

**JUNE 2008 CXC MATHEMATICS GENERAL PROFICIENCY (PAPER 2)
(TRINIDAD AND TOBAGO PAPER ONLY)**

Section I

1. a. **Required To Calculate:** $2\frac{1}{5} - 1\frac{1}{3}$
 $1\frac{5}{8}$

Calculation:

Numerator:

$$\begin{aligned} 2\frac{1}{5} - 1\frac{1}{3} &= \frac{11}{5} - \frac{4}{3} \\ &= \frac{3(11) - 5(4)}{15} \\ &= \frac{33 - 20}{15} \\ &= \frac{13}{15} \end{aligned}$$

Hence,

$$\begin{aligned} \frac{2\frac{1}{5} - 1\frac{1}{3}}{1\frac{5}{8}} &= \frac{\frac{13}{15}}{1\frac{5}{8}} \\ &= \frac{\frac{13}{15}}{\frac{13}{8}} \\ &= \frac{13}{15} \times \frac{8}{13} \\ &= \frac{8}{15} \text{ (in exact form)} \end{aligned}$$

b. **Data:** Table showing the tax allowances for Mr. Allen.

(i) **Required To Calculate:** His annual salary.

Calculation:

For 2007 Mr. Allen's monthly salary is \$7 500.

Hence, annual salary = \$7500 × 12

$$= \$90\,000$$

- (ii) **Required To Calculate:** Total allowances for 2007.

Calculation:

Total tax allowances for Mr. Allen according to the given table:

For self	\$10 000	
For wife	\$ 5 000	
For 3 children	\$ 7 500	(3 × 2500)
Total	\$22 500	

- (iii) **Required To Calculate:** Mr. Allen's income tax for 2007.

Calculation:

$$\begin{aligned} \text{Taxable income} &= \text{Gross salary} - \text{Tax allowances} \\ &= \$90\,000 - \$22\,500 \\ &= \$67\,500 \end{aligned}$$

$$\begin{aligned} \text{Mr. Allen's income tax} &= \frac{22}{100} \times \$67\,500 \\ &= \$14\,850 \end{aligned}$$

- (iv) **Required To Calculate:** Percentage of Mr. Allen's annual salary paid in income tax.

Calculation:

$$\begin{aligned} &\text{Percentage of Mr. Allen's annual salary paid in income tax} \\ &= \frac{\text{Income Tax}}{\text{Gross Salary}} \times 100 \\ &= \frac{14\,850}{90\,000} \times 100 \\ &= 16.5\% \end{aligned}$$

2. a. **Required To Simplify:** $(3a - 1)^2$

Solution:

$$\begin{aligned} (3a - 1)^2 &= (3a - 1)(3a - 1) \\ &= 9a^2 - 3a - 3a + 1 \\ &= 9a^2 - 6a + 1 \end{aligned}$$

b. **Data:** $q = \frac{5}{3+p}$

Required To Make: p the subject of the formula.

Solution:

$$q = \frac{5}{3+p}$$

$$\frac{q}{1} = \frac{5}{(3+p)}$$

$$q(3+p) = 5 \times 1$$

$$3q + pq = 5$$

$$pq = 5 - 3q$$

$$p = \frac{5-3q}{q}$$

c. **Required To Factorise (i)** $3mn - 6n^2$, **(ii)** $25p^2 - q^2$

Solution:

$$(i) \quad 3mn - 6n^2 = 3mn - 3 \times 2 \times n \times n \\ = 3n(m - 2n)$$

$$(ii) \quad 25p^2 - q^2 = (5p)^2 - (q)^2 \\ \text{(This is a difference of 2 squares)} \\ = (5p - q)(5p + q)$$

d. **Data:** $3x - 2y = 19$ and $2x + 3y = 4$

Required To Calculate: x and y

Calculation:

Let

$$3x - 2y = 19 \dots(1)$$

$$2x + 3y = 4 \dots(2)$$

From equation (1)

$$3x = 2y + 19$$

$$x = \frac{2y + 19}{3}$$

Substituting in equation (2)

$$2\left(\frac{2y + 19}{3}\right) + 3y = 4$$

$\times 3$

$$2(2y + 19) + 3(3y) = 3(4)$$

$$4y + 38 + 9y = 12$$

$$13y = 12 - 38$$

$$13y = -26$$

$$y = \frac{-26}{13}$$

$$= -2$$

When $y = -2$

$$3x - 2(-2) = 19$$

$$3x = 19 - 4$$

$$3x = 15$$

$$x = 5$$

Hence, $x = 5$ and $y = -2$.

OR

Let

$$3x - 2y = 19 \dots(1)$$

$$2x + 3y = 4 \dots(2)$$

Equation (1) $\times 3$

$$9x - 6y = 57 \dots(3)$$

Equation (2) $\times 2$

$$4x + 6y = 8 \dots(4)$$

$$9x - 6y = 57$$

$$4x + 6y = 8 \quad +$$

$$\hline 13x \quad = 65$$

$$x = \frac{65}{13}$$

$$= 5$$

Substitute $x = 5$ in equation (1).

$$3(5) - 2y = 19$$

$$-2y = 19 - 15$$

$$-2y = 4$$

$$y = \frac{4}{-2}$$

$$= -2$$

Hence, $x = 5$ and $y = -2$.

OR

$$3x - 2y = 19$$

$$2y = 3x - 19$$

Finding the coordinates of 2 points on the line.

x	y
3	-5
7	1

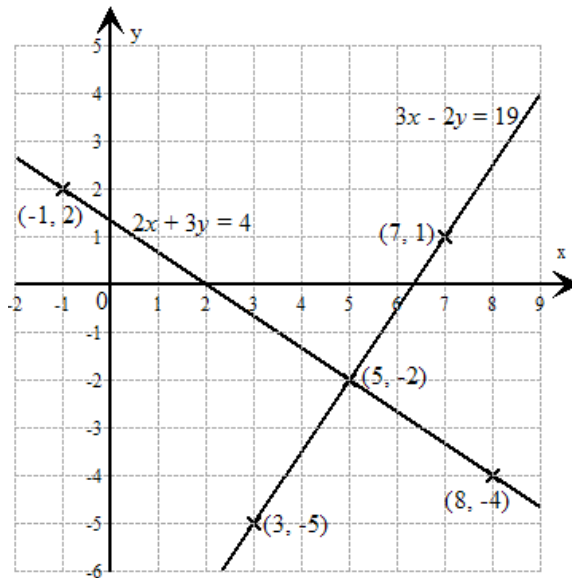
$$2x + 3y = 4$$

$$3y = -2x + 4$$

Find the coordinates of 2 points on the line.

x	y
-1	2
8	-4

If we draw the two straight lines on the same axes,



The point of intersection is $(5, -2)$ from which we deduce $x = 5$ and $y = -2$.

