JUNE 2007 CXC MATHEMATICS GENERAL PROFICIENCY (PAPER 2)

Section I

1. a. **Required To Calculate:** $(3.7)^2 - (6.24 \div 1.3)$ in exact form.

Calculation:

$$(3.7)^2 - (6.24 \div 1.3) = 13.69 - 4.8$$
$$= 8.89 \text{ (in exact form)}$$

b. **Data:** School of 1200 students with teacher : student ratio $1:30$.

(i) **Required To Calculate:** The number of teachers at the school.

Calculation:

Let no. of teachers = x

$$x : 1200 = 1 : 30$$
$$\frac{x}{1200} = \frac{1}{30}$$
$$30x = 1200$$
$$x = 40$$

The number of teachers = 40.

(ii) **Data:** $\frac{2}{5}$ of the students own personal computers.

Required To Calculate: No. of students not owning personal computers.

Calculation:

$\frac{2}{5}$ of the students own personal computers.

Fraction who do not own personal computers $= 1 - \frac{2}{5}$
$$= \frac{3}{5}$$

No. of students who do not personal computers $= \frac{3}{5} \times 1200$
$$= 720$$

(iii) **Data**: 30% of the students who own computers also own play stations.

Required To Calculate: Fraction of students who own play stations.

Fraction who also own play stations $= 30\% \text{ of } \frac{2}{5}$
$$= \frac{30}{100} \times \frac{2}{5}$$
$$= \frac{3}{25}$$

(in its lowest terms)

2. a. **Data:** $a * b = ab - \frac{b}{a}$

(i) **Required To Calculate:** $4 * 8$

Calculation:

$$4 * 8 = (4 \times 8) - \frac{8}{4}$$
$$= 32 - 2$$
$$= 30$$

(ii) **Required To Calculate:** $2 * (4 * 8)$

Calculation:

$$2 * (4 * 8) = 2 * 30$$
$$= (2 \times 30) - \frac{30}{2}$$
$$= 60 - 15$$
$$= 45$$

b. **Required To Simplify:** $\frac{5p}{3q} \div \frac{4p^2}{q}$

Solution:

$$\frac{5p}{3q} \div \frac{4p^2}{q} = \frac{5p}{3q} \times \frac{q}{4p^2}$$
$$= \frac{5}{12p} \text{(in its lowest form)}$$

c. **Data:** Stadium with section A seats \$ *a* each and section B seats \$ *b* each.

(i) **Required To Find:** Equations in *a* and *b* for the information given.

Solution:

For Johanna

5 section A and 3 section B cost \$105.

Hence,

$$(5 \times a) + (3 \times b) = 105$$
$$5a + 3b = 105 \ldots(1)$$

For Raiyah

4 section A seats and 1 section B seat cost \$63.

Hence,

$$(4 \times a) + (1 \times b) = 63$$
$$4a + b = 63 \ldots(2)$$

(ii) **Required To Calculate**: a and b

Calculation:

$5a+3b=105\ldots(1)$

$4a+b=63\ldots(2)$

From (2)

$b=63-4a$

Substitute in (1)

$$5a+3(63-4a)=105$$
$$5a+189-12a=105$$
$$189-7a=105$$
$$84=7a$$
$$a=12$$

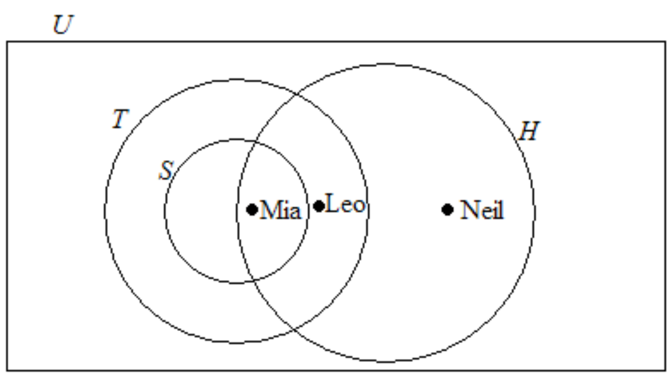
When $a=12$ $\quad b=63-4(12)$

$=63-48$

$=15$

Hence, when $a=12$ and $b=15$.

3. a. **Data:** Venn diagram showing the games played by members of a club.



(i) (a) **Required To State:** Game(s) played by Leo.

Solution:

Leo belongs to both *T* and *H*. Hence, Leo plays both tennis (*T*) and hockey (*H*).

(b) **Required To State:** Game(s) played by Mia.

Solution:

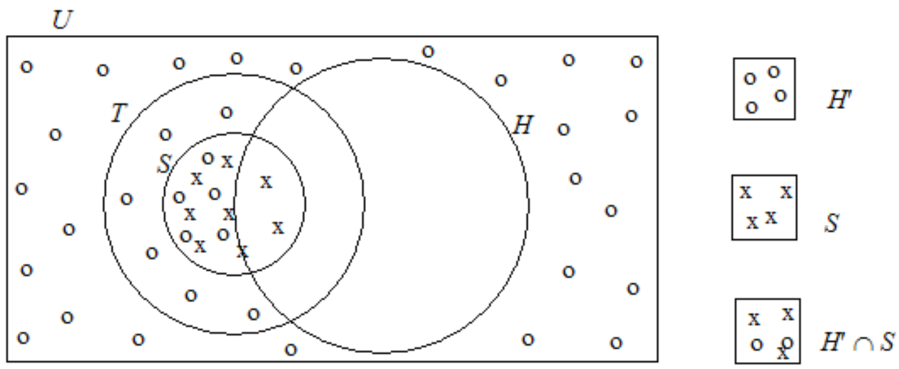
Mia belongs to all sets *H*, *S* and *T*. Therefore, Mia plays hockey (*H*), squash (*S*) and tennis (*T*).

(c) **Required To State:** Game(s) played by Neil.

Solution:

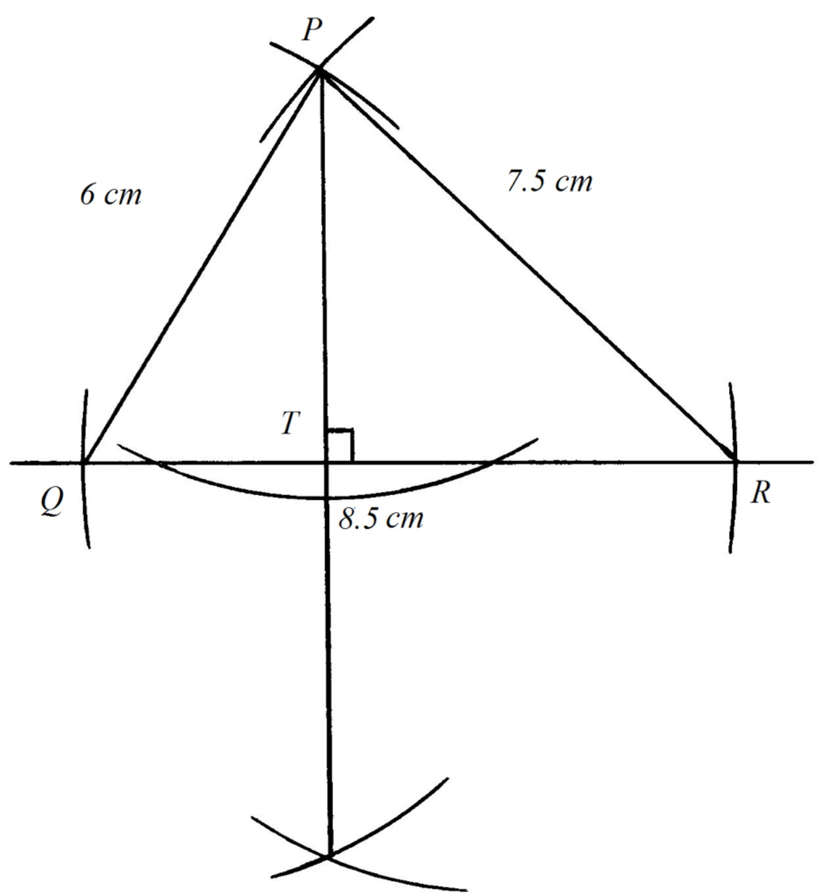
Neil belongs to the set *H* only. Hence, Neil plays hockey (*H*) only.

