CSEC JANUARY 2008 MATHEMATICS GENERAL PROFICIENCY (PAPER 2)

Section I

1\. a. (i) **Required To Calculate:** $\dfrac{1\frac{1}{7}-\frac{3}{4}}{2\frac{1}{2}\times\frac{1}{5}}$

Calculation:
Numerator:

$$1\frac{1}{7}-\frac{3}{4}=\frac{8}{7}-\frac{3}{4}$$
$$=\frac{4(8)-7(3)}{28}$$
$$=\frac{32-21}{28}$$
$$=\frac{11}{28}$$

Denominator:

$$2\frac{1}{2}\times\frac{1}{5}=\frac{5}{2}\times\frac{1}{5}$$
$$=\frac{1}{2}$$

Hence,

$$\frac{1\frac{1}{7}-\frac{3}{4}}{2\frac{1}{2}\times\frac{1}{5}}=\frac{\frac{11}{28}}{\frac{1}{2}}$$
$$=\frac{11}{28}\times\frac{2}{1}$$
$$=\frac{11}{14}\text{ (in exact form)}$$

(ii) **Required To Calculate:** $2-\dfrac{0.24}{0.15}$

Calculation:

$$2-\frac{0.24}{0.15}=2-1.6$$
$$=0.4\text{ (in exact form)}$$

b. **Data:** Bicycle with prices for cash or hire purchase (H.P.) payments.

(i) **Required To Calculate:** Total hire purchase price for the bicycle.
Calculation:
Hire purchase price of the bicycle = Deposit + Total for 10 installments
$= \$69.00 + 10(\$28.50)$
$= \$354.00$

(ii) **Required To Calculate:** Difference between hire purchase price and cash price.
Calculation:
Difference between the hire purchase price and the cash price
$= \$354.00 - \$319.95$
$= \$34.05$

(iii) **Required To Express:** Difference between 2 prices as a percentage of the cash price.
Solution:
The difference in price as a percentage of the cash price
$= \left(\frac{34.05}{319.95} \times 100\right)\%$
$= 10.642\%$
$= 10.64\% \text{(to 2 decimal places)}$

2. a. (i) **Required To Solve:** $3 - 2x < 7$.
Solution:
$3 - 2x < 7$
$-2x < 7 - 3$
$-2x < 4$
$\times -1$
$2x > -4$
$\div 2$
$x > -2$

(ii) **Required To Determine:** Smallest value of x that satisfies the above inequality.
Solution:
$W = \{0, 1, 2, 3, ...\}$
If $x \in W$, then the smallest x is 0.

b. **Required To Factorise:** *(i)* $x^2 - xy, (ii)\, a^2 - 1, (iii)\, 2p - 2q - p^2 + pq$

Solution:

(i) $x^2 - xy = x.x - x.y$

$= x(x-y)$

(ii) $a^2 - 1 = (a)^2 - (1)^2$

This is a difference of 2 squares

And $a^2 - 1 = (a-1)(a+1)$

(iii) $2p - 2q - p^2 + pq = 2(p-q) - p(p-q)$

$= (p-q)(2-p)$

c. **Data:** Table showing types of cakes, cost and the number sold.

(i) **Required To Find:** An expression, in terms of *k*, for the amount of money collected for the sale of sponge cakes.

Solution:

The cost of 2 sponge cakes at $\$(k+5)$ each $= 2 \times (k+5)$

$= \$(2k+10)$

(ii) **Required To Find:** An expression in terms of *k*, for the total amount of collected.

Solution:

Cost of sponge cakes $= \$(2k+10)$

Cost of 10 chocolate cakes at $\$k$ each $= k \times 10$

$= \$10k$

Cost of 4 fruit cakes at $\$2k$ each $= 2k \times 4$

$= \$8k$

Total amount of money collected $= (2k+10) + 10k + 8k$

$= \$(20k+10)$

(iii) **Data:** Total amount of money collected = \$140.00

Required To Calculate: *k*

Calculation:

Total amount of money collected = \$140.00

$$\therefore 20k + 10 = 140$$

$$20k = 140 - 10$$

$$20k = 130$$

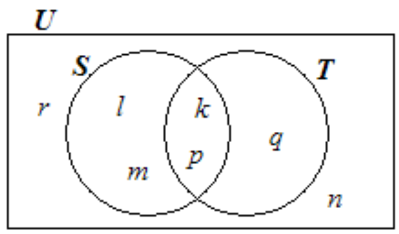
$$k = \frac{130}{20}$$

$$= 6.5$$

3. a. **Data:** Given the elements of universal set, *U*, *S* and *T*.

(i) **Required To Draw:** Venn diagram to represent the information given.
Solution:

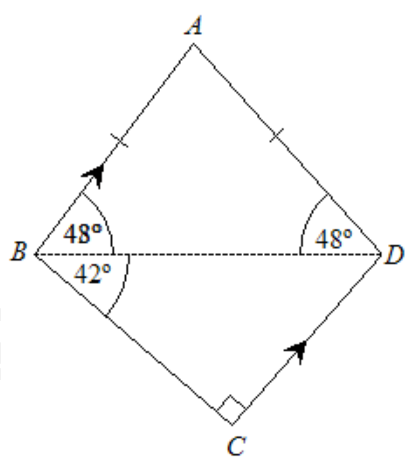
$$S = \{k, l, m, p\}$$
$$T = \{k, p, q\}$$
$$S \cap T = \{k, p\}$$



(ii) (a) **Required To List:** Members of $S \cup T$.
Solution:
$S \cup T = \{l, m, k, p, q\}$ as shown.

(b) **Required To List:** Members of S'.
Solution:
$S' = \{r, q, n\}$ as shown.

b. **Data:** Quadrilateral *ABCD* with $AB = AD$, $B\hat{C}D = 90°$, $D\hat{B}C = 42°$ and *AB* parallel to DC.



