CXC JUNE 2008 MATHEMATICS GENERAL PROFICIENCY (PAPER 2)
(REST OF THE CARIBBEAN BESIDES TRINIDAD & TOBAGO)

Section I

1. a. (i) **Required To Calculate:** $(3.9\times 0.27)+\sqrt{0.6724}$

Calculation:

$$(3.9\times 0.27)+\sqrt{0.6724}=1.053+0.82 \quad \text{(by use of calculator)}$$
$$=1.873 \text{ (in exact form)}$$

(ii) **Required To Calculate:** $\dfrac{2\frac{1}{2}-\frac{4}{5}}{\frac{3}{4}}$

Calculation:

Numerator:

$$2\frac{1}{2}-\frac{4}{5}=\frac{5}{2}-\frac{4}{5}$$
$$=\frac{25-8}{10}$$
$$=\frac{17}{10}$$

Hence,

$$\frac{2\frac{1}{2}-\frac{4}{5}}{\frac{3}{4}}=\frac{\frac{17}{10}}{\frac{3}{4}}$$
$$=\frac{17}{10}\times\frac{4}{3}$$
$$=\frac{34}{15} \text{ (as a single fraction in its lowest terms)}$$

b. (i) **Data:** CAN\$1.00 ≡ JA\$72.50

Required To Calculate: CAN\$250 in JA\$.

Calculation:

Cost of camera = CAN\$250

Hence cost of camera in JA\$ $=250\times 72.50$

= JA\$18 125

(ii) **Required To Calculate:** Remaining money on credit card in CAN$.
Calculation:
Limit on credit card = JA$30 000
Available remainder after buying the camera $= \text{JA\$}(30\,000 - \$18125)$

The equivalent in CAN$ $= \dfrac{30\,000 - 18125}{72.50}$

2. a. **Data:** $a = 2$, $b = -1$ and $c = 3$

(i) **Required To Calculate:** $a(b+c)$
Calculation:

$$a(b+c) = 2(-1+3)$$
$$= 2(2)$$
$$= 4$$

(ii) **Required To Calculate:** $\dfrac{4b^2 - 2ac}{a+b+c}$
Calculation:

$$\frac{4b^2 - 2ac}{a+b+c} = \frac{4(-1)^2 - 2(2)(3)}{2+(-1)+3}$$
$$= \frac{4(1) - 12}{4}$$
$$= \frac{-8}{4}$$
$$= -2$$

b. (i) **Required To Find:** Algebraic expression for the statement given.
Solution:
Four times the sum of x and 5.

$4\times$ $\quad\quad$ $(x+5)$

$= 4(x+5)$

(ii) **Required To Find:** Algebraic expression for the statement given.
Solution:
16 larger than the product of a and b.

$16 +$ $\quad\quad$ $(a \times b = ab)$

$= 16 + ab$

c. **Data:** $15-4x=2(3x+1)$
Required To Calculate: x
Calculation:

$$15-4x=2(3x+1)$$
$$15-4x=6x+2$$
$$15-2=6x+4x$$
$$10x=13$$
$$x=\frac{13}{10}$$
$$=1\frac{3}{10}$$

d. **Required To Factorise:** (i) $6a^2b^3+12a^4b$, $(ii)\,2m^2+9m-5$
Solution:

(i) $6a^2b^3+12a^4b=\underline{6}\times\underline{a}^2\times\underline{b}\times b^2+2\times\underline{6}\times\underline{a}^2\times a^2\times\underline{b}$
$$=6a^2b(b^2+2a^2)$$
$$=6a^2b(2a^2+b^2)$$

(ii) $2m^2+9m-5=(2m-1)(m+5)$

3. **Data:** Results of 1 080 students' choices in a career guidance seminar.

Career	Lawyer	Teacher	Doctor	Artist	Salesperson
Number of students	240	189	t	216	330

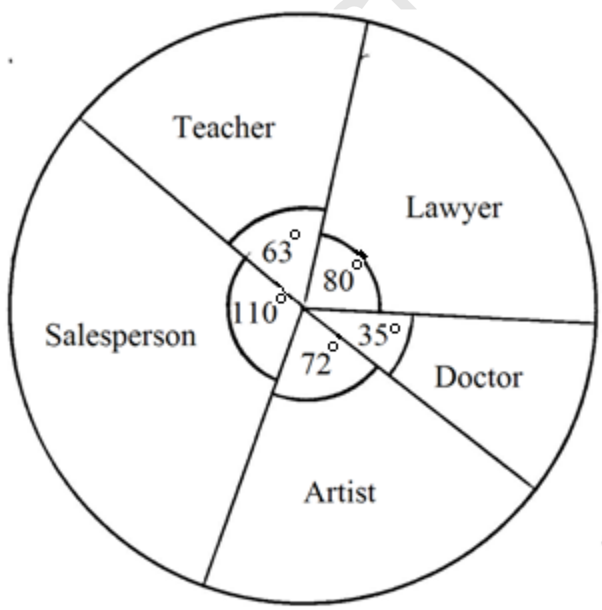
a. **Required To Calculate:** t
Calculation:

$240+189+t+216+330=1080$ (data)
$$t=1080-(240+189+216+330)$$
$$=105$$

b. (i) **Required To Calculate:** Size of the angles of the sectors in the pie chart.
Calculation:

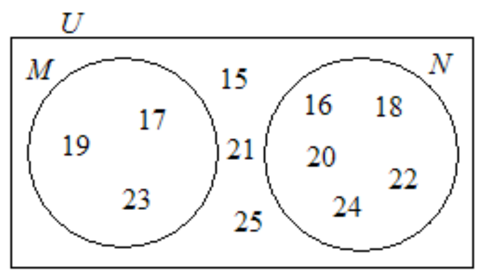
Sector illustrating	Angle of sector
Lawyer	$\frac{240}{1080} \times 360° = 80°$
Teacher	$\frac{189}{1080} \times 360° = 63°$
Doctor	$\frac{105}{1080} \times 360° = 35°$
Artist	$\frac{216}{1080} \times 360° = 72°$
Salesperson	$\frac{330}{1080} \times 360° = 110°$

(ii) **Required To Draw:** Pie chart to represent the information given, using a circle of radius = 4 cm.
Solution:



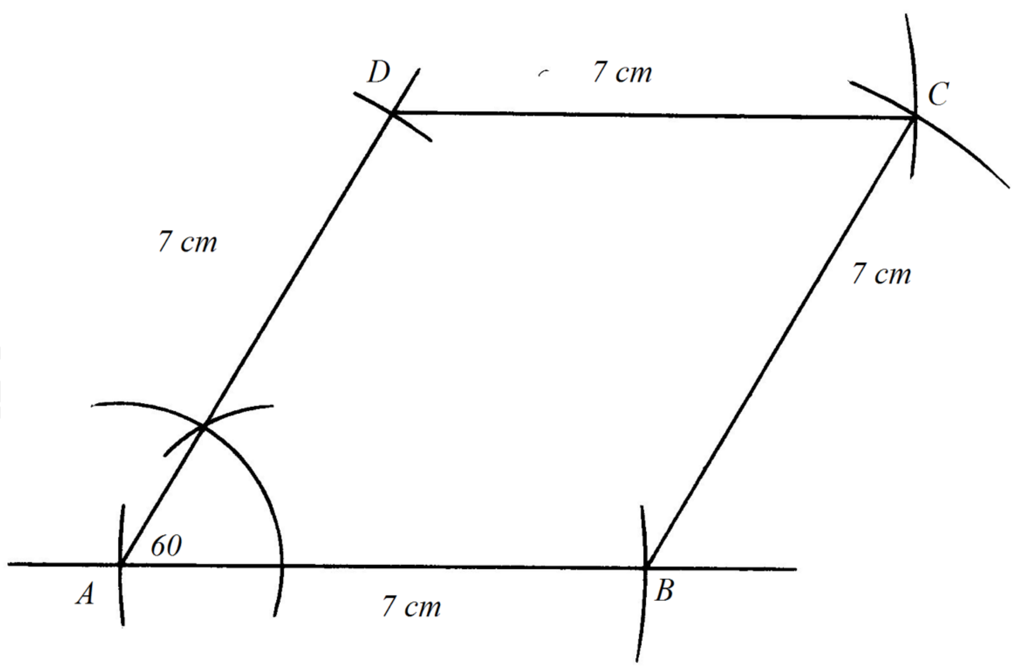
4. a. **Data:** Given the universal set U and sets M and N are defined.

(i) **Required To Draw:** Venn diagram for the information given.
Solution:



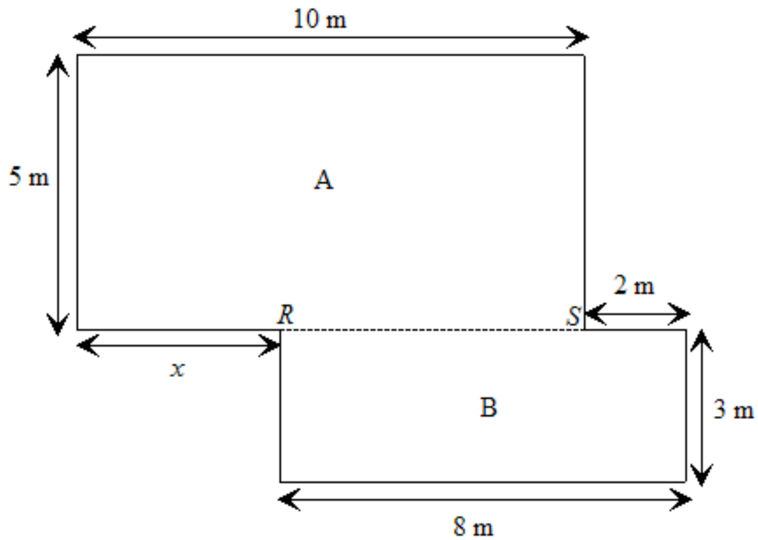
(ii) **Required To List:** Elements of the set $(M \cup N)'$.
Solution:
$(M \cup N)' = \{15, 21, 25\}$ (as illustrated on the Venn diagram)

b. (i) **Required To Construct:** Parallelogram $ABCD$, in which $AB = AD = 7$ cm and $B\hat{A}D = 60°$.
Solution:



(ii) **Required To Find:** Length of diagonal AC.
Solution:
$AC = 12.1$ cm (by measurement).

5. **Data:** Plan of a floor with given dimensions.



a. (i) **Required To Calculate:** Length RS.
Calculation:

$$RS + 2 = 8 \quad \text{(From diagram)}$$
$$RS = 8 - 2$$
$$\text{Length of } RS = 6\,\text{m}$$

(ii) **Required To Calculate:** x
Calculation:

$$x + RS = 10\,\text{m} \quad \text{(From diagram)}$$
$$x + 6 = 10$$
$$x = 4\,\text{m}$$

b. **Required To Calculate:** Perimeter of the entire floor.
Calculation:
Using R as the starting point and checking the total distance of all the edges, to find the perimeter of the floor.

$$\text{Perimeter } = (4 + 5 + 10 + 5 + 2 + 3 + 8 + 3)\,\text{m}$$
$$= 40\,\text{m}$$

c. **Required To Calculate:** Area of the entire floor.
Calculation:
Area of the entire floor = Area of region A + Area of region B

$$= \{(5\times 10)+(3\times 8)\}$$
$$= 50+24$$
$$= 74\,\text{m}^2$$

d. **Data:** Part A is to covered with flooring boards measuring 1 m by 20 cm.
Required To Calculate: Number of flooring boards needed to cover A.
Calculation:
Area of 1 flooring board $= 1\,\text{m}\times 20\,\text{cm}$

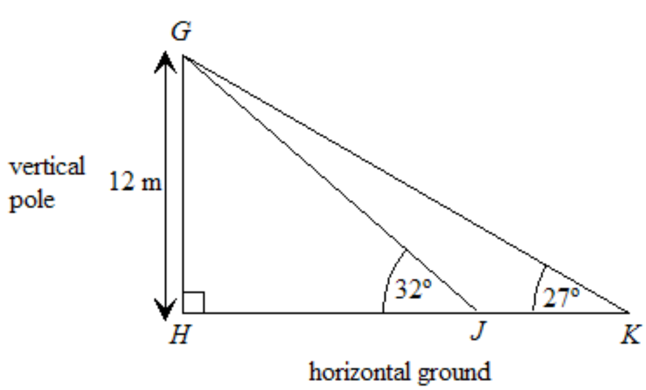
$$= 1\,\text{m}\times\frac{20}{100}\,\text{m}$$
$$= \frac{20}{100}\,\text{m}^2$$
$$= \frac{1}{5}\,\text{m}^2$$

Number of flooring boards needed to cover A $= \dfrac{\text{Area of A}}{\text{Area of 1 board}}$

$$= \frac{5\times 10}{\frac{1}{5}}$$
$$= 250$$

No. of boards = 250

6. a. **Data:** Diagram showing a vertical pole and *H*, *J* and *K*, points on the horizontal ground.
(i) **Required To Complete:** Diagram by inserting the angles of elevation.
Solution:



(ii) (a) **Required To Calculate:** Length of *HJ*.