(ii) **Required To Describe:** The members of $H' \cap S$.
Solution:



$\therefore H' \cap S$ describe the members who play squash (*S*) and tennis (*T*) only.

b. (i) **Required To Construct:** ΔPQR with $QR = 8.5$ cm, $PQ = 6$ cm and $PR = 7.5$ cm and the line PT such that PT is perpendicular to QR and meets QR at T.
Solution:



(ii) (a) **Required To State:** Size of $P\hat{Q}R$

Solution:

$P\hat{Q}R = 59°$ (by measurement)

(b) **Required To State:** Length of PT

Solution:

$PT = 5.2$ cm (by measurement)

4\. a. **Data:** Diagram of a golf course map with a scale of $1:4\,000$.

(i) **Required To Find:** Distance from South Gate to east Gate.

Solution:

Distance from South Gate to East Gate = 3 cm (from map)

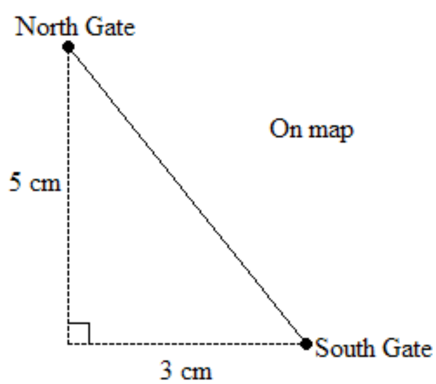
$= 3 \times 4\,000\text{ cm}$

$= \dfrac{3 \times 4\,000}{100}\text{ m}$

$= 120\text{ m}$ (to the nearest m)

(ii) **Required To Find:** Distance from North Gate to South Gate.

Solution:



Distance from North Gate to South Gate is exactly

$= \sqrt{(5)^2 + (3)^2}$ (by Pythagoras' Theorem)

$= \sqrt{34}$

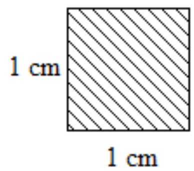
$\therefore$ Actual distance $= \dfrac{\sqrt{34} \times 4\,000}{100}\text{ m}$

$= 233.2\text{ m}$

$= 233\text{ m}$ to the nearest m

(iii) **Required To Find:** Area on the ground represented by 1 cm^2 on the map.

Solution:



The area represented by 1 cm^2 $= \dfrac{1 \times 4\,000}{100}$ *by* $\dfrac{1 \times 4\,000}{100}$

$= (40 \times 40)\,\text{m}^2$

$= 1600\,\text{m}^2$

(iv) **Required To Calculate:** Actual area of the golf course.

Calculation:

Since the map is not a definite shape, we have to estimate the area. Check 'whole squares' as 1 cm^2. Blocks that are more than 'half square' are considered as 'whole squares' = 1 cm^2. Blocks that are less than 'half square' are ignored.

No. of whole block/squares = 17

$\therefore 17 \times 1 = 17\,\text{cm}^2$

No. of blocks that are more than 'half square' = 10

$\therefore 10 \times 1 = 10\,\text{cm}^2$

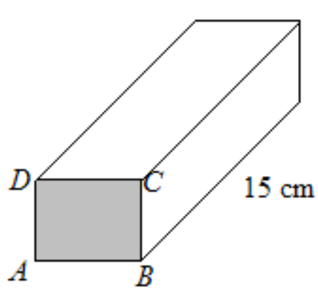
Total estimated area $= 17 + 10$

$= 27\,\text{cm}^2$

Actual estimated area $= 27 \times 1600$

$= 43\,200\,\text{m}^2$

b. **Data:** Diagram illustrating a prism of length 15 cm, volume 960 cm^3 and has a square cross-section *ABCD*.



(i) **Required To Calculate:** Length of *AB*.
Calculation:
Area of cross-section *ABCD* $\times$ Length of 15 cm $\equiv$ Volume of 960 cm^3

$$\text{Area of } ABCD = \frac{960}{15}$$
$$= 64 \text{ cm}^2$$

$$\text{Length of } AB = \sqrt{64} \text{ cm}$$
$$= 8 \text{ cm}$$

(ii) **Required To Calculate:** Total surface area of the prism.
Calculation:

$$\text{Surface area of the 2 square faces } = 64 \times 2$$
$$= 128 \text{ cm}^2$$

$$\text{Area of the 4 rectangular faces } = (8 \times 15) \times 4$$
$$= 480 \text{ cm}^2$$

$$\therefore \text{Total surface area } = 128 + 480$$
$$= 608 \text{ cm}^2$$

5. **Data:** Variables *x* and *y* where *y* varies inversely as the square of *x*.

a. **Required To Find:** Equation in *x*, *y* and *k* to represent the inverse variation.
Solution:

$$y \propto \frac{1}{x^2}$$

$$y = k \times \frac{1}{x^2} \text{ (} k \text{ is the constant of variation)}$$

$$y = \frac{k}{x^2}$$

b. **Data:** Table of values of *x* and corresponding values of *y*.

(i) **Required To Calculate:** *k*
Calculation:
From the data $x = 3$ when $y = 2$.

$$2 = \frac{k}{(3)^2}$$

$$k = 2 \times (3)^2$$
$$= 18$$

and

$$y = \frac{18}{x^2}$$

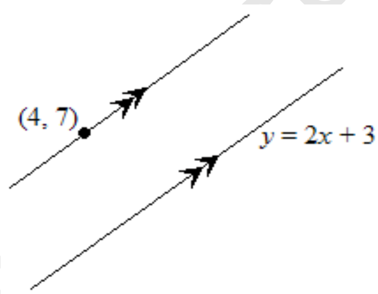
(ii) **Data**: $x = 1.8$
Required To Calculate: r
Calculation:

$$y = \frac{18}{(1.8)^2}$$
$$= 5.55$$
$$\therefore r = 5.5\underline{5}$$
$$= 5.6 \text{ (to 1 decimal place)}$$

(iii) **Data:** $y = 8$
Required To Calculate: f
Solution:

$$8 = \frac{18}{x^2}$$
$$x^2 = \frac{18}{8}$$
$$x^2 = 2.25$$
$$x = \sqrt{2.25}$$
$$x = \pm 1.5$$
$$\therefore f = \pm 1.5$$

c. **Required To Find:** Equation of the straight line passing through (4, 7) and which is parallel to $y = 2x + 3$.
Solution:



The line $y = 2x + 3$ is of the form $y = mx + c$ where $m = 2$ is the gradient.
The gradient of the required line is 2.
(Parallel lines have the same gradient).
Equation of the required line is

$$\frac{y-7}{x-4} = 2$$
$$y - 7 = 2(x - 4)$$
$$y - 7 = 2x - 8$$
$$y = 2x - 1$$

6. a. **Data:** Diagram showing $L'M'N'$, the enlargement of LMN.

