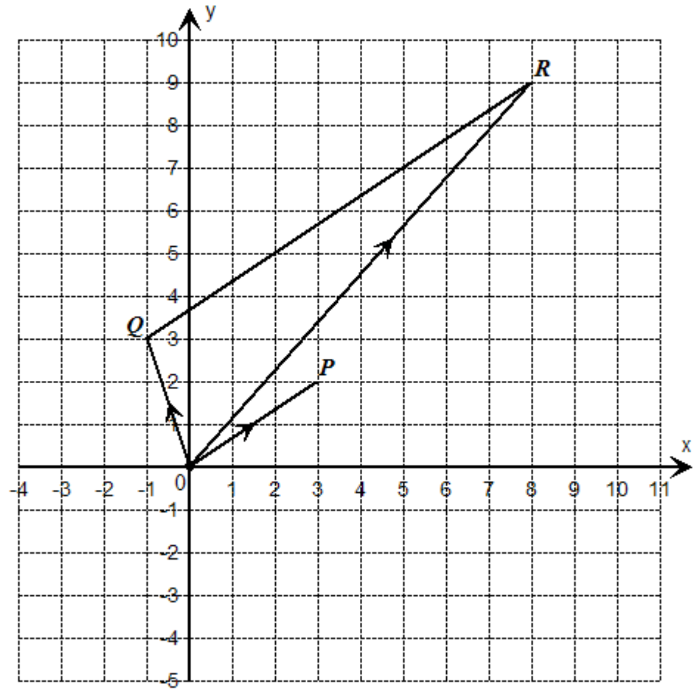
$$\overrightarrow{QR} = \overrightarrow{QO} + \overrightarrow{OR}$$

$$= -\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 8 \\ 9 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ where $x = 9$ and $y = 6$.

(ii) **Required To Prove:** $\overrightarrow{OP}$ is parallel to $\overrightarrow{QR}$.
Proof:



$\overrightarrow{OP} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\overrightarrow{QR} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$

$= 3\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\overrightarrow{QR}$ is a scalar multiple (3) of $\overrightarrow{OP}$, hence $\overrightarrow{OP}$ and $\overrightarrow{QR}$ are parallel.

(iii) **Required To Find:** $\left|\overrightarrow{PR}\right|$.

Solution:

$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR}$

$= -\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ 9 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 7 \end{pmatrix}$

Magnitude of $\overrightarrow{PR} = \left|\overrightarrow{PR}\right|$

$= \sqrt{(5)^2 + (7)^2}$

$= \sqrt{24 + 49}$

$= \sqrt{74}\text{units}$

c. **Data:** $S = (a, b)$.

(i) **Required To Find:** $\overrightarrow{QS}$ in terms of a and b.

Solution:

$$\overrightarrow{OS} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\overrightarrow{QS} = \overrightarrow{QO} + \overrightarrow{OS}$$

$$= -\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

$$= \begin{pmatrix} a+1 \\ b+3 \end{pmatrix}$$

in terms of a and b.

(ii) **Data:** $\overrightarrow{QS} = \overrightarrow{OP}$

Required To Calculate: a and b.

Calculation:

If

$\overrightarrow{QS} = \overrightarrow{OP}$, then

$$\begin{pmatrix} a+1 \\ b+3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

Equating components.