OR

Let

$$3x - 2y = 19 \dots (1)$$

$$2x + 3y = 4 \dots (2)$$

Expressing in matrix form

$$\begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 19 \\ 4 \end{pmatrix}$$

Let $A = \begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix}$

$$\begin{aligned} \text{Det } A &= (3 \times 3) - (-2 \times 2) \\ &= 13 \end{aligned}$$

$$\begin{aligned} A^{-1} &= \frac{1}{13} \begin{pmatrix} 3 & -(-3) \\ -(2) & 3 \end{pmatrix} \\ &= \begin{pmatrix} \frac{3}{13} & \frac{2}{13} \\ -\frac{2}{13} & \frac{3}{13} \end{pmatrix} \end{aligned}$$

Matrix equation A^{-1}

$$A \times A^{-1} \times \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{13} & \frac{2}{13} \\ -\frac{2}{13} & \frac{3}{13} \end{pmatrix} \begin{pmatrix} 19 \\ 4 \end{pmatrix}$$

$$I \times \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \left(\frac{3}{13} \times 19 \right) + \left(\frac{2}{13} \times 4 \right) \\ \left(-\frac{2}{13} \times 19 \right) + \left(\frac{3}{13} \times 4 \right) \end{pmatrix}$$

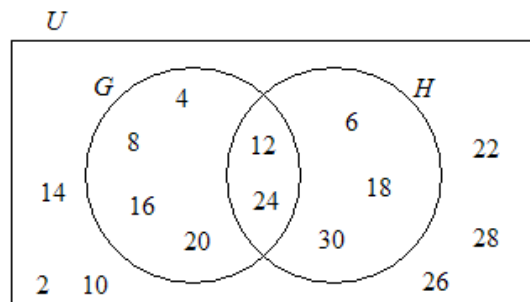
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{57}{13} + \frac{8}{13} \\ -\frac{38}{13} + \frac{12}{13} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{65}{13} \\ -\frac{26}{13} \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

Equating corresponding entries.
 $x = 5$ and $y = -2$.

3. a. **Data:** Venn diagram.



(i) (a) **Required To List:** Members of $G \cap H$.
Solution:
 $G \cap H = \{12, 24\}$ as seen on the given diagram.

(b) **Required To List:** Members of $G \cap H'$.
Solution:
 $H' = \{2, 4, 8, 10, 14, 16, 20, 22, 26, 28\}$
 $G = \{4, 8, 12, 16, 20, 24\}$
 $\therefore G \cap H' = \{4, 8, 16, 20\}$

(ii) **Required To Find:** $n(G \cup H)$.
Solution:
 $(G \cup H) = \{4, 8, 16, 20, 12, 24, 6, 18, 30\}$
 $\therefore n(G \cup H) = 9$

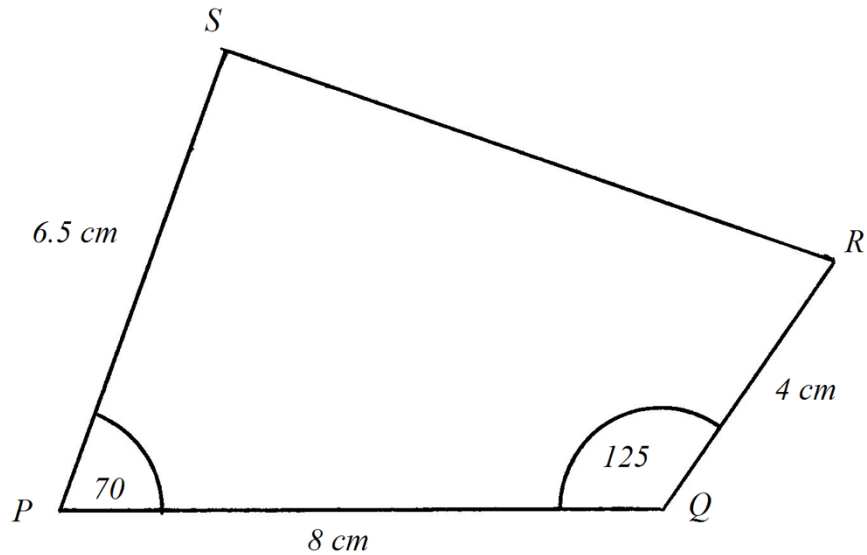
(iii) (a) **Required To Describe:** U in words.
Solution:
 $U = \{\text{Even numbers from 2 to 30 inclusive}\}$

(b) **Required To Describe:** H in words.
Solution:
 $H = \{\text{multiples of 6}\}$
(Since the universal set is from 2 to 30 we need not say ‘multiples of 6 from 2 to 30’).

b. (i) **Required To Construct:** Quadrilateral $PQRS$, in which $PQ = 8$ cm, $QR = 4$ cm, $PS = 6.5$ cm, $\hat{PQR} = 125^\circ$ and $\hat{QPS} = 70^\circ$.

NOTE: The angles 125° and 70° cannot be constructed and are drawn using the protractor.

Solution:



(ii) **Required To Find:** Length of RS .

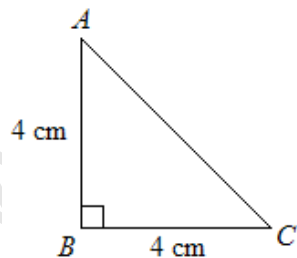
Solution:

$RS = 8.6$ cm (by measurement)

4. **Data:** Given a triangular prism with right-angled isosceles triangles at both ends.

a. **Required To Calculate:** Area of $\triangle ABC$

Calculation:



$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{4 \times 4}{2} \\ &= 8 \text{ cm}^2 \end{aligned}$$

- b. **Required To Calculate:** Length of edge CD .

Calculation:

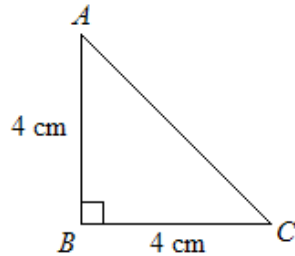
$$\text{Volume of prism} = \text{Cross-sectional area} \times \text{Length } CD$$

$$8 \times CD = 72$$

$$CD = 9 \text{ cm}$$

- c. **Required To Calculate:** Length of edge AC .

Calculation:



$$(AC)^2 = (4)^2 + (4)^2 \quad (\text{Pythagoras' Theorem})$$

$$AC^2 = 32$$

$$AC = \sqrt{32}$$

$$= 5.66$$

$$= 5.7 \text{ cm (to 1 decimal place)}$$

- d. **Required To State:** Number of faces, edges and vertices of the prism.

Solution:

No. of faces:	2	triangular
	1	rectangular side
	1	rectangular base
	<u>1</u>	rectangular sloping
Total	<u>5</u>	

No. of edges: $AB, BC, AC, EF, FD, ED, AE, BE, CF$

Total = 9

No. of vertices: A, B, C, D, E, F

Total = 6

5. **Data:** $\triangle LMN$ drawn on labeled axes.

- a. **Required To Draw:** Line $x = 4$ on the diagram.

Solution:

Line $x = 4$ is shown on the diagram.

- b. **Required To Draw:** $\Delta L'M'N'$, the image when ΔLMN is reflected in the line $x = 4$.