(i) **Required To Calculate:** $A\hat{B}C$
Calculation:

$$A\hat{B}C = 180° - 90°$$
$$= 90°$$

(Co-interior angles are supplementary).

(ii) **Required To Calculate:** $A\hat{B}D$
Calculation:

$$A\hat{B}D = 90° - 42°$$
$$= 48°$$

(iii) **Required To Calculate:** $B\hat{A}D$
Calculation:

$AB = AD$ (data)

$\therefore A\hat{D}B = 48°$

(Base angles of an isosceles triangle are equal).

$$B\hat{A}D = 180° - (48° + 48°)$$
$$= 84°$$

(Sum of angles in a triangle = 180°).

4. a. **Data:** John left Port A at 07:30 hours and travelled to Port B in the same time zone.
He arrives at Port B at 14:20 hours.

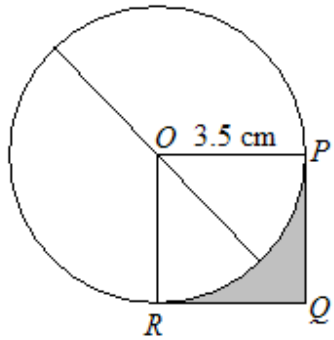
(i) **Required To Find:** Duration of the journey.
Solution:
Departure time from A is 07:30 hours.
Arrival time at B is 14:20 hours.

$$\text{Duration of the journey} = \begin{array}{r} 14:20 \\ -\ 7:30 \\ \hline 6:50 \\ \hline \end{array}$$
$$= 6 \text{ hours } 50 \text{ minutes}$$
$$= 6\frac{5}{6} \text{ hours}$$

(ii) **Data:** John travelled 410 km.
Required To Calculate: Average speed.
Calculation:

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$
$$= \frac{410\text{km}}{6\frac{5}{6}\text{ hours}}$$
$$= 60\,\text{kmh}^{-1}$$

b. **Data:** Diagram showing a circle of radius 3.5 cm, centre O and square $OPQR$.



(i) **Required To Calculate:** Area of the circle.
Calculation:

$$\text{Area of circle} = \frac{22}{7}(3.5)^2$$
$$= 38.5\,\text{cm}^2$$

(ii) **Required To Calculate:** Area of square *OPQR*.
Calculation:

$$\text{Area of square } OPQR = 3.5 \times 3.5$$
$$= 12.25\,\text{cm}^2$$

(iii) **Required To Calculate:** Area of the shaded region.
Calculation:

Area of the shaded region = Area of square *OPQR* – Area of sector *OPR*.

$$= 12.25 - \frac{1}{4}(38.5)$$
$$= 2.62\underline{5}\,\text{cm}^2$$
$$= 2.63\,\text{cm}^2 \text{ (to 2 decimal places)}$$

5. **Data:** Bar graph illustrating the number of books read by the boys of a book club.

a. **Required To Draw:** The frequency data representing the information given.
Solution:
From the bar graph

No. of books read, *x*	No. of boys, *f*
0	2
1	6
2	17
3	8
4	3

$$\sum f = 36$$

b. **Required To Find:** The number of boys in the book club.

Solution:

No. of boys in the book club $= \sum f$

$= 36$

c. **Required To Find:** Modal number of books read.

Solution:

Modal number of books read = 2, as $x = 2$ corresponds to the maximum frequency, 17.

d. **Required To Calculate:** Total number of books read.

Calculation:

Total number of books read $= (2\times 0)+(6\times 1)+(17\times 2)+(8\times 3)+(3\times 4)$

$= 6+34+24+12$

$= 76$

e. **Required To Calculate:** The mean number of books read.

Calculation:

The mean number of books read, $\bar{x}$.

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$= \frac{76}{36}$$

$$= 2.1$$

This value is a whole number and may be represented by the integer 2.

f. **Required To Calculate:** Probability a randomly chose boy reads at least books.

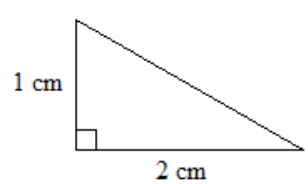
Calculation:

$$P(\text{Randomly chosen boy reads} \geq 3 \text{ books}) = \frac{\text{No. of boys reading} \geq 3 \text{ books}}{\text{Total no. of boys}}$$

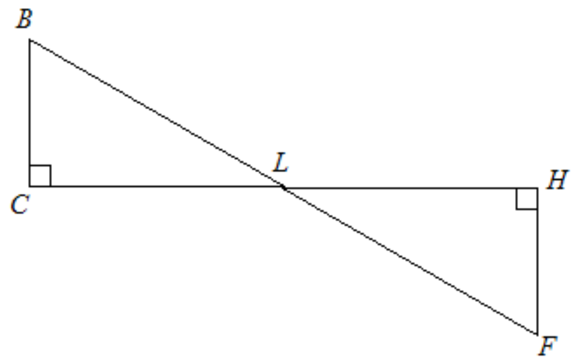
$$= \frac{8+3}{\sum f = 36}$$

$$= \frac{11}{36}$$

6. a. **Data:** Diagram showing a pattern of congruent right angled triangles as



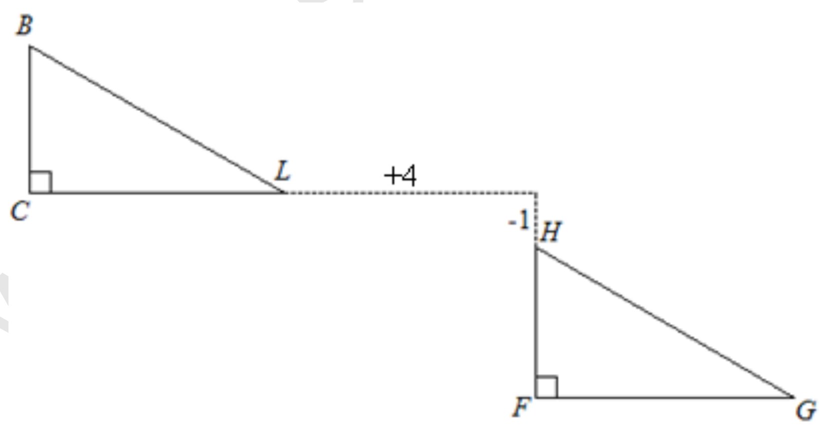
(i) **Required To Describe:** Transformation that maps ΔBCL onto ΔFHL.
Solution:



$\Delta BCL \to \Delta HFL$
L is a common point.
Image is re-oriented and congruent.
BLF is a straight line.
$\therefore$ *BCL* is mapped onto *HFL* by a rotation of 180° (clockwise or anti-clockwise) about *L*.

