JANUARY 2005 CXC MATHEMATICS GENERAL PROFICIENY (PAPER 2)

Section I

1. a. **Required To Calculate:** $\sqrt{\dfrac{13.2}{0.33}}$ to 3 decimal places.

Calculation:

$\sqrt{\dfrac{13.2}{0.33}} = \sqrt{40}$ (using a calculator)

$= 6.324\underline{5}$

$= 6.325$ to 3 decimal places (as required).

b. **Data:** Table illustrating telephone rates.
Required To Prove: The cost of using the land line is less than the cost for using the cellular phone.
Proof:

(i) Duration of calls in the month is 1 hour 5 minutes = (60 + 5)
= 65 minutes

Cost of using the land line phone = Rental fee + Charges per min.

$= \$45.00 + (65 \times 0.15)$

$= \$54.75$

Cost using the cellular phone $= \$0.85 \times 65$

$= \$55.25$

Hence the cost of using the land line telephone is less (\$54.75) than using the cellular phone (\$55.25).

Required To Calculate: Duration of the calls for the month of March

(ii) In March, the cost of using the land line phone accounted for a bill of \$54.60.

Cost of only the calls = \$54.60 – Rental fee
= \$54.60 – \$45.00
= \$9.60

$\therefore$ Duration of the land line calls $= \dfrac{9.60}{0.15} = 64$ minutes.

2. a. **Data:** $r = \dfrac{2p^2}{q-3}$

Required To Calculate: The value of r when $p = 6$ and $q = 12$

Solution:

(i) When $p = 6$ and $q = 12$,

$$r = \frac{2(6)^2}{12-3} = \frac{2(36)}{9} = \frac{72}{9} = 8$$

Required To : Make q the subject

(ii) $r = \dfrac{2p^2}{q-3}$

$$\frac{r}{1} = \frac{2p^2}{(q-3)}$$

$$r(q-3) = 2p^2 \times 1$$

$$rq - 3r = 2p^2$$

$$rq = 2p^2 + 3r$$

$$q = \frac{2p^2 + 3r}{r} \qquad \left(= \frac{2p^2}{r} + \frac{3r}{r} \right)$$

OR $\dfrac{2p^2}{r} + 3$

b. **Required To Factorise Completely:**

(i) $3g - 3t + 2mg - 2mt$

(ii) $3x^2 + 2x - 8$

(iii) $3x^2 - 27$

Solution:

(i) $3g - 3t + 2mg - 2mt$

$= 3(g-t) + 2m(g-t)$

$= (g-t)(3+2m)$

(ii) $3x^2 + 2x - 8$

$(3x-4)(x+2)$

(iii) $3x^2 - 27$

$= 3(x^2 - 9)$

$= 3\{(x)^2 - (3)^2\}$

This is a difference of 2 squares and

$= 3(x-3)(x+3)$

c. **Data:** Table of values of *x* and *y* and that *y* varies inversely as *x*.
Required To Calculate: The value of *a*
Solution:

$y \propto \frac{1}{x}$

$\therefore y = k \times \left(\frac{1}{x}\right)$ (*k* = the constant of proportionality)

And $y = \frac{k}{x}$

From the table $x = 2$ when $y = 8$

$\therefore 8 = \frac{k}{2}$

$\therefore k = 16$

and $y = \frac{16}{x}$

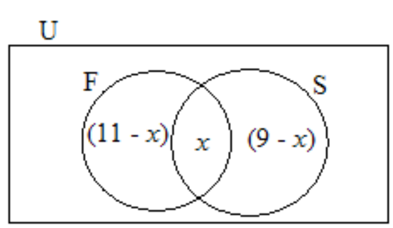
When $x = 32$

$y = \frac{16}{32}$

$y = \frac{1}{2}$

$\therefore a = \frac{1}{2}$

3. a. **Data:** Information to complete the given sketch of a Venn diagram.
Required To Complete: The Venn diagram to represent the information given
Solution:
(i)



(ii) **Required To Find:** an equation in *x* for the number of candidates in U.
Solution:

$n(U) = (11 - x) + x + (9 - x) + 18 = 32$

$\therefore 38 - x = 32$

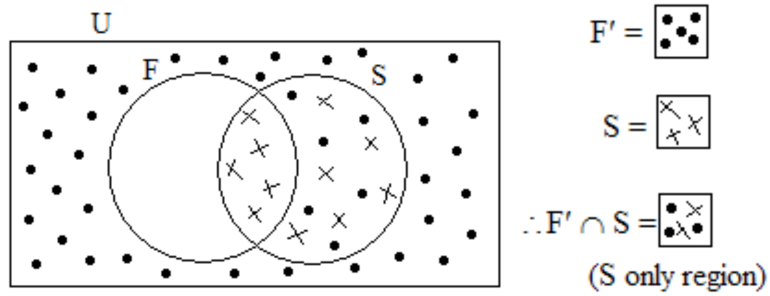
(iii) **Required to Calculate:** the value of x.
Solution:

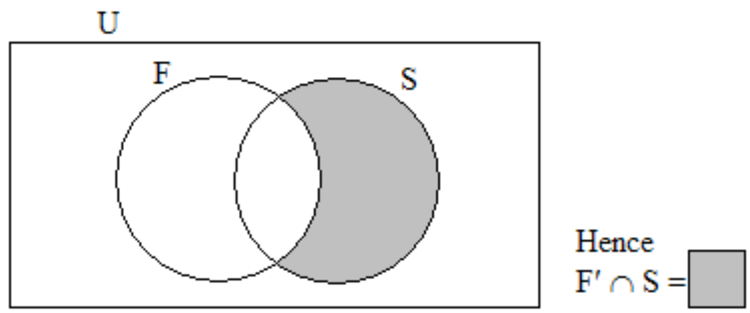
$38 - x = 32$

$\therefore 38 - 32 = x$

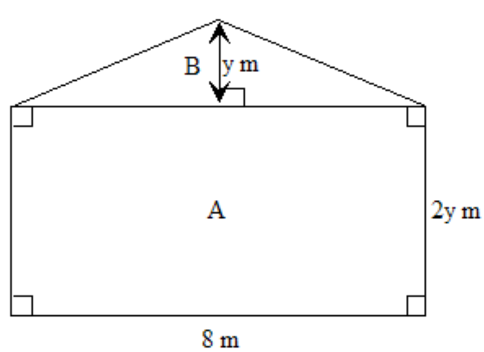
$x = 6$

(iv) **Required:** To shade the region $F' \cap S$
Solution:





b. **Data:** Diagram showing the cross-section of a shed.



(i) **Required To Find:** an expression in terms of y for the area of the figure.
Solution:
The cross section is divided into 2 regions, A and B, as shown on the diagram.

Area of rectangle A $= 8 \times 2y$

$= 16y \quad \text{m}^2$

Area of triangle B $= \dfrac{8 \times y}{2}$

$= 4y \, \text{m}^2$

$\therefore$ Cross sectional area of the figure $= 16y + 4y$

$= 20y \quad \text{m}^2.$

(ii) **Required To Calculate:** the value of y

Solution:

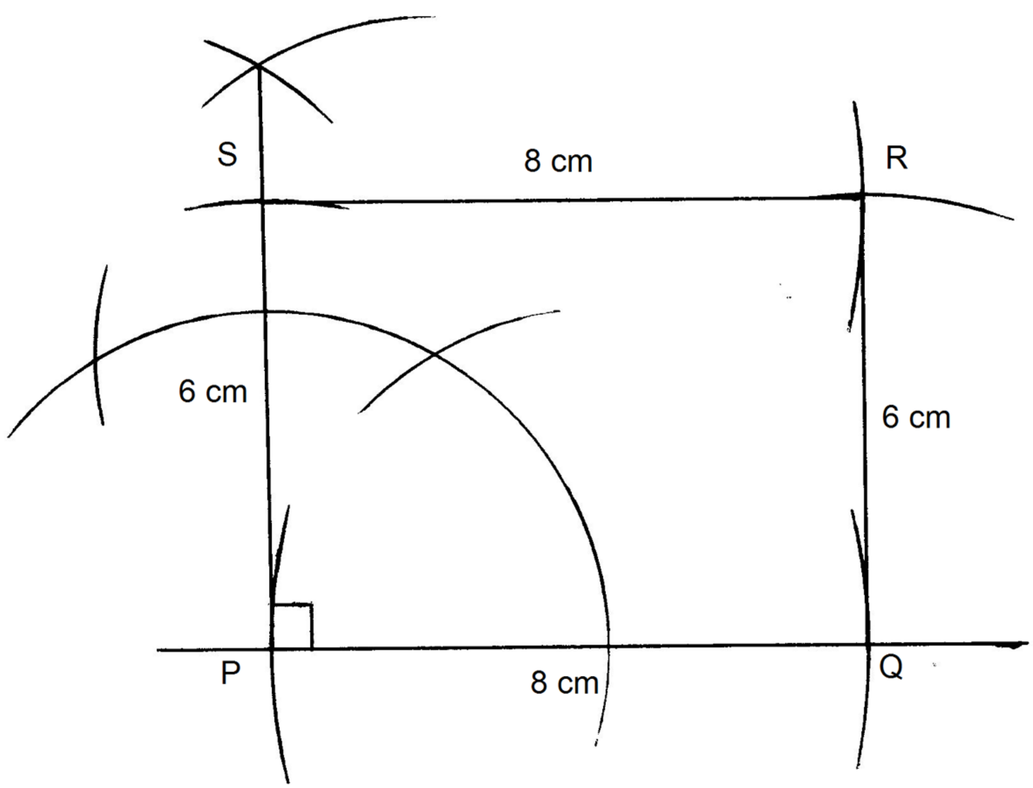
Total area $= 28 \, \text{m}^2$

$\therefore 20y = 28$

$\therefore y = 1\dfrac{8}{20}$

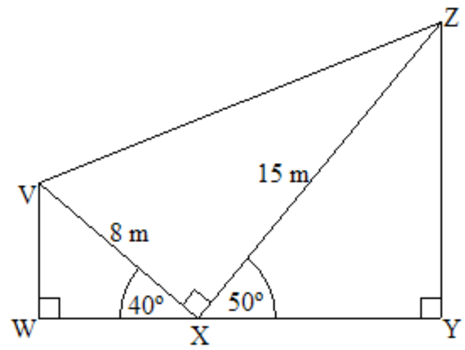
$y = 1\dfrac{2}{5}$

4. a. Constructing a rectangle $PQRS$ with $PQ = 8$ cm and $PS = 6$ cm



Diagonal $PR = 10.0$ cm (by measurement)

b. **Data:** Diagram with Y due east of W and V north of W.