(i) (a) **Required To Find:** Scale factor for the enlargement.

Solution:

$$\frac{\text{Image length}}{\text{Object length}} = \text{scale factor}$$

$$\frac{L'M'}{LM}$$

From the diagram

$L = (1, 4)$, $M = (2, 2)$, $L' = (2, 8)$ and $M' = (4, 4)$

$$\text{Length of } LM = \sqrt{(2-1)^2 + (2-4)^2}$$
$$= \sqrt{(1)^2 + (-2)^2}$$
$$= \sqrt{5}$$

$$\text{Length of } L'M' = \sqrt{(4-2)^2 + (4-8)^2}$$
$$= \sqrt{(2)^2 + (-4)^2}$$
$$= \sqrt{20}$$

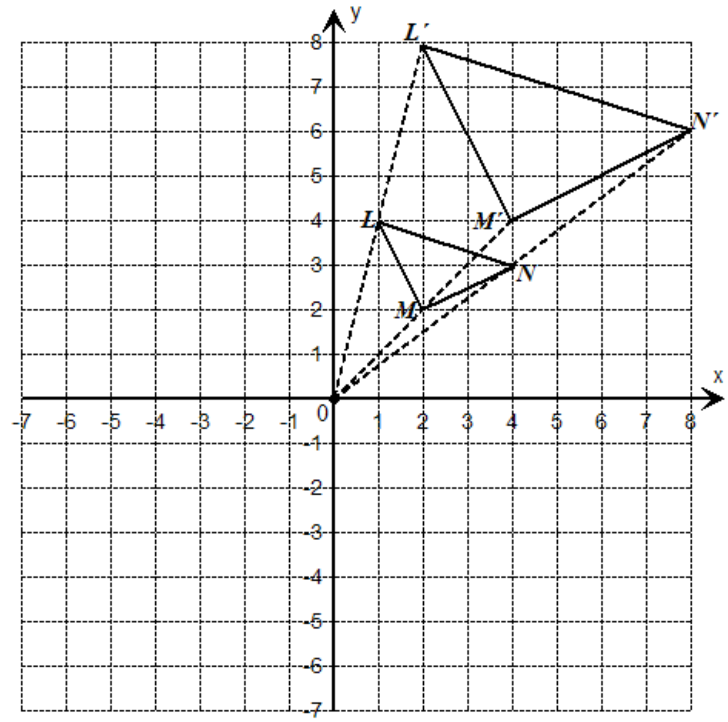
$$\therefore \text{Scale factor} = \frac{\sqrt{20}}{\sqrt{5}}$$
$$= \frac{\sqrt{4}\sqrt{5}}{\sqrt{5}}$$
$$= \sqrt{4}$$
$$= 2$$

(b) **Required To Find:** Coordinates of the centre of enlargement.

Solution:

$L'L$, $M'M$ and $N'N$ when produced backwards intersect at the same point O.

$\therefore$ The centre of enlargement is (0, 0).



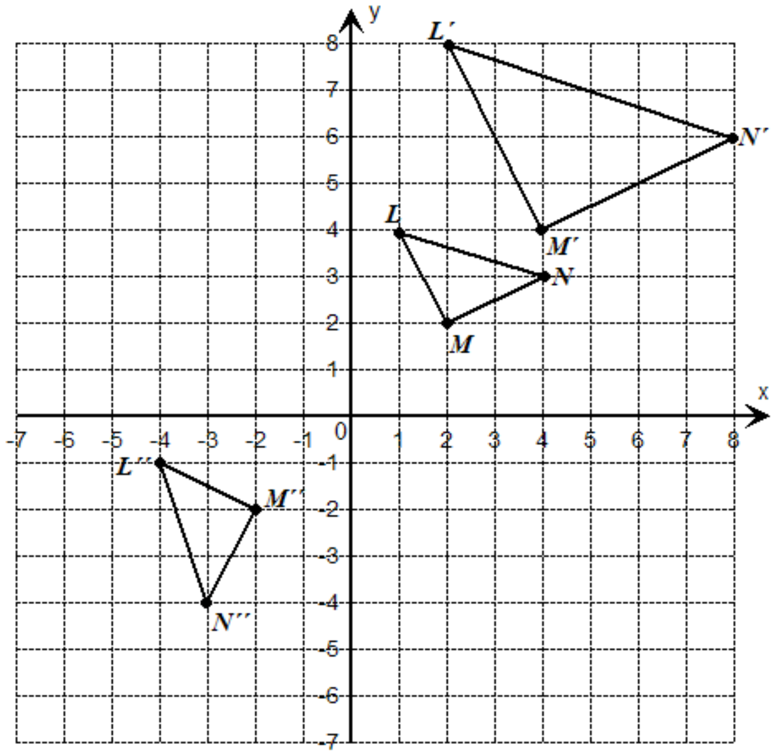
(ii) **Data:** $L''M''N''$ is the image LMN under a reflection in the line $y=-x$.
Required To Draw: The triangle $L''M''N''$.
Solution:

The matrix that identifies a reflection in the line $y=-x$ is $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

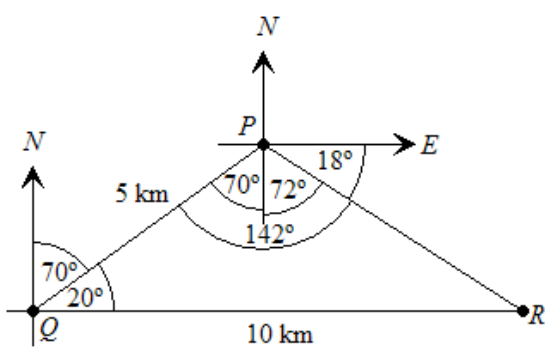
$\therefore LMN \xrightarrow{\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}} L''M''N''$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} L & M & N \\ 1 & 2 & 4 \\ 4 & 2 & 3 \end{pmatrix} = \begin{pmatrix} L'' & M'' & N'' \\ -4 & -2 & -3 \\ -1 & -2 & -4 \end{pmatrix}$$

$\therefore L''=(-4,-1)$, $M''=(-2,-2)$ and $N''=(-3,-4)$



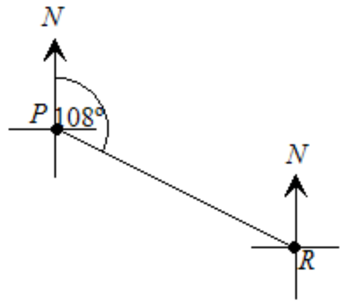
b. **Data:** Diagram illustrating three towns *P*, *Q* and *R*, bearings and relative distances.



