CXC JUNE 2006 MATHEMATICS GENERAL PROFICIENCY (PAPER 2)

Section I

1. a. (i) **Required To Calculate:** $(12.3)^2 - (0.246 \div 3)$ exactly.
Calculation:

$$(12.3)^2 - (0.246 \div 3) = 151.29 - 0.082$$
$$= 151.208 \text{ exactly}$$

(ii) **Required To Calculate:** $(12.3)^2 - (0.246 \div 3)$ to 2 significant figures.
Calculation:
The number $15\underline{1}.208 = 150$ to 2 significant figures.

b. **Data:** Table showing the depreciation of vehicles over a period.

(i) **Required To Calculate:** The values of *p* and *q*.
Calculation:
Taxi depreciates by 12% per year.

$\therefore$ Depreciation of taxi costing \$40 000 after 1 year $= \frac{12}{100} \times 40\,000$
$= \$4\,800$

Hence, value after 1 year $= \$40\,000 - \$4\,800$
$= \$35\,200$
$p = 35\,200$

Depreciation of private car $= \$25\,000 - \$21\,250$
$= \$3\,750$

$$\% \text{ Depreciation} = \frac{3\,750}{25\,000} \times 100$$
$$= 15\%$$
$$q = 15$$

(ii) **Required To Calculate:** Value of taxi after 2 years.
Calculation:
Depreciation of taxi in the 2nd year is 12% of its value after 1st year.

Depreciation in 2nd year $= \frac{12}{100} \times 35\,200$
$= \$4\,224$

$\therefore$ Value of taxi after 2 years $= \$35\,200 - \$4\,224$
$= \$30\,976$

OR

$$A = P\left(1 - \frac{R}{100}\right)^n$$

$P = 40\,000$ $\quad R = -12$ $\quad n = 2$

$$A = 40\,000\left(1 - \frac{12}{100}\right)^2$$
$$= \$30\,976$$

c. **Data:** $\text{GUY}\,\$1.00 \equiv \text{US}\,\$0.01$ and $\text{EC}\,\$1.00 \equiv \text{US}\,\$0.37$

(i) **Required To Calculate:** Value of GUY \$60 000 in US \$.
Calculation:

$$\text{GUY}\,\$1.00 \equiv \text{US}\,\$0.01$$
$$\text{GUY}\,\$60\,000 = \text{US}\,\$0.01 \times 60\,000$$
$$= \text{US}\,\$600.00$$

(ii) **Required To Calculate:** Value of US \$925 in EC \$.
Calculation:

$$\text{US}\,\$0.37 \equiv \text{US}\,\$1.00$$
$$\text{US}\,\$1.00 = \text{EC}\,\$\frac{1.00}{0.37}$$
$$\text{US}\$925.00 = \text{EC}\,\$\frac{1.00}{0.37} \times 925$$
$$= \text{EC}\,\$2\,500.00$$

2. a. **Required To Simplify:** $\frac{x-3}{3} - \frac{x-2}{5}$

Solution:
Simplifying

$$\frac{x-3}{3} - \frac{x-2}{5}$$
$$= \frac{5(x-3) - 3(x-2)}{15}$$
$$= \frac{5x - 15 - 3x + 6}{15}$$
$$= \frac{2x - 9}{15}$$

b. (i) **Required To Factorise:** (a) $x^2 - 5x$, (b) $x^2 - 81$

Factorising:

(a) $x^2 - 5x = x.x - 5.x$

$= x(x-5)$

(b) $x^2 - 81 = (x)^2 - (9)^2$

Difference of 2 squares.

$= (x-9)(x+9)$

(ii) **Required To Simplify:** $\frac{a^2+4a}{a^2+3a-4}$

Solution:

Simplifying

$$\frac{a^2+4a}{a^2+3a-4} = \frac{a(a+4)}{(a-1)(a+4)}$$

$$= \frac{a}{a-1}$$

c. **Data:** 2 cassettes and 3 CD's cost \$175 and 4 cassettes and 1 CD cost \$125. One cassette costs \$*x* and one CD costs \$*y*.

(i) **Required To Find:** Expression in x and y for the information given.

Solution:

2 cassettes at \$*x* each and 3 CD's at \$*y* each cost $(2 \times x) + (3 \times y)$,

Hence, $2x + 3y = 175 \ldots(1)$

4 cassettes and 1 CD cost $(4 \times x) + (1 \times y)$,

Hence, $4x + y = 125 \ldots(2)$

(ii) **Required To Calculate**: Cost of one cassette.

Calculation:

From (2)

$y = 125 - 4x$

Substitute in (1)

$$2x + 3(125 - 4x) = 175$$

$$2x + 375 - 12x = 175$$

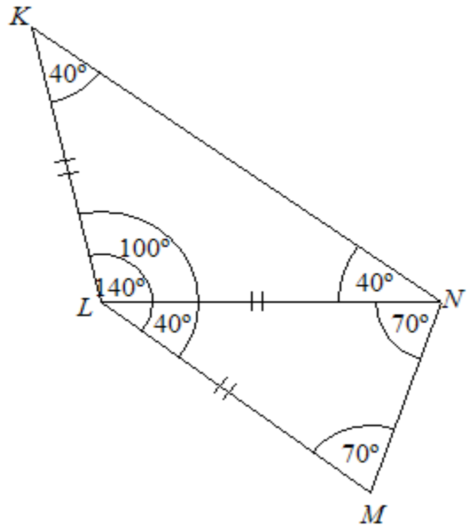
$$375 - 175 = 12x - 2x$$

$$10x = 200$$

$$x = 20$$

$\therefore$ Cost of one cassette is \$20.

3. a. **Data:** Diagram of a quadrilateral $KLMN$ with $LM = LN = LK$, $K\hat{L}M = 140°$ and $L\hat{K}N = 40°$.



(i) **Required To Calculate:** $L\hat{N}K$
Calculation:

$LK = LN$ (data)

$L\hat{N}K = 40°$

(Base angles of an isosceles triangle are equal).

(ii) **Required To Calculate:** $N\hat{L}M$
Calculation:

$$N\hat{L}K = 180° - (40° + 40°)$$
$$= 100°$$

(Sum of angles in a triangle = 180°).

$$N\hat{L}M = 140° - 100°$$
$$= 40°$$

(iii) **Required To Calculate:** $K\hat{N}M$
Calculation:

$LN = LM$ (data)

$$L\hat{N}M = L\hat{M}N$$
$$= \frac{180° - 40°}{2}$$
$$= 70°$$

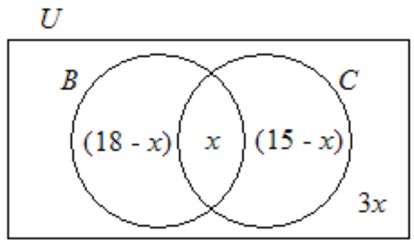
(Base angles in an isosceles triangle are equal and sum of angles in a triangle = 180°).

$$K\hat{N}M = 40° + 70°$$
$$= 110°$$

b. **Data:** Survey done on 39 students on the ability to ride a bike and /or drive a car.

