FAS-PASS Maths

JANUARY 2006 MATHEMATICS GENERAL PROFICIENCY (PAPER 2)

Section I

1. a. (i) **Required To Calculate:** $\dfrac{2\frac{1}{4}\times\frac{4}{5}}{\frac{3}{5}-\frac{1}{2}}$

Calculation:

Numerator

$2\frac{1}{4}\times\frac{4}{5}$

$=\frac{9}{4}\times\frac{4}{5}$

$=\frac{9}{5}$

Denominator

$\frac{3}{5}-\frac{1}{2}$

$\frac{6-5}{10}=\frac{1}{10}$

Hence, $\dfrac{2\frac{1}{4}\times\frac{4}{5}}{\frac{3}{5}-\frac{1}{2}}=\dfrac{\frac{9}{5}}{\frac{1}{10}}$

$=\frac{9}{5}\times\frac{10}{1}$

$=18$ (in exact form)

(ii) **Required To Calculate:** $18.75-(2.11)^2$

Calculation:

$18.75-(2.11)^2=18.75-4.4521$

$=14.2\underline{9}79$

$=14.3$ to 3 significant figures

b. **Data:** Amount of loan = \$12 000

Rate = 14% per annum

(i) **Required To Calculate:** Interest on the loan at the end of the first year.

Calculation:

Interest on loan at the end of 1st year $=\frac{14}{100}\times 12000$

$=\$1680$

(ii) **Required To Calculate**: Total amount owing at the end of 1st year.
Calculation:
Total amount owing at the end of 1st year
= Original amount borrowed + Interest after 1 year.
= \$12 000 + \$1 680
= \$13 680

Data: Repayment at start of 2nd year = \$7 800

(iii) **Required To Calculate:** Amount outstanding at the start of 2nd year.
Calculation:
Amount owed at start of second year = \$13 680 - \$7 800
= \$5 880

(iv) **Required To Calculate:** Interest on the outstanding amount at the end of 2nd year.
Calculation:
Interest on the outstanding amount at the end of 2nd year

$$= \frac{14}{100} \times 5880$$
$$= \$823.20$$

2. a. **Data:** $m = -2$ and $n = 4$
Required To Calculate: $(2m + n)(2m - n)$
Calculation:

$$\begin{aligned}(2m+n)(2m-n) &= (2(-2)+4)(2(-2)-4) \\ &= (-4+4)(-4-4) \\ &= 0 \times -8 \\ &= 0\end{aligned}$$

b. **Data:** $5x + 6y = 37$, $2x - 3y = 4$
Required To Calculate: The value of x and of y
Calculation:
Let $5x + 6y = 37 \ldots(1)$ and $2x - 3y = 4 \ldots(2)$
Equation (2) $\times 2$
$4x - 6y = 8 \ldots(3)$
Equation (1) + (3)

$$\begin{aligned} 5x + 6y &= 37 \quad (1) \\ 4x - 6y &= 8 \quad (3) \\ \hline 9x \quad &= 45 \end{aligned}$$

$$x = \frac{45}{9}$$

$$x = 5$$

When $x = 5$ Substitute in equation (1)

$$5x + 6y = 37$$

$$5(5) + 6y = 37$$

$$6y = 37 - 25$$

$$6y = 12$$

$$y = 2$$

Hence $x = 5$ and $y = 2$

OR

Let $5x + 6y = 37 \ldots(1)$ and $2x - 3y = 4 \ldots(2)$

From (2)

$$2x - 3y = 4$$

$$2x = 3y + 4$$

$$x = \frac{3y + 4}{2}$$

Substituting in (1)

$$5\left(\frac{3y + 4}{2}\right) + 6y = 37$$

$$5(3y + 4) + 2(6y) = 2(37)$$

$$15y + 20 + 12y = 74$$

$$27y = 54$$

$$y = 2$$

Substitute $y = 2$ in $x = \frac{3y + 4}{2}$

$$x = \frac{3(2) + 4}{2}$$

$$= \frac{10}{2}$$

$$= 5$$

Hence, $x = 5$ and $y = 2$

OR

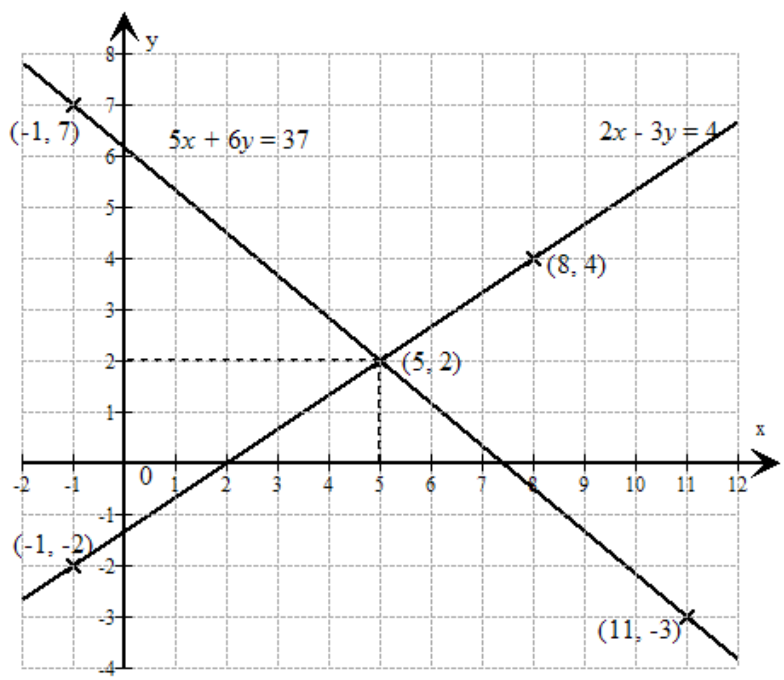
Obtaining 2 points on the straight line $5x+6y=37$

x	y
-1	7
11	-3

Obtaining 2 points on the straight line $2x-3y=4$

x	y
-1	-2
8	4

Plotting both straight lines on the same axes.



Point of intersection is (5, 2), therefore, $x=5$ and $y=2$.

OR

$5x+6y=37$ and $2x-3y=4$ and

$$\begin{pmatrix}5 & 6\\ 2 & -3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}37\\ 4\end{pmatrix}$$ …matrix equation

Let $A=\begin{pmatrix}5 & 6\\ 2 & -3\end{pmatrix}$

$$\begin{aligned}\det A &= (5\times -3)-(6\times 2)\\ &= -15-12\\ &= -27\end{aligned}$$

$$\therefore A^{-1} = -\frac{1}{27}\begin{pmatrix} -3 & -(6) \\ -(2) & 5 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{27} & \frac{6}{27} \\ \frac{2}{27} & -\frac{5}{27} \end{pmatrix}$$

Matrix equation $\times A^{-1}$

$$A \times A^{-1} \times \begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} 37 \\ 4 \end{pmatrix}$$

$$I \times \begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} 37 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{27} & \frac{6}{27} \\ \frac{2}{27} & -\frac{5}{27} \end{pmatrix}\begin{pmatrix} 37 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} \left(\frac{3}{27} \times 37\right) + \left(\frac{6}{27} \times 4\right) \\ \left(\frac{2}{27} \times 37\right) + \left(-\frac{5}{27} \times 4\right) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{135}{27} \\ \frac{54}{27} \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

Equating corresponding entries

$x = 5$ and $y = 2$

c. **Required To Factorise:** (i) $4x^2 - 25$, (ii) $6p - 9ps + 4q - 6qs$, (iii) $3x^2 + 4x - 4$

Factorising:

(i) $4x^2 - 25$

$= (2x)^2 - (5)^2$

Difference of 2 squares

$= (2x - 5)(2x + 5)$

(ii) $6p - 9ps + 4q - 6qs$

$= 3p(2 - 3s) + 2q(2 - 3s)$

$= (2 - 3s)(3p + 2q)$

(iii) $3x^2 + 4x - 4$
$(3x-2)(x+2)$

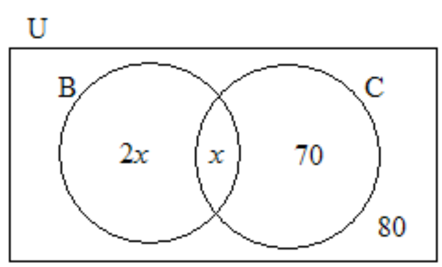
3. a. **Data:** $s = \frac{1}{2}(u+v)t$

Required To Express: u in terms of v, s and t.
Solution:

$$s = \frac{1}{2}(u+v)t$$
$$\frac{s}{1} = \frac{(u+v)t}{2}$$
$$2s = (u+v)t$$
$$2s = ut + vt$$
$$2s - vt = ut$$
$$\frac{2s - vt}{t} = u$$
$$\therefore u = \frac{2s - vt}{t} \text{ or } \frac{2s}{t} - v$$

b. **Data:** Venn diagram illustrating the customers' purchases at a bakery.