Solution:

$$\Delta LMN \xrightarrow{\text{Reflection in } x=4} \Delta L'M'N'$$

$$L' = (7, 1), M' = (5, 6) \text{ and } N' = (5, 2)$$

as shown on the diagram.

- c. **Data:** $L'' = (-2, -2)$, $M'' = (-6, -12)$, $N'' = (-6, -4)$

- (i) **Required To Draw:** $\Delta L''M''N''$.

Solution:

L'' , M'' and N'' plotted and $\Delta L''M''N''$ drawn, as shown on the diagram.

- (ii) **Required To Describe:** The transformation that maps ΔLMN onto $\Delta L''M''N''$.

Solution:

LL'' , MM'' and NN'' pass through O .

$$\frac{M''N''}{MN} = \frac{8}{4}$$

$$= 2$$

$$= \frac{N''L''}{NL}$$

$$= \frac{M''L''}{ML}$$

$\therefore \Delta LMN \rightarrow \Delta L''M''N''$ by an enlargement, centre O and scale factor 2.

- d. **Required To Calculate:** $\frac{\text{Area of } \Delta L''M''N''}{\text{Area of } \Delta LMN}$

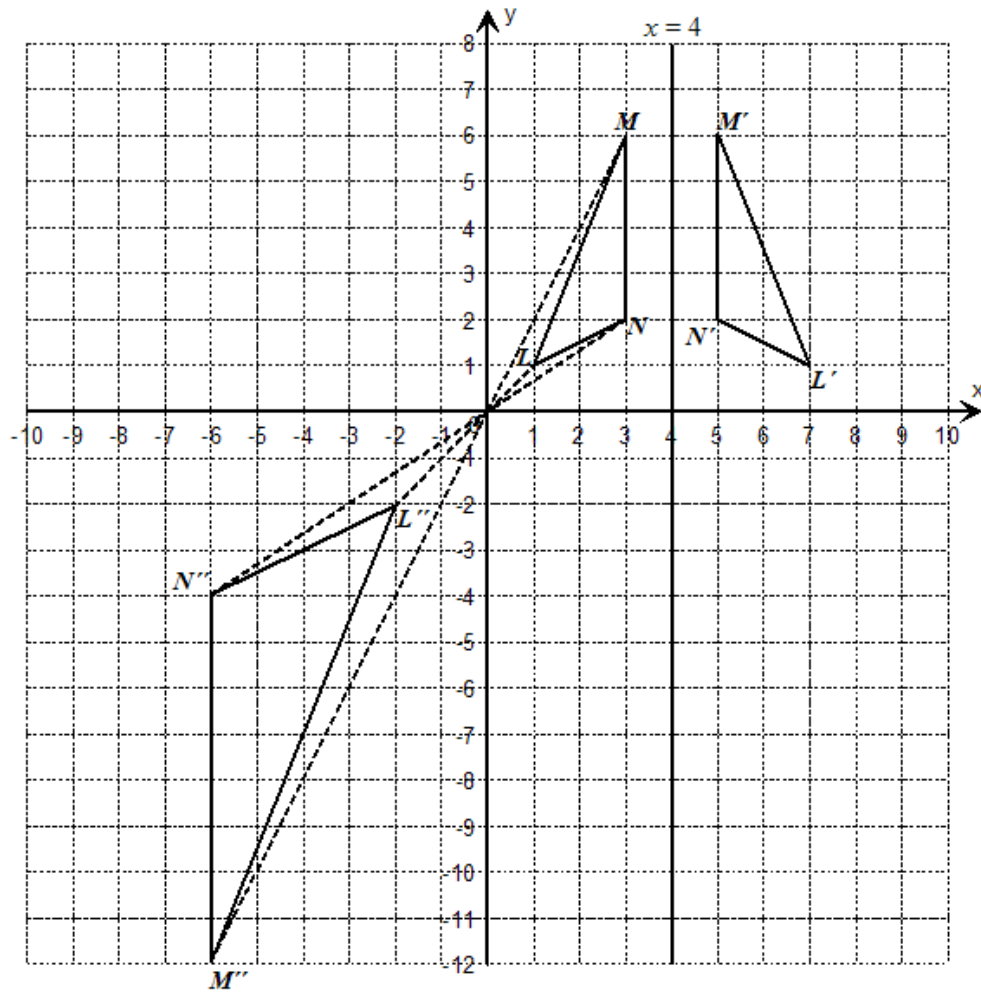
Calculation:

Since, $L''M'' : LM = 2 : 1$

Then,

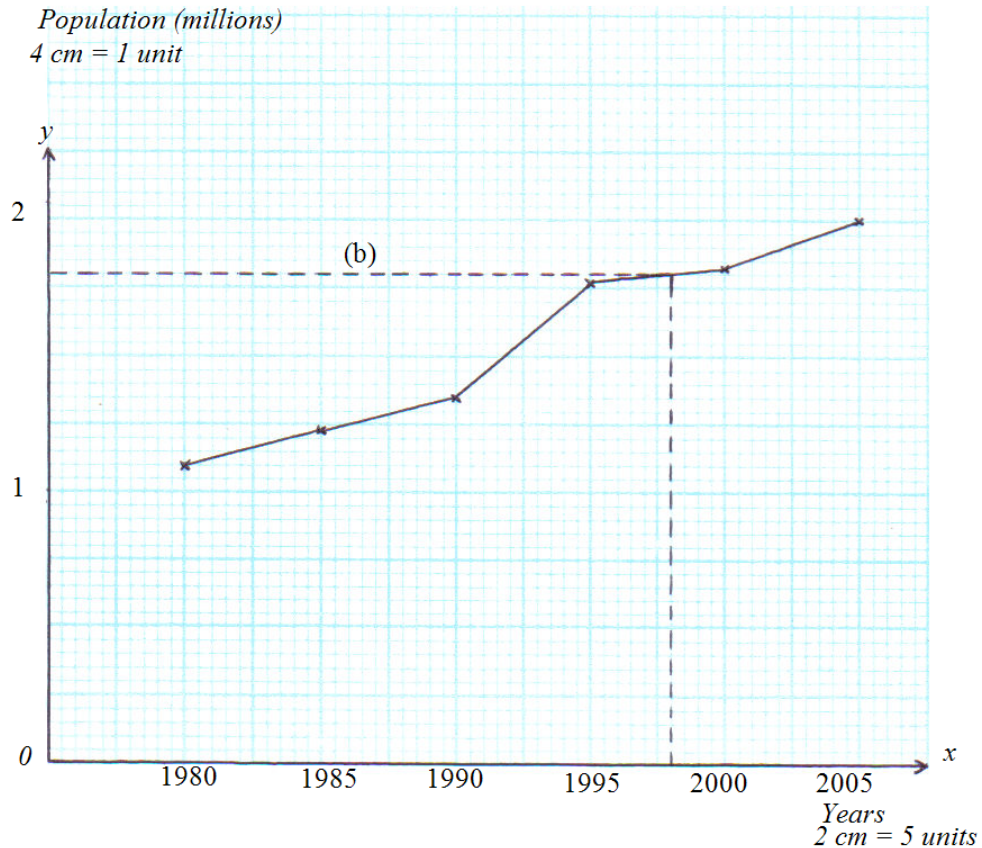
$$\frac{\text{Area of } \Delta L''M''N''}{\text{Area of } \Delta LMN} = \frac{(2)^2}{(1)^2}$$

$$= \frac{4}{1}$$



6. **Data:** Table showing the population of country in five year periods from 1980 to 2005.
a. **Required To Draw:** Line graph to represent the information given.