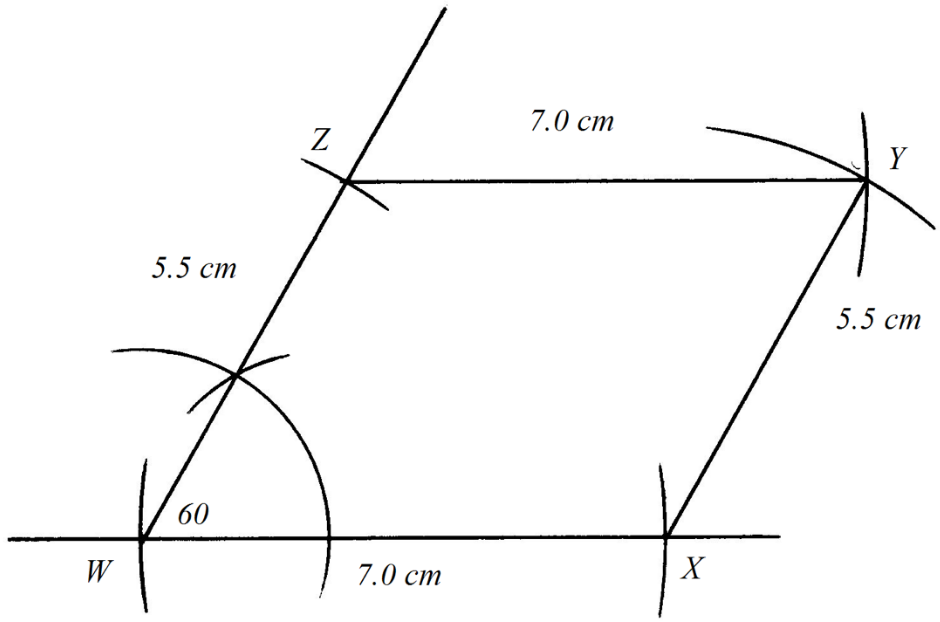
(ii) **Required To Describe**: Transformation that maps ΔBCL onto ΔHFG.
Solution:

ΔBCL and ΔHFG have the same 'orientation'.



ΔBCL is mapped onto ΔHFG by a horizontal shift 4 units to the right and 1 unit vertically downwards. This may be represented by the translation, *T* where $T = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

b. (i) **Required To Construct:** Parallelogram *WXYZ*, in which, *WX* = 7.0 cm, *WZ* = 5.5 cm and $X\hat{W}Z = 60°$.
Solution:



(ii) **Required To Find:** The length of *WY*.
Solution:
$WY = 10.9$ cm (by measurement.)

7. **Data:** Incomplete table for $y = x^2 - 4x$.

a. **Required To Complete:** The table of values for $y = x^2 - 4x$.
Solution:

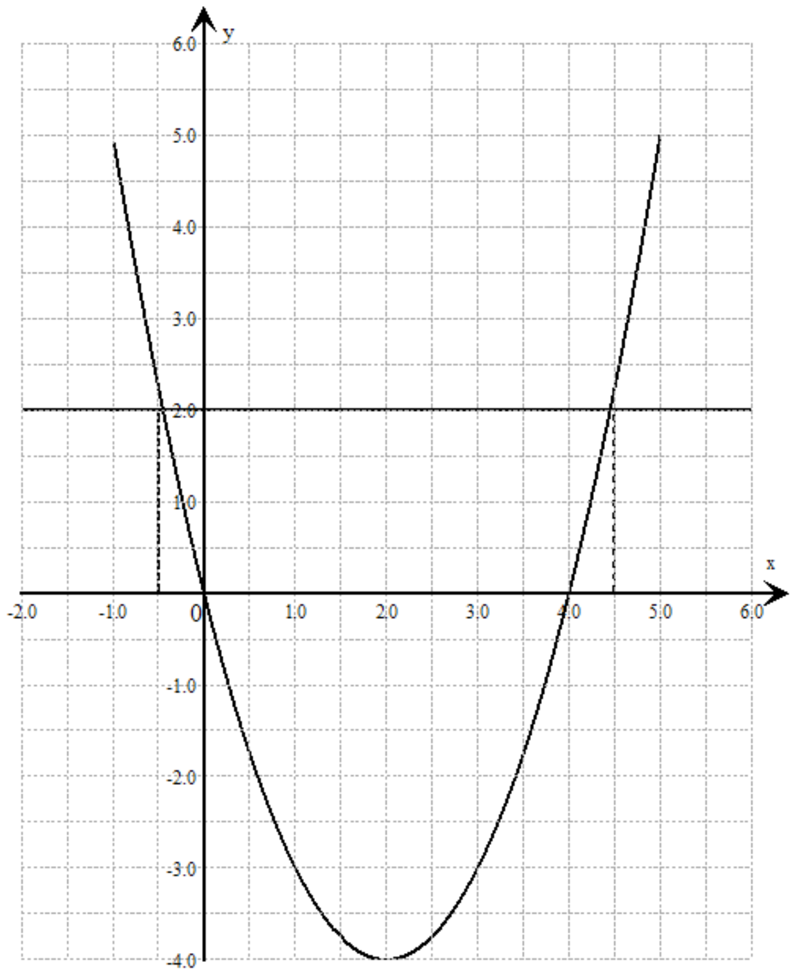
x	-1	0	1	2	3	4	5
y	5	0	-3	-4	-3	0	5

When $x = 0$ $\quad y = (0)^2 - 4(0)$
$= 0$

When $x = 3$ $\quad y = (3)^2 - 4(3)$
$= -3$

When $x = 5$ $\quad y = (5)^2 - 4(5)$
$= 5$

b. **Required To Draw:** The graph of $y = x^2 - 4x$.
Solution:



c. (ii) **Required To Find:** Points at which the curve meets the line.
Solution:
The line $y = 2$ and the curve $y = x^2 - 4x$ meet at $x = -0.5$ and $x = 4.5$.

(iii) **Required To Find:** The equation whose roots are the coordinates found above.
Solution:
$x = -0.5$ and $x = 4.5$ are the roots of the equation $x^2 - 4x = 2$ or $x^2 - 4x - 2 = 0$.

8. a. **Data:** Table showing the sum of rational numbers.
Required To Complete: The table given.

n	**SERIES**	**SUM**	**FORMULA**
1	1	1	$\frac{1}{2}(1)(1+1)$
2	1 + 2	3	$\frac{1}{2}(2)(2+1)$
3	1 + 2 + 3	6	$\frac{1}{2}(3)(3+1)$
4	1 + 2 + 3 + 4	10	$\frac{1}{2}(4)(4+1)$
5	1 + 2 + 3 + 4 + 5	15	$\frac{1}{2}(5)(5+1)$
(i) 6	1 + 2 + 3 + 4 + 5 + 6	21	$\frac{1}{2}(6)(6+1)$
⋮	⋮	⋮	⋮
8	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	36	$\frac{1}{2}(8)(8+1)$
⋮	⋮	⋮	⋮
(ii) *n*	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + …+ *n*		$\frac{1}{2}n(n+1)$

The formula for the sum is a constant, $\frac{1}{2}n(n+1)$, example

When $n=5$ Sum is $=\frac{1}{2}(5)(5+1) \quad =15$

$\therefore \text{Sum} = 15$

When $n=6$ Sum $=\frac{1}{2}(6)(6+1) \quad = 21$

$\therefore$ Sum = 21

For (ii), the formula is $\frac{1}{2}n(n+1)$.

b. **Data:**

1 + 2 + 3 = 6 and $1^3+2^3+3^3 = 6^2 = 36$

So too,

$1+2+3+4=10$ and $1^3+2^3+3^3+4^3 = 10^2$

$= 100$

(i) **Required To Find:** $1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3$

Solution:

$1+2+3+4+5+6+7+8=36$

From the table

$$1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3=36^2$$
$$=1296$$

(ii) **Required To Find:** $1^3+2^3+3^3+...+n^3$

Solution:

$$1+2+3+...+n=\frac{1}{2}n(n+1)$$
$$\therefore 1^3+2^3+3^3+...+n^3=\left\{\frac{1}{2}n(n+1)\right\}^2$$

c. **Required To Find:** $1^3+2^3+3^3+4^3+...+12^3$

Solution:

$$1+2+3+4+...+12=\frac{1}{2}(12)(12+1)$$
$$=78$$
$$\therefore 1^3+2^3+3^3+4^3=(78)^2$$
$$=6084$$

9. a. **Data:** Volume, V of a gas varies inversely as the pressure, P, with temperature constant.

(i) **Required To Find:** Equation relating V to P.

Solution:

$$V\propto\frac{1}{P}$$
$$V=k\times\frac{1}{P}$$

(k is constant of proportion or variation)

(ii) **Required To Find:** k when $V=12.8$ and $P=500$.

Solution:

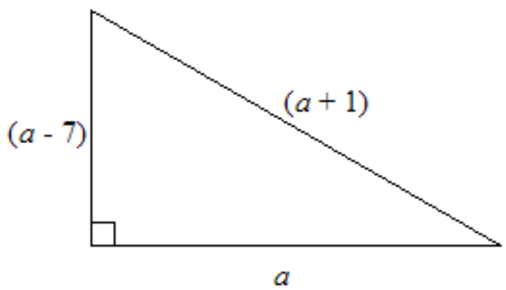
$V=12.8$ when $P=500$

$$12.8=k\times\frac{1}{500}$$
$$k=12.8\times500$$
$$=6400$$

(iii) **Required To Calculate:** V when $P = 480$.
Calculation:
When
$P = 480$

$$V = 6400 \times \frac{1}{480}$$
$$= 13\frac{1}{3}$$

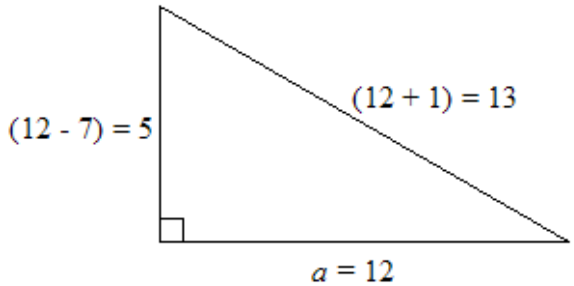
b. **Data:** Right-angled triangle of sides $a, (a-7)$ and $(a+1)$.



(i) **Required To Find:** An equation in terms of a to relate the three sides, using Pythagoras' Theorem.
Solution:
$$(a)^2 + (a-7)^2 = (a+1)^2 \quad \text{Pythagoras' Theorem}$$
$$a^2 + a^2 - 14a + 49 = a^2 + 2a + 1$$
$$a^2 - 16a + 48 = 0$$

(ii) **Required To Calculate:** a
Calculation:
$$a^2 - 16a + 48 = 0$$
$$(a-12)(a-4) = 0$$
$$a = 4 \text{ or } 12$$
If $a = 4$, the side $(a-7)$ would compute to be a negative value. Hence, $a = 12$ only.