(i) **Required to Calculate:** $Z\hat{X}V$
Solution:

$$Z\hat{X}V = 180° - (40° + 50°) \quad (\angle \text{ in a straight line})$$
$$= 90°$$

(ii) **Required To Calculate:** $Z\hat{V}X$
Solution:

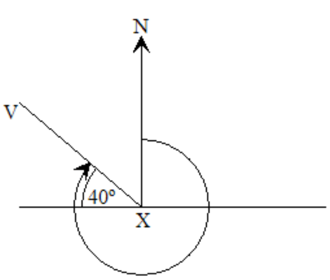
$$\tan Z\hat{V}X = \frac{15}{8}$$
$$\therefore Z\hat{V}X = \tan^{-1}\left(\frac{15}{8}\right)$$
$$= 61.9° \quad \text{(to the nearest } 0.1^0\text{)}$$

(iii) **Required To Calculate:** The length VZ
Solution:

$$VZ^2 = (8)^2 + (15)^2 \quad \text{(Pythagoras' Theorem)}$$
$$\therefore VZ = \sqrt{64 + 225}$$
$$= \sqrt{289}$$
$$= 17 \text{ m}$$

(iii) **Required To Calculate:** the bearing from V to X
Solution:

The bearing of V from $X = 270° + 40°$
$= 310°$



5. a. **Data:** $f(x)=\frac{2x+5}{x-4}$ and $g(x)=2x-3$

(i) **Required To Calculate:** The value of $g(4)$

Solution:

$$g(4)=2(4)-3$$
$$=8-3$$
$$=5$$

(ii) **Required To Calculate:** The value of $fg(2)$

Solution:

$$g(2)=2(2)-3$$
$$=4-3$$
$$=1$$
$$\therefore fg(2)=f(1)$$
$$=\frac{2(1)+5}{1-4}$$
$$=\frac{7}{-3}$$
$$=-\frac{7}{3}$$

(iii) **Required To Calculate:** $g^{-1}(7)$

Solution:

$$g(x)=2x-3$$

Let $y=2x-3$

$$y+3=2x$$
$$x=\frac{y+3}{2}$$

Replace y by x

and $g^{-1}(x)=\frac{x+3}{2}$

and $g^{-1}(7)=\frac{7+3}{2}$

$$=\frac{10}{2}$$
$$=5$$

b. **Required To Express:** $\frac{3}{x}+\frac{4}{x+1}$ as a single fraction in its simplest form

Solution:

$$\frac{3}{x}+\frac{4}{x+1}$$

$$\frac{3(x+1)+4(x)}{x(x+1)}=\frac{3x+3+4x}{x(x+1)}$$

$$=\frac{7x+3}{x(x+1)} \text{ as a single fraction}$$

c. **Required To Find:** The value of $9^{\frac{1}{2}} \times 8^{\frac{2}{3}} \times 4^0$

Solution:

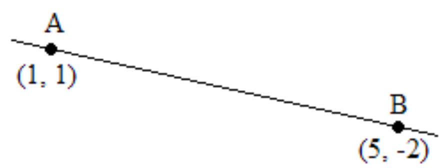
$$9^{\frac{1}{2}} \times 8^{\frac{2}{3}} \times 4^0 = \sqrt{9} \times \sqrt[3]{8^2} \times 1$$

$$= 3 \times \sqrt[3]{64} \times 1$$

$$= 3 \times 4 \times 1$$

$$= 12$$

6. a. **Data:** Straight line drawn through *A* (1, 1) and *B* (5, -2).



(i) **Required To Calculate:** The gradient of the line *AB*

Solution:

$$\text{Gradient of } AB = \frac{1-(-2)}{1-5}$$

$$= \frac{3}{-4}$$

$$= -\frac{3}{4}$$

(ii) **Required To Find:** The gradient of the line perpendicular to *AB*

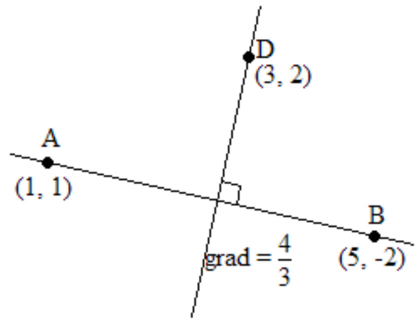
Solution:

Gradient of ANY line perpendicular to *AB* $= \frac{4}{3}$

(Product of the gradients of perpendicular lines = - 1)

(iii) **Required To Find:** the equation of the line passing through *D* (3, 2) which is perpendicular to *AB*.

Solution:



Equation of the line through D and perpendicular to AB is

$$\frac{y-2}{x-3}=\frac{4}{3}$$

$$y-2=\frac{4}{3}x-4$$

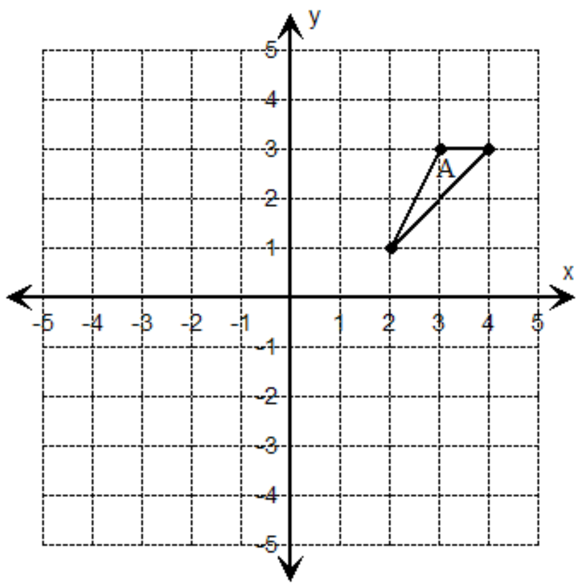
$$y=\frac{4}{3}x-2$$

is of the form $y=mx+c$, where $m=\frac{4}{3}$ and $c=-2$.

b. **Data:** Coordinates of the 3 vertices of a triangle named A.