Calculation:

$$\tan 32^\circ = \frac{12}{HJ}$$

$$\therefore HJ = \frac{12}{\tan 32^\circ}$$

$$= 19.2\underline{0}\text{ m}$$

$$= 19.2\text{ m (to 1 decimal place)}$$

(b) **Required To Calculate:** The length of *JK*.

Calculation:

$$\tan 27^\circ = \frac{12}{HK}$$

$$\therefore HK = \frac{12}{\tan 27^\circ}$$

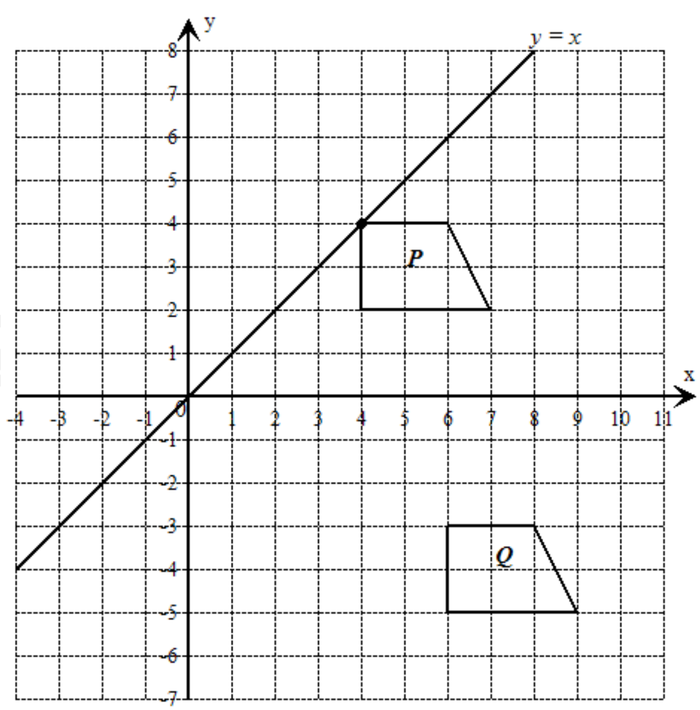
$$= 23.55\text{ m}$$

$$\text{Length of } JK = 23.55 - 19.2$$

$$= 4.3\underline{5}$$

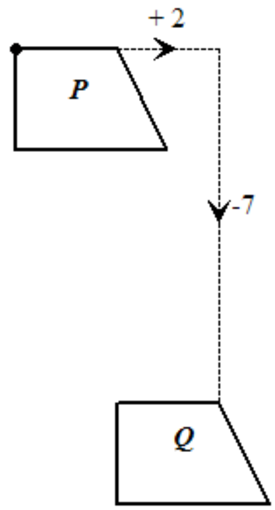
$$= 4.4\text{ m (to 1 decimal place)}$$

b. **Data:** Diagram on axes illustrating figure *P* and congruent figure *Q*.



(i) **Required To Describe:** The transformation that P undergoes to produce Q.

Solution:



P is mapped onto Q by a horizontal shift of 2 units to the right and a vertical shift of 7 units downwards.

This is a translation, T, where $T = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$.

(ii) (a) **Required To Draw:** The line $y = x$ on the answer sheet.

(b) **Required To Draw:** S, the image of P under a reflection in the line $y = x$.

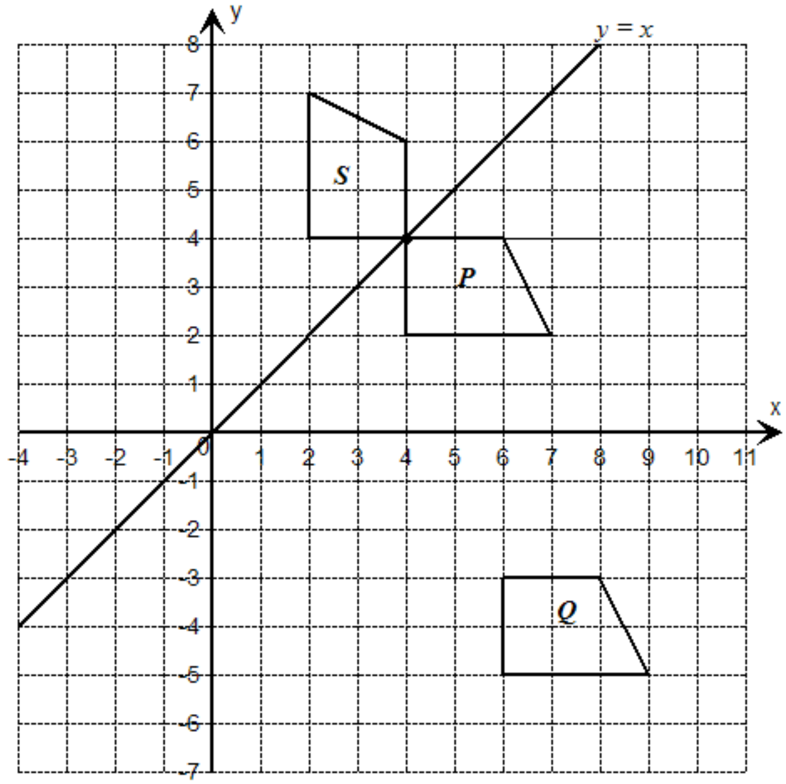
Solution:

P onto S by a reflection in the line $y = x$.