(i) **Required To Complete:** Venn diagram to represent the information given.

Solution:



(ii) **Required To Find:** Expression in x for the number of students in the survey.

Solution:

No. of students in the survey $= (18 - x) + x + (15 - x) + 3x$

$= 33 + 2x$

(iii) **Required To Calculate:** x

Calculation:

Hence,

$33 + 2x = 39$

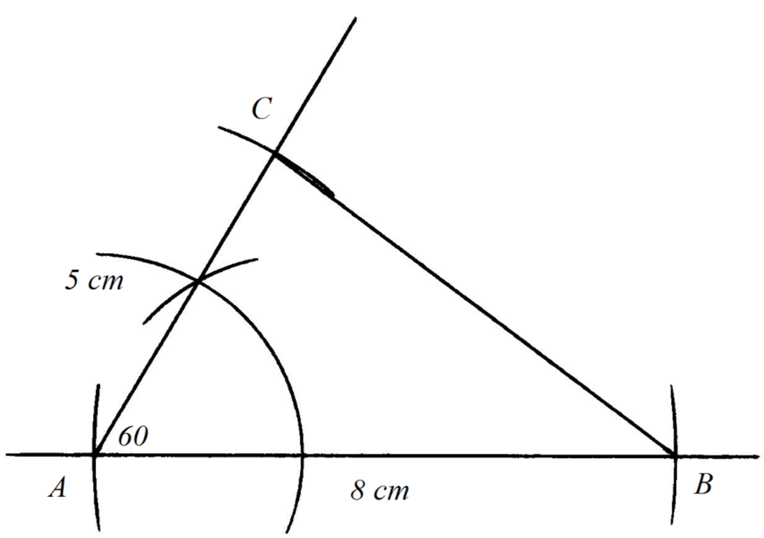
$2x = 39 - 33$

$x = 3$

4. **Data:** $AB = 8$ cm, $B\hat{A}C = 60°$ and $AC = 5$ cm

a. **Required To Construct:** Triangle ABC based on the information given.

Solution:



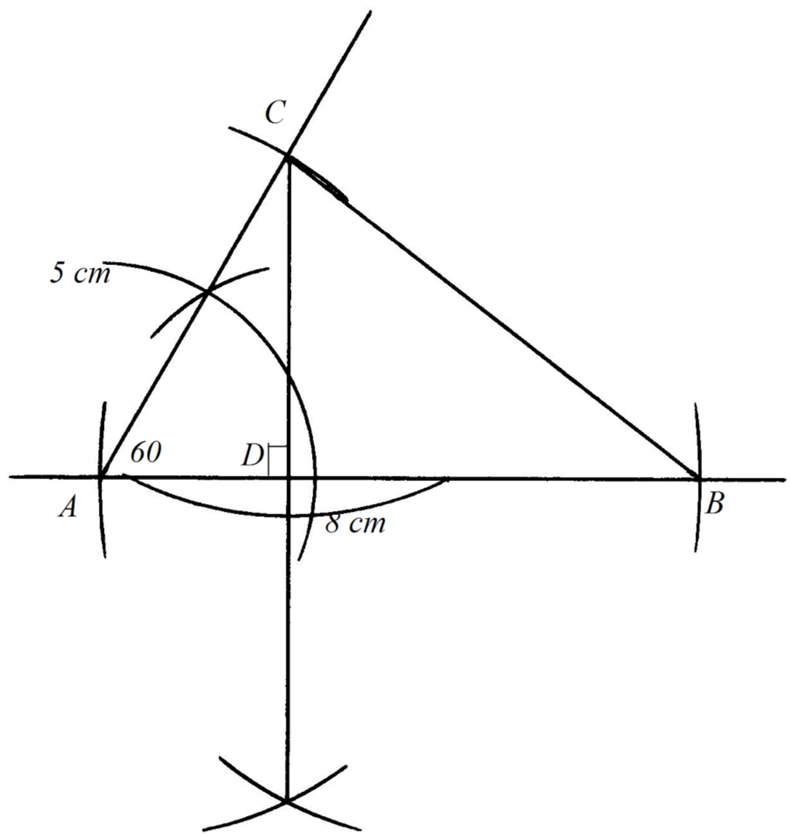
b. **Required To Find:** Length of *BC*
Solution:
$BC = 7$ cm (by measurement)

c. **Required To Calculate:** Perimeter of ΔABC
Calculation:

$$\text{Perimeter of } \Delta ABC = 5 \text{ cm} + 8 \text{ cm} + 7 \text{ cm}$$
$$= 20 \text{ cm}$$

d. **Required To Draw:** Line *CD* which is perpendicular to *AB* and meets *AB* at *D*.

Solution:



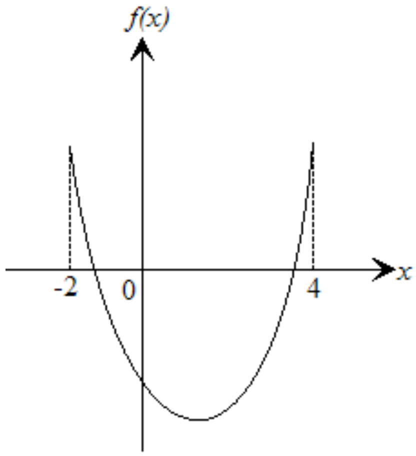
e. **Required To Find:** The length of *CD*.
Solution:
$CD = 4.3$ cm (by measurement)

f. **Required To Calculate:** Area of ΔABC
Calculation:

$$\text{Area of } \Delta ABC = \frac{8 \times 4.3}{2} = 17.2\,\text{cm}^2$$

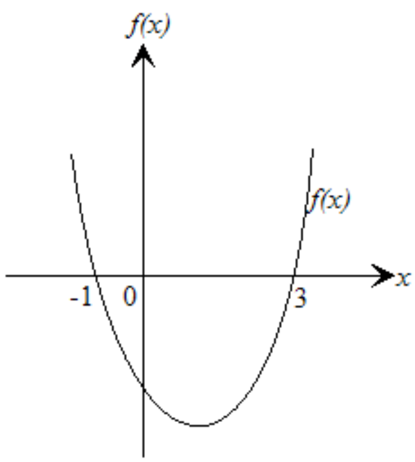
5. **Data:** Diagram illustrating the graph of the function $f(x) = x^2 - 2x - 3$ for $a \le x \le b$ and the tangent at (2, -3).

a. **Required To Find:** *a* and *b*.
Solution:



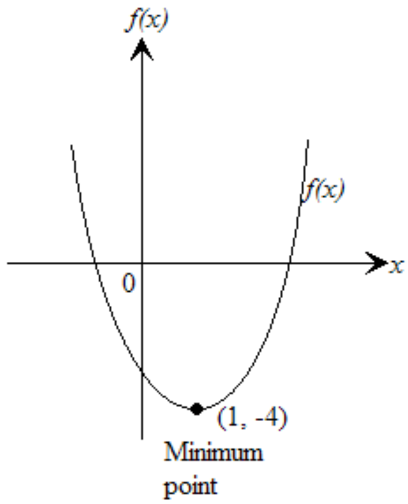
$x \ge -2$ and $x \le 4$.
$\therefore a = -2$ and $b = 4$ from the diagram, that is $-2 \le x \le 4$.