(iii) **Required To State:** The lengths of the 3 sides of the triangle.
Solution:
The lengths of the 3 sides are



The sides are 5, 12 and 13 units.

10. a. **Data:** School buys x balls and y bats.

(i) **Required To Find:** Inequality for the information given.
Solution:
Total number of balls and bats is no more than 30.

$x+y \qquad \leq 30$

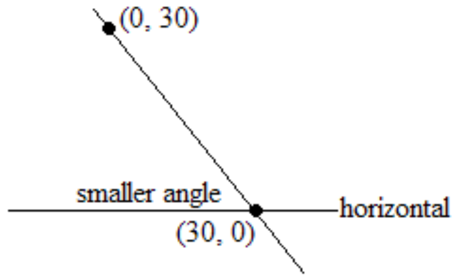
Hence, $x+y \leq 30 \ldots(1)$

(ii) **Data:** School allows no more than \$360 to be spent on bats and balls. The cost of a ball is \$6 and the cost of a bat is \$24.
Required To Find: Inequality to represent the information given.
Solution:
The cost of x balls at \$6 each and y bats at \$24 each is
$(x \times 6)+(y \times 24)=6x+24y$
Budget allows no more than \$360. Similarly,
$6x+24y \leq 360$
$\div 6$
$x+4y \leq 60 \ldots(1)$

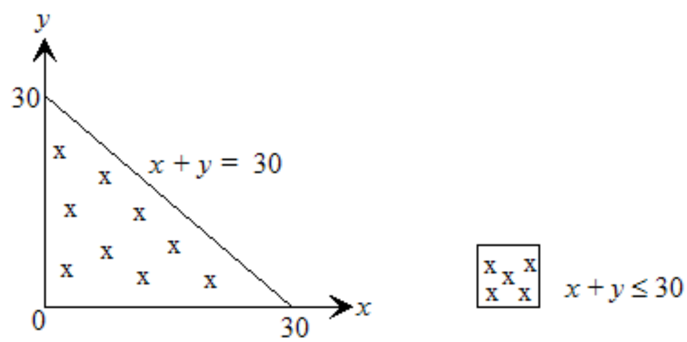
b. **Required To Draw:** The graphs of the inequalities shown above, shade the region that satisfies the inequalities and state the vertices of the feasible region.
Solution:
$x \geq 0$ and $y \geq 0$ identifies the 1st quadrant.

Obtaining 2 points on the line $x+y=30$
When $x=0$ $\qquad 0+y=30$
$y=30$
The line $x+y=30$ passes through the point (0, 30).

When $y=0$ $\qquad x+0=30$
$x=30$
The line $x+y=30$ passes through the point (30, 0).



The region with the smaller angle corresponds to the $\leq$ region.
The region which satisfies $x + y \leq 30$ is



Obtaining 2 points on the line $x + 4y = 60$.

When $x = 0$ $\quad 0 + 4y = 60$

$$4y = 60$$

$$y = \frac{60}{4}$$

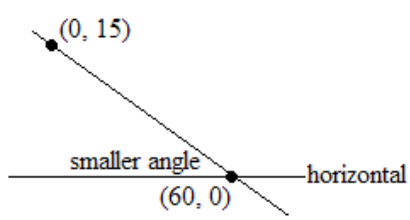
$$= 15$$

The line $x + 4y = 60$ passes through the point (0, 15).

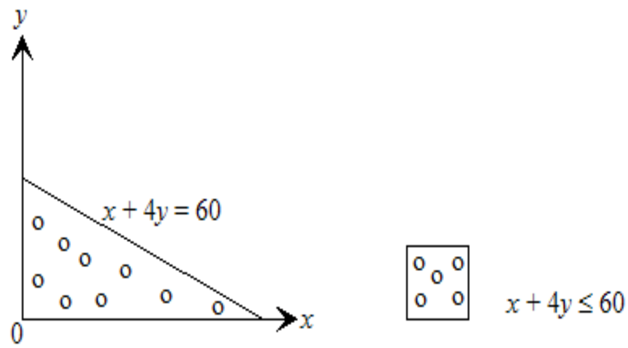
When $y = 0$ $\quad x + 4(0) = 60$

$$x = 60$$

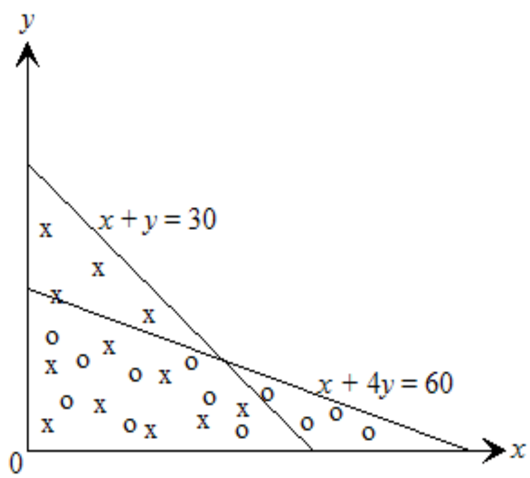
The line $x + 4y = 60$ passes through the point (60, 0).