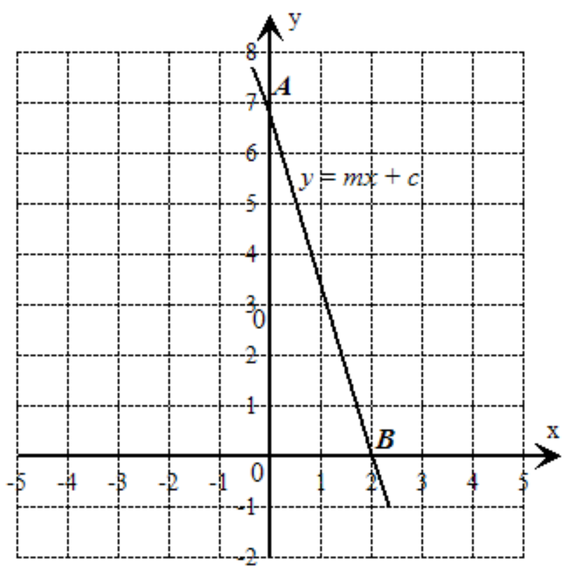
$$\therefore P \xrightarrow{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} S$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 6 & 7 & 4 \\ 4 & 4 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 4 & 2 & 2 \\ 4 & 6 & 7 & 4 \end{pmatrix}$$

Coordinates of the vertices of P | Coordinates of the vertices S



7. **Data:** Diagram of a straight line cutting the axes at *A* and *B*.



a. (i) **Required To Calculate:** c

Calculation:

From the diagram, the straight line cuts the vertical axis at *A* (0, 7),
Hence, $c = 7$.

(ii) **Required To Calculate:** m
Calculation:
From the diagram $B = (2, 0)$

$$\text{Gradient of } AB = \frac{7-0}{0-2} = m$$
$$m = -\frac{7}{2}$$

(iii) **Required To Calculate:** Midpoint of AB.
Calculation:

$$\text{Midpoint of } AB = \left(\frac{0+2}{2}, \frac{7+0}{2}\right)$$
$$= \left(1, 3\frac{1}{2}\right)$$

b. **Data:** The point (-2, k) lies on the line.
Required To Calculate: k
Calculation:

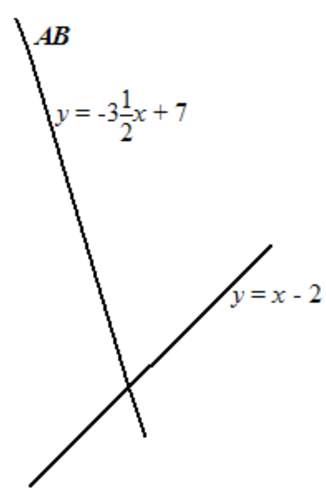
Equation of AB is $y = mx + c$, where $m = -3\frac{1}{2}$ and $c = 7$, that is

$$y = -3\frac{1}{2}x + 7$$

(-2, k) lies on the graph.

$$k = \left(-3\frac{1}{2}\right)(-2) + 7$$
$$= 14$$

c. **Required To Calculate:** Point of intersection of $y = x - 2$ and the given line.
Calculation:



To determine the point of intersection of $y = x - 2$ and AB, we solve the equations simultaneously.
Let

$y = -3\frac{1}{2}x + 7 \ldots(1)$

$y = x - 2 \qquad \ldots(2)$

Equating (1) and (2)

$$-3\frac{1}{2}x + 7 = x - 2$$

$$7 + 2 = x + 3\frac{1}{2}x$$

$$9 = 4\frac{1}{2}x$$

$$x = \frac{9}{4\frac{1}{2}}$$

$$= 2$$

When $x = 2$ $\qquad y = 2 - 2$
$\qquad = 0$

$\therefore$ The point of intersection of the two lines is (2, 0).

8. **Data:** Annie bought 6 each of four different kind of stamps.

a. **Required To Calculate:** Total cost of the stamps.

Calculation:

Cost of 6 stamps at \$4.00 each = \$24.00
Cost of 6 stamps at \$2.50 each = \$15.00
Cost of 6 stamps at \$1.20 each = \$ 7.20
Cost of 6 stamps at \$1.00 each = \$ 6.00
Total = \$52.20

Hence, the total cost of the stamps that Annie bought = \$52.20

b. **Data:** Cost of Annie posting a parcel was \$25.70

(i) **Required To Calculate:** Stamps that she can select using as many \$4.00 stamps as possible.

Calculation:

Cost of posting a parcel is \$25.70
Using
5 stamps at \$4.00 each = \$20.00
1 stamp at \$2.50 each = \$ 2.50
1 stamp at \$1.20 each = \$ 1.20
2 stamps at \$1.00 each = \$ 2.00
Total is exactly = \$ 25.70

(ii) **Required To Calculate:** Stamps that she can select using all her \$1.00 stamps.
Calculate:
Using
All 6 stamps at \$1.00 each = \$ 6.00
4 stamps at \$4.00 each = \$16.00
1 stamp at \$2.50 each = \$ 2.50
1 stamp at \$1.20 each = \$ 1.20
Total is exactly = \$25.70

c. (i), (ii) **Required To Calculate:** The largest number of stamps she can use and list the selection of stamps she can use.
Calculation:
To use the most number of stamps, Annie must use
6 stamps at \$1.00 = \$ 6.00
6 stamps at \$1.20 = \$ 7.20
5 stamps at \$2.50 = \$12.50
Total is exactly = \$25.70

And the maximum number of stamps $= 6 + 6 + 5 = 17$

9. a. (i) **Required To Simplify:** $x^2 \times x^3 \div x^4$
Solution:
$$\begin{aligned} x^2 \times x^3 \div x^4 &= x^{2+3-4} \\ &= x^1 \\ &= x \end{aligned}$$

(ii) **Required To Simplify:** $a^{\frac{3}{2}} b^{\frac{5}{2}} \times \sqrt{ab^3}$
Solution:
$$\begin{aligned} a^{\frac{3}{2}} b^{\frac{5}{2}} \times \sqrt{ab^3} &= a^{\frac{3}{2}} b^{\frac{5}{2}} \times \left(ab^3\right)^{\frac{1}{2}} \\ &= a^{\frac{3}{2}} b^{\frac{5}{2}} a^{\frac{1}{2}} b^{\frac{3}{2}} \\ &= a^{\frac{3}{2}+\frac{1}{2}} b^{\frac{5}{2}+\frac{3}{2}} \\ &= a^2 b^4 \end{aligned}$$

b. **Data:** $f(x) = 2x - 3$
(i) **Required To Calculate:** $f(2)$
Calculation:
$$\begin{aligned} f(2) &= 2(2) - 3 \\ &= 4 - 3 \\ &= 1 \end{aligned}$$

(ii) **Required To Calculate:** $f^{-1}(0)$

Calculation:

Let

$$y = 2x - 3$$
$$y + 3 = 2x$$
$$x = \frac{y+3}{2}$$

Replace y by x

$$f^{-1}(x) = \frac{x+3}{2}$$
$$f^{-1}(0) = \frac{0+3}{2}$$
$$= 1\frac{1}{2}$$

(iii) **Required To Calculate:** $f^{-1}f(2)$

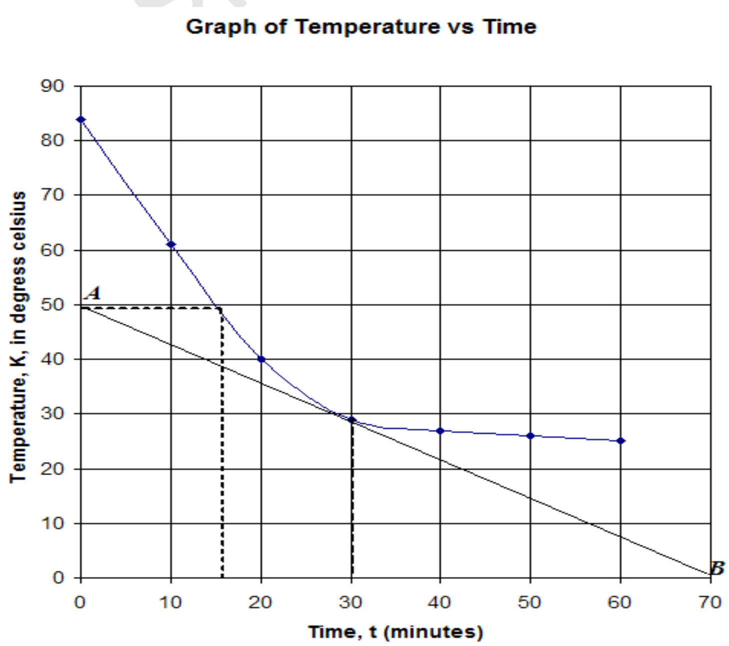
Calculation:

$$f(2) = 1$$
$$\therefore f^{-1}f(2) = f^{-1}(1)$$
$$= \frac{1+3}{2}$$
$$= 2$$

c. **Data:** Table of values of temperature vs time for a liquid as its cools.

(i) **Required To Draw:** The curve to represent the information given.

Solution:



Note:
Scale is
Horizontal axis, 2 cm $\equiv$ 10 minutes, and not 10 seconds as printed.

(ii) (a) **Required To Find:** Temperature of the liquid after 15 minutes.
Solution:
The temperature of the liquid after 15 minutes is approximately 49.5°C. (Read off).

(b) **Required To Find:** Rate of cooling of the liquid at $t = 30$.
Solution:
The tangent to the curve at $t = 30$ cuts the axes at A and B as shown.
A is (0, 50) and B is (70, 0)

$$\text{Gradient } AB = -\frac{50}{70}$$
$$= -0.7\underline{1}$$
$$= -0.7°C \text{ min}^{-1}$$

(The negative sign $\Rightarrow$ temperature is decreasing.)
The rate of cooling of the liquid a $t = 30$ minutes is 0.7 °C min^{-1}.

10. a. **Data:** $y + 4x = 27$ and $xy + x = 40$
Required To Calculate: x and y
Calculation:
Let
$y + 4x = 27 \ldots(1)$
$xy + x = 40 \ \ldots(2)$
From (1)
$y = 27 - 4x$
Substitute in (2)

$$x(27 - 4x) + x = 40$$
$$27x - 4x^2 + x = 40$$
$$4x^2 - 28x + 40 = 0$$
$\div 4$
$$x^2 - 7x + 10 = 0$$
$$(x - 2)(x - 5) = 0$$
$$x = 2 \text{ or } 5$$

When $x = 2$ $\quad y = 27 - 4(2) \quad = 19$
When $x = 5$ $\quad y = 27 - 4(5) \quad = 7$
Hence, $x = 2$ and $y = 19$ **OR** $x = 5$ and $y = 7$.

b. (i) **Data:** A shaded region illustrating a set of inequalities representing boys and girls in a cricket club

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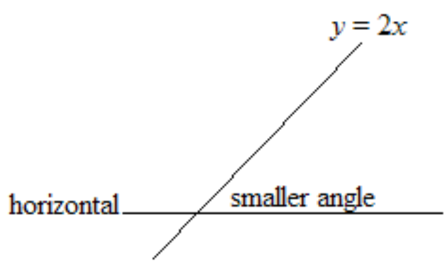
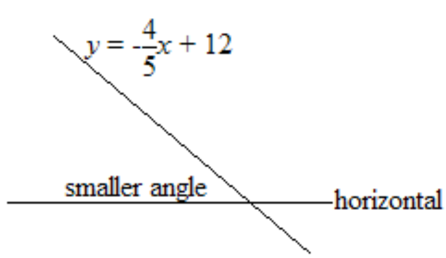
(a) **Required To State:** Whether the cricket club can have 10 boys and 5 girls.
Solution:
x = no. of boys and y = no. of girls
If $x = 10$ and $y = 5$, we note that the point (10, 5) does not lie in the shaded region. Hence, the club cannot have 10 boys and 5 girls.

(b) **Required To State:** Whether the cricket club can have 6 boys and 6 girls.
Solution:
If $x = 6$ and $y = 6$ we note that the point (6, 6) lies within the shaded region. Hence, the club can have 6 boys and 6 girls.

(ii) **Required To Find:** The inequalities that defines the shaded region.
Solution:



The region with the shaded is on the side with the smaller angle. Hence, the region is $y < 2x$. (It may be $y \le 2x$ if the line is included).

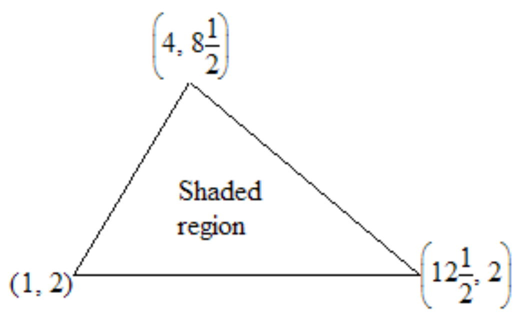


The region shaded is on the side with the smaller angle. Hence, the region is $y < -\frac{4}{5}x + 12$. (It may be $y \le -\frac{4}{5}x + 12$ if the line is included).

The region shaded is above the horizontal $y = 2$. Hence, the region is $y > 2$. (It may be $y \ge 2$ if the line is included).