b. **Required To Find:** *x* for $x^2 - 2x - 3 = 0$.
Solution:



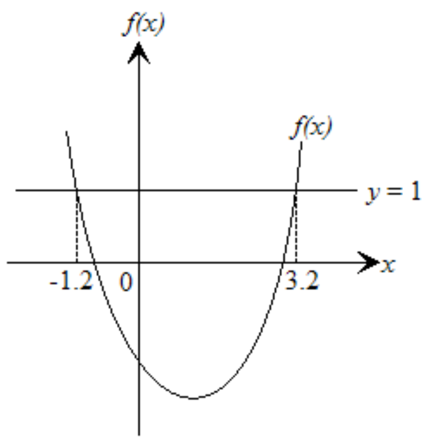
$x^2 - 2x - 3 = 0$ cuts the x – axis at – 1 and 3 as seen on the diagram. Therefore, the values of *x* are – 1 and 3.

c. **Required To Find:** Coordinates of the minimum point on the graph.
Solution:



The minimum point of $f(x)$ is (1, -4) as seen on the diagram.

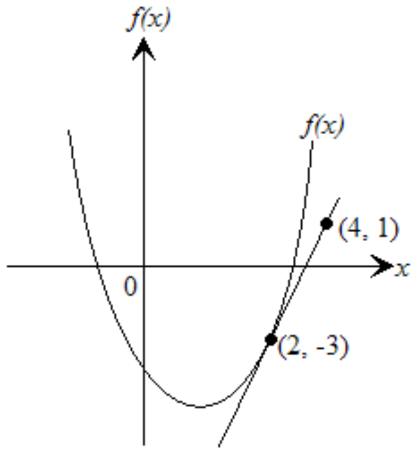
d. **Required To Find:** Whole number values of x for which $x^2 - 2x - 3 < 1$.
Solution:



From the diagram, $x^2 - 2x - 3 < 1$ for $x > -1.2$ and $x < 3.2$, that is $-1.2 < x < 3.2$.

$x \in W \qquad \therefore x = \{0, 1, 2, 3\}$

e. **Required To Find:** gradient of $f(x)=x^2-2x-3$ at $x=2$.
Solution:



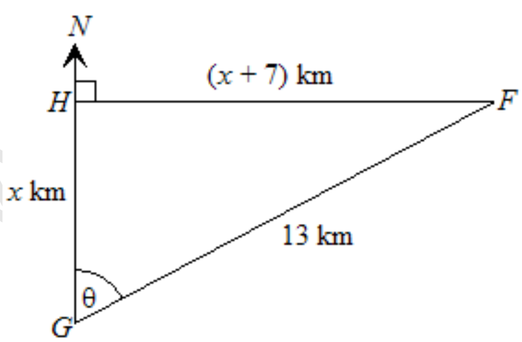
Choosing (2, -3) and (4, 1) as 2 points on the tangent to $f(x)$ at (2, -3).

$$\text{Gradient } = \frac{1-(-3)}{4-2} = \frac{4}{2} = 2$$

$\therefore$ Gradient of $f(x)$ at (2, -3) is 2.

6. **Data:** Diagram showing the direction and distance of a man walking.

a. **Required To Complete:** The diagram given showing distances x km, $(x+7)$km and 13 km.
Solution:



b. **Required To Find:** Equation in x that satisfies Pythagoras' Theorem and that simplifies to $x^2 + 7x - 60 = 0$.

Solution:

$$(x)^2 + (x+7)^2 = (13)^2 \quad \text{(Pythagoras' Theorem)}$$

$$x^2 + (x^2 + 14x + 49) = 168$$

$$2x^2 + 14x - 120 = 0$$

$\div 2$

$$x^2 + 7x - 60 = 0$$

Q.E.D.

c. **Required To Find:** Distance *GH*.

Solution:

$$x^2 + 7x - 60 = 0$$

$$(x+12)(x-5) = 0$$

$$x = -12 \text{ or } 5$$

$x \neq -12$ (since *GH* and *HF* would be negative)

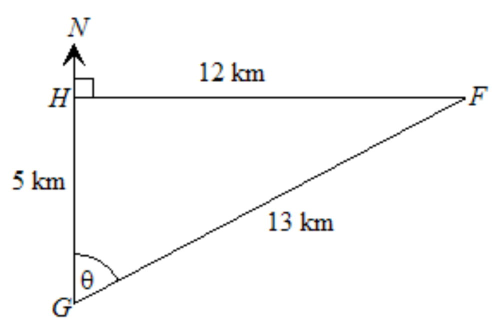
$x = 5$ only

GH = 5 km

d. **Required To Find:** Bearing of *F* from *G*.

Solution:

The bearing of *F* from *G* is illustrated by θ.



$$\tan\theta = \frac{12}{5}$$

$$\theta = \tan^{-1}\left(\frac{12}{5}\right)$$

$$\theta = 67.4°$$

$\therefore$ The bearing of *F* from *G* is 067.4°

7. **Data:** Table showing the gains in mass of 100 cows over a certain period.

a. **Required To Complete:** Table of information given.

Solution:

Modifying the table for the data of the continuous variable

Gain in mass in kg Continuous variable	**L.C.B U.C.B.**	**Mid-class Interval, *x*** $\frac{\text{L.C.B.}+\text{U.C.B.}}{2}$	**Frequency, *f***
		2	0
5 – 9	$4.5 \leq x < 9.5$	$\frac{4.5+9.5}{2}=7$	2
10 – 14	$9.5 \leq x < 14.5$	$\frac{9.5+14.5}{2}=12$	29
15 – 19	$14.5 \leq x < 19.5$	$\frac{14.5+19.5}{2}=17$	37
20 – 24	$19.5 \leq x < 24.5$	$\frac{19.5+24.5}{2}=22$	16
25 – 29	$24.5 \leq x < 29.5$	$\frac{24.5+29.5}{2}=27$	14
30 – 34	$29.5 \leq x < 34.5$	$\frac{29.5+34.5}{2}=32$	2
		37	0

b. (i) **Required To Estimate:** Mean gain in mass of the 100 cows.