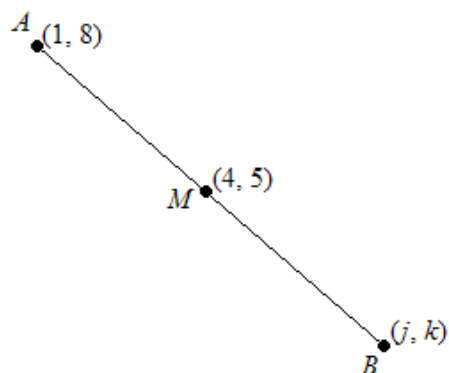
Solution:



- b. **Required To Find:** Population in 1998.
Solution:
From the graph, the population in 1998 is 1.8 million.
- c. (i) **Required To Find:** The five year period with the greatest increase in population.
Solution:
The greatest increase is 1990 to 1995 (line has the highest positive grad.)
- (ii) **Required To Find:** The five year period with the smallest increase in population.
Solution:
The smallest increase is 1995 to 2000. (line has the smallest gradient)
- d. **Required To Explain:** How the answers above are shown on the graph.
Solution:
The greatest increase would be shown by the steepest line segment or the line segment with the greatest positive gradient, that is 1990 to 1995.
The smallest increase would be shown by the least steep line segment or the line with the smallest positive gradient, that is 1995 to 2000.

7. a. **Data:** $M(4, 5)$ is the midpoint of AB , with $A(1, 8)$ and $B(j, k)$.
Required To Calculate: j and k .

Calculation:



Using the midpoint formula.

$$(4, 5) = \left(\frac{1+j}{2}, \frac{8+k}{2} \right)$$

Equating

$$\frac{1+j}{2} = 4$$

$$1+j = 8$$

$$j = 7$$

and

$$\frac{8+k}{2} = 5$$

$$8+k = 10$$

$$k = 2$$

$$\therefore j = 7 \text{ and } k = 2$$

- b. **Data:** $f : x \rightarrow 5x - 2$ and $g : x \rightarrow \frac{4}{x+1}$

- (i) **Required To Calculate:** $f(0)$.

Calculation:

$$\begin{aligned} f(0) &= 5(0) - 2 \\ &= -2 \end{aligned}$$

- (ii) **Required To Calculate:** $g(2)$

Calculation:

$$\begin{aligned} g(2) &= \frac{4}{2+1} \\ &= \frac{4}{3} \end{aligned}$$

(iii) **Required To Calculate:** $f^{-1}(x)$

Calculation:

Let

$$y = 5x - 2$$

$$y + 2 = 5x$$

$$x = \frac{y + 2}{5}$$

Replace y by x

$$f^{-1}(x) = \frac{x + 2}{5}$$

(iv) **Required To Calculate:** x when $fg(x) = 1$

Calculation:

$$fg(x) = 5\left(\frac{4}{x+1}\right) - 2$$

$$1 = \frac{20}{x+1} - 2 \quad (fg(x) = 1)$$

$$\frac{20}{x+1} = 3$$

$$20 = 3x + 3$$

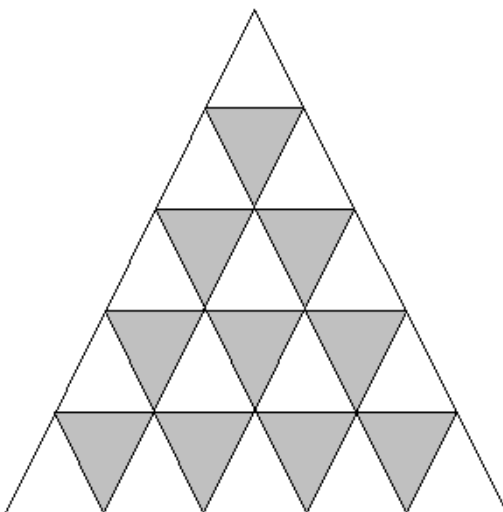
$$17 = 3x$$

$$x = 5\frac{2}{3}$$

8. **Data:** Geometrical pattern of grey and white triangles.

a. **Required To Sketch:** Geometrical pattern whose side is 5 cm long.

Solution:



b. (i) **Required To Complete:** Table for a pattern whose side is 6 cm.

Solution:

Length of one side of the pattern (cm)	Total number of triangular shapes used to make pattern	Number of white triangular shapes used	Number of grey triangular shapes used
2	4	3	1
3	9	6	3
4	16	10	6

So far it is obvious that from the figures and the tables,
Total number of triangular shapes used (column 2) = No. of white triangular shapes (column 3) + No. of grey triangular shapes used (column 4).

Example:

$$4 = 3 + 1$$

$$9 = 6 + 3$$

$$16 = 10 + 6$$

Also, column 2 is the square of the column 1 values.

2	4	$(= 2^2)$
3	9	$(= 3^2)$
4	16	$(= 4^2)$

When $n = 6$ (column 1)

$$\begin{aligned} \text{Column 2} &= (6)^2 \\ &= 36 \end{aligned}$$

$$\begin{aligned} \text{Column 4} &= \frac{n(n-1)}{2} \\ &= \frac{6(6-1)}{2} \\ &= 15 \end{aligned}$$

$$\begin{aligned} \text{Column 3} &= \text{Column 2} - \text{Column 4} \\ &= 36 - 15 \\ &= 21 \end{aligned}$$

The 4th row is

6	36	21	15
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(ii) **Required To Complete:** Table for a pattern whose side is 20 cm.

Solution:

When $n = 20$

$$\begin{aligned} \text{Column 2} &= (20)^2 \\ &= 400 \end{aligned}$$

$$\begin{aligned} \text{Column 4} &= \frac{n(n-1)}{2} \\ &= \frac{20(20-1)}{2} \\ &= 190 \end{aligned}$$

The 5th row is

20 400 210 190

c. **Required To Complete:** Table for a pattern whose side is n cm long.