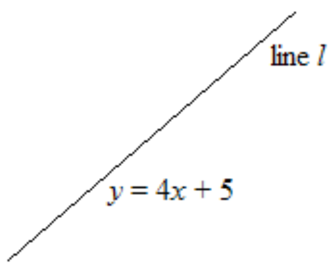
(i) **Required To Complete:** The Venn diagram to represent the data given.
Solution:



(ii) **Required To Find:** An expression in terms of x to represent the total number of customers to visit the bakery that day.
Solution:
Total no. of customers who visited the bakery $= 2x + x + 70 + 80$
$= 3x + 150$

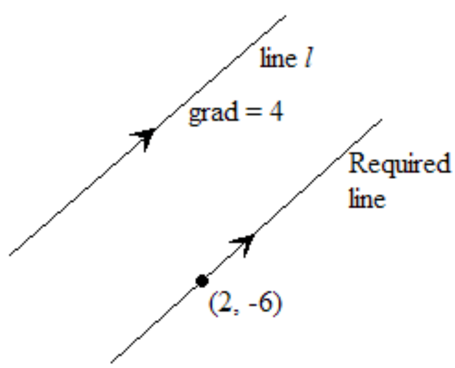
(iii) **Required To Calculate:** The number or customers who bought bread only.
Calculation:
$3x+150=300$ (data)
$\therefore 3x=150$
$x=50$
$\therefore$ No. of customers who bought bread only $=2x$
$=2(50)$
$=100$

4. a. **Data:** Line, *l*, with equation $y=4x+5$
(i) **Required To Find:** The gradient of and line parallel to l.
Solution:



$y=4x+5$ is of the form $y=mx+c$, where $m=4$ is the gradient.

(ii) **Required To Find:** The equation of the line parallel to *l* that passes through (2, -6)
Solution:



The gradient of the required line = 4
(Parallel lines have the same gradient).
Hence equation of the required line is

$$\frac{y-(-6)}{x-2}=4$$
$$y+6=4(x-2)$$
$$y+6=4x-8$$
$$y=4x-14$$

b. **Data:** Map showing the position of three cities A, B and C with a scale of 1 : 20 000 000.

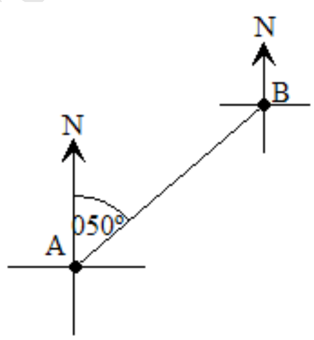
(i) **Required To Find:** The length of line segment BC.
Solution:

$BC = 6.7\,\text{cm}$ (by measurement)

(ii) **Required To Calculate:** The actual shortest distance from B to C.
Calculation:
The shortest distance between B and C $= 6.7\times 20\ 000\ 000\,\text{cm}$

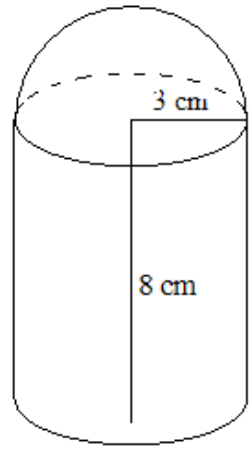
$$= \frac{6.7\times 20\ 000\ 000}{1000\times 100}\text{km}$$
$$= 1340\ \text{km}$$

(iii) **Required To Find:** The bearing of B from A.
Solution:



By measurement, the bearing of B from A = 050°

5. a. **Data:** Diagram illustrating a solid glass paper weight consisting of a hemisphere mounted on a cylinder.



(i) **Required To Calculate:** The curved surface area of the cylinder
Calculation:
The area of the curved surface of the cylinder is

$$2\pi rh = 2(3.14)(3)(8)\text{cm}^2$$
$$= 150.72 \text{ cm}^2$$

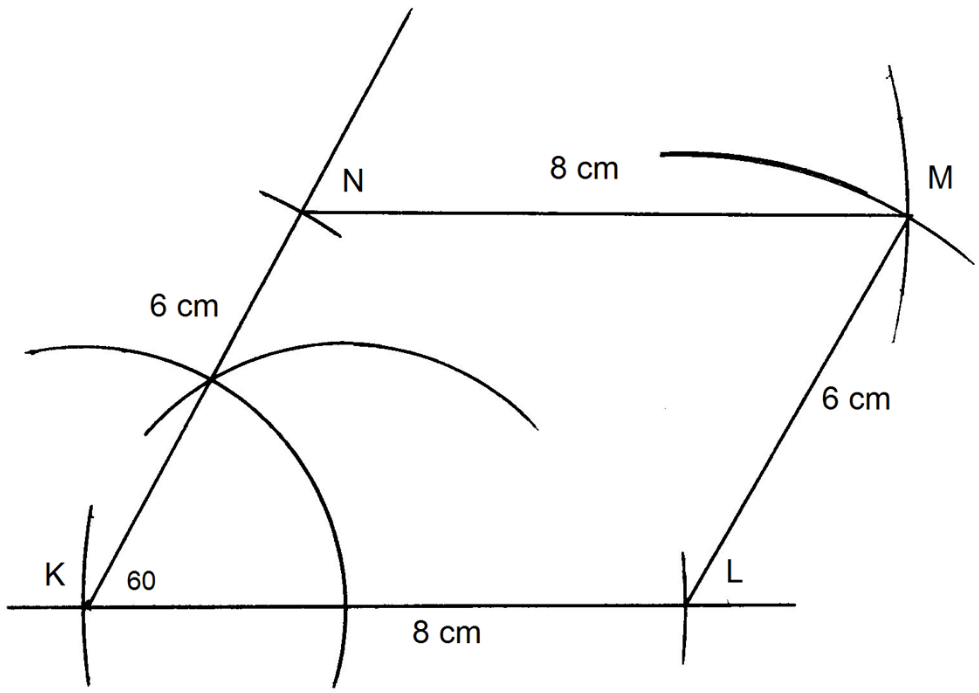
(ii) **Required To Calculate:** the surface area of the hemisphere.
Calculation:

$$\text{Surface area of the hemisphere } = \frac{1}{2}\left(4\pi r^2\right)$$
$$= \frac{1}{2}\left(4 \times 3.14 \times (3)^2\right)$$
$$= 56.52 \text{ cm}^2$$