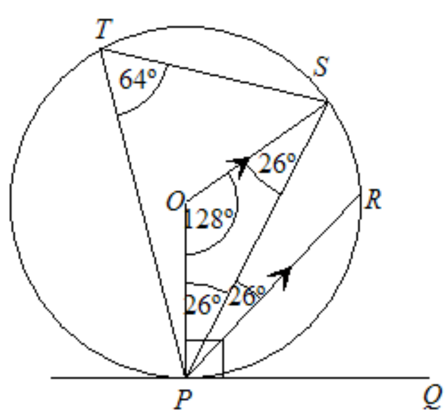
(iii) (a) **Data:** Profit of \$3.00 made on boy's uniform and profit of \$5.00 made of a girl's uniform.
Required To Find: An expression in x and y for the total profit made.
Solution:
The profit on x boy's uniforms at \$3 each and y girls uniforms at \$5 each $=(x\times 3)+(5\times y)$
Let the total profit be P.
$$P = 3x + 5y$$

(b) **Required To Calculate:** The minimum profit the company can make.
Calculation:



The minimum profit will be made when $x=1$ and $y=2$.
$$\therefore P_{\min} = 3(1)+5(2)$$
$$= \$13$$

11. a. **Data:** Diagram, shown below, with circle centre O, $S\hat{P}R = 26^\circ$ and OS parallel to PR.



(i) **Required To Calculate:** $P\hat{T}S$

Calculation:

$O\hat{S}P = 26°$

(alternate angles).

$OS = OP$ (radii)

$O\hat{P}R = 26°$

(Base angles of an isosceles triangle are equal).

Hence,

$$S\hat{O}P = 180° - (26° + 26°)$$
$$= 128°$$

(Sum of angles in a triangle = 180°).

$$P\hat{T}S = \frac{1}{2}(128°)$$
$$= 64°$$

(The angle subtended by a chord at the centre of the circle is twice the angle subtended at the circumference, standing on the same arc).

(ii) **Required To Calculate:** $R\hat{P}Q$

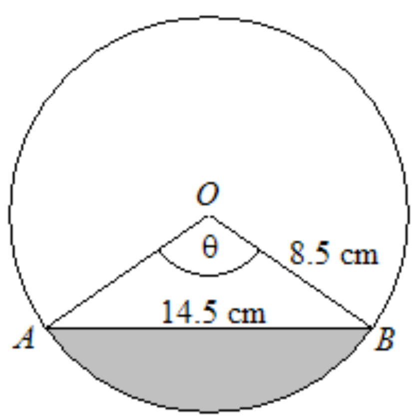
Calculation:

$O\hat{P}Q = 90°$

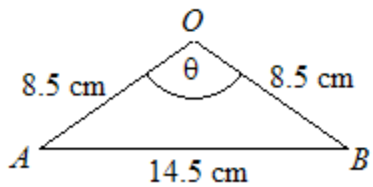
(The angle made by a tangent to a circle with radius, at the point of contact = 90°).

$$R\hat{P}Q = 90° - (26° + 26°)$$
$$= 38°$$

b. **Data:** Diagram showing a circle, centre O and chord AB = 14.5 cm.

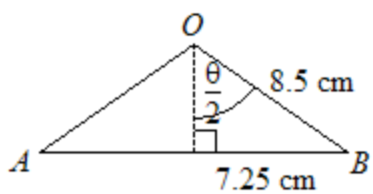


(i) **Required To Calculate:** The value of θ.
Calculation:



$$(14.5)^2 = (8.5)^2 + (8.5)^2 - 2(8.5)(8.5)\cos\theta \ (\cos law)$$
$$\theta = \cos^{-1}(-0.454) = 117.06$$
$$= 117°(\text{ to the nearest degree})$$

OR

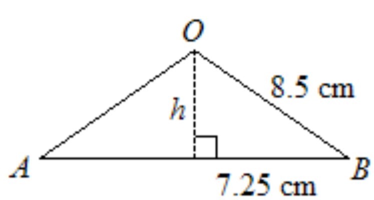


$$\sin\frac{\theta}{2} = \frac{7.25}{8.5}$$
$$\frac{\theta}{2} = 58.533$$
$$\theta = 58.533 \times 2$$
$$= 117.\underline{0}6$$
$$= 117°(\text{ to the nearest degree})$$

(ii) **Required To Calculate:** Area of triangle *AOB*.
Calculation:

$$\text{Area of } \Delta AOB = \frac{1}{2}(8.5)(8.5)\sin 117°$$
$$= 32.2 \text{ cm}^2$$

OR



$$h = \sqrt{(8.5)^2 - (7.25)^2}$$
$$= \sqrt{19.6875}$$
$$= 4.437$$

$$\text{Area } = \frac{14.5 \times 4.437}{2}$$
$$= 32.2\,\text{cm}^2$$

OR

$$s = \frac{8.5 + 8.5 + 14.5}{2}$$
$$= 15.75$$

$$\text{Area } = \sqrt{15.75(15.75 - 8.5)(15.75 - 8.5)(15.75 - 14.5)} \text{ (Heron's Formula)}$$
$$= 32.2 \text{ cm}^2$$

(iii) **Required To Calculate:** Area of the shaded region.
Calculation:
Area of the shaded segment = Area of sector AOB – Area of ΔAOB

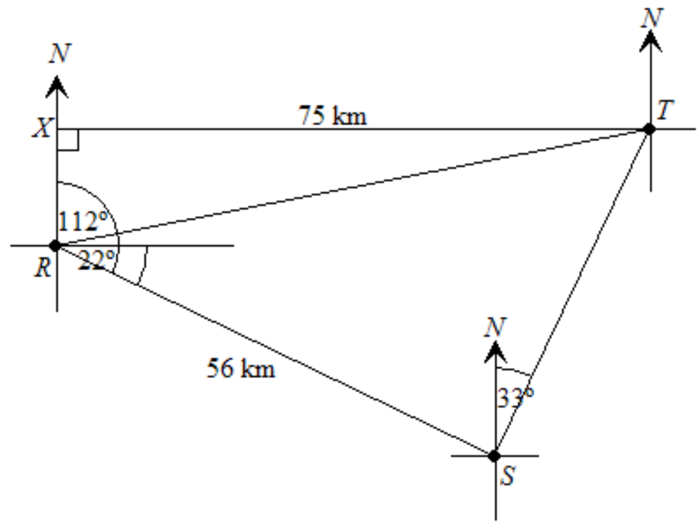
$$= \frac{117°}{360°} \times \pi(8.5)^2 - 32.2$$
$$= 41.5\,\text{cm}^2$$