Solution:

The mean gain , $\bar{x}$

$$\bar{x}=\frac{\sum fx}{\sum f}$$

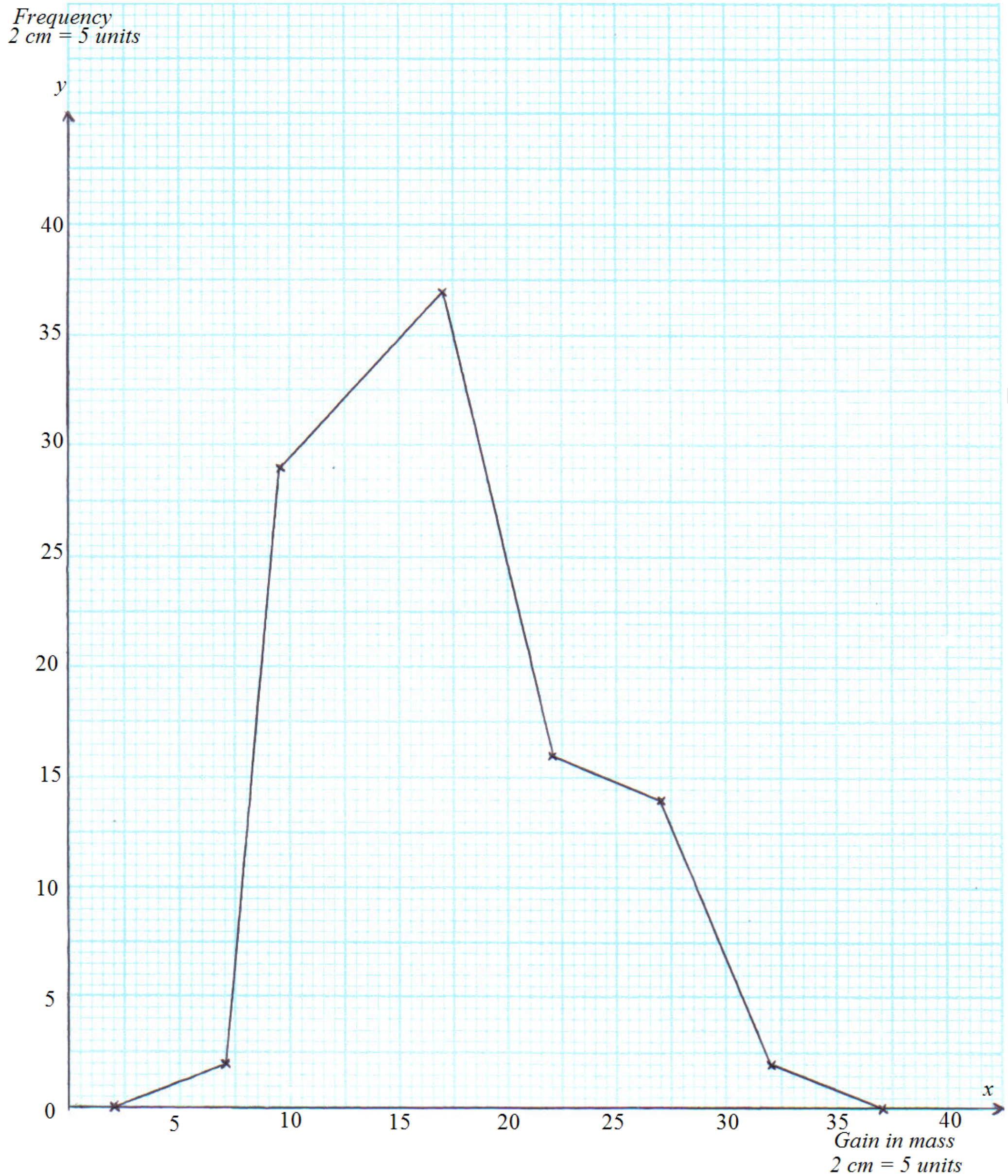
$$=\frac{(2\times 7)+(29\times 12)+(37\times 17)+(16\times 22)+(14\times 27)+(2\times 32)}{\sum f=100}$$

$$=17.85\text{ kg}$$

(ii) **Required To Draw:** The frequency polygon for the information given.

Solution:

The points (2, 0) and (37, 0) are obtained by extrapolation as the frequency polygon is to be bounded by the horizontal axis.



c. **Required To Calculate:** Probability that a randomly chosen cow gained 20 kg or more.

Solution:

$$P(\text{cow gained} \geq 20 \text{ kg}) = \frac{\text{No. of cows gaining} \geq 20 \text{ kg}}{\text{Total no. of cows}}$$

$$= \frac{16 + 14 + 2}{\sum f = 100}$$

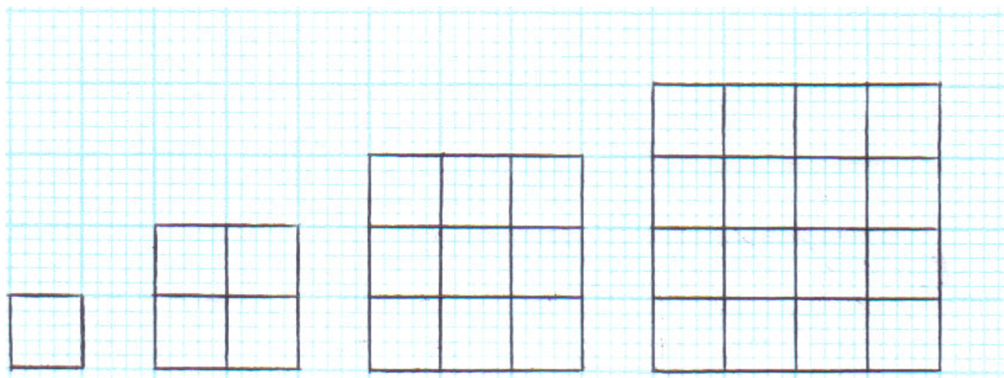
$$= \frac{32}{100}$$

$$= \frac{8}{25}$$

8. **Data:** Drawings showing a sequence of squares made from toothpicks.

a. (i) **Required To Draw**: Next shape in the sequence.

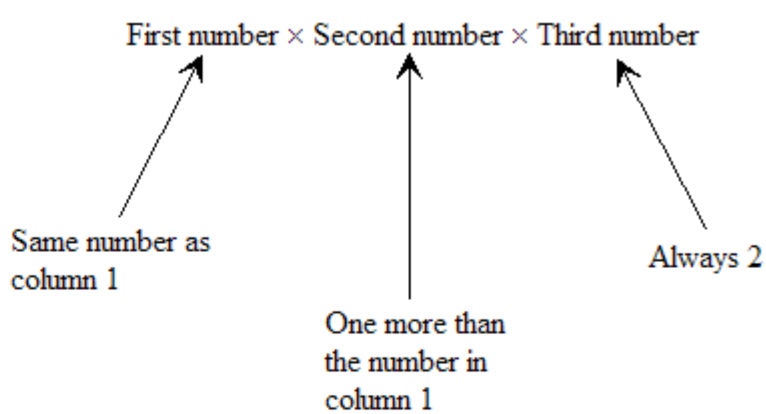
Solution:



(ii)

Column 1	Column 2	Column 3
Length, *n*, of one side of square	**Pattern for calculating number of toothpicks in square**	**Total number of toothpicks in square**
1	$1\times2\times2$	4
2	$2\times3\times2$	12
3	$3\times4\times2$	24
4	$4\times5\times2$	40
7	$7\times8\times2$	112
n	$r=n\times(n+1)\times2$	$2n(n+1)$
$s=10$	$10\times11\times2$	220

(ii) The column 2 is a product of three numbers , that is



a) **Required To Complete:** Table when $n=4$

Solution:
When column 1 is 4

$$\text{Column } 2 = 4\times(4+1)\times 2$$
$$= 4\times 5\times 2$$

Column 3 is the result = 40 of column 2.

b) **Required To Complete:** The table when $n = 7$

Solution:
When column 1 is 7

$$\text{Column } 2 = 7\times(7+1)\times 2$$
$$= 7\times 8\times 2$$

And column 3 is 112.

b. (i) **Required To Complete:** The table for length of side *n*.