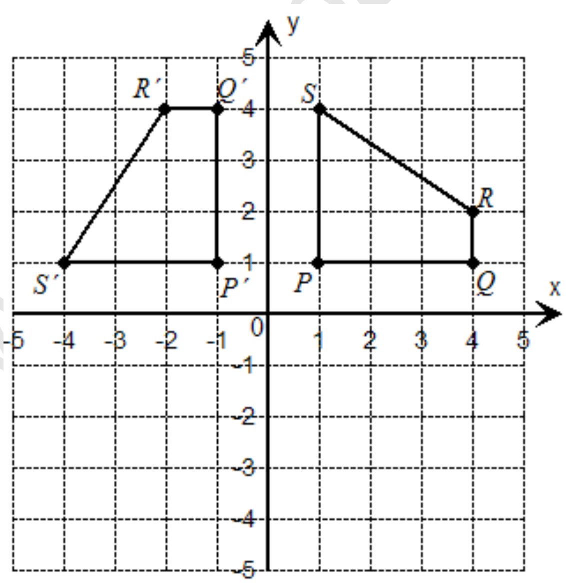
(iii) **Required To Calculate:** total surface area of the solid paper weight
Calculation:
Total surface area of the paper weight
= Area of curved surface of cylinder + Area of curved surface of hemisphere + Area of the circular base.

$$= 150.72 + 56.52 + 3.14(3)^2$$
$$= 235.50 \text{ cm}^2$$

b. **Required to Construct:** Parallelogram KLMN with KL = 8 cm, KN = 6 cm and $L\hat{K}N = 60°$
Solution:



6. **Data:** Diagram illustrating *PQRS* and its image $P'Q'R'S'$ under a rotation.



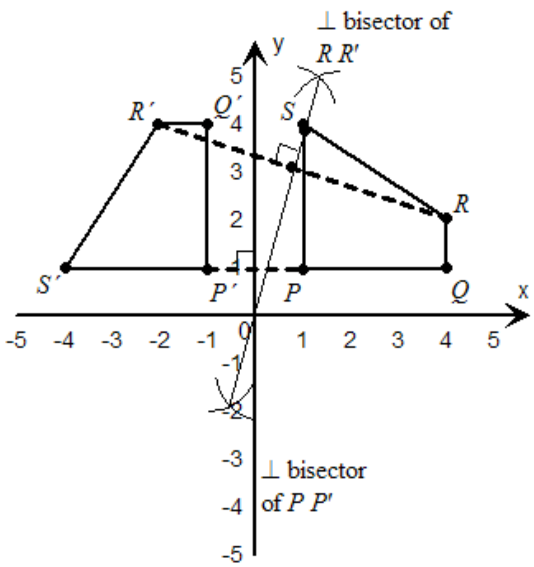
a. **Required To Find:** the coordinates of R' and S'
Solution:
$R' = (-2, 4)$

$S' = (-4, 1)$ (from the given diagram)

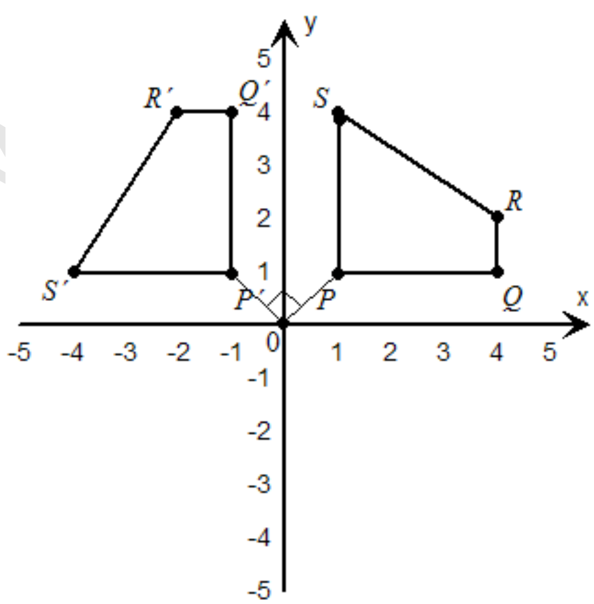
b. **Required To Describe:** the rotation completely.

Solution:

Two object vertices – P and R
Two image vertices - P' and R'
Broken line segments join P to P' and R to R'
Perpendicular bisectors of these line segments were constructed and found to intersect at the origin (0, 0).
$\therefore$ The centre of rotation is (0, 0).



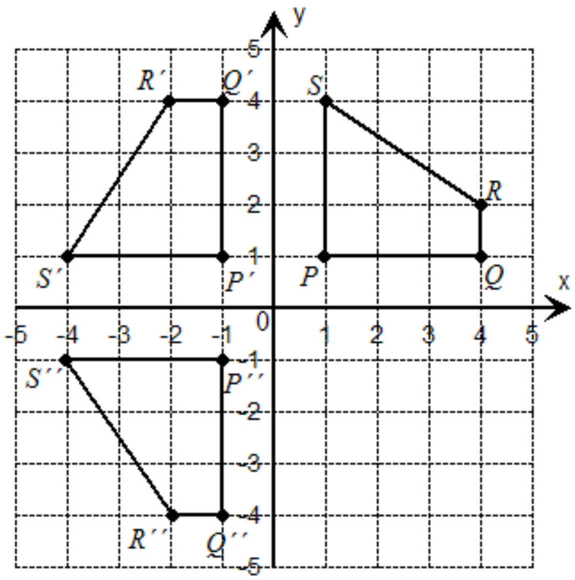
Line segments are drawn from an object vertex and the image vertex to the centre of rotation, example OP and OP'. The angle between both line segments is the angle of rotation.



$\therefore$ The angle of rotation is 90°.

Quadrilateral $PQRS$ is mapped onto $P'Q'R'S'$ under a rotation of 90° anti-clockwise about O. This may be represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

c. **Data:** $P''Q''R''S''$ is a reflection of the image $P'Q'R'S'$ in the x – axis.
Required To Draw: $P''Q''R''S''$
Solution:



d. **Required To Describe:** the transformation that maps $PQRS$ onto $P''Q''R''S''$
Solution:

$$PQRS \xrightarrow[\text{about } O]{\text{Rotation of } 90° \text{ anti-clockwise}} P'Q'R'S' \xrightarrow{\text{Reflection in } x\text{-axis}} P''Q''R''S''$$

$$PQRS \xrightarrow{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}} P'Q'R'S' \xrightarrow{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}} P''Q''R''S''$$

$$PQRS \xrightarrow{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}} P''Q''R''S''$$

$\therefore$ The simple transformation that maps $PQRS$ onto $P''Q''R''S''$ is $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. This is equivalent to a reflection in the line $y = -x$.

7. **Data:** The lengths of the right foot of 25 students.

a. **Required to complete:** the grouped frequency table for the given data.
Solution:

Modifying and completing the table of values for the continuous variable.

Length of right foot, ***x* cm**	L.C.B U.C.B.	Frequency	Mid-class Interval $\frac{\text{L.C.B}+\text{U.C.B.}}{2}$
14 – 16	$13.5 \le x < 16.5$	4	$\frac{13.5+16.5}{2}=15$
17 – 19	$16.5 \le x < 19.5$	6	$\frac{16.5+19.5}{2}=18$
20 – 22	$19.5 \le x < 22.5$	8	$\frac{19.5+22.5}{2}=21$
23 – 25	$22.5 \le x < 25.5$	5	$\frac{22.5+25.5}{2}=24$
26 – 28	$25.5 \le x < 28.5$	2	$\frac{25.5+28.5}{2}=27$

$$\sum f = 25$$

Frequency for class interval 17 – 19 $= 25-(4+8+5+2)$
$= 6$

b. **Required To Find:** The lower class boundary of the class interval 14 – 16
Solution:
The lower class boundary of the class interval 14 – 16 is 13.5 (as shown in the above table).

c. **Required To Find:** The width of class interval 20 – 22.
Solution:
The width of class interval 20 – 22 is
$\text{U.C.B} - \text{L.C.B.} = 22.5 - 19.5$
$= 3$

d. **Required To Find:** The class interval in which a measurement of 16.8 cm would lie.
Solution:
A measurement of 16.8 cm lies in the class interval 17 – 19.

e. **Required To Calculate:** The probability that a randomly chosen student has a right foot measuring greater than or equal to 20 cm.