(i) **Required To Draw:** Triangle A with coordinates (2, 1), (3, 3) and (4, 3).

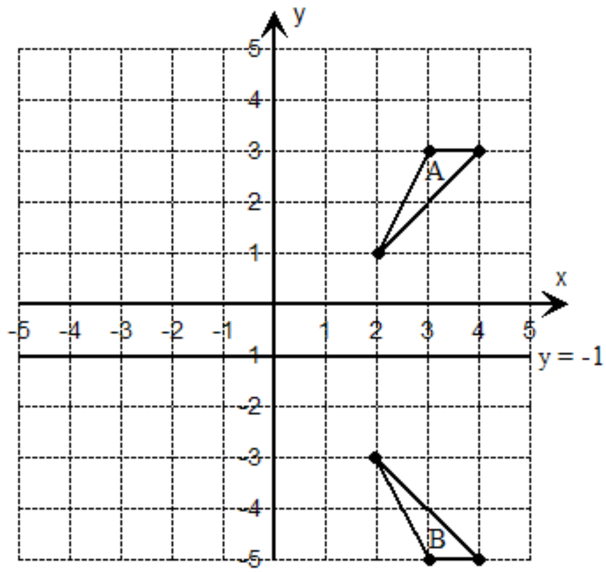
Solution:



(ii) **Required To Draw:** Triangle B, which is the reflection of Triangle A in the line $y=-1$.

Solution:

Coordinates of the vertices of B are (2, -3), (3, -5) and (4, -5)



(iii) **Required To Draw:** Triangle C which is the translation of Triangle A by the vector $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$.

Solution:

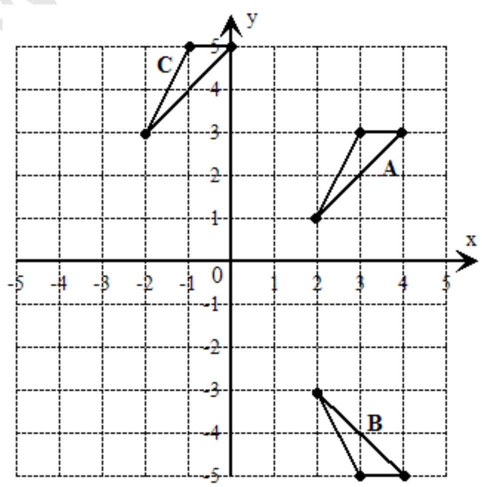
$$A \xrightarrow{T=\begin{pmatrix} -4 \\ 2 \end{pmatrix}} C$$

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \xrightarrow{\begin{pmatrix} -4 \\ 2 \end{pmatrix}} \begin{pmatrix} 2-4 \\ 1+2 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 3 \end{pmatrix} \xrightarrow{\begin{pmatrix} -4 \\ 2 \end{pmatrix}} \begin{pmatrix} 4-4 \\ 3+2 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$$

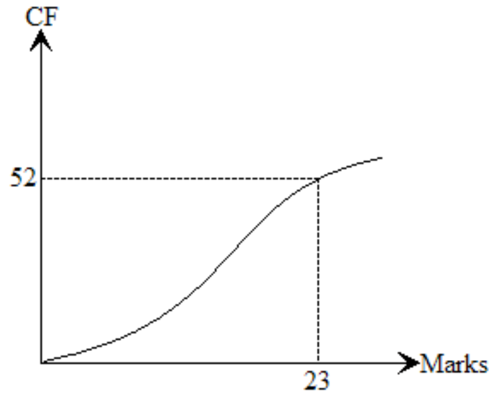
$$\begin{pmatrix} 3 \\ 3 \end{pmatrix} \xrightarrow{\begin{pmatrix} -4 \\ 2 \end{pmatrix}} \begin{pmatrix} 3-4 \\ 3+2 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$$

$\therefore$ Coordinates of the vertices of C are (-2, 3), (-1, 5) and (0, 5)



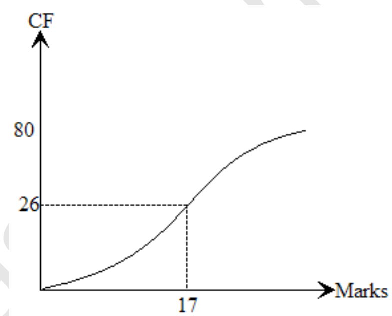
7. **Data:** Diagram of a cumulative frequency curve for the marks on a test by 80 students.

(i) **Required to Find:** The number of students who scored less than 23 marks
Solution:



The vertical at 23 corresponds to the horizontal at 52. Hence 52 candidates scored less than 23 marks.

(ii) **Required To Find:** the number of students who scored more than 17 marks.
Solution:

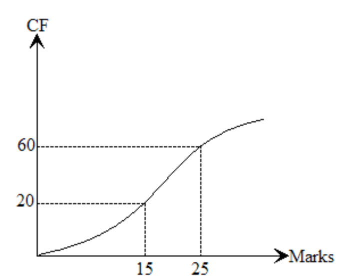


The vertical at 17 corresponds to the horizontal at 26.
$\therefore$ Number of candidates who scored more than 17 marks = 80 – 26 = 54.