Solution:
When column 1 is *n*, column 2 is *r*.

$$\therefore r = n\times(n+1)\times 2$$
$$= 2n(n+1)$$
$$Col\ 3 = 2n(n+1)$$

(ii) **Required To Complete:** The table when column 3 is 220.

Solution:
Column 3 is 220.

$$n\times(n+1)\times 2 = 220$$
$$2n(n+1) = 220$$
$$n(n+1) = 110$$
$$n^2 + n - 110 = 0$$
$$(n+11)(n-10) = 0$$
$$n = -11\ or 10$$

$n \neq -ve$

$n = 10$

Therefore, in (b) (ii) $s = 10$ and

Column 2 $= 10 \times (10 + 1) \times 2$

$= 10 \times 11 \times 2$

9. a. **Data:** $y = x + 2$ and $y = x^2$

Required To Calculate: x and y

Calculation:

Let $y = x + 2 \ldots (1)$ and $y = x^2 \ldots (2)$

Equating

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$\therefore x = 2 \text{ or } -1$

When $x = 2$ $\quad y = 2 + 2$

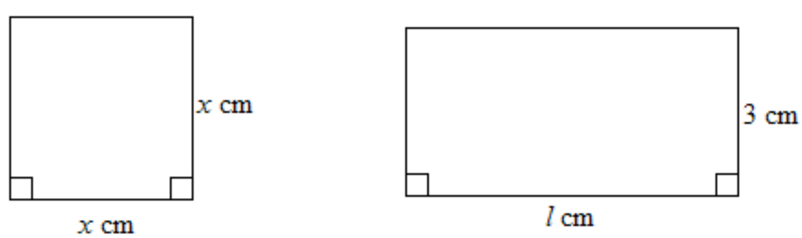
$= 4$

When $x = -1$ $\quad y = (-1)^2$

$= 1$

Hence, $x = 2$ and $y = 4$ **OR** $x = -1$ and $y = 1$.

b. **Data:** Strip of wire 32 m long is cut into 2 pieces and formed into a square and a rectangle.



(i) **Required To Find:** Expression in terms of x and l for the length of the strip of wire.

Solution:

Perimeter of square $= (x \times 4)$

$= 4x$ cm

Perimeter of rectangle $= 2(l + 3)$

$= 2l + 6$ cm

$\therefore 4x + 2l + 6 = 32$

(ii) **Required To Prove:** $l = 13 - 2x$
Proof:

$4x + 2l + 6 = 32$

$4x + 2l = 32 - 6$

$4x + 2l = 26$

$\div 2$

$2x + l = 13$

$l = 13 - 2x$

(iii) **Required To Prove:** $S = x^2 - 6x + 39$.
Proof:

$S = (x^2) + (3)(l)$

$S = x^2 + 3l$

$S = x^2 + 3(13 - 2x)$

$= x^2 + 39 - 6x$

$= x^2 - 6x + 39$

Q.E.D.

(iv) **Required To Calculate:** x for which $S = 30.25$
Calculation:

$x^2 - 6x + 39 = 30.25$

$x^2 - 6x + 8.75 = 0$

$\times 4$

$4x^2 - 24x + 35 = 0$

$(2x - 5)(2x - 7) = 0$

$$x = 2\frac{1}{2} \text{ or } 3\frac{1}{2}$$

Hence, when $S = 30.25$, $x = 2\frac{1}{2}$ or $3\frac{1}{2}$.

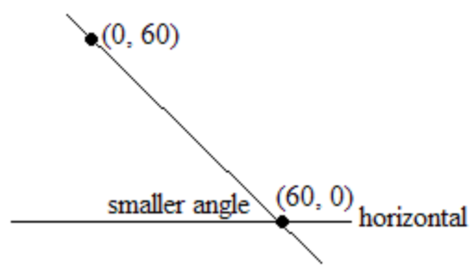
10. **Data:** Conditions for the parking of x vans and y cars at a lot.

(i) **Required To Find:** Inequality for the information given.
Solution:
No. of vans = x
No. of cars = y
Lot has space for no more than 60 vehicles. Therefore,
$\therefore x + y \leq 60 \ldots(1)$

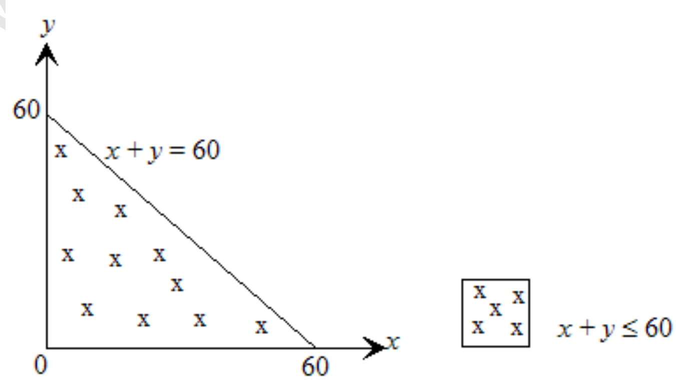
(ii) **Data:** Owner must part at least 10 cars.
Required To Find: Inequality for the information given.
Solution:
No. of cars is at least 10.
$\therefore y \geq 10 \ldots(2)$

(iii) **Data:** Number of cars parked must be fewer than or equal to twice the number of vans parked.
Required To Find: Inequality for the information given.
Solution:
The no. of cars parked must be fewer than or equal to twice the number of vans.
$y \qquad\qquad \leq \qquad\qquad 2x$
$\therefore y \leq 2x \ldots(3)$

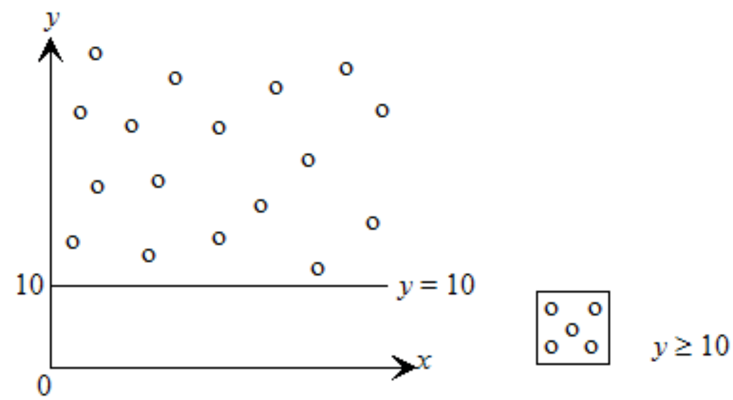
(iv) **Required To Draw:** The graphs of the lines associated with the inequalities and shaded the region which satisfies all three.
Solution:
Obtaining 2 points on the line $x + y = 60$.
When $x = 0$ $\qquad 0 + y = 60$
$\qquad\qquad\qquad y = 60$
The line $x + y = 60$ passes through the point (0, 60).
When $y = 0$ $\qquad x + 0 = 60$
$\qquad\qquad\qquad x = 60$
The line $x + y = 60$ passes through the point (60, 0).