Calculation:

$$P(\text{Length of student's foot} \geq 20\,\text{cm}) = \frac{\text{No. of students with foot} \geq 20\,\text{cm}}{\text{Total number of students in the class}}$$
$$= \frac{15}{25}$$
$$= \frac{3}{5}$$

f. **Required to find:** The modal length of a student's right foot.
Solution:
Modal class is 20 – 22 since this class corresponds to the greatest frequency.

$$\therefore \text{Modal length of student's foot} = \frac{\text{U.C.B} + \text{L.C.B.}}{2}$$
$$= \frac{19.5 + 22.5}{2}$$
$$= 21\,\text{cm}$$

g. **Required To Estimate:** the mean length of a student's foot using the mid-class intervals for the data given.
Solution:
Mean length of student's foot $= \bar{x}$ where

$$\bar{x} = \frac{\sum fx}{\sum f}$$

f = frequency
x = mid-class interval of class

$$\bar{x} = \frac{(4\times 15) + (6\times 18) + (8\times 21) + (5\times 24) + (2\times 27)}{25}$$
$$= 20.4\,\text{cm}$$

8. **Data:** Path of a ball follows the equation $h = 20t - 5t^2$, h = height in m and t = time in seconds.

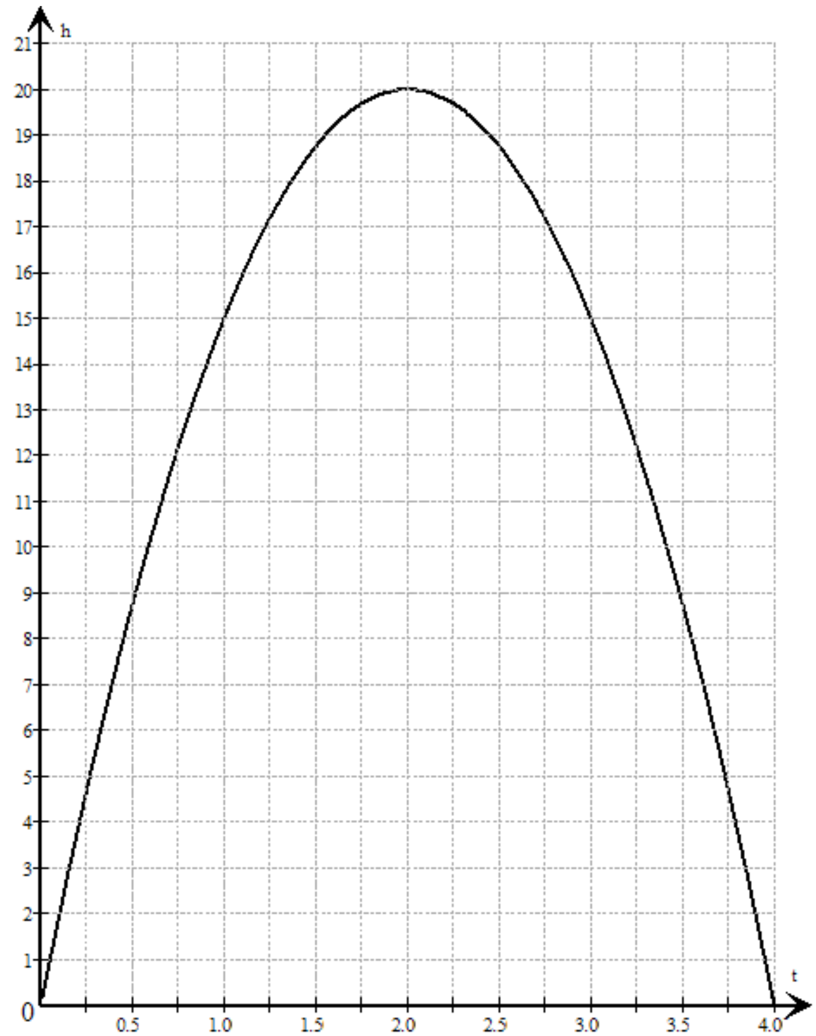
a. **Required To Complete:** the table of values of t and corresponding values of h.
Solution:

t	0	0.5	1	1.5	2	2.5	3	3.5	4
h	0.0	8.8	15	18.8	(20)	18.8	(15)	8.8	0

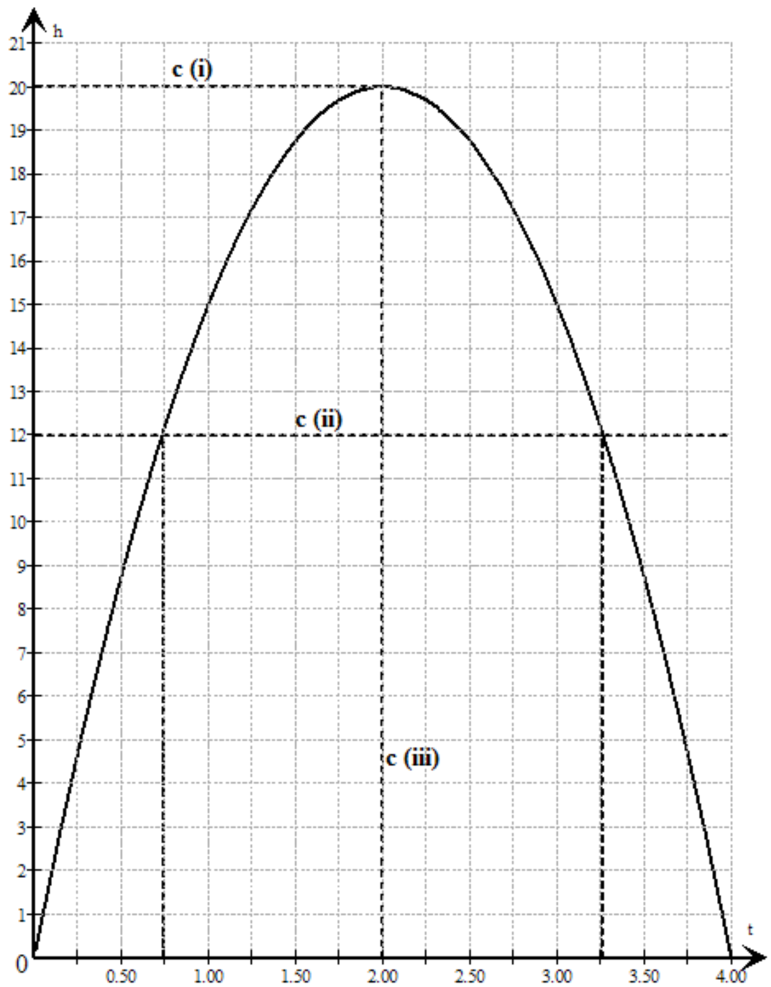
When $t = 2$ $\quad h = 20(2) - 5(2)^2$
$\quad = 20$

When $t = 3$ $\quad h = 20(3) - 5(3)^2$
$\quad = 15$

b. **Data:** Scale of 2 cm to represent 0.5 seconds on $t-$ axis for $0 \le t \le 4$.
Scale of 1 cm to represent 1 metre on the $h-$ axis for $0.0 \le h \le 21.0$.
Required To Draw: the graph of $h = 20t - 5t^2$ for $0 \le t \le 4$
Solution:



c.



(i) **Required To Find:** the greatest height above the ground reached by the ball.
Solution:
The greatest height reached by the ball is 20 m.

(ii) **Required To Find:** the duration for which the ball was more than 12 m above the ground.
Solution:
$h > 12\text{ m}$ from the ground
$t = 0.74$ to $t = 3.26$
Time $= 3.26 - 0.74$
$= 2.52$ seconds
That is, $h > 12\text{ m}$ for 2.52 seconds.

(iii) **Required To Find:** the time interval during which the ball was moving upwards.
Solution:
The ball is moving upwards from $t = 0$ to $t = 2$, that is for $2 - 0 = 2$ seconds.