(iii) **Required To Find:** the inter-quartile range of the marks scored.
Solution:

$\frac{1}{4}$ of 80 = 20

$\frac{3}{4}$ of 80 = 60



The cumulative frequency value of 20 corresponds to a mark of 15 (lower quartile, Q_1).

The cumulative frequency value of 60 corresponds to a mark of 25 (upper Quartile, Q_3).

$\therefore$ Interquartile range = $Q_3 - Q_1$

(Upper quartile – Lower quartile)

= 25 – 15

= 10

(iv) **Required To Calculate:** the probability that a randomly chosen student scored between 17 and 23 marks.

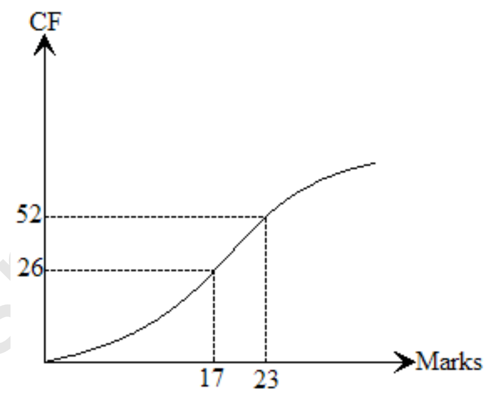
Solution:

$$P(\text{student scored between 17 and 23}) = \frac{\text{No. of students scoring between 17 and 23}}{\text{Total no. of students}}$$

$$= \frac{52-26}{80}$$

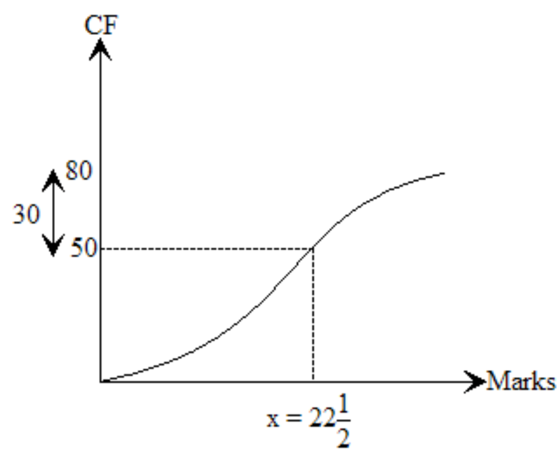
$$= \frac{26}{80}$$

$$= \frac{13}{40}$$



(v) **Required to Find:** the value of x if 30 students scored more than x marks

Solution:



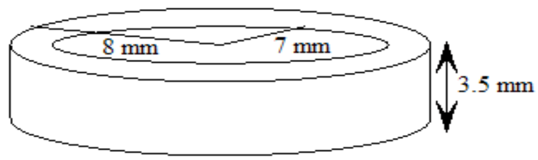
If 30 students scored more than x, then x is the horizontal value that

corresponds to a vertical (cumulative frequency) value of
$(80 - 50) = 30$

$$x \approx 22\frac{1}{2}$$

8. **Data:** Diagrams showing the link from a chain.

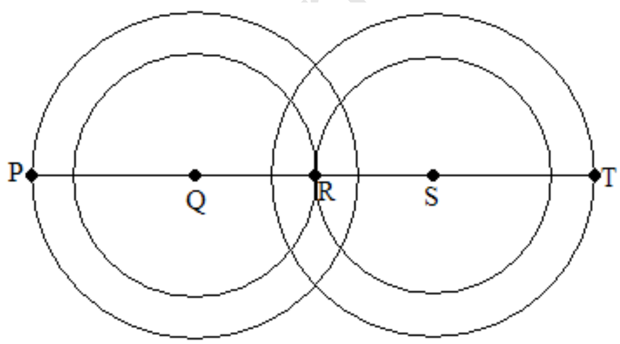
a. **Required To Calculate:** the volume of a single link of chain
Solution:



Volume of a single link = External volume – Internal volume

$= 164.\underline{9}\,\text{mm}^3$
$= 165\ \text{mm}^3$ to 3 significant figures

b. **Required To Prove:** the length of chain is 16 mm + 14 mm.
Solution:



Length of link = *PQRST (as shown on the diagram)*
$QR = RS = 7\text{ mm}$ (radius of inner circle of link)
$PQ = ST = 8\text{ mm}$ (radius of outer circle of link)
$\therefore$ Length of link $= 2(8) + 2(7)\text{mm}$
$= 16\text{ mm} + 14\text{ mm}$

c. **Required To Complete:** a table showing the length of the chain formed when rings are linked in a straight line.
Solution:

No. of rings, *n*	Length of chain, *L*
1	16
2	30
3	44

Trying to obtain a pattern or sequence between L and n.