The side with the smaller angle satisfies the $\leq$ region.
The region which satisfies $x + y \leq 60$ is



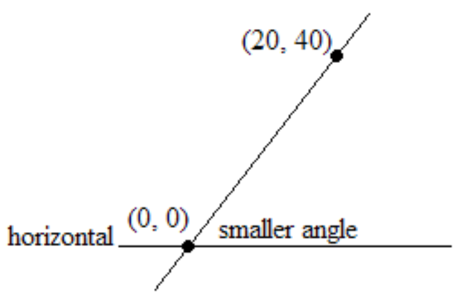
The line $y=10$ is a horizontal straight line.
The region which satisfies $y \geq 10$ is



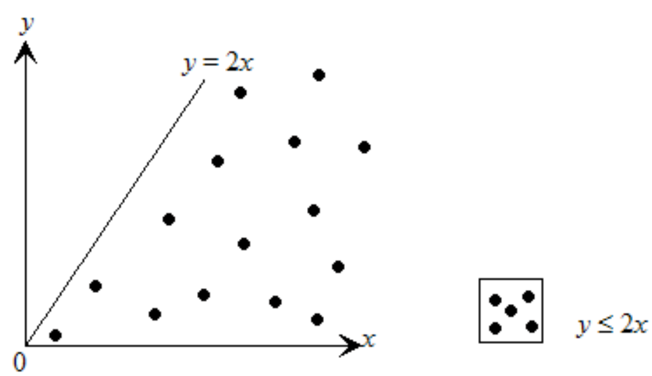
Obtaining 2 points on the line $y = 2x$.
The line $y = 2x$ passes through the origin (0, 0).
When $x = 20$ $\quad y = 2(20)$

$y = 40$

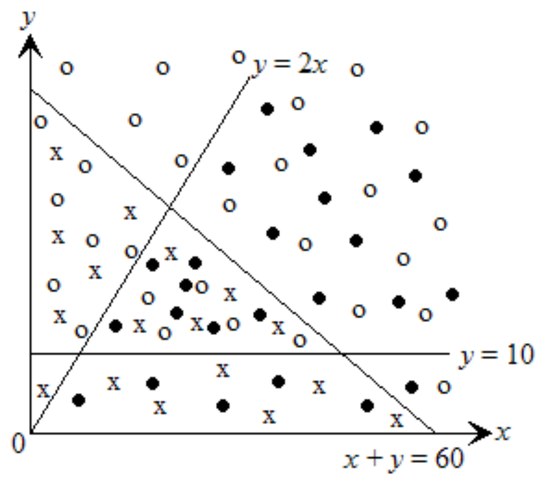
The line $y = 2x$ passes through the point (20, 40).

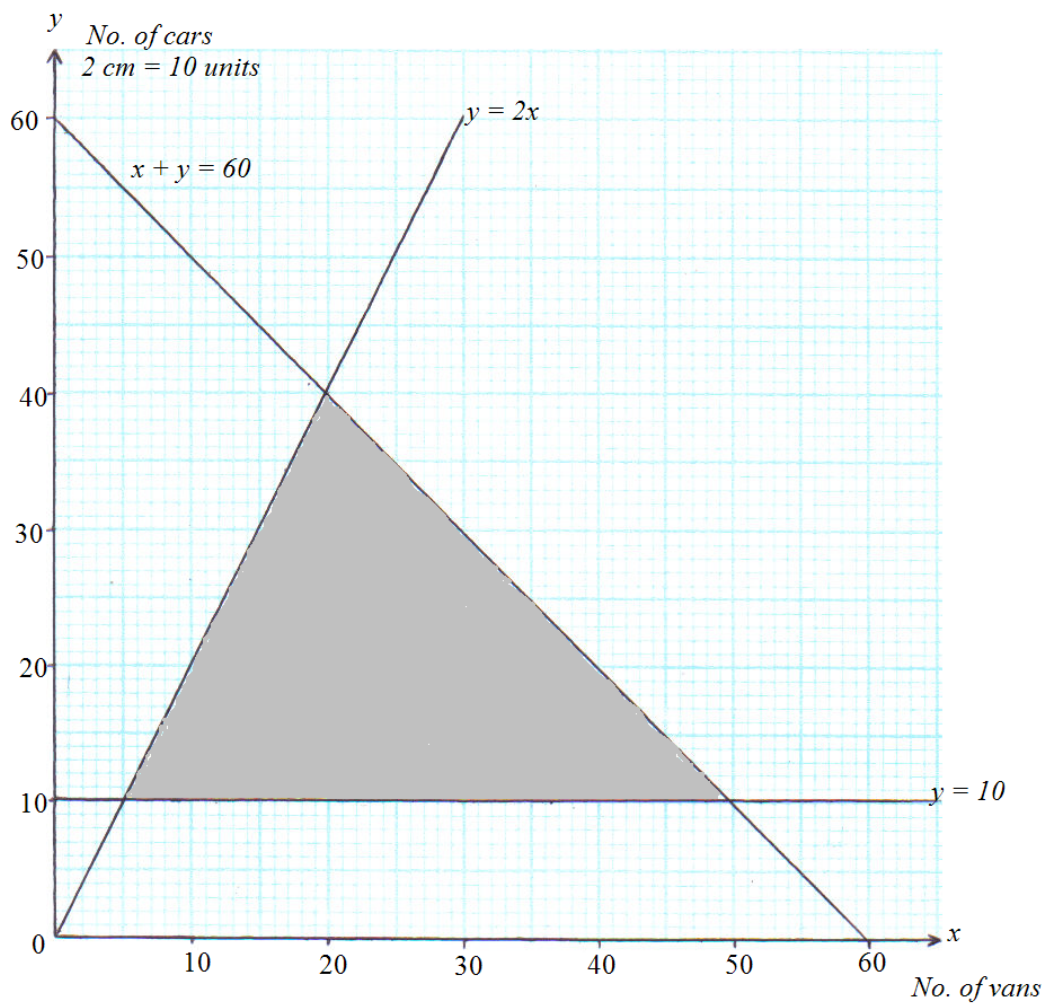


The side with the smaller angle satisfies the $\leq$ region.
The region which satisfies $y \leq 2x$ is



The region which satisfies all three inequalities is the area in which all three shaded regions overlap.





(v) **Data:** Parking fee for a van is \$6 and parking fee for a car is \$5.

Required To Find: Expression in x and y for total fees charged for parking x vans and y cars.
Solution:
The total fees on x vans at \$6 each and y cars at \$5 each $= (x \times 6) + (y \times 5)$
$= 6x + 5y$

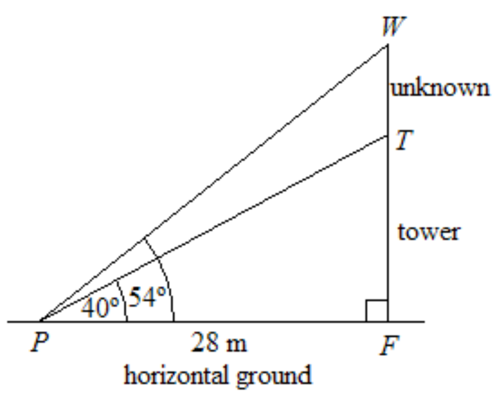
(vi) **Required To Find:** Vertices of the shaded region.
Solution:
The vertices are (5, 10), (20, 40) and (50, 10).