Section II

9. a. **Data:** $2x + y = 7$ and $x^2 - xy = 6$
Required To Calculate: x and y
Calculation:
Let $2x + y = 7 \ldots(1)$ and $x^2 - xy = 6 \ldots(2)$
From (1)

$$2x + y = 7$$
$$y = 7 - 2x$$

Substitute in (2)

$$x^2 - x(7 - 2x) - 6 = 0$$
$$x^2 - 7x + 2x^2 - 6 = 0$$
$$3x^2 - 7x - 6 = 0$$
$$(3x + 2)(x - 3) = 0$$
$$x = -\frac{2}{3} \text{ or } 3$$

When $x = -\frac{2}{3}$ $\quad y = 7 - 2\left(-\frac{2}{3}\right)$

$$= 7 + \frac{4}{3}$$
$$= 8\frac{1}{3}$$

When $x = 3$ $\quad y = 7 - 2(3)$

$$= 1$$

Hence, $x = -\frac{2}{3}$ and $y = 8\frac{1}{3}$ or $x = 3$ and $y = 1$

b. **Required To Express:** $4x^2 - 12x - 3$ in the form $a(x + h)^2 + k$
Solution:

$$4x^2 - 12x - 3 = 4(x^2 - 3x) - 3$$

Half the coefficient of x is

$$\frac{1}{2}(-3) = -\frac{3}{2}$$

$$4x^2 - 12x - 3 = 4\left(x - \frac{3}{2}\right)^2 + ?$$

$$4\left(x - \frac{3}{2}\right)^2 = 4\left(x^2 - 3x + \frac{9}{4}\right)$$

$$and\ 4x^2 - 12x + 9$$

$$\underline{-12}$$

$$\underline{-3}\ and\ ? = -12$$

$\therefore 4x^2 - 12x - 3 \equiv 4\left(x - \frac{3}{2}\right)^2 - 12$ is of the form $a(x+h)^2 + k$ where

$$a = 4 \in \Re$$

$$h = -\frac{3}{2} \in \Re$$

$$k = -12 \in \Re$$

OR

$$4x^2 - 12 - 3 = a(x+h)^2 + k$$
$$= a\left(x^2 + 2hx + h^2\right) + k$$
$$= ax^2 + 2ahx + ah^2 + k$$

Equating coefficient of x^2

$$a = 4 \in \Re$$

Equating coefficient of x

$$-12 = 2(4)h$$

$$h = \frac{-12}{8}$$

$$= -\frac{3}{2} \in \Re$$

Equating constants

$$-3 = 4\left(-\frac{3}{2}\right)^2 + k$$

$$-3 = 9 + k$$

$$k = -12 \in \Re$$

$$\therefore 4x^2 - 12x - 3 \equiv 4\left(x - \frac{3}{2}\right)^2 - 12$$

c. (i) **Required To Find:** the minimum value of $4x^2-12x-3$

Solution:

$$4x^2-12x-3 \equiv 4\left(x-\frac{3}{2}\right)^2-12$$

$$4\left(x-\frac{3}{2}\right)^2 \geq 0 \quad \forall x$$

$\therefore$ Minimum value of $4x^2-12x-3 = 0-12$
$= -12$

(ii) **Required To Find:** the value of x at which the minimum point occurs.

Solution:

Minimum value occurs at

$$4\left(x-\frac{3}{2}\right)^2=0$$

$$\left(x-\frac{3}{2}\right)^2=0$$

$$x-\frac{3}{2}=0$$

$$x=\frac{3}{2}$$

OR

Let $y=4x^2-12x-3$.

The axis of symmetry passes through the minimum point and occurs at

$$x=\frac{-(-12)}{2(4)}$$

$$=\frac{12}{8}$$

$$=\frac{3}{2}$$

Therefore,

$$y_{\min}=4\left(\frac{3}{2}\right)^2-12\left(\frac{3}{2}\right)-3$$

$$=-12$$

Therefore, $y_{\min}=-12$ at $x=\frac{3}{2}$

(iii) **Required To Solve:** $4x^2 - 12x - 3 = 0$
Solution:
$4x^2 - 12x - 3 = 0$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(-3)}}{2(4)}$$
$$= \frac{12 \pm \sqrt{144 + 48}}{8}$$
$$= \frac{12 \pm \sqrt{192}}{8}$$
$$= 3.23\underline{2} \text{ or } -0.232\underline{0}$$
$$= 3.23 \text{ or } -0.232 \text{ to 3 significant figures.}$$

OR

$$4x^2 - 12x - 3 = 0$$
$$4\left(x - \frac{3}{2}\right)^2 - 12 = 0$$
$$\left(x - \frac{3}{2}\right)^2 = \frac{12}{4}$$
$$\left(x - \frac{3}{2}\right)^2 = 3$$
$$x - \frac{3}{2} = \pm\sqrt{3}$$
$$x = \frac{3}{2} \pm \sqrt{3}$$
$$= 1.5 \pm 1.732$$
$$= 3.23\underline{2} \text{ or } -0.232\underline{0}$$
$$= 3.23 \text{ or } -0.232 \text{ to 3 significant figures}$$

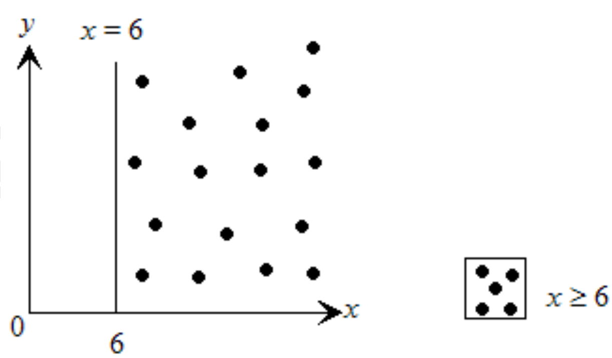
10. a. **Data:** No. of Sonix radios $= x$
No. of Zent radios $= y$
Space on shelf for up to 20 radios.

(i) **Required To Find:** The inequality to represent the data given.
Solution:
Therefore, total must be less than or equal to 20.
Hence, $x + y \le 20 \ldots(1)$

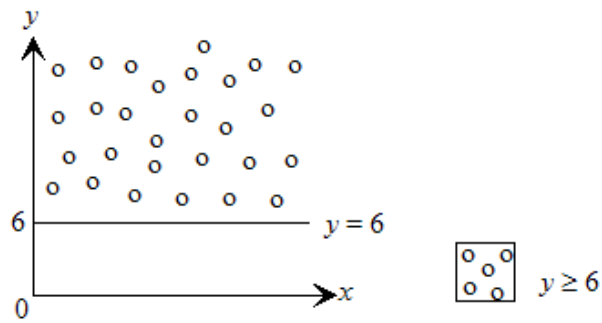
(ii) **Data:** Cost of Sonix radio = \$150
Cost of Zent radio = \$ 300
Shop's owner has \$4500
Required To Find: The inequality to represent the data given
Solution:
Cost of x Sonix radios at \$150 each and y Zent radios at \$300 each
$= x(150) + y(300)$
Total available to spent is \$4500.
$\therefore 150x + 300y \leq 4500$
$\div 150$
$$x + 2y \leq 30$$

(iii) **Data:** Owner decides to stock at least 6 Sonix and at least 6 Zent radios.
Required To Find: The two inequalities to represent the data given.
Solution:
Stock is at least 6 Sonix and at least 6 Zent radios.
$\therefore x \geq 6$ and $y \geq 6$

b. **Data:** Scale of 2 cm to represent 5 Sonix radios and 2 cm to represent 5 Zent radios.
Horizontal axis - $0 \leq x \leq 30$
Vertical axis - $0 \leq y \leq 25$
Required To: Draw the boundary lines for all four above inequalities, shade the region that satisfies all four inequalities and state the vertices of the shaded region.
Solution:
The line $x = 6$ is a straight vertical line.
The region $x \geq 6$ is represented as



The line $y = 6$ is a straight horizontal line.
The region $y \geq 6$ is represented as



Obtaining two points on the line $x + y = 20$.