(i) **Required To Calculate:** Length *PR*.
Calculation:

$$P\hat{Q}R = 90° - 70°$$
$$= 20°$$
$$PR^2 = (5)^2 + (10)^2 - 2(5)(10)\cos 20° \quad \text{(Cosine Rule)}$$
$$= 25 + 100 - 100\cos 20°$$
$$= 31.031$$
$$PR = \sqrt{31.031}$$
$$= 5.57 \text{ km}$$
$$= 5.6 \text{ km (to one decimal place)}$$

(ii) **Required To Calculate:** The bearing of R from P.
Calculation:



QP makes an angle of $70°$ with the South line. Hence, RP makes $142° - 70° = 72°$ with the South line. PR makes $90° - 72° = 18°$ with the East line.

The bearing of R from P $= 90° + 18°$
$= 108°$

7. **Data:** Results of the time taken by 32 students in a race.
   a. **Required To Complete:** The frequency table to represent the data given.
   Solution:
   THE TABLE OF VALUES FOR THE CONTINUOUS VARIABLE

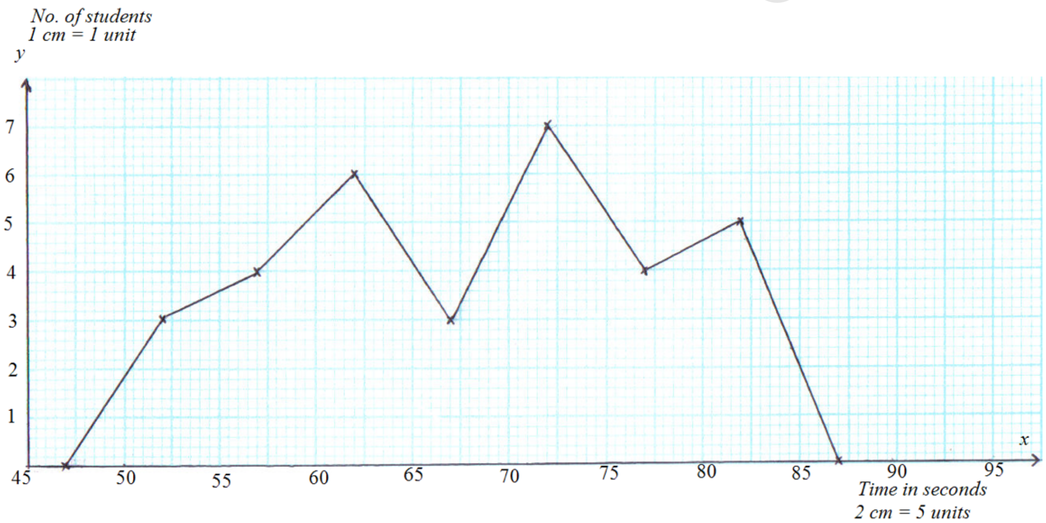
Time in seconds, *t*	**L.C.B U.C.B**	**Mid-class Interval** $\frac{\text{L.C.B} + \text{U.C.B}}{2}$	**Frequency**	**Points to plot**
				(47, 0)
50 – 54	$49.5 \le t < 54.5$	$\frac{49.5 + 54.5}{2} = 52$	3	(52, 3)
55 – 59	$54.5 \le t < 59.5$	$\frac{54.5 + 59.5}{2} = 57$	4	(57, 4)
60 – 64	$59.5 \le t < 64.5$	$\frac{59.5 + 64.5}{2}$ 62	6	(62, 6)
65 – 69	$64.5 \le t < 69.5$	$\frac{64.5 + 69.5}{2} = 67$	3	(67, 3)
70 – 74	$69.5 \le t < 74.5$	$\frac{69.5 + 74.5}{2} = 72$	7	(72, 7)
75 – 79	$74.5 \le t < 79.5$	$\frac{74.5 + 79.5}{2} = 77$	4	(77, 4)
80 – 84	$79.5 \le t < 84.5$	$\frac{79.5 + 84.5}{2} = 82$	5	(82, 5)
				(87, 0)

A frequency polygon must start from the horizontal axis so by extrapolation the points (47, 0) are obtained to start and (87, 0) to end the frequency polygon.

b. **Required To Find:** Range of the data.
Solution:
From the raw data,
Highest score = 83
Lowest score = 51

$$\therefore \text{Range} = 83 - 51$$
$$= 32$$

c. **Required To Draw:** Frequency polygon for the data using a scale of 2 cm to represent 5 seconds on the horizontal axis and 1 cm to represent 1 student on the vertical axis.
Solution:



d. **Required To Calculate:** Probability that a student from the class will qualify for the finals.
Solution:

$$P(\text{student qualifies for the finals}) = \frac{\text{No. of students finishing race before 60 seconds}}{\text{Total no. of participants}}$$
$$= \frac{3+4}{32}$$
$$= \frac{7}{32}$$

8. **Data:** Diagram showing a whole unit rectangle divided into seven smaller parts $A - G$.

a. **Required To Complete:** The table showing what fraction of the rectangle each part represents.
Solution:
Rectangle is 3 units $\times$ 12 units = 36 square units.

Part	Fraction	Perimeter to 1 decimal place
A	$\frac{\text{Area of } A}{36}=\frac{3\times 3}{36}=\frac{9}{36}=\frac{1}{4}$	$4\times 3=12$
B	$\frac{\text{Area of } B}{36}=\frac{2\times 3}{36}=\frac{6}{36}=\frac{1}{6}$	$2(2+3)=10$
C	$\frac{1}{24}$	$3+1+\sqrt{10}=7.2$ (to 1 dp)
D	$\frac{\text{Area of } D}{36}=\frac{\frac{1}{2}(2+3)\times 3}{36}=\frac{15}{72}=\frac{5}{24}$	$2+3+3+\sqrt{10}=11.2$ (to1 dp)
E	$\frac{\text{Area of } E}{36}=\frac{\frac{1}{2}(1+3)\times 2}{36}=\frac{4}{36}=\frac{1}{9}$	$1+2+3+\sqrt{8}=8.8$ (to 1 dp)
F	$\frac{\text{Area of } F}{36}=\frac{\frac{1}{2}(2+4)\times 2}{36}=\frac{6}{36}=\frac{1}{6}$	$2+2+4+\sqrt{8}=10.8$ (to 1 dp)
G	$\frac{1}{18}$	$2(1+2)=6$

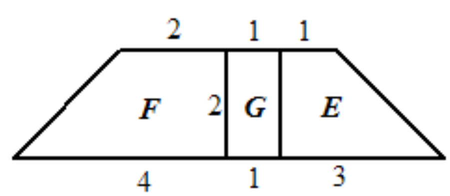
b. **Required To Write:** The parts in order of the size of their perimeters.
Solution:
In order of the size of the perimeters, with the smallest written first
G *C* *E* *B* *F* *D* *A*

c. **Data:** The area of *G* is 2 square units. *E*, *F* and *G* are rearranged to form a trapezium.