Solution:

The last row of the table:

$$n \qquad n^2 \qquad * \qquad \frac{n(n-1)}{2}$$

$$n^2 = * + \frac{n(n-1)}{2}$$

$$n^2 = * + \frac{n^2 - n}{2}$$

$$n^2 = * + \frac{1}{2}n^2 - \frac{1}{2}n$$

$$* = \frac{1}{2}n^2 + \frac{1}{2}n$$

$$= \frac{1}{2}n(n+1)$$

$$= \frac{n(n+1)}{2}$$

The last row may now be completed as

$$n \qquad n^2 \qquad \frac{n(n+1)}{2} \qquad \frac{n(n-1)}{2}$$

And the completed table is

Length of one side of pattern (cm)	Total number of triangular shapes used to make pattern	Number of white triangular shapes used	Number of grey triangular shapes used
2	4	3	1
3	9	6	3
4	16	10	6
⋮	⋮	⋮	⋮
6	36	21	15
⋮	⋮	⋮	⋮
20	400	210	190
⋮	⋮	⋮	⋮
n	n^2	$\frac{1}{2}n(n+1)$	$\frac{1}{2}n(n-1)$

9. a. **Data:** $y = x^2 - 3x - 12$ and $y = 2x - 16$

Required To Calculate: x and y .

Calculation:

Let

$$y = x^2 - 3x - 12 \quad \dots(1)$$

$$y = 2x - 16 \quad \dots(2)$$

Equating (1) and (2)

$$x^2 - 3x - 12 = 2x - 16$$

$$x^2 - 5x + 4 = 0$$

$$(x-1)(x-4) = 0$$

$$x = 1 \text{ or } 4$$

$$\begin{aligned} \text{When } x = 1 \quad y &= 2(1) - 16 \\ &= -14 \end{aligned}$$

$$\begin{aligned} \text{When } x = 4 \quad y &= 2(4) - 16 \\ &= -8 \end{aligned}$$

$$x = 1 \text{ and } y = -14, \quad x = 4 \text{ and } y = -8$$

b. **Data:** $y = x^2 - 3x - 12$

(i) **Required To Express:** In the form $y = (x-h)^2 + k$

Solution:

$$x^2 - 3x - 12 = \left(x - \frac{3}{2}\right)^2 + *$$

$$\text{Half the coefficient of } -3x = \frac{1}{2}(-3) = -\frac{3}{2}$$

$$\begin{array}{r}
 x^2 - 3x + 2\frac{1}{4} \\
 -14\frac{1}{4} \quad (= *) \\
 \hline
 -12
 \end{array}$$

is of the form $(x - h)^2 + k$, where $h = \frac{3}{2}$ and $k = -14\frac{1}{4}$.

OR

$$\begin{aligned}
 y &= x^2 - 3x - 12 = (x - h)^2 + k \\
 &= x^2 - 2hx + h^2 + k
 \end{aligned}$$

Equating coefficient of x .

$$-3 = -2h$$

$$h = \frac{3}{2}$$

Equating constants.

$$-12 = \left(-\frac{3}{2}\right)^2 + k$$

$$-12 = 2\frac{1}{4} + k$$

$$k = -14\frac{1}{4}$$

$$\therefore x^2 - 3x - 12 = \left(x - \frac{3}{2}\right)^2 - 14\frac{1}{4}$$

(ii) **Required To Calculate:** Minimum value of $y = x^2 - 3x - 12$.

Calculation:

$$x^2 - 3x - 12 = \left(x - \frac{3}{2}\right)^2 - 14\frac{1}{4}$$

$(x - 3/2)(x - 3/2)$ is ≥ 0 for $\forall x$

$$\therefore y_{\min} = 0 - 14\frac{1}{4}$$

$$= -14\frac{1}{4}$$

(iii) **Required To Calculate:** The roots of $x^2 - 3x - 12 = 0$.

Calculation:

If

$$x^2 - 3x - 12 = 0$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-12)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9 + 48}}{2}$$

$$= \frac{3 \pm \sqrt{57}}{2}$$

$$= 5.27 \text{ or } -2.27$$

$$= 5.3 \text{ or } -2.3 \text{ (to 1 decimal place)}$$

OR

$$x^2 - 3x - 12 = 0$$

$$\left(x - \frac{3}{2}\right)^2 - 14\frac{1}{4} = 0$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{57}{4}$$

$$x - \frac{3}{2} = \pm \sqrt{\frac{57}{4}}$$

$$= \pm \frac{\sqrt{57}}{2}$$

$$x = \frac{3}{2} \pm \frac{\sqrt{57}}{2}$$

$$= \frac{3 \pm \sqrt{57}}{2}$$

$$= 5.27 \text{ or } -2.27$$

$$= 5.3 \text{ or } -2.3 \text{ to 1 decimal place}$$

c. **Required To Sketch:** The graph of $y = x^2 - 3x - 12$.

Solution:

$$\text{When } x = 0 \quad y = (0)^2 - 3(0) - 12$$

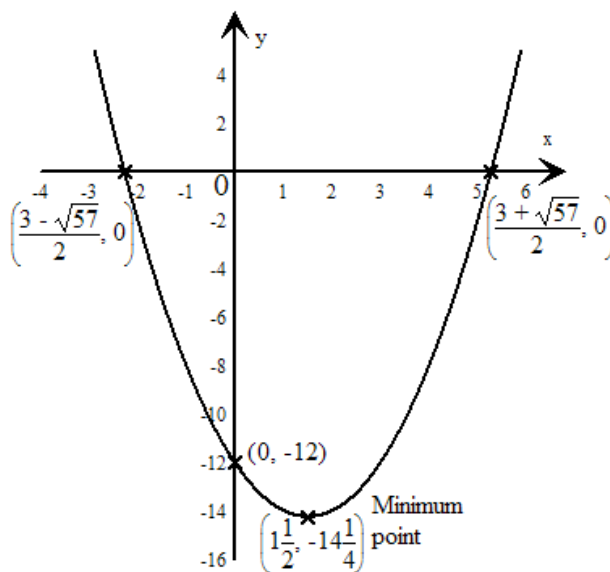
$$= -12$$

Curve cuts y - axis at -12 .

$$y_{\min} = -14\frac{1}{4} \text{ at } x = 1\frac{1}{2}$$

The minimum point is $\left(1\frac{1}{2}, -14\frac{1}{4}\right)$.

Curve cuts the x -axis at $\frac{3+\sqrt{57}}{2}$ and $\frac{3-\sqrt{57}}{2}$.



10. **Data:** Curve with equation $y = \frac{20}{x^2}$ for $2 \leq x \leq 7$.

a. **Required To Complete:** Table of values given.