When $x = 0$ $\quad 0 + y = 20$

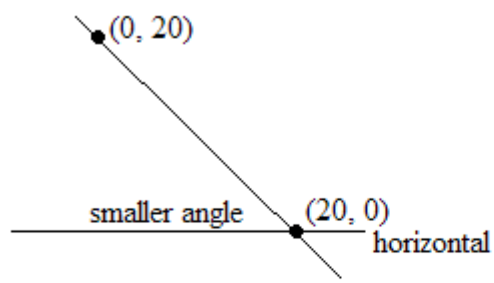
$y = 20$

The line $x + y = 20$ passes through the point (0, 20).

When $y = 0$ $\quad x + 0 = 20$

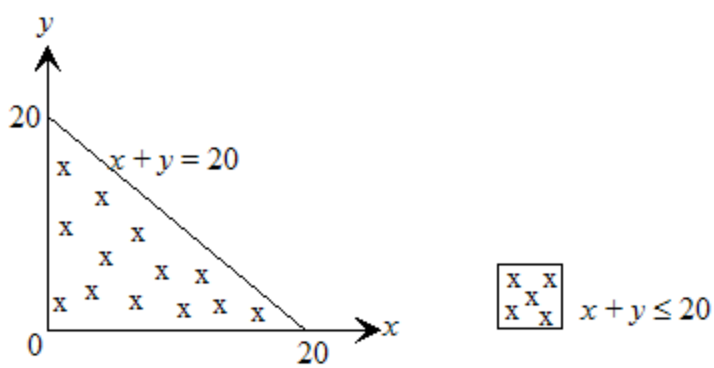
$x = 20$

The line $x + y = 20$ passes through the point (20, 0).



The region with the smaller angle satisfies the $\leq$ region.

The region $x + y \leq 20$ is



Obtaining two points on the line $x + 2y = 30$

When $x = 0$ $\quad 0 + 2y = 30$

$2y = 30$

$y = \frac{30}{2}$

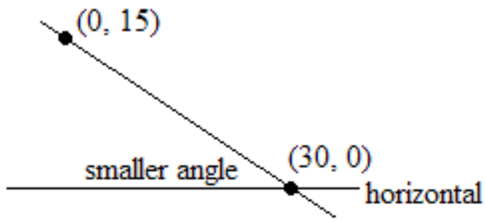
$= 15$

The line $x + 2y = 30$ passes through the point (0, 15).

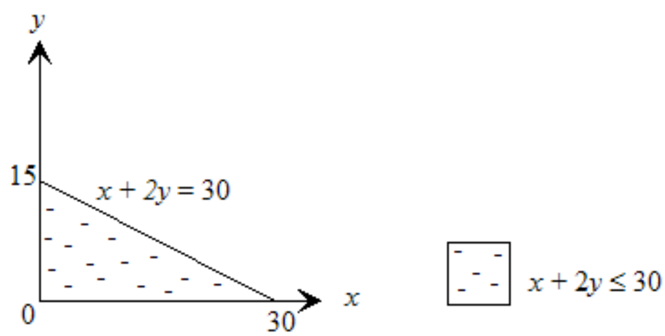
When $y = 0$ $\qquad x + 2(0) = 30$

$$x = 30$$

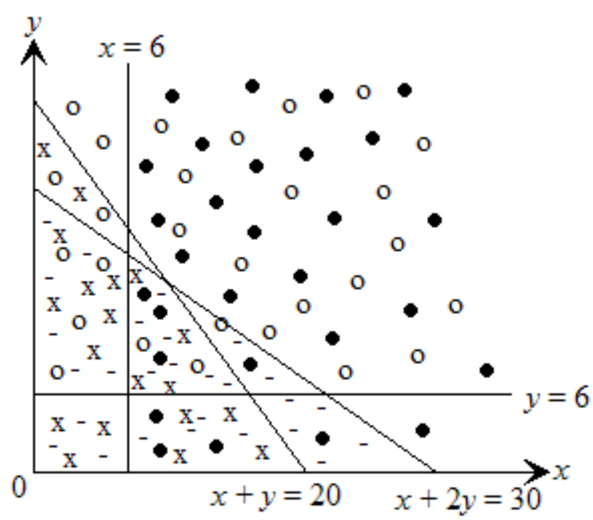
The line $x + 2y = 30$ passes through the point (30, 0).