(i) **Required To Find:** The area of the trapezium.
Solution:

$$\text{Area of the trapezium } =\frac{1}{2}\left(\{(2+1+1)+(4+1+3)\}\times 2\right)$$
$$=\frac{1}{2}(4+8)\times 2$$
$$=12 \text{ square units}$$

(ii) **Required To Sketch:** The trapezium.
Solution:



Section II

9. a. **Data:** $g(x)=\frac{2x+1}{5}$ and $f(x)=x+4$

(i) **Required To Calculate:** $g(-2)$

Calculation:

$$g(-2)=\frac{2(-2)+1}{5}$$
$$=\frac{-4+1}{5}$$
$$=-\frac{3}{5}$$

(ii) **Required To Find:** Expression for $gf(x)$ in its simplest form.

Solution:

$$gf(x)=\frac{2(x+4)+1}{5}$$
$$=\frac{2x+8+1}{5}$$
$$=\frac{2x+9}{5}$$

(iii) **Required To Find:** $g^{-1}(x)$

Solution:

$$g(x)=\frac{2x+1}{5}$$

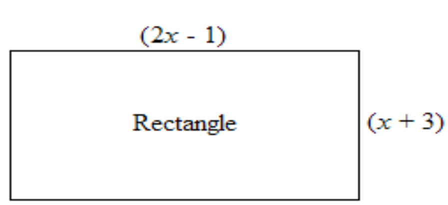
Let

$$y=\frac{2x+1}{5}$$
$$5y=2x+1$$
$$5y-1=2x$$
$$x=\frac{5y-1}{2}$$

Replace y by x

$$g^{-1}(x)=\frac{5x-1}{2}$$

b. **Data:** Diagram of a rectangle with length $(2x-1)$ cm and width $(x+3)$cm.



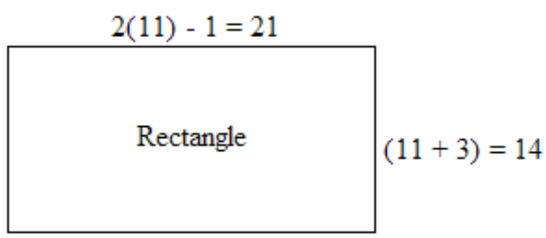
(i) **Required To Find:** Expression for area of the rectangle.
Solution:
Area of rectangle $= (2x-1)(x+3)$
$= 2x^2 - x + 6x - 3$
$= 2x^2 + 5x - 3$
is of the form $ax^2 + bx + c$, where $a = 2$, $b = 5$ and $c = -3$.

(ii) **Data:** Area of rectangle = 294 cm^2
Required To Calculate: x
Calculation:
Area = 294 cm^2
Hence,

$$2x^2 + 5x - 3 = 294$$
$$2x^2 + 5x - 297 = 0$$
$$(2x+27)(x-11) = 0$$
$$x = -13\frac{1}{2} \text{ or } 11$$

$x \neq -\text{ve}$, $\therefore x = 11$ only.

(iii) **Required To Find:** The dimensions of the rectangle.
Solution:



Hence, the rectangle is 21 cm long and 14 cm wide, as illustrated.

10. **Data:** The conditions for packaging of packets of gold and silver stars.

a. **Required To Find:** Inequalities to represent the conditions given.
Solution:

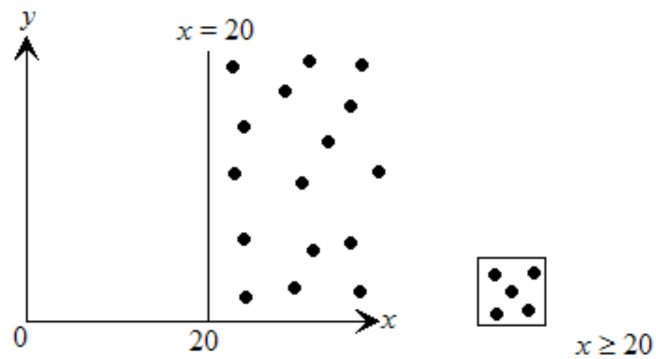
(2) Each packet must have at least 15 silver stars.
No. of silver stars is y, which must be at least 15. Hence,
$y \geq 15$

(3) Total number of stars in each packet must not be more than 60.
Total number of gold and silver stars is $x + y$, must not be more than 60.
Hence,
$x + y \leq 60$

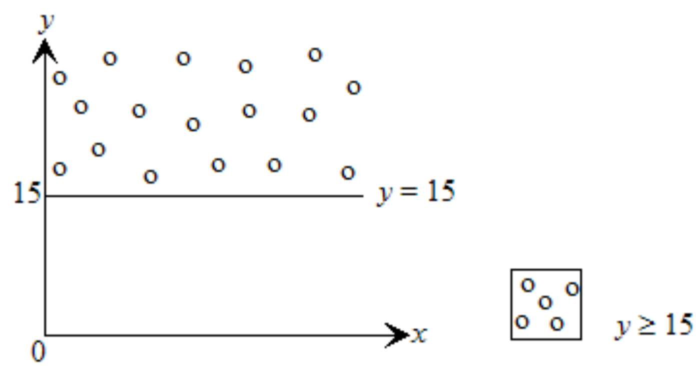
b. **Required To Describe:** The condition $x < 2y$ in words.

Solution:
$x < 2y$
The number of gold stars is less than twice the number of silver stars.
x $<$ 2 x y

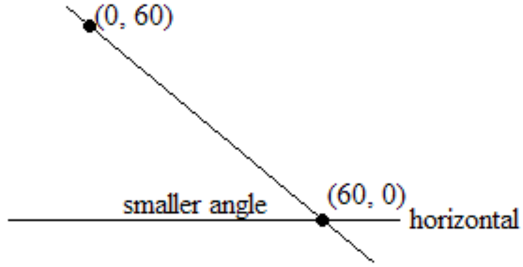
c. **Required To Draw:** The graphs for all 4 inequalities
Solution:
The line $x = 20$ is a straight vertical line.
The region which satisfies $x \geq 20$ is



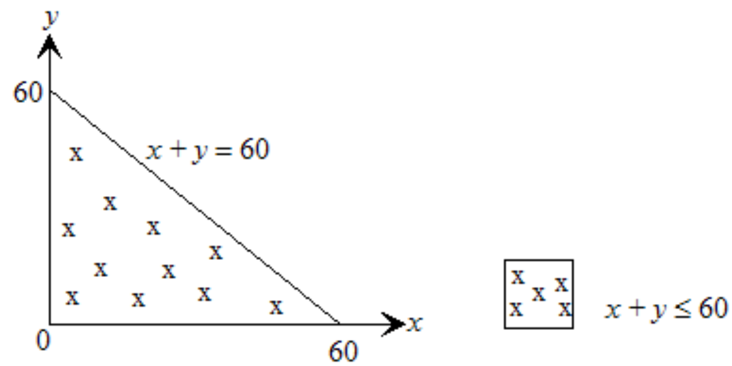
The line $y = 15$ is a horizontal straight line.
The region which satisfies $y \geq 15$ is



Obtaining 2 points on the line $x + y = 60$.
When $x = 0$ $\quad 0 + y = 60$
$y = 60$
The line $x + y = 60$ passes through the point (0, 60).
When $y = 0$ $\quad x + 0 = 60$
$x = 60$
The line $x + y = 60$ passes through the point (60, 0).