$n =$	1	2,	3,…
$L =$	16	30,	44,…
$L =$	16+ 14(0)	16 + 14(1),	16 + 14(2),…

$\therefore$ When $n = 6$ $\quad L = 16 + 14(6 - 1) = 86$

When $L = 170$ $\quad 170 = 16 + 14(n - 1)$

$$14(n-1) = 154$$

$$n - 1 = \frac{154}{14}$$

$$n = 1 + 11$$

$$n = 12$$

No. of rings, n	Length of chain, L
1	16
2	30
3	44
⋮	⋮
6	(86)
⋮	⋮
(12)	170

Section II

9. a. **Data:** $x^2 = 4 - y$

$x = y + 2$

Required to Calculate: x and y

Solution:

Let $x^2 = 4 - y$ …(1) and $x = y + 2$…(2)

$\therefore y = 4 - x^2$

Substitute in (2)

$x = (4 - x^2) + 2$

$x - (4 - x^2) - 2 = 0$

$x^2 + x - 4 - 2 = 0$

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

And $x = -3$ or 2

When $x = -3$ $\quad y = 4 - (-3)^2 = 4 - 9 = 5$

When $x = 2$ $\quad y = 4 - (2)^2 = 4 - 4 = 0$

Hence, $x = -3$ and $y = -5$ **OR** $x = 2$ and $y = 0$

b. **Required to Prove:** $(2x-3)(2x+3)-(x-4)^2 \equiv 3x^2+8x-25$

Proof: L.H.S

$$(2x-3)(2x+3)-(x-4)^2$$
$$=(4x^2-6x+6x-9)-(x^2-4x-4x+16)$$
$$=4x^2-9-x^2+8x-16$$
$$=3x^2+8x-25$$
$$=\text{RHS}$$

Q.E.D

c.

(i) **Required to Express:** $3x^2+8x-25$ in the form $a(x+h)^2+k$

Solution:

$$3x^2+8x-25$$

$$3\left(x^2+\frac{8}{3}x\right)-25$$

$$3\left(x+\frac{4}{3}\right)^2+?$$

$$=3\left(x^2+\frac{8}{3}x+\frac{16}{9}\right)+?$$

$$=\ 3x^2+8x+5\frac{1}{3}+$$

$$\underline{-30\frac{1}{3}}$$

$$\underline{-25}$$

$$=3\left(x+\frac{4}{3}\right)^2-30\frac{1}{3}$$

is of the form $a(x+h)^2+k$

where $a=3\in\Re$

$h=\frac{4}{3}\in\Re$

and $k=-30\frac{1}{3}\in\Re$

(ii) **Required To Find:** the minimum value of $3x^2+8x-25$

Solution:

$$3x^2+8x-25\equiv 3\left(x+\frac{4}{3}\right)^2-30\frac{1}{3}$$

$$3\left(x+\frac{4}{3}\right)^2 \geq 0 \quad \forall x$$

$\therefore$ Minimum value of the function is $-30\frac{1}{3}$.

ALTERNATIVE METHOD

For a quadratic ax^2+bx+c, a maximum or minimum value of the function occurs at $x=\frac{-b}{2a}$.

The minimum value of the function occurs at $x=\frac{-(8)}{2(3)}=-\frac{4}{3}$

When $x=-\frac{4}{3}$, the minimum value is $3\left(-\frac{4}{3}\right)^2+8\left(-\frac{4}{3}\right)-25=-30\frac{1}{3}$

d. $3x^2+8x-25=0$

$$x=\frac{-8\pm\sqrt{(8)^2-4(3)(-25)}}{2(3)}$$

$$=\frac{-8\pm\sqrt{64+300}}{6}$$

$$=\frac{-8\pm\sqrt{364}}{6}$$

$$=1.8\underline{4} \text{ or } -4.5\underline{1}$$

$x=1.8$ or -4.5 to 1 decimal place

OR

$$3x^2+8x-25=0$$

$$\therefore 3\left(x+\frac{4}{3}\right)^2-30\frac{1}{3}=0$$

$$3\left(x+\frac{4}{3}\right)^2=30\frac{1}{3}$$

$$\left(x+\frac{4}{3}\right)^2=\frac{91}{9}$$

$$x+\frac{4}{3}=\pm\sqrt{\frac{91}{9}}$$

$$x=-\frac{4}{3}\pm\frac{\sqrt{91}}{3}$$

$$x=\frac{-4\pm\sqrt{91}}{3}$$

$$=1.8\underline{4} \text{ or } -4.5\underline{1}$$

$$x=1.8 \text{ or } -4.5 \text{ to 1 decimal place}$$

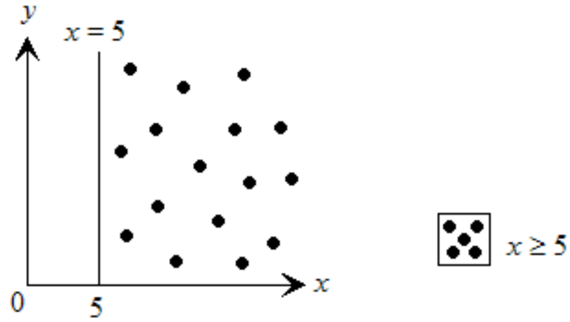
10. **Data:** Information about the numbers, prices and special conditions involving calculators and folders bought for a school.

(i) **Required To Find:** An inequality to represent the information given.
Solution:
No. of calculators bought = x
No. of calculators bought is at least 5
$\therefore x \geq 5 \ldots(1)$

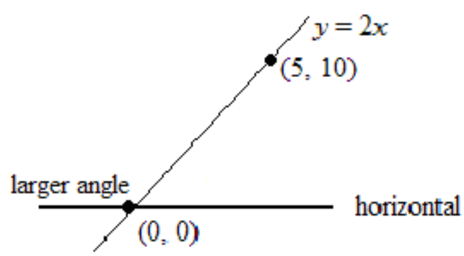
(ii) **Required To Find:** An inequality to represent the information given.
Solution:
The number of folders must be at least twice the number of calculators.
$y \qquad \geq \qquad 2\times x$
and $y \geq 2x \ldots(2)$