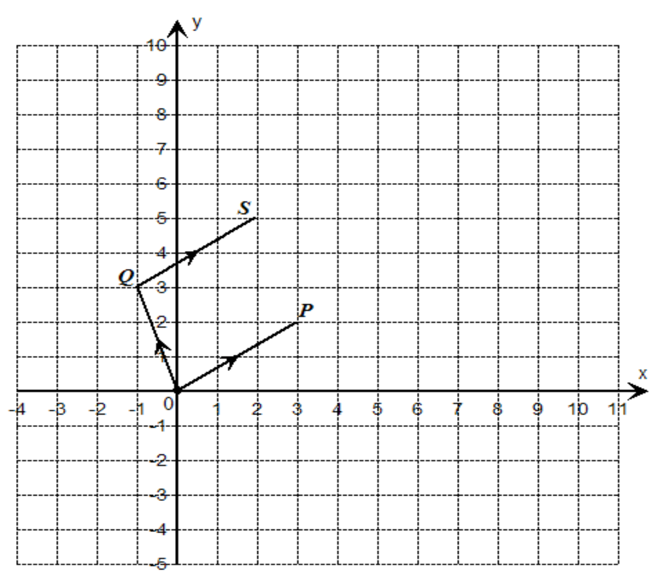
$$a + 1 = 3$$

$$a = 2$$

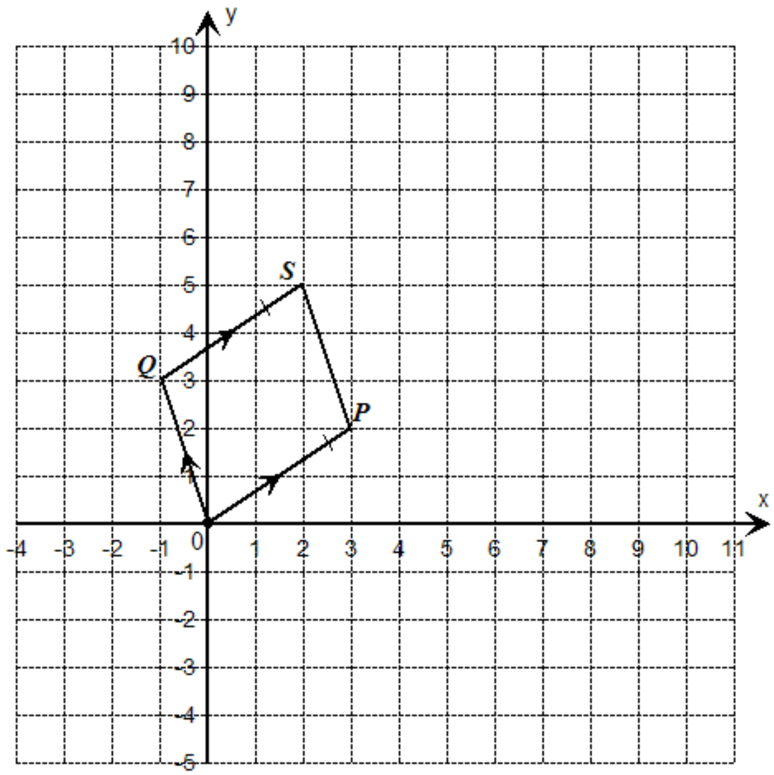
and

$$b + 3 = 2$$

$$b = 5$$



(iii) **Required To Prove:** *OPSQ* is a parallelogram.
Proof:



If $\overrightarrow{QS} = \overrightarrow{OP}$ then $\left|\overrightarrow{QS}\right| = \left|\overrightarrow{OP}\right|$ and $\overrightarrow{QS}$ is parallel to $\overrightarrow{OP}$. If one pair of opposite sides of a quadrilateral is both parallel and equal, then the quadrilateral is a parallelogram. Therefore, *OPSQ* is a parallelogram.

14. a. **Data:** $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$.
Required To Calculate: $3AB$.
Calculation:

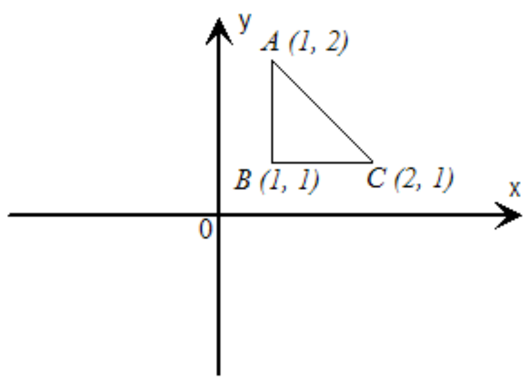
$$A_{2\times 2} \times B_{2\times 2} = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$$

$$e_{11} = (1\times 1) + (2\times 2)$$
$$= 5$$
$$e_{12} = (1\times 3) + (2\times 5)$$
$$= 13$$
$$e_{21} = (2\times 1) + (1\times 2)$$
$$= 4$$
$$e_{22} = (2\times 3) + (1\times 5)$$
$$= 11$$

$$AB = \begin{pmatrix} 5 & 13 \\ 4 & 11 \end{pmatrix}$$

$$3AB = 3\begin{pmatrix} 5 & 13 \\ 4 & 11 \end{pmatrix}$$
$$= \begin{pmatrix} 15 & 39 \\ 12 & 33 \end{pmatrix}$$

b. **Data:**



and matrices $V = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ and $W = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(i) **Required To State:** Effect of V on ΔABC.
Solution:
$\Delta ABC \xrightarrow{V} ?$
The image of ΔABC under V is an enlargement, centre O and scale factor 2.

(ii) **Required To Find:** 2×2 matrix that represents the combined transformation of V followed by W.

Solution:
The combined transformation V followed by W is expressed as WV.

$$WV = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} (-1\times 2)+(0\times 0) & (-1\times 0)+(0\times 2) \\ (0\times 2)+(1\times 0) & (0\times 0)+(1\times 2) \end{pmatrix}$$
$$= \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$$

(iii) **Required To Find:** Coordinates of the image of ΔABC under the combined transformation.
Solution:
The image of ΔABC under WV

$$\begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -2 & -2 & -4 \\ 4 & 2 & 2 \end{pmatrix}$$

A is mapped onto (-2, 4).
B is mapped onto (-2, 2).
C is mapped onto (-4, 2).

c. **Data:** $11x+6y=6$ and $9x+5y=7$.

(i) **Required To Express:** The two equations in the form $AX=B$.
Solution:
$11x+6y=6$
$9x+5y=7$
may be expressed as $\begin{pmatrix} 11 & 6 \\ 9 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$ is of the form $AX=B$, where
$A = \begin{pmatrix} 11 & 6 \\ 9 & 5 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$.

(ii) **Required To Calculate:** x and y.
Calculation:

$$A = \begin{pmatrix} 11 & 6 \\ 9 & 5 \end{pmatrix}$$

$$\begin{aligned} \det A &= (11 \times 5) - (6 \times 9) \\ &= 55 - 54 \\ &= 1 \end{aligned}$$

$$A^{-1} = \frac{1}{1}\begin{pmatrix} 5 & -(6) \\ -(9) & 11 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -6 \\ -9 & 11 \end{pmatrix}$$

$$AX = B$$

$$\times A^{-1}$$

$$A \times A^{-1} \times X = A^{-1} \times B$$

$$I \times X = A^{-1}B$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -6 \\ -9 & 11 \end{pmatrix}\begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} (5 \times 6) + (-6 \times 7) \\ (-9 \times 6) + (11 \times 7) \end{pmatrix}$$

$$= \begin{pmatrix} -12 \\ 23 \end{pmatrix}$$

Equating corresponding entries.
$x = -12$ and $y = 23$.