The region with the smaller angle represents the $\leq$ region.



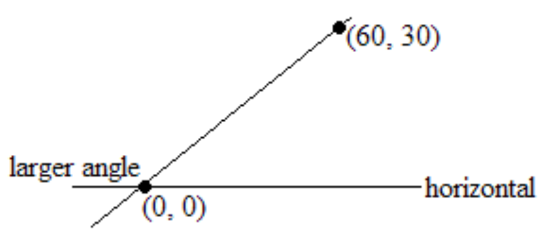
Obtaining 2 points on the line $x = 2y$ or $y = \frac{1}{2}x$.

The line $y = \frac{1}{2}x$ passes through the origin (0, 0).

When $x = 60$ $\qquad y = \frac{1}{2}(60)$

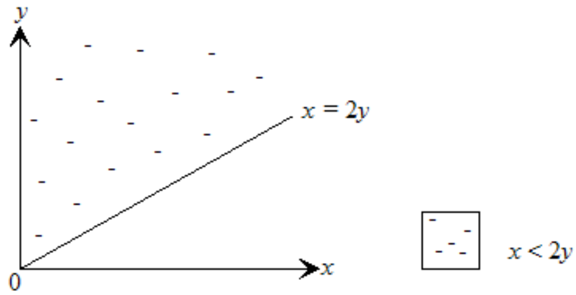
$\qquad\qquad\qquad = 30$

The line $y = \frac{1}{2}x$ passes through the point (60, 30).

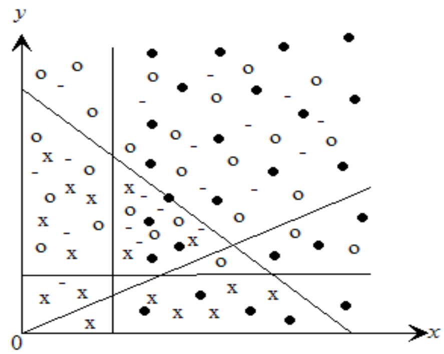


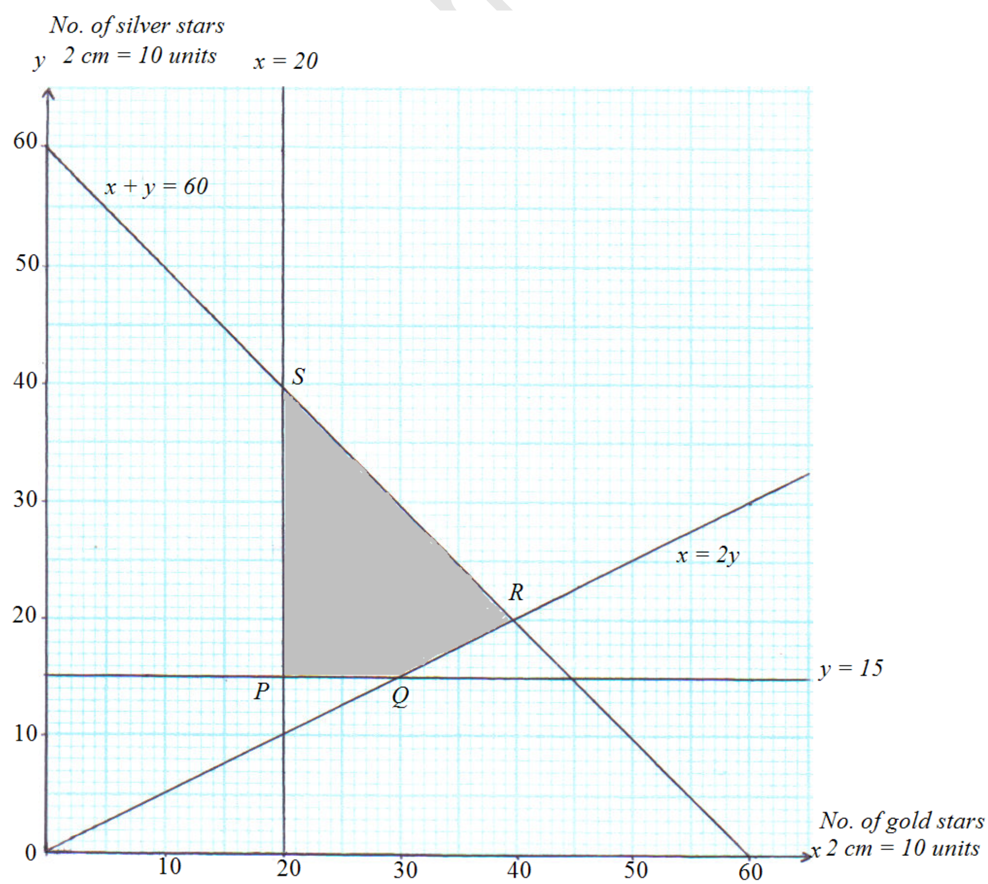
The region with the larger angle represents the $\geq$ region.

The region which satisfies y or $x < 2y$ is



The region which satisfies all four inequalities is the area in which all four shaded regions overlap.





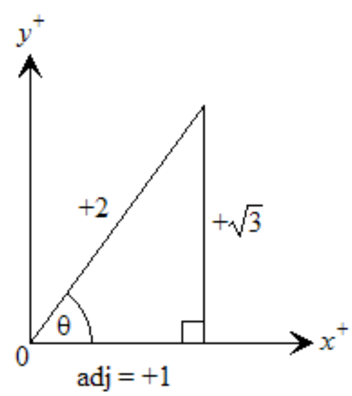
d. **Data:** Table showing the number of gold and silver stars which three packets contain.
Required To Determine: Which of the 3 packets satisfy all the conditions.
Solution:
The feasible region that satisfies all four inequalities is shown by *PQRS* on the diagram.
For *A*
When $x = 25$ and $y = 20$, the point *A*, (25, 20) lies within *PQRS* and so packet *A* satisfies all the conditions.

For *B*
When $x = 35$ and $y = 15$, the point *B* (35, 15) does not lie within *PQRS* and so packet *B* does not satisfy all the conditions.

For *C*
When $x = 30$ and $y = 25$, the point *C* (30, 25) lies within *PQRS* and so packet *C* satisfies all the conditions.