(vii) Required To Calculate: Maximum fees charged.
Calculation:
Testing (20, 40) and (50, 10)
$x = 20 \quad y = 40$
Fees $= 6(20) + 5(40)$
$= 320$

$x = 50 \quad y = 10$
Fees $= 6(50) + 5(10)$
$= 350$
$\therefore$ Maximum fee charged is \$350, when there are 50 vans and 10 cars.

11. a. **Data:** Diagram of a vertical tower and antenna mounted atop. Point P lies on horizontal ground.

(i) **Required To Complete:** The diagram given, showing the distance 28 m, angles 40° and 54° and any right angles.
Solution:



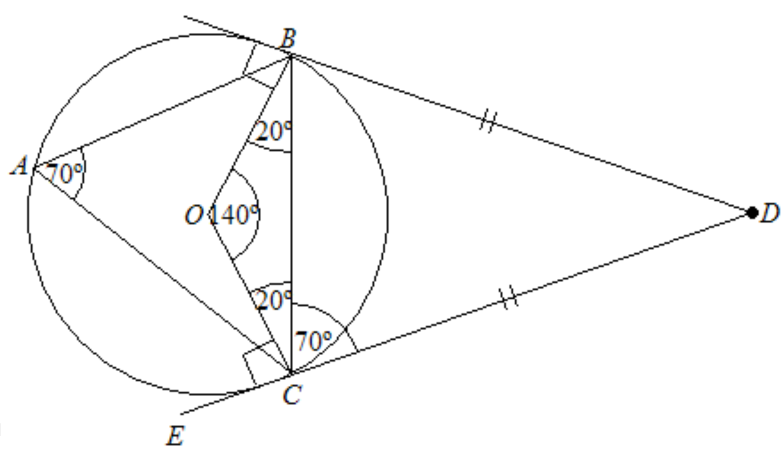
(ii) **Required To Calculate:** Length of antenna TW.
Calculation:

$$\frac{TF}{28} = \tan 40°$$
$$TF = 28\tan 40°$$
$$\frac{WF}{28} = \tan 54°$$
$$WF = 28\tan 54°$$

Length of antenna = Length of WF – Length of TF

$$= 28\tan 54° - 28\tan 40°$$
$$= 15.0\underline{4} \text{ m}$$
$$= 15.0 \text{ m}$$

b. **Data:** Diagram showing a circle centre O and tangents BD and DCE. $B\hat{C}D = 70°$.



(i) **Required To Calculate:** $O\hat{C}E$

Calculation:

$O\hat{C}E = 90°$

(Angles made by tangent to a circle and radius, at point of contact = 90°).

(ii) **Required To Calculate:** $B\hat{A}C$

Calculation:

$$B\hat{A}C = \frac{1}{2}(140°)$$
$$= 70°$$

(Angles subtended by a chord at the centre of the circle equal twice the angle is subtends at the circumference, standing on the same arc).

(iii) **Required To Calculate:** $B\hat{O}C$

Calculation:

$O\hat{C}B = 180° - (70° + 90°)$
$= 20°$

(Angles in a straight line).

$OB = OC$ (radii)

$O\hat{B}C = 20°$

(Base angles of an isosceles triangle are equal).

$B\hat{O}C = 180° - (20° + 20°)$
$= 140°$

(Sum of angles in a triangle = 180°).

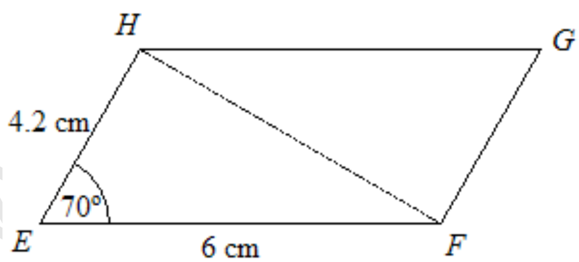
(iv) **Required To Calculate:** $B\hat{D}C$

Calculation:

$B\hat{D}C = 360° - (90° + 90° + 140°)$
$= 40°$

(Sum of angles in a quadrilateral is 360°).

12. a. **Data:** Parallelogram *EFGH* with *EH* = 4.2 cm, *EF* = 6 cm and $H\hat{E}F = 70°$



(i) **Required To Calculate:** Length of *HF*.

Calculation:

$HF^2 = (4.2)^2 + (6)^2 - 2(4.2)(6)\cos 70°$ (Cosine Rule)
$= 36.402$

$HF = \sqrt{36.402}$
$= 6.03\underline{3}$
$= 6.03$ to 2 decimal places

(ii) **Required To Calculate:** Area of parallelogram *EFGH*.
Calculation:

Area of $\Delta HEF = \frac{1}{2}(4.2)(6)\sin 70°$

Diagonal *HF* bisects the parallelogram *EFGH*.

$\therefore$ Area of parallelogram $EFGH = 2\left(\frac{1}{2}(4.2)(6)\sin 70°\right)$

$= 23.68\underline{0}$

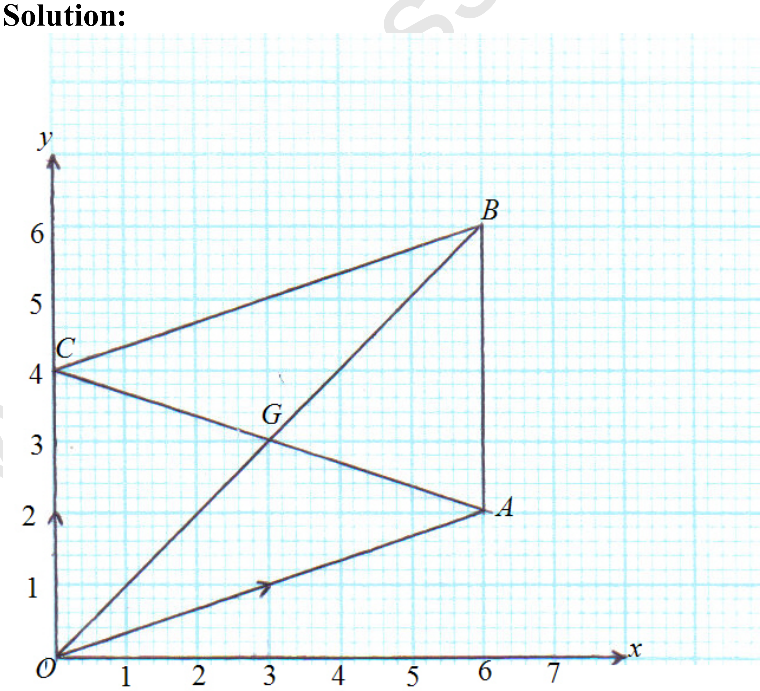
$= 23.68$ to 2 decimal places

b. This part of the question has not been solved as it involves Earth Geometry which has since been removed from the syllabus.