(iii) **Required To Find:** An inequality to represent the information given.
Solution:
Cost of x calculators at \$20 each and y folders at \$5 each
$=(20\times x)+(5\times y)=20x+5y$
Amount available for spending is not more than \$300
$\therefore 20x+5y \leq 300$
$\div 5$
$4x+y \leq 60 \ldots(3)$

(iii) **Required To Draw:** the lines of the three equalities and hence shade the region that satisfies all three inequalities, stated above.
Solution:
The line $x=5$ is a vertical straight line.
The region which satisfies $x \geq 5$ is

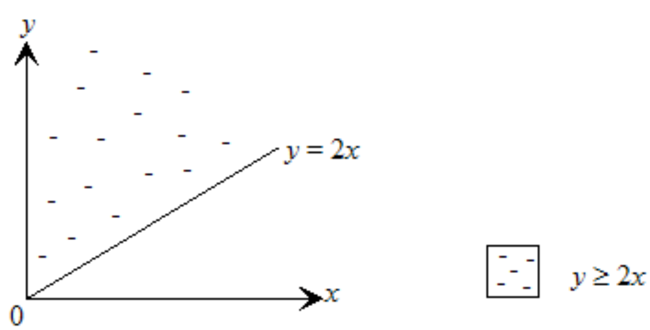


Finding two points on the line $y = 2x$ so as to draw the line.
The line $y = 2x$ passes through the origin (0, 0).
When $x = 5$ $\qquad y = 2(5)$
$\qquad\qquad\qquad = 10$
The line $y = 2x$ passes through (5, 10).



The side that makes the larger angle satisfies the ≥ region.

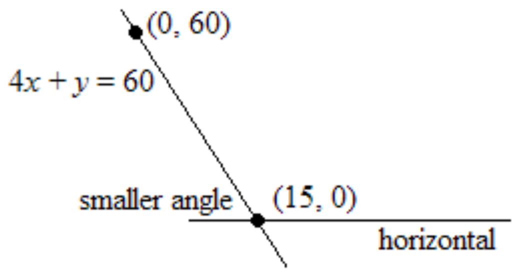
Therefore, the region which satisfies $y \geq 2x$ is



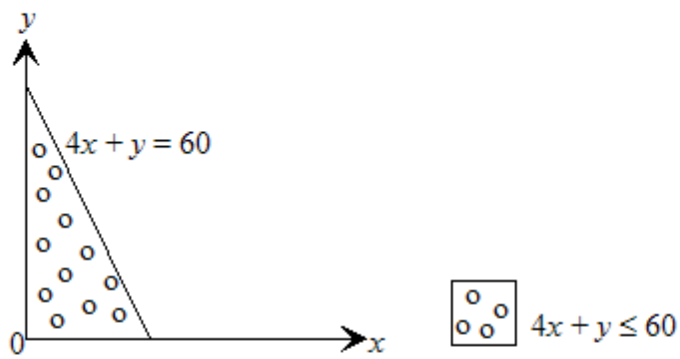
Finding two points on the line $4x + y = 60$.
When $x = 0$ $\qquad 4(0) + y = 60$
$\qquad\qquad\qquad y = 60$
The line $4x + y = 60$ passes through the point (0, 60).
When $y = 0$ $\qquad 4x + 0 = 60$

$$x = \frac{60}{4}$$
$$= 15$$

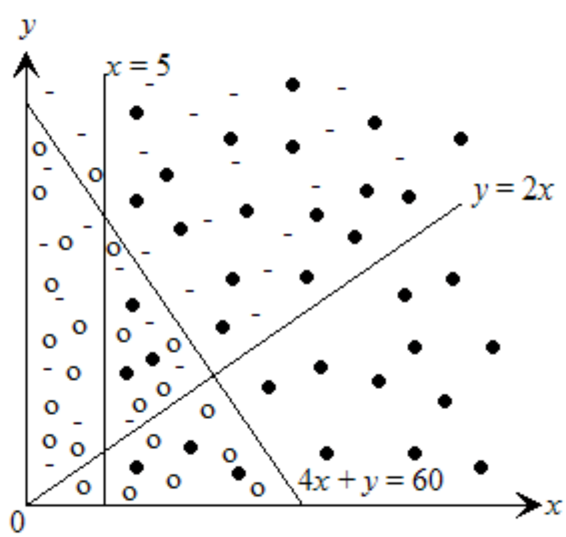
The line $4x + y = 60$ passes through the point (15, 0).