11. a. **Data:** $\sin\theta = \frac{\sqrt{3}}{2}$

(i) (a) **Required To Calculate:** $\cos\theta$
Solution:
Note: The question should have indicated whether θ is acute or obtuse.
Assuming θ is acute.



$$\text{adj} = \sqrt{(2)^2 - (\sqrt{3})^2}$$
$$= \pm 1$$

In this case adj = +1

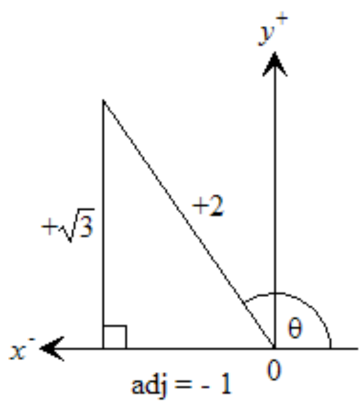
$$\cos\theta = \frac{+1}{+2}$$
$$= \frac{1}{2}$$

(b) **Required To Calculate:** $\tan\theta$

Solution:

$$\tan\theta = \frac{+\sqrt{3}}{+1}$$
$$= \sqrt{3}$$

Assuming θ is obtuse when $\sin\theta = \frac{\sqrt{3}}{2}$.



In this case adj $= \sqrt{1}$
$= \pm 1$
$= -1$

(i) (a) $\cos\theta = \frac{-1}{+2}$
$$= -\frac{1}{2}$$

(b) $\tan\theta = \frac{+\sqrt{3}}{-1}$
$$= -\sqrt{3}$$

(ii) **Required To Find:** $\frac{\sin\theta}{\tan\theta}$

Solution:

Assuming θ is acute.

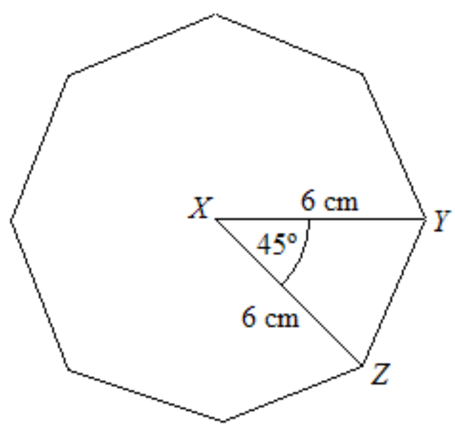
$$\frac{\sin\theta}{\tan\theta} = \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}}$$
$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}$$
$$= \frac{1}{2} \text{ in exact form}$$

Assuming θ is obtuse.

$$\frac{\sin\theta}{\tan\theta} = \frac{\frac{\sqrt{3}}{2}}{-\sqrt{3}}$$
$$= -\frac{1}{2}$$

b. This part of the question has not been solved as it involves Earth Geometry which has since been removed from the syllabus.

12. a. **Data:** Diagram with centre X and $XY = 6$ cm



(i) **Required To Calculate:** $Y\hat{X}Z$
Calculation:
Total angle at the centre of a circle = 360°

$$\therefore Y\hat{X}Z = \frac{360°}{8}$$
$$= 45°$$

(ii) **Required To Calculate:** Area of ΔYXZ
Calculation:

Area of $\Delta YXZ = \frac{1}{2}(6)(6)\sin 45°$

$= 12.7\underline{3}\,\text{cm}^2$

$= 12.7\,\text{cm}^2$ (to 1 decimal place)

(iii) **Required To Calculate:** Area of the octagon.
Calculation:

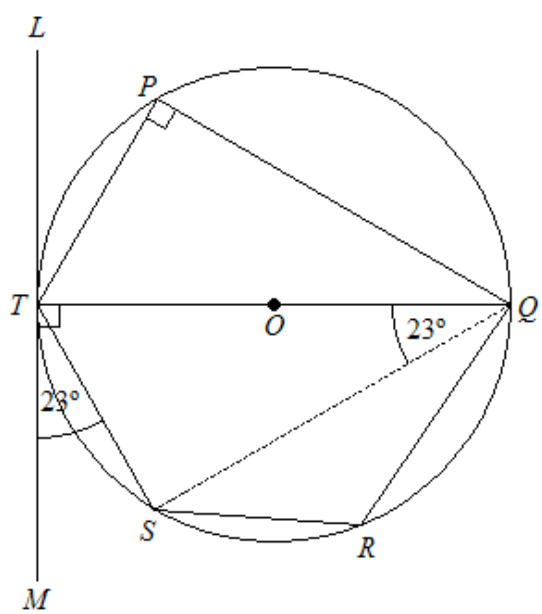
XYZ represents $\frac{1}{8}$ x (the area of the octagon).

Area of the octagon $= 8 \times 12.73$

$= 101.8\underline{4}$

$= 101.8\ \text{cm}^2$

b. **Data:** Diagram of a circle centre O. LM is a tangent to the circle $PQRST$ at T and $M\hat{T}S = 23°$.



(i) **Required To Calculate:** $T\hat{P}Q$
Calculation:
$T\hat{P}Q = 90°$
(Angle in a semi-circle is a right angle).

(ii) **Required To Calculate:** $M\hat{T}Q$
Calculation:

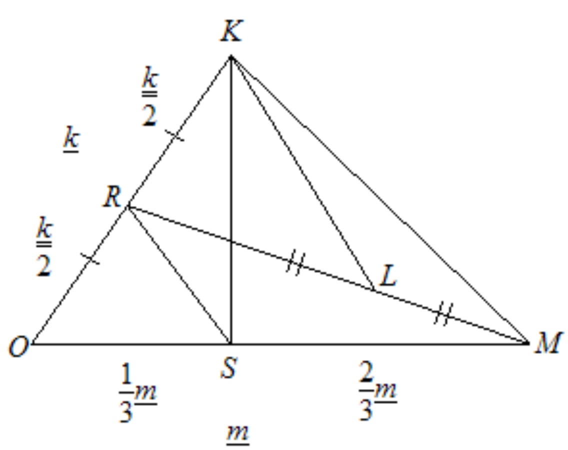
$M\hat{T}O = 90°$
(Angle made by a tangent to a circle and a radius, at the point of contact = 90°).
TOQ is a straight line.
$\therefore M\hat{T}Q = 90°$

(iii) **Required To Calculate:** $T\hat{Q}S$
Calculation:
$T\hat{Q}S = 23°$
(Angle made by a tangent to a circle and a chord, at the point of contact = angle in the alternate segment).

(iv) **Required To Calculate:** $S\hat{R}Q$
Calculation:
$S\hat{T}Q = 90° - 23°$
$= 67°$
$S\hat{R}Q = 180° - 67°$
$= 113°$
(Opposite angles in a cyclic quadrilateral are supplementary).

13. **Data:** Vector diagram with $\overrightarrow{OK} = \underline{k}$ and $\overrightarrow{OM} = \underline{m}$.

a. **Required To Sketch:** Diagram of the information given.
Solution:



b.

R is the midpoint of $\overrightarrow{OK}$.

$\therefore \overrightarrow{OR} = \overrightarrow{RK}$

$= \frac{1}{2}\underline{k}$

$\overrightarrow{OS} = \frac{1}{3}\overrightarrow{OM}$

$\therefore \overrightarrow{OS} = \frac{1}{3}\underline{m}$

And

$\overrightarrow{SM} = \frac{2}{3}\underline{m}$

(i) **Required To Express:** $\overrightarrow{MK}$ in terms of $\underline{k}$ and $\underline{m}$.