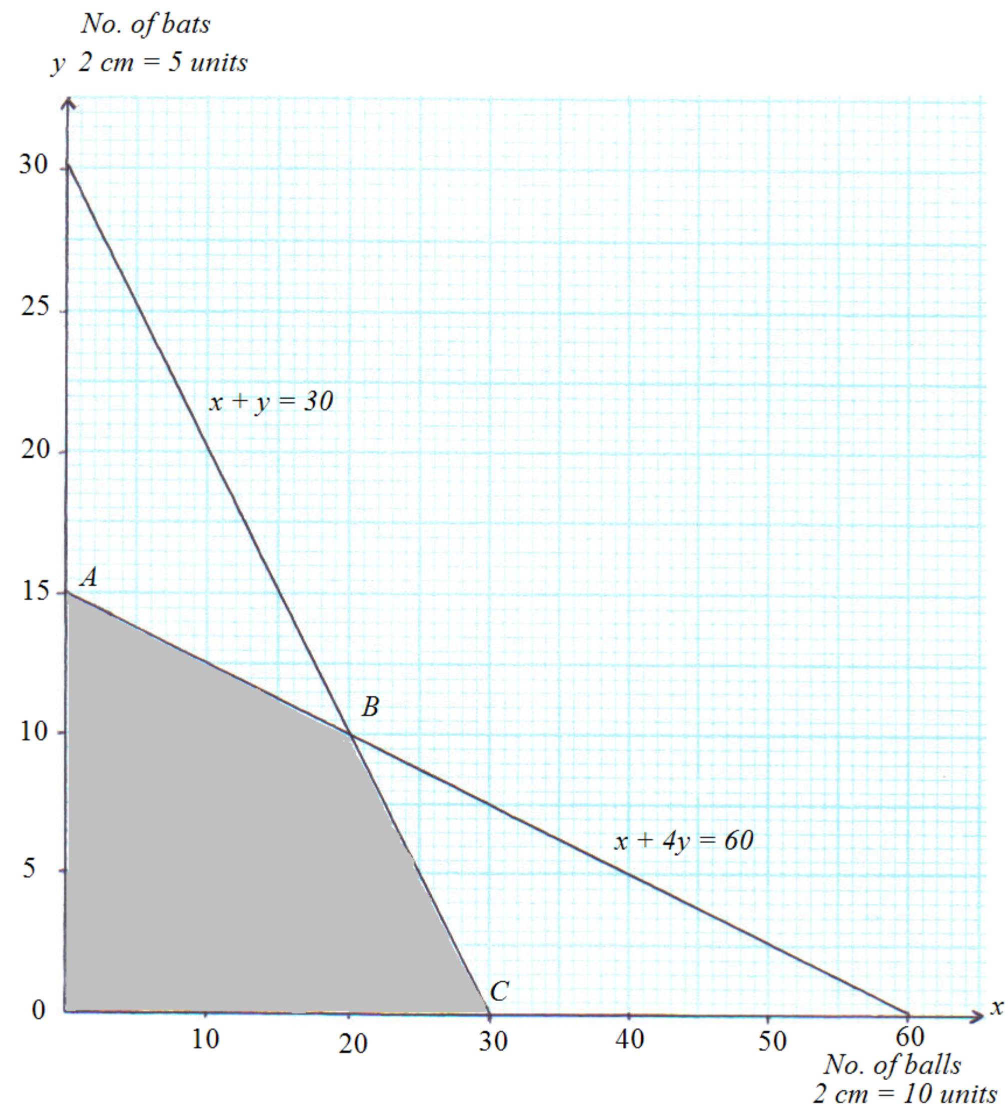


The region with the smaller angle corresponds to the $\leq$ region.
The region which satisfies $x + 4y \leq 60$ is



The feasible region is the area in which both previously shaded regions overlap.





The vertices of the feasible region are *O* (0, 0), *A* (0, 15), *B* (20, 10) and *C* (30, 0).

c. **Data:** Profit made of each ball is \$1 and profit made on each bat is \$3.

(i) **Required To Find:** The profit for reach of the combinations above.

Solution:

$P = x + 3y$

Testing the point (0, 15), (30, 0) and (20, 10).

When $x = 0$ and $y = 15$

$P = 0 + 3(15)$

$= \$45$

When $x = 30$ and $y = 0$

$P = 30 + 3(0)$

$= \$30$

When $x = 20$ and $y = 10$

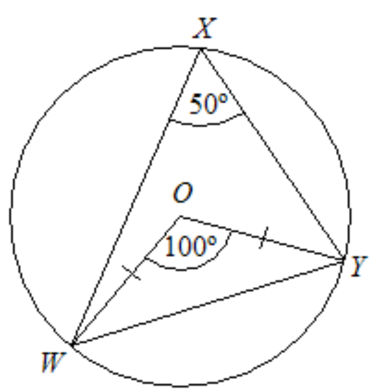
$P = 20 + 3(10)$

$= \$50$

As a point of interest, the only point to be considered should be (20, 10), where $x = 20$ and $y = 10$ since the question specifically indicates – a school buys x balls **AND** y balls. (They cannot buy 0 bats or 0 balls, that is x and $y \in Z^+$.)

(ii) **Required To Find:** Maximum profit that may be made.
Solution:
The maximum profit, $P_{\max} = \$50$, which occurs when $x = 20$ and $y = 10$.

11. a. **Data:** O is the centre of the circle WXY and $W\hat{X}Y = 50°$



(i) **Required To Calculate:** $W\hat{O}Y$
Calculation:

$W\hat{O}Y = 2(50°)$

$= 100°$

(The angle subtended by a chord at the centre of the circle is twice the angle subtended at the circumference, standing on the same arc).