(iv) **Required To Calculate:** Length of major arc AB.
Calculation:
Angle of major sector $= 360° - 117°$

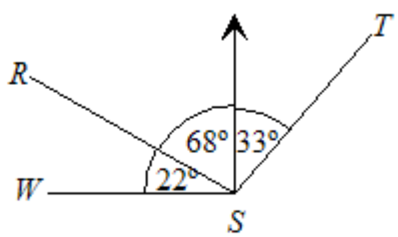
$$\text{Length of major arc } = \left(\frac{360° - 117°}{360°}\right) \times 2\pi(8.5)$$
$$= 36.05 \text{ cm}$$

12. **Data:** Ship sails from point R to S and then to T.

a. **Required To Draw:** Diagram showing the journey of the ship from R to S to T.
Solution:



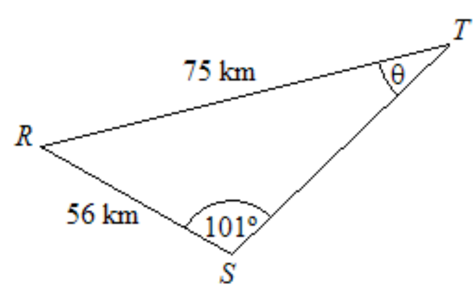
b. (i) **Required To Calculate:** $R\hat{S}T$
Calculation:



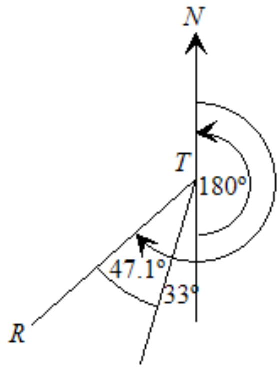
$$R\hat{S}T = 68° + 33°$$
$$= 101°$$

(ii) **Required To Calculate:** $R\hat{T}S$
Calculation:

$$Let\ R\hat{T}S = \theta$$
$$\frac{56}{\sin\theta} = \frac{75}{\sin 101°}\ (\sin rule)$$
$$\sin\theta = \frac{56\sin 101°}{75}$$
$$= 0.733$$
$$\theta = \sin^{-1} 0.733$$
$$= 47.13$$
$$= 47.1°\ (to\ the\ nearest\ \ 0.1\ of\ a\ degree)$$

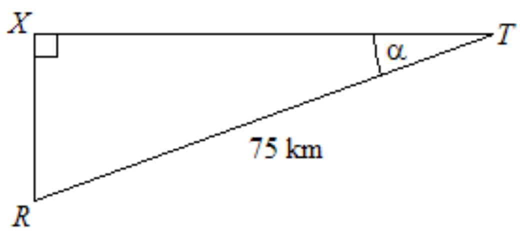


(iii) **Required To Calculate:** Bearing from R to T.
Calculation:



$\therefore$ The bearing from R to $T = 180° + 33° + 47.1°$
$= 260.1°$

c. (ii) **Required To Calculate:** Distance *TX*.
Calculation:

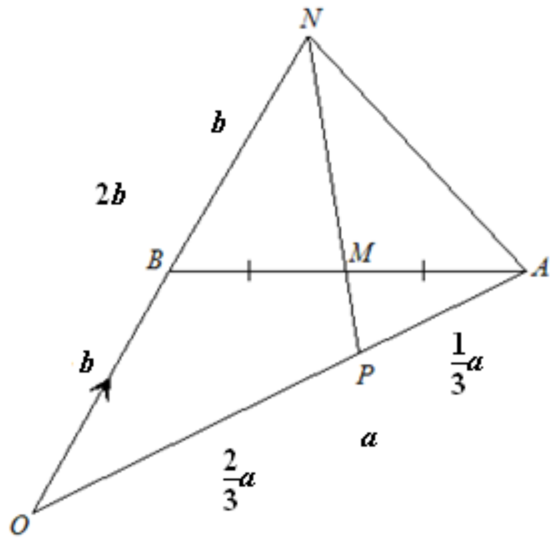


$\alpha = 90° - (47.1° + 33°)$
$= 9.9°$

$\cos 9.9° = \frac{TX}{75}$

$TX = 75 \cos 9.9°$
$= 73.88 \text{ km}$

13. **Data:** $\overrightarrow{OA} = a$, $\overrightarrow{OB} = b$, P is on OA such that $\overrightarrow{OP} = 2PA$ and *M* is on *BA* such that *BM* = *MA* and *OB* is produced to *N* such that *OB* = *BN*.
a. **Required To Draw:** The diagram showing the points *P* and *M*.
Solution:



b. **Required To Express:** $\overrightarrow{AB}$, $\overrightarrow{PA}$ and $\overrightarrow{PM}$ in terms of ***a*** and ***b***.

Solution:

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -(a) + b$

$= -a + b$

If

$\overrightarrow{OP} = 2PA$, then

$\overrightarrow{OP} = \frac{2}{3}\overrightarrow{OA}$

$= \frac{2}{3}a$

and

$\overrightarrow{PA} = \frac{1}{3}a$

$\overrightarrow{PM} = \overrightarrow{PA} + \overrightarrow{AM}$

$= \frac{1}{3}a + \overrightarrow{AM}$

$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB}$

$\overrightarrow{PM} = \frac{1}{3}a + \frac{1}{2}(-a + b)$

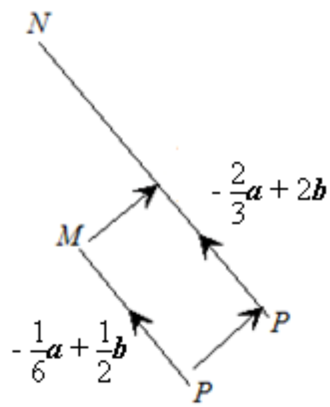
$= \frac{1}{3}a - \frac{1}{2}a + \frac{1}{2}b$

$= -\frac{1}{6}a + \frac{1}{2}b$

c. **Required To Prove:** P, M and N are collinear.