Solution:

$\overrightarrow{MK} = \overrightarrow{MO} + \overrightarrow{OK}$

$= -\underline{m} + \underline{k}$

$= \underline{k} - \underline{m}$

(ii) **Required To Express:** $\overrightarrow{RM}$ in terms of $\underline{k}$ and $\underline{m}$.

Solution:

$\overrightarrow{RM} = \overrightarrow{RO} + \overrightarrow{OM}$

$= -\frac{\underline{k}}{2} + \underline{m}$

(iii) **Required To Express:** $\overrightarrow{KS}$ in terms of $\underline{k}$ and $\underline{m}$.

Solution:

$\overrightarrow{KS} = \overrightarrow{KO} + \overrightarrow{OS}$

$= -\underline{k} + \frac{1}{3}\underline{m}$

(iv) **Required To Express:** $\overrightarrow{RS}$ in terms of $\underline{k}$ and $\underline{m}$.

Solution:

$\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS}$

$= -\frac{\underline{k}}{2} + \frac{1}{3}\underline{m}$

c. **Required To Prove:** *RS* is parallel to *KL*.

Proof:

$$\overrightarrow{RL} = \frac{1}{2}\overrightarrow{RM}$$
$$= \frac{1}{2}\left(-\frac{\underline{k}}{2} + \underline{m}\right)$$
$$\overrightarrow{KL} = \overrightarrow{KR} + \overrightarrow{RL}$$
$$= -\frac{\underline{k}}{2} + \frac{1}{2}\left(-\frac{\underline{k}}{2} + \underline{m}\right)$$

$$= -\frac{3}{4}\underline{k} + \frac{1}{2}\underline{m}$$
$$= \frac{3}{2}\left(-\frac{\underline{k}}{2} + \frac{1}{3}\underline{m}\right)$$
$$= \frac{3}{2}\overrightarrow{RS}$$

$\therefore \overrightarrow{KL}$ is a scalar multiple , $\left(\frac{3}{2}\right)$, of $\overrightarrow{RS}$, hence $\overrightarrow{KL}$ and $\overrightarrow{RS}$ are parallel.

14. a. **Data:** $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $B = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 14 & 0 \\ -9 & 5 \end{pmatrix}$

(i) **Required To Calculate:** $3A$
Calculation:

$$3A = 3\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$
$$= \begin{pmatrix} 3a & 3b \\ 3c & 3d \end{pmatrix}$$

(ii) **Required To Calculate:** B^{-1}
Calculation:

$$\text{Det } B = (5 \times 2) - (3 \times 3)$$
$$= 10 - 9$$
$$= 1$$
$$B^{-1} = \frac{1}{1}\begin{pmatrix} 2 & -(3) \\ -(3) & 5 \end{pmatrix}$$
$$= \begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix}$$

(iii) **Required To Calculate:** $3A + B^{-1}$

Calculation:

$$3A + B^{-1} = \begin{pmatrix} 3a & 3b \\ 3c & 3d \end{pmatrix} + \begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 3a+2 & 3b-3 \\ 3c-3 & 3d+5 \end{pmatrix}$$

(iv) **Data:** $3A + B^{-1} = C$

Required To Calculate: a, b, c and d.

Solution:

$$3A + B^{-1} = C$$

$$\begin{pmatrix} 3a+2 & 3b-3 \\ 3c-3 & 3d+5 \end{pmatrix} = \begin{pmatrix} 14 & 0 \\ -9 & 5 \end{pmatrix}$$

Equating corresponding entries.

$$3a + 2 = 14 \qquad 3b - 3 = 0$$
$$3a = 12 \qquad 3b = 3$$
$$a = 4 \qquad b = 1$$

$$3c - 3 = -9 \qquad 3d + 5 = 5$$
$$3c = -6 \qquad 3d = 0$$
$$c = -2 \qquad d = 0$$

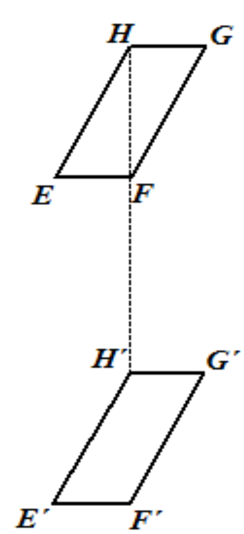
b. **Data:** Diagram showing a parallelogram *EFGH* and its images after undergoing 2 successive transformations.

(i) (a) **Required To Describe:** In words the transformation *J*.

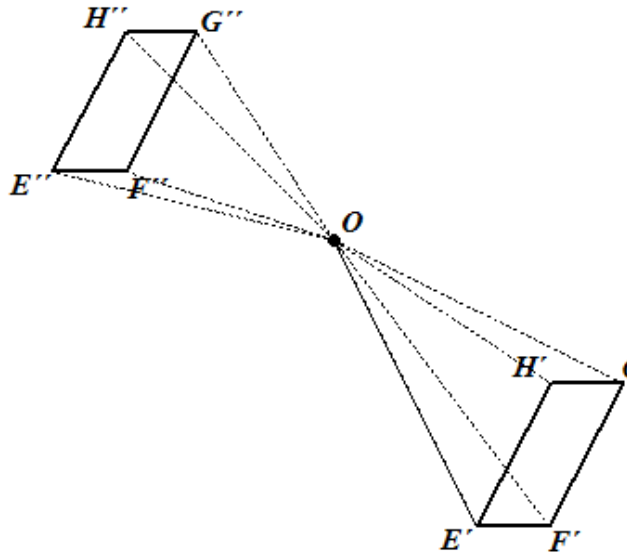
Solution:

EFGH is mapped onto $E'F'G'H'$ by a vertical shift of 4 units downwards.

$\therefore J$ describes a translation $T = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$.



(b) **Required To Describe:** In words transformation *K*.
Solution:



$E'OE''$, $F'OF''$, etc are all 180° and pass through *O*. Hence, $E'F'G'H'$ is mapped onto $E''F''G''H''$ by a rotation of 180° about *O* (clockwise or anti-clockwise), which describes *K*.

(ii) (a) **Required To Find:** Matrix which represents *J*.
Solution:

$$J = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

(b) **Required To Find:** Matrix which represents *K*.
Solution:

$$K = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

(iii) **Data:** *P* (6, 2) is mapped onto P' by *J*.
Required To Find: Coordinates of P'.
Solution:

$$P \xrightarrow{J} P'$$

$$\begin{pmatrix} 6 \\ 2 \end{pmatrix} \xrightarrow{\begin{pmatrix} 0 \\ -4 \end{pmatrix}} P'$$

$$\begin{pmatrix} 6+0 \\ 2+(-4) \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$$

$$\therefore P' = (6, -2)$$

(iv) **Data:** Q (5, -4) is mapped onto Q' by K.
Required To Find: Coordinates of Q'.
Solution:

$$Q \xrightarrow{K} Q'$$

$$\begin{pmatrix} 5 \\ -4 \end{pmatrix} \xrightarrow{\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}} Q'$$

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} (-1\times 5)+(0\times -4) \\ (0\times 5)+(-1\times -4) \end{pmatrix}$$

$$= \begin{pmatrix} -5 \\ 4 \end{pmatrix}$$

$$\therefore Q' = (-5, 4)$$