(ii) **Required To Calculate:** $O\hat{W}Y$

Calculation:

$OW = OY$ (radii)

$O\hat{W}Y = O\hat{Y}W$ (*the base angles of an isosceles triangle are equal*)

$= \dfrac{180° - 100°}{2}$

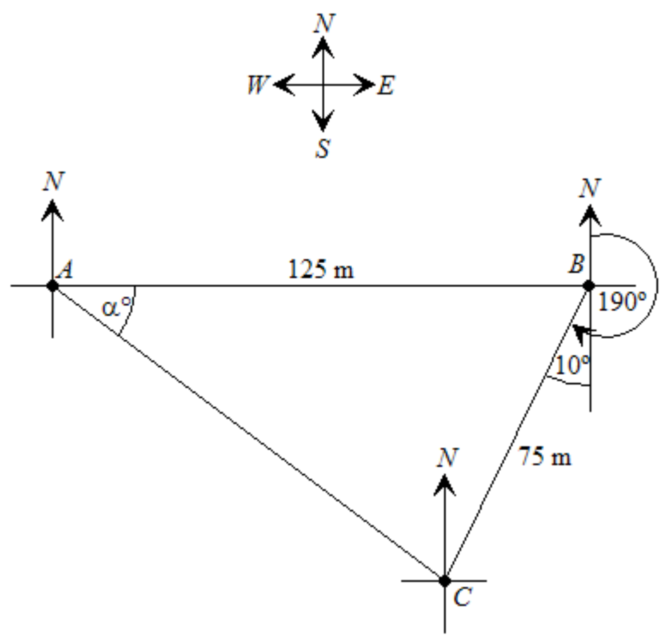
$= 40°$

(Sum of the angles in a triangle = 180°).

b. **Data:** Three buoys A, B and C, their relative distances apart and their positions.

(i) **Required To Sketch:** Diagram showing the information given.

Solution:



(ii) **Required To Calculate:** Distance AC.

Calculation:

$A\hat{B}C = 90° - 10°$

$= 80°$

$AC^2 = (125)^2 + (75)^2 - 2(125)(75)\cos 80°$ (cos law)

$AC = 134.14\text{ m}$

$AC = 134.1\text{ m}$

(iii) **Required To Calculate:** Bearing of C from A.
Calculation:
Let

$$B\hat{A}C = \alpha°$$

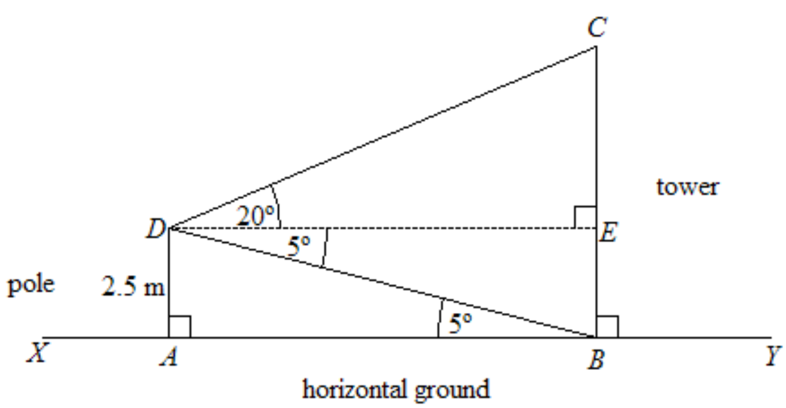
$$\frac{75}{\sin\alpha} = \frac{134.1}{\sin 80°} \qquad \text{(Sine Law)}$$

$$\sin\alpha = \frac{75\sin 80°}{134.1}$$

$$\alpha = 33.4°$$

The bearing of C from A $= 90° + 33.4°$
$= 123°$ to the nearest degree

12. a. **Data:** Diagram illustrating a vertical pole and tower standing on horizontal ground.



(i) **Required To Calculate:** Horizontal distance AB.
Calculation:

$$D\hat{B}A = 5°$$

(alternate angles).

$$\tan 5° = \frac{2.5}{AB}$$

$$AB = \frac{2.5}{\tan 5°}$$

$$= 28.5\underline{7}\text{ m}$$

$$= 28.6\text{ m (to 1 decimal place)}$$

(ii) **Required To Calculate:** Height of the tower BC.

Calculation:

$DE = 28.57$

(Opposite sides of a rectangle).

$$\tan 20° = \frac{CE}{28.57}$$

$$CE = 28.57 \tan 20°$$

$$= 10.39 \text{ m}$$

Height of tower $BC = 2.5 + 10.39$

$$= 12.89 \text{ m}$$

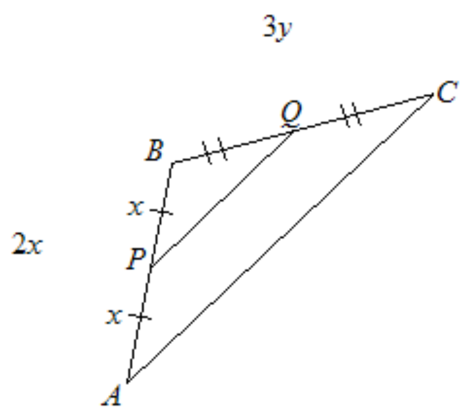
$$= 12.9 \text{ m}$$

b. **No solution has been offered for this question as it is based on latitude and longitude (Earth Geometry) which has been removed from the syllabus)**

13. a. **Data:** P and Q are the midpoints of AB and BC of vector triangle ABC.

(i) **Required To Sketch:** Diagram to show the information given.

Solution:



$$\overrightarrow{AB} = 2x$$

$$\therefore \overrightarrow{AP} = \overrightarrow{PB}$$

$$= x$$

$$\overrightarrow{BC} = 3y$$

$$\therefore \overrightarrow{BQ} = \overrightarrow{QC}$$

$$= \frac{1}{2}(3y)$$

$$= \frac{3}{2}y$$

$$= 1\frac{1}{2}y$$

(ii) (a) **Required To Find:** Expression in terms of x and y for $\overrightarrow{AC}$.

Solution:

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$
$$= 2x + 3y$$

(b) **Required To Find:** Expression in terms of x and y for $\overrightarrow{PQ}$.