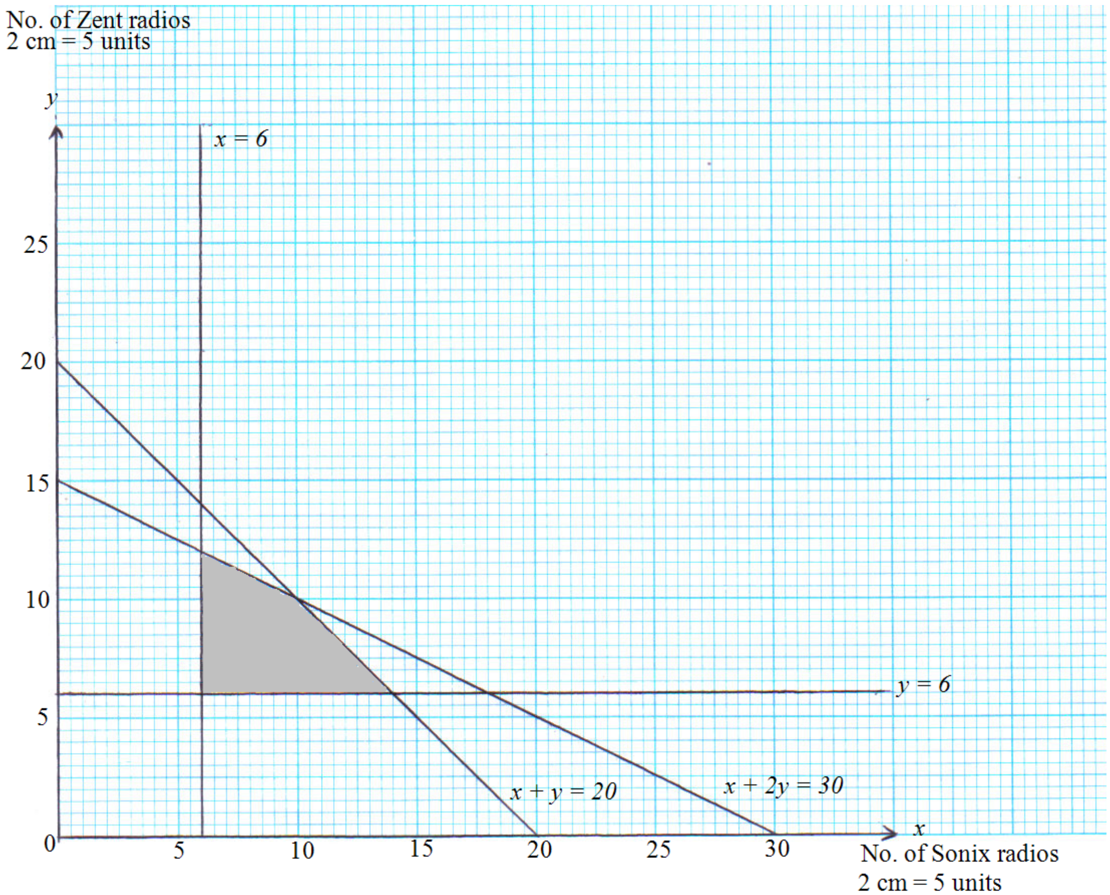


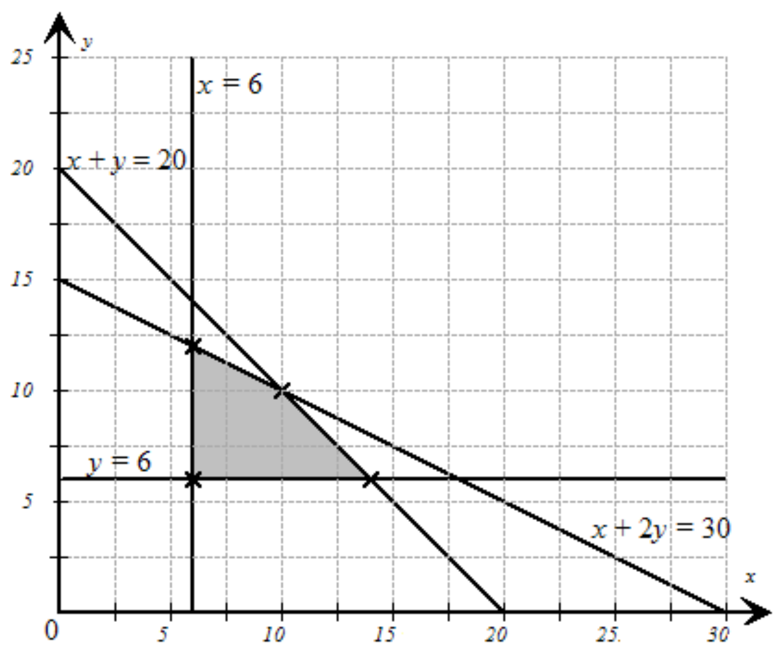
The region with the smaller angle satisfies the $\leq$ region.
The region $x + 2y \leq 30$ is



The region which satisfies all 4 inequalities is the area where all shaded regions overlap.







(iv) The vertices of the shaded region are (6, 6), (14, 6), (10, 10) and (6, 12).

c. **Data:** The owner of the shop sells to make a profit of \$80 on each Sonix radio and \$100 on each Zent radio.

(i) **Required To Express:** The total profit in terms of x and y

Solution:

Profit, P on x Sonix radios at \$80 each and y Zent radios at \$100 each is

$$P = (80 \times x) + (100 \times y)$$
$$= \$(80x + 100y)$$

(ii) **Required To Calculate:** The maximum profit

Calculation:

Test $x = 6 \quad y = 12$

$$P = 80(6) + 100(12)$$
$$= \$1680$$

Test $x = 10 \quad y = 10$

$$P = (80 \times 10) + (100 \times 10)$$
$$P = \$1800$$

Test $x = 14 \quad x = 6$

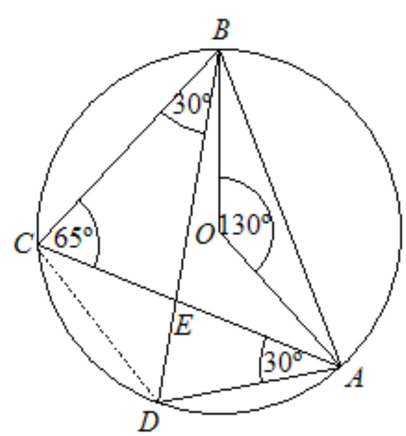
$$P = (80 \times 14) + (100 \times 6)$$
$$= \$1720$$

$\therefore$ Maximum profit = \$1800 when the owner sells 10 Sonix radios and 10 Zent radios.

11. **Data:** Circle, centre O with $A\hat{O}B = 130°$ and $D\hat{A}C = 30°$. AEC and BED are chords.

a. (i) **Required To Calculate:** $\angle ACB$

Solution:



$$A\hat{C}B = \frac{1}{2}(130°)$$
$$= 65°$$

(Angle subtended by chord (AB) at centre of circle is twice the angle that the chord subtends at the circumference, standing on the same arc.)

(ii) **Required To Find:** $\angle CBD$

Solution:

$C\hat{B}D = 30°$

(Chord CD subtends equal angles at the circumference, standing on the same arc).

(iii) **Required To Find:** $\angle AED$

Solution:

$C\hat{E}B = 180° - (60° + 35°)$

$= 85°$

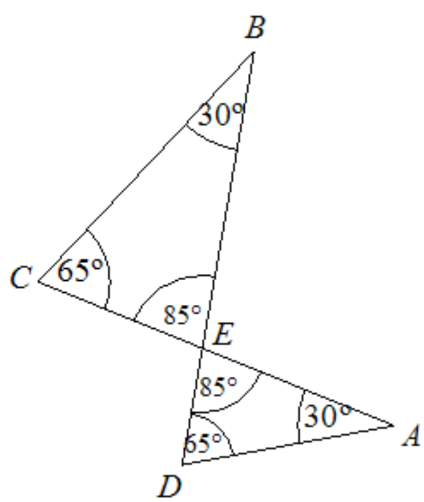
(Sum of angles in a triangle = 180°).

$A\hat{E}D = 85°$

(Vertically opposite angles).

b. **Required To Show:** ΔBCE is similar to ΔADE

Solution:



$E\hat{D}A = 65°$

(Sum of angles in a triangle = 180°).

$\hat{B} = \hat{A}$

Angle E is common

$\hat{C} = \hat{D}$

$\therefore \Delta BCE$ and ΔADE are equi-angular OR similar.

c. **Data:** $CE = 6$ cm, $EA = 9.1$ cm and $DE = 5$ cm

(i) **Required To Calculate:** length of EB.

Calculation:

If $\Delta BCE \sim \Delta ADE$, then the ratio of their corresponding sides are the same. That is,

$$\frac{BC}{AD} = \frac{CE}{DE} = \frac{BE}{AE}$$

$$\frac{6}{5} = \frac{EB}{9.1}$$

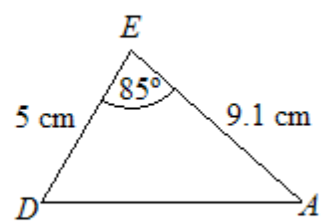
$$EB = \frac{6 \times 9.1}{5}$$

$$= \frac{54.6}{5}$$

$$= 10.92 \text{ cm}$$

(ii) **Required To Calculate:** the area of ΔAED

Calculation:



Area of $\Delta AED = \frac{1}{2}(5)(9.1)\sin 85°$

$= 22.6\underline{6} \text{ cm}^2$

$= 22.7 \text{ cm}^2$ to 1 decimal place

12. This question is not done since it involves latitude and longitude (Earth Geometry) which has been removed from the syllabus.

13. **Data:** A (1, 2), B (5, 2), C (6, 4) and D (2, 4) are the vertices of a quadrilateral $ABCD$.

a. (i)**Required To Express:** The position vectors of $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution:

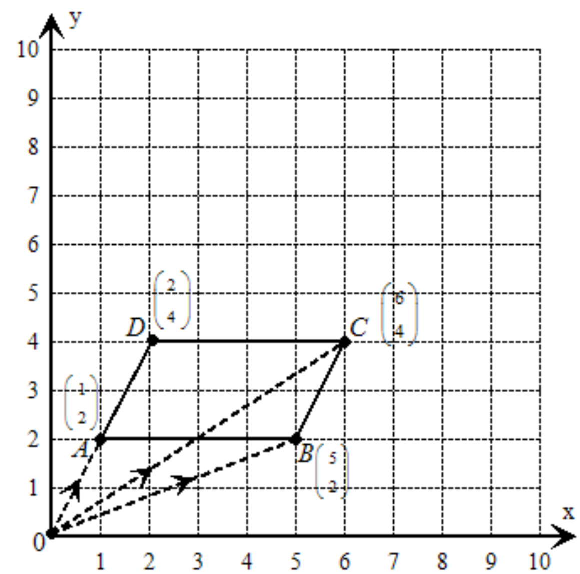
If $A = (1,\ 2)$ then $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ where $x = 1$ and $y = 2$.

If $B=(5,\ 2)$ then $\overrightarrow{OB}=\begin{pmatrix}5\\2\end{pmatrix}$ is of the form $\begin{pmatrix}x\\y\end{pmatrix}$ where $x=5$ and $y=2$.