Solution:

x	2	3	4	5	6	7
y	5.0	2.2	1.3	0.8	0.6	0.4

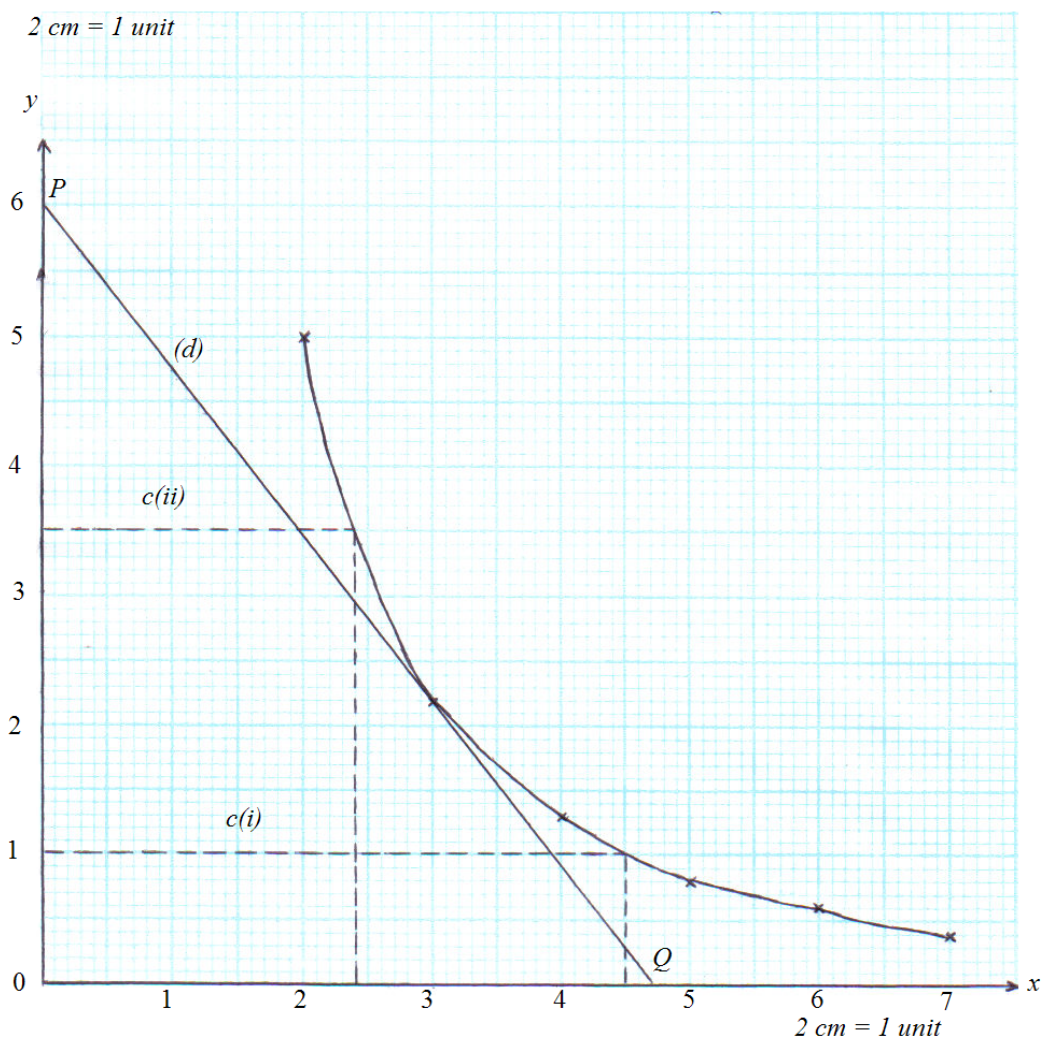
$$\begin{aligned} \text{When } x = 2 \quad y &= \frac{20}{(2)^2} \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{When } x = 5 \quad y &= \frac{20}{(5)^2} \\ &= 0.8 \end{aligned}$$

$$\begin{aligned} \text{When } x = 7 \quad y &= \frac{20}{(7)^2} \\ &= 0.4 \end{aligned}$$

- b. **Required To Plot:** The points above and draw a smooth curve.

Solution:



- c. (i) **Required To Find:** y when $x = 4.5$

Solution:

When $x = 4.5$, $y = 1$ (Read off from graph).

- (ii) **Required To Find:** x when $y = 3.5$

Solution:

When $y = 3.5$, $x = 2.4$ (Read off from graph).

- d. **Required To Estimate:** Gradient of the curve at $(3, 2.2)$.

Solution:

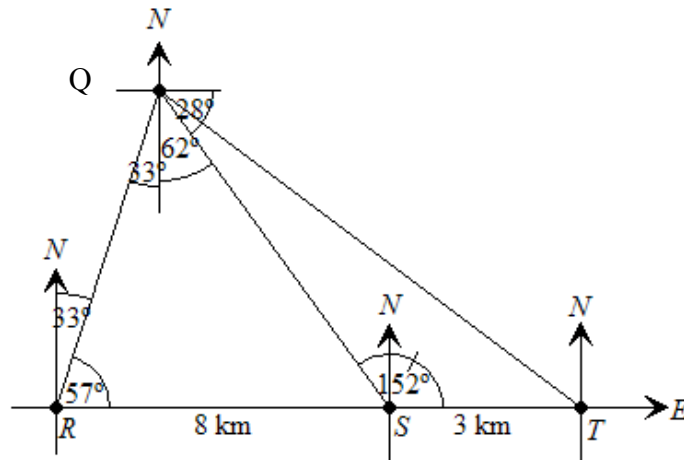
$P = 6$ and $Q = 4.7$

$$\begin{aligned} \text{Gradient of the curve at } (3, 2.2) &= -\frac{6}{4.7} \\ &= -1.277 \\ &= -1.28 \end{aligned}$$

11. **Data:** Given R, S and T lying on a straight line. The bearing of Q from R is 033° and the bearing of S from Q is 118° .

a, b **Required To Draw:** Diagram showing the information given.

Solution:



c. (i) **Required To Calculate:** $\hat{Q}RS$.

Calculation:

$$\begin{aligned} \hat{Q}RS &= 90^\circ - 33^\circ \\ &= 57^\circ \end{aligned}$$

(ii) **Required To Calculate:** $\hat{R}QS$

Calculation:

$$\begin{aligned} \hat{R}QS &= 33^\circ + (90^\circ - 28^\circ) \\ &= 33^\circ + 62^\circ \\ &= 95^\circ \end{aligned}$$

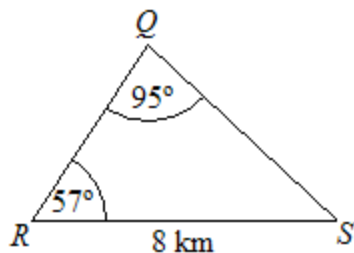
(iii) **Required To Calculate:** $\hat{Q}ST$

Calculation:

$$\begin{aligned} \hat{Q}ST &= 57^\circ + 95^\circ \\ &= 152^\circ \end{aligned}$$

(Exterior angles of a triangle = sum of the interior opposite angles).