Solution:

$$\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ}$$
$$= x + 1\frac{1}{2}y$$

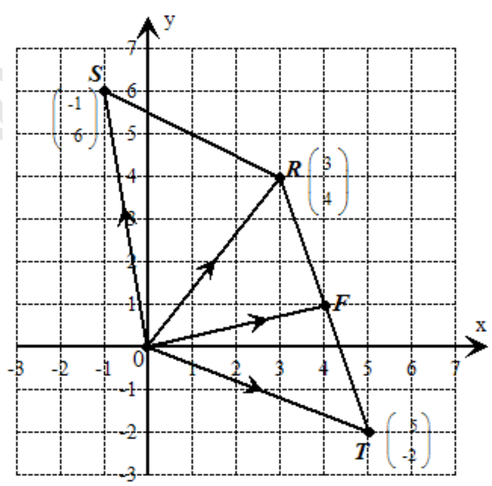
(iii) **Required To Prove:** $\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{AC}$

Proof:

$$\overrightarrow{PQ} = x + 1\frac{1}{2}y$$
$$\overrightarrow{AC} = 2x + 3y$$
$$= 2\left(x + 1\frac{1}{2}y\right)$$
$$= 2\overrightarrow{PQ}$$
$$\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{AC}$$

Q.E.D.

b. **Data:** $\overrightarrow{OR} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $\overrightarrow{OS} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$ and $\overrightarrow{OT} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$



(i) (a) **Required To Express:** $\overrightarrow{RT}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

Solution:

$$\overrightarrow{RT} = \overrightarrow{RO} + \overrightarrow{OT}$$
$$= -\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$
$$= \begin{pmatrix} 2 \\ -6 \end{pmatrix}$$

(b) **Required To Express:** $\overrightarrow{SR}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

Solution:

$$\overrightarrow{SR} = \overrightarrow{SO} + \overrightarrow{OR}$$
$$= -\begin{pmatrix} -1 \\ 6 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$
$$= \begin{pmatrix} 4 \\ -2 \end{pmatrix}$$

(ii) (a) **Required To Find:** The position vector of F.

Solution:

If $RF = FT$, then F is the midpoint of $\overrightarrow{RT}$.

$$\overrightarrow{OF} = \begin{pmatrix} \frac{3+5}{2} \\ \frac{4-2}{2} \end{pmatrix}$$
$$= \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

OR

$$\overrightarrow{RF} = \frac{1}{2}\overrightarrow{RT}$$
$$= \frac{1}{2}\begin{pmatrix} 2 \\ -6 \end{pmatrix}$$
$$= \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$
$$\overrightarrow{OF} = \overrightarrow{OR} + \overrightarrow{RF}$$
$$= \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$
$$= \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

(b) **Required To State:** Coordinates of *F*.
Solution:
If $\overrightarrow{OF} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, then *F* = (4, 1).

14. a. **Data:** $A = \begin{pmatrix} 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & x \\ y & -2 \end{pmatrix}$ and $C = \begin{pmatrix} 5 & 6 \end{pmatrix}$

$AB = C$.
Required To Calculate: *x* and *y*.
Calculation:
$$AB = \begin{pmatrix} 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & x \\ y & -2 \end{pmatrix}$$
$$= \begin{pmatrix} (2\times 1)+(1\times y) & (2\times x)+(1\times -2) \end{pmatrix}$$
$$= \begin{pmatrix} 2+y & 2x-2 \end{pmatrix}$$
If $AB = C$ then

$$\begin{pmatrix} 2+y & 2x-2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \end{pmatrix}$$
Equating corresponding entries.
$$2+y=5$$
$$y=3$$
$$2x-2=6$$
$$2x=8$$
$$x=4$$

$\therefore x=4$ and $y=3$

b. **Data:** $R = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$.

(i) **Required To Show:** R is a non-singular matrix.
Proof:
$\det R = (2 \times 3) - (-1 \times 1) = 6 + 1 = 7$
Since $R \neq 0$, then R is non-singular.

(ii) **Required To Find:** R^{-1}
Solution:

$$R^{-1} = \frac{1}{7}\begin{pmatrix} 3 & -(-1) \\ -(1) & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{7} & \frac{1}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{pmatrix}$$

(iii) **Required To Prove:** $RR^{-1} = I$
Proof:

$$R \times R^{-1} = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} \frac{3}{7} & \frac{1}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{pmatrix}$$

$$= \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$$

$$e_{11} = \left(2 \times \frac{3}{7}\right) + \left(-1 \times -\frac{1}{7}\right)$$
$$= 1$$

$$e_{12} = \left(2 \times \frac{1}{7}\right) + \left(-1 \times \frac{2}{7}\right)$$
$$= 0$$

$$e_{21} = \left(1 \times \frac{3}{7}\right) + \left(3 \times -\frac{1}{7}\right)$$
$$= 0$$

$$e_{22} = \left(1 \times \frac{1}{7}\right) + \left(3 \times \frac{2}{7}\right)$$
$$= 1$$

$$\therefore RR^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$= I$$

Q.E.D.

(iv) **Data:** $2x - y = 0$ and $x + 3y = 7$

Required To Calculate: x and y by matrix method.

Calculation:

$2x - y = 0 \ldots(1)$

$x + 3y = 7 \ldots(2)$

$$\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \end{pmatrix}$$

$$R\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \end{pmatrix}$$

$\times R^{-1}$

$$R \times R^{-1}\begin{pmatrix} x \\ y \end{pmatrix} = R^{-1}\begin{pmatrix} 0 \\ 7 \end{pmatrix}$$

$$I \times \begin{pmatrix} x \\ y \end{pmatrix} = R^{-1}\begin{pmatrix} 0 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{7} & \frac{1}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{pmatrix}\begin{pmatrix} 0 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} \left(\frac{3}{7} \times 0\right) + \left(\frac{1}{7} \times 7\right) \\ \left(-\frac{1}{7} \times 0\right) + \left(\frac{2}{7} \times 7\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Equating corresponding entries.

$x = 1$ and $y = 2$