If $C=(6,\ 4)$ then $\overrightarrow{OC}=\begin{pmatrix}6\\4\end{pmatrix}$ is of the form $\begin{pmatrix}x\\y\end{pmatrix}$ where $x=6$ and $y=4$.

If $D=(2,\ 4)$ then $\overrightarrow{OD}=\begin{pmatrix}2\\4\end{pmatrix}$ is of the form $\begin{pmatrix}x\\y\end{pmatrix}$ where $x=2$ and $y=4$.

(ii) **Required To Find:** $\overrightarrow{AB}$ and $\overrightarrow{DC}$
Solution:



$\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}$

$=-\begin{pmatrix}1\\2\end{pmatrix}+\begin{pmatrix}5\\2\end{pmatrix}$

$=\begin{pmatrix}4\\0\end{pmatrix}$

$\overrightarrow{DC}=\overrightarrow{DO}+\overrightarrow{OC}$

$=-\begin{pmatrix}2\\4\end{pmatrix}+\begin{pmatrix}6\\4\end{pmatrix}$

$=\begin{pmatrix}4\\0\end{pmatrix}$

b. **Required To Find:** $\left|\overrightarrow{AB}\right|$ and the unit vector in the direction of $\overrightarrow{AB}$

Solution:

$\left|\overrightarrow{AB}\right| = \sqrt{(4)^2 + (0)^2}$

$= 4$ units

Any vector in the direction of $\overrightarrow{AB} = \alpha\begin{pmatrix}4\\0\end{pmatrix}$, where α is a scalar.

If the vector is unit then the magnitude is 1 unit.

$\therefore \begin{vmatrix}4\alpha\\0\end{vmatrix} = 1$

$\sqrt{(4\alpha)^2 + (0)^2} = 1$

$\alpha = \frac{1}{4}$

The unit vector in the direction of $\overrightarrow{AB} = \frac{1}{4}\begin{pmatrix}4\\0\end{pmatrix}$

$= \begin{pmatrix}1\\0\end{pmatrix}.$

c. (i)**Required To Find:** Two geometrical relationships between the line segments *AB* and *DC*.

Solution:

$\overrightarrow{AB} = \begin{pmatrix}4\\0\end{pmatrix}$

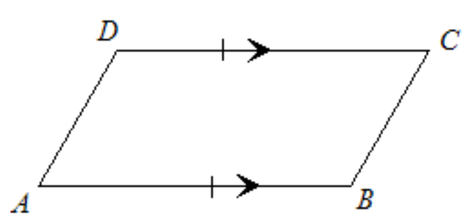
$\overrightarrow{DC} = \begin{pmatrix}4\\0\end{pmatrix}$

$= 1\begin{pmatrix}4\\0\end{pmatrix}$

Hence, $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{DC}\right|$ and $\overrightarrow{AB}$ is parallel to $\overrightarrow{DC}$.

(ii) **Required To Explain:** Why *ABCD* is a parallelogram.

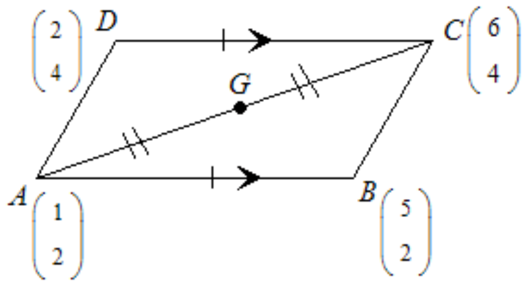
Solution:



If one pair of opposite sides of a quadrilateral are both parallel and equal then the quadrilateral is a parallelogram.

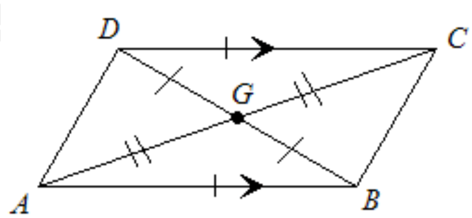
Hence, *ABCD* is a parallelogram.

c. **Required To Find**: The position vector of G, the midpoint of line AC and the coordinates of the point of intersection of the diagonals AC and BD.
Solution:



$$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$$
$$= -\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$
$$= \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$
$$\overrightarrow{AG} = \frac{1}{2}\begin{pmatrix} 5 \\ 2 \end{pmatrix}$$
$$\overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{AG}$$
$$= \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2\frac{1}{2} \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 3\frac{1}{2} \\ 3 \end{pmatrix}$$

Position vector of G is $\overrightarrow{OG} = \begin{pmatrix} 3\frac{1}{2} \\ 3 \end{pmatrix}$



The diagonals of a parallelogram bisect each other.
AC and *BD* intersect at *G*.

$\therefore$ Intersection of *AC* and *BD* is $G = \left(3\frac{1}{2},\ 3\right)$.

14. a. **Data:** $L = \begin{pmatrix} x & 4 \\ 1 & x \end{pmatrix}$

(i) **Required To Calculate:** The determinant of L.
Calculation:

$$\text{Det } L = (x \times x) - (4 \times 1)$$
$$= x^2 - 4$$

(ii) **Required To Calculate:** The values of x given that L is singular.
Calculation:
If L is singular, then det $L = 0$

When $x^2 - 4 = 0$
$$x^2 = 4$$
$$x = \pm 2$$

b. **Data:** $M = \begin{pmatrix} 3 & 1 \\ 2 & 6 \end{pmatrix}$

(i) **Required To Calculate:** M^{-1}
Calculation:

$$\det M = (3 \times 6) - (1 \times 2)$$
$$= 18 - 2$$
$$= 16$$

$$M^{-1} = \frac{1}{16}\begin{pmatrix} 6 & -(1) \\ -(2) & 3 \end{pmatrix}$$
$$= \begin{pmatrix} \frac{6}{16} & -\frac{1}{16} \\ -\frac{2}{16} & \frac{3}{16} \end{pmatrix}$$

(ii) **Data:** $M\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ -8 \end{pmatrix}$

Required To Calculate: The value of x and of y
Calculation:

$$M\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ -8 \end{pmatrix}$$

$\times M^{-1}$

$$MM^{-1}\begin{pmatrix} x \\ y \end{pmatrix} = M^{-1}\begin{pmatrix} 12 \\ -8 \end{pmatrix}$$

$$I\begin{pmatrix} x \\ y \end{pmatrix} = M^{-1}\begin{pmatrix} 12 \\ -8 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{6}{16} & -\frac{1}{16} \\ -\frac{2}{16} & \frac{3}{16} \end{pmatrix}\begin{pmatrix} 12 \\ -8 \end{pmatrix}$$

$$= \begin{pmatrix} \left(\frac{6}{16} \times 12\right) + \left(-\frac{1}{16} \times -8\right) \\ \left(-\frac{2}{16} \times 12\right) + \left(\frac{3}{16} \times -8\right) \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Equating corresponding entries.
$x = 5$ and $y = -3$

c. **Data:** $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

(i) **Required To Calculate:** $(x',\ y')$, the image of (3, -1) under N.
Calculation:

When $(x,\ y) = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} (2 \times 3) + (0 \times -1) \\ (0 \times 3) + (2 \times -1) \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -2 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 11 \\ -4 \end{pmatrix}$$

$\therefore$ The image of (3, -1) is (11, -4).

(ii) **Required To Calculate:** (x, y) when (x', y') is (7, 4)

Calculation:

$$\begin{pmatrix} 7 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 4 \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 4 \end{pmatrix} = \begin{pmatrix} 2x+5 \\ 2y-2 \end{pmatrix}$$

Equation corresponding entries

$7 = 2x + 5$

$2x = 2$

$x = 1$

and

$4 = 2y - 2$

$6 = 2y$

$y = 3$

$\therefore$ The point which is mapped onto (7, 4) is (1, 3).