- d. (i) **Required To Calculate:** Distance QS .
Calculation:



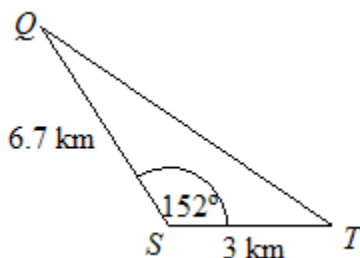
$$\frac{QS}{\sin 57^\circ} = \frac{8}{\sin 95^\circ} \quad (\text{Sine rule})$$

$$\therefore QS = \frac{8 \sin 57^\circ}{\sin 95^\circ}$$

$$= 6.7 \text{ km}$$

$$= 7 \text{ km to the nearest km}$$

- (ii) **Required To Calculate:** Distance QT .
Calculation:



$$QT^2 = (6.7)^2 + (3)^2 - 2(6.7)(3)\cos 152^\circ \quad (\text{Cosine Rule})$$

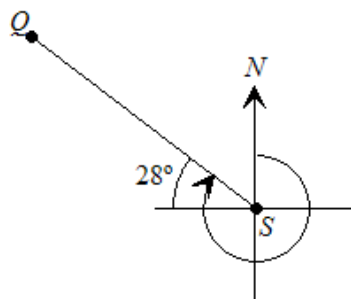
$$QT^2 = 89.384$$

$$QT = \sqrt{89.384}$$

$$= 9.4 \text{ km}$$

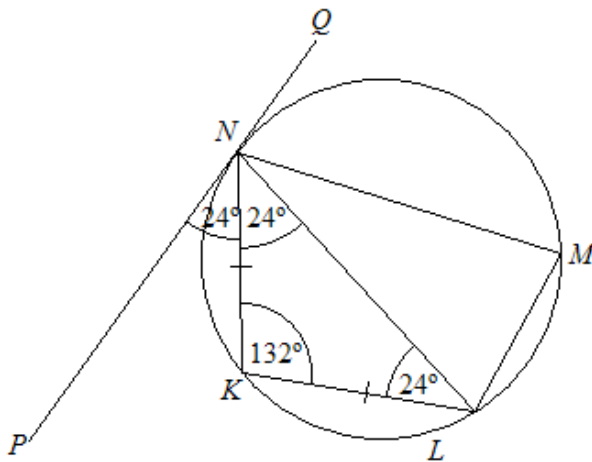
$$= 9 \text{ km to the nearest km}$$

- e. **Required To Calculate:** Bearing of Q from S .
Calculation:



$$\text{Bearing of } Q \text{ from } S = 270^\circ + 28^\circ = 298^\circ$$

12. a. **Data:** Diagram, as shown below, with $\hat{PNK} = 24^\circ$, PNQ is the tangent at N and $KN = KL$.



- (i) **Required To Calculate:** \hat{KLN}

Calculation:

$$\hat{KLN} = 24^\circ$$

(The angle made by the tangent to a circle and a chord, at the point of contact equals the angle in the alternate segment).

- (ii) **Required To Calculate:** \hat{NKL}

Calculation:

$$\hat{KNL} = 24^\circ$$

(The base angles of an isosceles triangle are equal).

$$\begin{aligned}\hat{NKL} &= 180^\circ - (24^\circ + 24^\circ) \\ &= 132^\circ\end{aligned}$$

(Sum of the angles in a triangle = 180°).

- (iii) **Required To Calculate:** \hat{LMN}

Calculation:

$$\begin{aligned}\hat{LMN} &= 180^\circ - 132^\circ \\ &= 48^\circ\end{aligned}$$

(Opposite angles of a cyclic quadrilateral are equal).

- b. This part of the question has not been solved as it involves Earth Geometry which has since been removed from the syllabus.

13. **Data:** Points O, A, B and C and their coordinates.

- a. (i) **Required To Express:** \overrightarrow{OA} in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:

$$A = (-1, 3)$$

$$\therefore \overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \text{ is of the form } \begin{pmatrix} x \\ y \end{pmatrix} \text{ where } x = -1 \text{ and } y = 3.$$

- (ii) **Required To Express:** \overrightarrow{OB} in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:

$$B = (-5, 4)$$

$$\therefore \overrightarrow{OB} = \begin{pmatrix} -5 \\ 4 \end{pmatrix} \text{ is of the form } \begin{pmatrix} x \\ y \end{pmatrix} \text{ where } x = -5 \text{ and } y = 4.$$

- (iii) **Required Express:** \overrightarrow{OC} in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:

$$C = (7, p)$$

$$\therefore \overrightarrow{OC} = \begin{pmatrix} 7 \\ p \end{pmatrix} \text{ is of the form } \begin{pmatrix} x \\ y \end{pmatrix} \text{ where } x = 7 \text{ and } y = p.$$

- b. (i) **Required To Express:** \overrightarrow{BA} in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:

$$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA}$$

$$= -\begin{pmatrix} -5 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$$\text{is of the form } \begin{pmatrix} x \\ y \end{pmatrix}, \text{ where } x = 4 \text{ and } y = -1.$$

- (ii) **Required To Express:** \overrightarrow{AC} in terms of p , in the form $\begin{pmatrix} x \\ y \end{pmatrix}$

Solution:

$$\begin{aligned}\vec{AC} &= \vec{AO} + \vec{OC} \\ &= -\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 7 \\ p \end{pmatrix} \\ &= \begin{pmatrix} 8 \\ p-3 \end{pmatrix}\end{aligned}$$

is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ where $x = 8$ and $y = p - 3$.

c. **Data:** $|\vec{AC}| = 10$

Required To Calculate: p

Calculation:

If

$$|\vec{AC}| = 10,$$

then,

$$\sqrt{(8)^2 + (p-3)^2} = 10$$

$$(8)^2 + (p-3)^2 = 100$$

$$64 + p^2 - 6p + 9 = 100$$

$$p^2 - 6p - 27 = 0$$

$$(p-9)(p+3) = 0$$

$$p = 9 \text{ or } -3$$

d. **Data:** D is such that $\vec{OD} = \begin{pmatrix} 11 \\ 0 \end{pmatrix}$

Required To Prove: A , B and D are collinear.

Proof:

$$\vec{BA} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

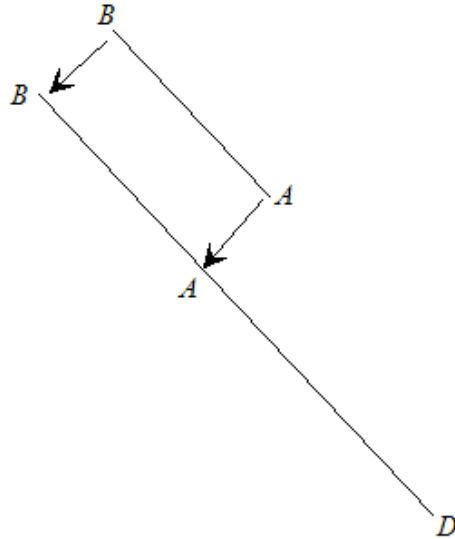
$$\vec{BD} = \vec{BO} + \vec{OD}$$

$$= -\begin{pmatrix} -5 \\ 4 \end{pmatrix} + \begin{pmatrix} 11 \\ 0 \end{pmatrix}$$