Proof:

$$OB = BN$$
$$\overrightarrow{ON} = 2b$$
$$\overrightarrow{PN} = \overrightarrow{PO} + \overrightarrow{ON}$$
$$= -\frac{2}{3}a + 2b$$
$$= 4(-\frac{1}{6}a + \frac{1}{2}b)$$



$\therefore \overrightarrow{PN} = 4\overrightarrow{PM}$, that is a scalar multiple. Hence, $\overrightarrow{PN}$ is parallel to $\overrightarrow{PM}$, *P* is a common point, *M* must lie on *PN* and *P*, *M* and *N* lie on the same straight line, that is they are collinear.

Q.E.D

d. **Data:** $a = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Required To Calculate: Length of *AN*.

Calculation:

$$\overrightarrow{AN} = \overrightarrow{AO} + \overrightarrow{ON}$$
$$= -a + 2b$$
$$= -\begin{pmatrix} 6 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$
$$= \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

$$\text{Length of } AN = \sqrt{(-4)^2 + (2)^2}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5} \text{ units}$$

14. a. **Data:** $X = \begin{pmatrix} -2 & 0 \\ 5 & 1 \end{pmatrix}$ and $Y = \begin{pmatrix} 4 & -1 \\ 3 & 7 \end{pmatrix}$.

Required To Calculate: $X^2 + Y$

Calculation:

$$X^2 = \begin{pmatrix} -2 & 0 \\ 5 & 1 \end{pmatrix}\begin{pmatrix} -2 & 0 \\ 5 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$$

$$= \begin{pmatrix} (-2 \times -2) + (0 \times 5) & (2 \times 0) + (0 \times 1) \\ (5 \times -2) + (1 \times 5) & (5 \times 0) + (1 \times 1) \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 \\ -5 & 1 \end{pmatrix}$$

and

$$X^2 + Y = \begin{pmatrix} 4 & 0 \\ -5 & 1 \end{pmatrix} + \begin{pmatrix} 4 & -1 \\ 3 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & -1 \\ -2 & 8 \end{pmatrix}$$

b. **Data:** $Q \xrightarrow{\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}} Q'$

Required To Find: 2×2 matrix that maps Q' back to Q.

Solution:

$$Q \xrightarrow{\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}} Q'$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \xrightarrow{\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}} \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

Let

$$M = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$$

If M maps Q onto Q', then M^{-1} maps Q' onto Q.

$$\det M = (1 \times 3) - (2 \times 1)$$

$$= 3 - 2$$

$$= 1$$

$$M^{-1} = \frac{1}{1}\begin{pmatrix} 3 & -(2) \\ -(1) & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$$

$\therefore Q' \to Q$ by $\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$

Note: The coordinates of Q and Q' are totally irrelevant in this question.

c. **Data:** $DEF \xrightarrow{\text{Enlargement}} D'E'F'$

(i) (a) **Required To Calculate:** Scale factor k.

Calculation:

Let the enlargement be L. L has a centre O and scale factor k. L may be represented by $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$.

$$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} 5 \\ 12 \end{pmatrix} = \begin{pmatrix} 7\frac{1}{2} \\ 18 \end{pmatrix}$$

$$\begin{pmatrix} 5k \\ 12k \end{pmatrix} = \begin{pmatrix} 7\frac{1}{2} \\ 18 \end{pmatrix}$$

Equating corresponding entries.

$5k = 7\frac{1}{2}$

$k = 1\frac{1}{2}$

OR

$12k = 18$

$k = 1\frac{1}{2}$

(b) **Required To Find:** Coordinates of E' and F'

Solution:

The translation matrix for L is

$$L = \begin{pmatrix} 1\frac{1}{2} & 0 \\ 0 & 1\frac{1}{2} \end{pmatrix}$$

and similarly

$$\begin{pmatrix} 1\frac{1}{2} & 0 \\ 0 & 1\frac{1}{2} \end{pmatrix}\begin{pmatrix} 2 & 8 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 12 \\ 10\frac{1}{2} & 6 \end{pmatrix} \text{ and}$$

$$E' = \left(3, 10\frac{1}{2}\right) \text{ and } F' = (12,\ 6)$$

(ii) **Data:** $D'E'F'$ undergoes a clockwise rotation of 90° about the origin.

(a) **Required To Find:** 2×2 matrix that represents the transformation.

Solution:

The 2×2 matrix that represents a 90° clockwise rotation about O is $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

(b) **Required To Find:** Coordinates of D'', E'' and F''

Solution:

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 7\frac{1}{2} & 3 & 12 \\ 18 & 10\frac{1}{2} & 6 \end{pmatrix}$$

$$= \begin{pmatrix} \left(0 \times 7\frac{1}{2}\right) + (1 \times 18) & (0 \times 3) + \left(1 \times 10\frac{1}{2}\right) & (0 \times 12) + (1 \times 6) \\ \left(-1 \times 7\frac{1}{2}\right) + (0 \times 18) & (-1 \times 3) + \left(0 \times -10\frac{1}{2}\right) & (-1 \times 12) + (0 \times 6) \end{pmatrix}$$

$$= \begin{pmatrix} 18 & 10\frac{1}{2} & 6 \\ -7\frac{1}{2} & -3 & -12 \end{pmatrix}$$

Hence $D'' = \left(18,\ -7\frac{1}{2}\right)$, $E'' = \left(10\frac{1}{2},\ -3\right)$ and $F'' = (6,\ -12)$.

(c) **Required To Find:** 2×2 matrix that maps DEF onto $D''E''F''$.

Solution:

$$\Delta DEF \xrightarrow{\begin{pmatrix} 1\frac{1}{2} & 0 \\ 0 & 1\frac{1}{2} \end{pmatrix}} \Delta D'E'F' \xrightarrow{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} \Delta D''E''F''$$

$$\Delta DEF \xrightarrow{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \times \begin{pmatrix} 1\frac{1}{2} & 0 \\ 0 & 1\frac{1}{2} \end{pmatrix}} \Delta D''E''F''$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1\frac{1}{2} & 0 \\ 0 & 1\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \left(0 \times 1\frac{1}{2}\right) + (1 \times 0) & (0 \times 0) + \left(1 \times 1\frac{1}{2}\right) \\ \left(-1 \times 1\frac{1}{2}\right) + (0 \times 0) & (-1 \times 0) + \left(0 \times 1\frac{1}{2}\right) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1\frac{1}{2} \\ -1\frac{1}{2} & 0 \end{pmatrix}$$

Hence, the single matrix that maps ΔDEF onto $\Delta D''E''F''$ is $\begin{pmatrix} 0 & 1\frac{1}{2} \\ -1\frac{1}{2} & 0 \end{pmatrix}$.