13. **Data:** Diagram showing the position vectors of 2 points A and C relative to O.

a. **Required To Complete:** The diagram to show B, such that OABC is a parallelogram and $\underline{u}$.

Solution:



$\overrightarrow{OA} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ from diagram

$\underline{u} = \overrightarrow{OA} + \overrightarrow{OC}$

$= \begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 6 \end{pmatrix}$

b. (i) **Required To Express:** $\overrightarrow{OA}$ in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:

Since A is (6, 2) then $\overrightarrow{OA} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ where $x = 6$ and $y = 2$.

(iii) **Required To Express:** $\overrightarrow{OC}$ in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:

Since C is (0, 4) then $\overrightarrow{OC} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ where $x = 0$ and $y = 4$.

(iv) **Required To Express:** $\overrightarrow{AC}$ in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$= -\begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$ is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ where $x = -6$ and $y = 2$.

c. **Data:** *G* is the midpoint of *OB*.

(i) **Required To Find:** Coordinates of *G*.

Solution:

$\overrightarrow{OB} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$

$\overrightarrow{OG} = \frac{1}{2}\overrightarrow{OB}$

$= \frac{1}{2}\begin{pmatrix} 6 \\ 6 \end{pmatrix}$

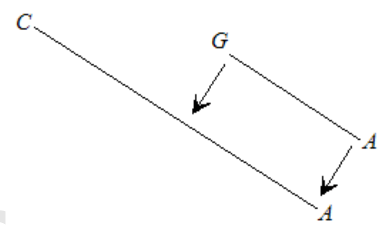
$= \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

Hence G is (3 , 3)

(ii) **Required To Prove:** A, G and C lie on a straight line.

Proof:

$\overrightarrow{AC} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$



$\overrightarrow{AG} = \overrightarrow{AO} + \overrightarrow{OG}$

$= -\begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

$= \frac{1}{2}\overrightarrow{AC}$

$\overrightarrow{AG}$ is a scalar multiple of $\overrightarrow{AC}$.$\therefore$ $\overrightarrow{AG}$ and $\overrightarrow{AC}$ are parallel. G is a common point, therefore, G lies on AC, hence, A, G and C lies on the same straight line, that they are collinear.

14. a. **Data:** $|M| = \begin{pmatrix} 2 & 3 \\ -1 & x \end{pmatrix} = 9$

(i) **Required To Calculate:** a

Calculation:

$$|M| = 9$$
$$(2 \times x) - (3 \times -1) = 9$$
$$2x + 3 = 9$$
$$2x = 6$$
$$x = 3$$

(ii) **Required To Calculate:** M^{-1}

Calculation:

$$M = \begin{pmatrix} 2 & 3 \\ -1 & 3 \end{pmatrix}$$

$$M^{-1} = \frac{1}{9}\begin{pmatrix} 3 & -(3) \\ -(-1) & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{9} & -\frac{3}{9} \\ \frac{1}{9} & \frac{2}{9} \end{pmatrix}$$

(iii) Required To Prove: $M^{-1}M = I$

Proof:

$$M_{2\times 2} \times M^{-1}{}_{2\times 2} = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$$

$$e_{11} = \left(2\times\frac{3}{9}\right) + \left(3\times\frac{1}{9}\right)$$

$$= \frac{9}{9}$$

$$= 1$$

$$e_{12} = \left(2\times-\frac{3}{9}\right) + \left(3\times\frac{2}{9}\right)$$

$$= \frac{0}{9}$$

$$= 0$$

$$e_{21} = \left(-1\times\frac{3}{9}\right) + \left(3\times\frac{1}{9}\right)$$

$$= \frac{0}{9}$$

$$= 0$$

$$e_{22} = \left(-1\times-\frac{3}{9}\right) + \left(3\times\frac{2}{9}\right)$$

$$= \frac{9}{9}$$

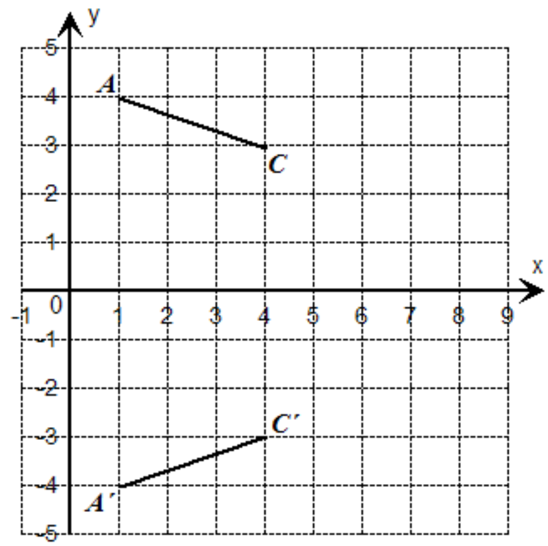
$$= 1$$

$$M \times M^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= I$$

Q.E.D.

b. **Data:** Graph showing line segment AC and its image $A'C'$ after a transformation $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$.



(i) (a) **Required To Express:** A and C as a single 2×2 matrix.
Solution:

Coordinates of A and C in matrix form is $\begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix}$.

(b) **Required To Express:** A' and C' as a single 2×2 matrix.
Solution:

Coordinates of A' and C' in matrix form is $\begin{pmatrix} 2 & 5 \\ -4 & -3 \end{pmatrix}$.

(ii) **Required To Find:** Equation to represent the transformation of AC onto $A'C'$.
Solution:

$$AC \rightarrow A'C'$$

$$\begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix} \xrightarrow{\begin{pmatrix} p & q \\ r & s \end{pmatrix}} \begin{pmatrix} 2 & 5 \\ -4 & -3 \end{pmatrix}$$

$$\begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 2p+4q & 5p+3q \\ 2r+4s & 5r+3s \end{pmatrix}$$

(iii) **Required To Calculate:** p, q, r and s
Calculation:

Equating corresponding entries

$2p+4q=2 \ldots(1)$
$\times 5$
$10p+20q=10$

$5p+3q=5 \ \ldots(2)$
$\times -2$
$-10p-6q=-10$

$$\begin{array}{r} 10p+20q = \ \ 10 \\ -10p - \ 6q = -10 \\ \hline 14q = 0 \\ \hline \end{array}$$

$\therefore q=0$ and $p=1$
Similarly,
$2r+4s=-4 \ldots(3)$
$\times 5$
$10r+20s=-20$

$5r+3s=-3 \ \ldots(4)$
$\times -2$
$-10r-6s=6$

$$\begin{array}{r} 10r+20s = -20 \\ -10r - \ 6s = \ \ 6 \\ \hline 14s = -14 \\ \hline \end{array}$$

$s=-1$ and $r=0$

$\therefore \ p=1, \ q=0, \ r=0$ and $s=-1$ and the matrix $\begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, which represents a reflection in the $x-$axis.
We may also deduce this by observing the object AC and its image $A'C'$.