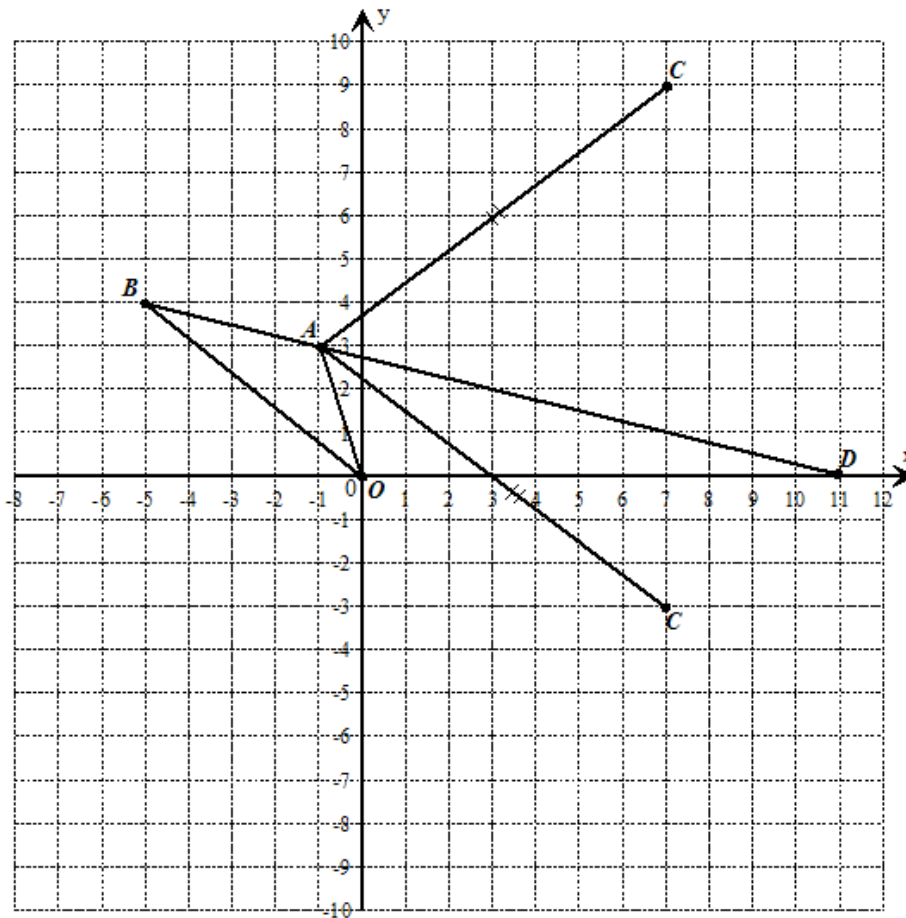
$$= \begin{pmatrix} 16 \\ -4 \end{pmatrix}$$

$$= 4 \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$\therefore \vec{BD}$ is a scalar multiple of \vec{BA} and hence they are parallel.



B is a common point to both line segments, therefore, B , A and D are collinear.



14. a. **Data:** $M = \begin{pmatrix} -6 & 5 \\ -12 & 10 \end{pmatrix}$

Required To Prove: M is singular.

Proof:

$$\begin{aligned} \text{Det } M &= (-6 \times 10) - (5 \times -12) \\ &= -60 + 60 \\ &= 0 \end{aligned}$$

Hence, M is singular.

b. **Data:** $\begin{pmatrix} 2 & 1 \\ a & 4 \end{pmatrix} \begin{pmatrix} 5 \\ b \end{pmatrix} = \begin{pmatrix} 8 \\ 7 \end{pmatrix}$

Required To Calculate: a and b .

Calculation:

$$\begin{aligned} \begin{pmatrix} 2 & 1 \\ a & 4 \end{pmatrix} \begin{pmatrix} 5 \\ b \end{pmatrix} &= \begin{pmatrix} 8 \\ 7 \end{pmatrix} \\ \begin{pmatrix} (2 \times 5) + (1 \times b) \\ (a \times 5) + (4 \times b) \end{pmatrix} &= \begin{pmatrix} 8 \\ 7 \end{pmatrix} \\ \begin{pmatrix} 10 + b \\ 5a + 4b \end{pmatrix} &= \begin{pmatrix} 8 \\ 7 \end{pmatrix} \end{aligned}$$

Equating corresponding entries.

$$10 + b = 8$$

$$b = -2$$

and

$$5a + 4(-2) = 7$$

$$5a = 15$$

$$a = 3$$

Hence, $a = 3$ and $b = -2$.

c. (i) **Data:** $E(5, 2)$ is mapped onto $E'(2, -5)$ under $E \xrightarrow{S} E'$ where

$$S = \begin{pmatrix} 0 & x \\ y & 0 \end{pmatrix}.$$

Required To Calculate: x and y .

Calculation:

$$\begin{pmatrix} 0 & x \\ y & 0 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

$$\begin{pmatrix} (0 \times 5) + (x \times 2) \\ (y \times 5) + (0 \times 2) \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 2x \\ 5y \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

Equating corresponding entries.

$$2x = 2$$

$$x = 1$$

$$5y = -5$$

$$y = -1$$

Therefore, $x = 1$ and $y = -1$.

- (ii) **Data:** $G(7, 1)$ is mapped onto G' under the translation, T , where

$$T = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \text{ and } H \text{ is mapped onto } H'.$$

- (a) **Required To Calculate:** Coordinates of G' .

Calculation:

$$\begin{pmatrix} 7 \\ 1 \end{pmatrix} \xrightarrow{T = \begin{pmatrix} -3 \\ 4 \end{pmatrix}} \begin{pmatrix} 7 + (-3) \\ 1 + 4 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 1 \end{pmatrix} \xrightarrow{T = \begin{pmatrix} -3 \\ 4 \end{pmatrix}} \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$$\therefore G' = (4, 5)$$

- (b) **Required To Calculate:** p and q

Calculation:

$$H \xrightarrow{T} H'$$

$$\begin{pmatrix} p \\ q \end{pmatrix} \xrightarrow{T = \begin{pmatrix} -3 \\ 4 \end{pmatrix}} \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} p + (-3) \\ q + 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

Equating corresponding entries.

$$p - 3 = 2$$

$$p = 5$$

$$q + 4 = 6$$

$$q = 2$$

Hence, $p = 5$ and $q = 2$.

- (iii) **Data:** $P(10, 12)$ undergoes the combined translation S followed by T .
Required To Calculate: Image of P .

Calculation:

Let

$$P \xrightarrow{S} P' \xrightarrow{T} P''$$

$$\begin{pmatrix} 10 \\ 12 \end{pmatrix} \xrightarrow{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} P'$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 10 \\ 12 \end{pmatrix} = \begin{pmatrix} (0 \times 10) + (1 \times 12) \\ (-1 \times 10) + (0 \times 12) \end{pmatrix}$$

$$= \begin{pmatrix} 12 \\ -10 \end{pmatrix}$$

$$\therefore P' = \begin{pmatrix} 12 \\ -10 \end{pmatrix}$$

and

$$\begin{pmatrix} 12 \\ -10 \end{pmatrix} \xrightarrow{T = \begin{pmatrix} -3 \\ 4 \end{pmatrix}} \begin{pmatrix} 12 + (-3) \\ -10 + 4 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \\ -6 \end{pmatrix}$$

The image of P under the combined transformation is $(9, -6)$.