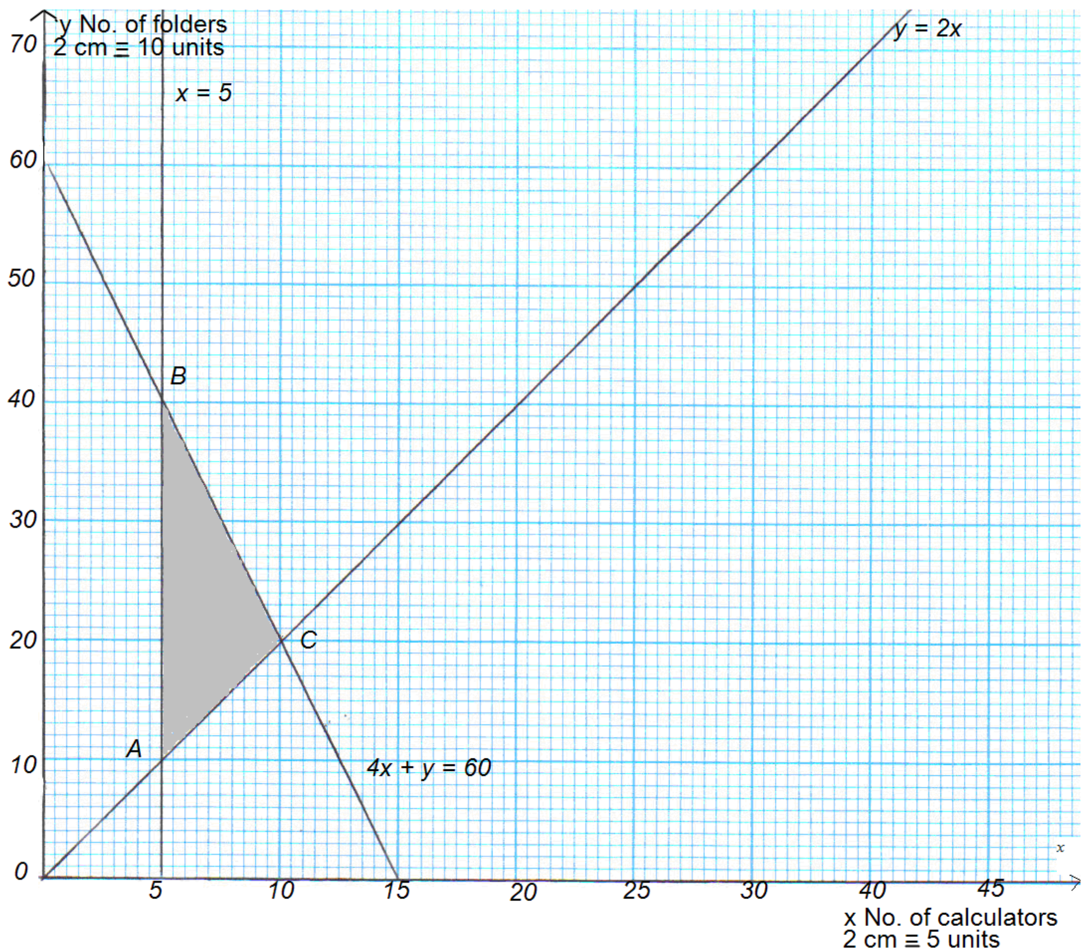


The side that makes the smaller angle satisfies the ≤ region.
Therefore, the region which satisfies $4x + y \le 60$ is



Therefore, the region which satisfies all three inequalities is the area where all three shaded regions, shown above, overlap.





This is identified as the feasible region is *ABC*.

(v) **Required To Find:** an expression in terms of x and y for the total profit, P.

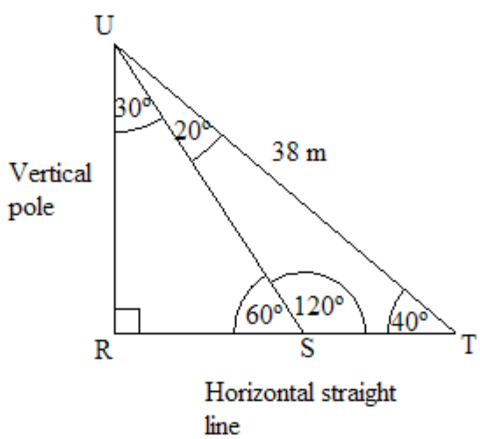
Solution:
Total profit $= P$
Profit of x calculators at \$6 each and y folders at \$2 each is
$(x \times 6) + (y \times 2) = 6x + 2y$
$\therefore P = 6x + 2y$

(vi) **Required To Find:** the coordinates of the vertices of the shaded region.
Solution:
Coordinates of the vertices of the shaded region are
$A\,(5, 10)$, $B\,(5, 40)$, $C\,(10, 20)$

(vii) **Required To Calculate:** maximum profit.

Solution:
$P_{\max}$ is to be found by testing the coordinates of the vertices of the feasible region.
The point *A* obviously need not be tested as it has both lower *x* and *y* values than the other points.
Point *B*
When $x = 5$ and $y = 40$
$P = 6(5) + 2(40)$
$= \$110$
Point *C*
When $x = 10$ and $y = 20$
$P = 6(10) + 2(20)$
$= \$100$
$\therefore$ Maximum profit is \$110, when 10 calculators and 20 folders are sold.

11. a. **Data:** Diagram illustrating a vertical pole standing on horizontal ground.



(i) **Required To Calculate:** the angle of elevation from *U* to *S*.
Solution:
$U\hat{S}R = 180° - 120°$ (angles in a straight line)
$= 60°$
$\therefore$ Angle of elevation of *U* from *S* is 60° (as illustrated)

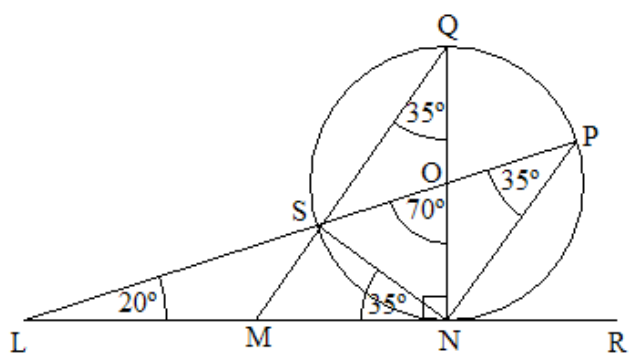
(ii) **Required To Calculate:** the length to *UT*
Solution:
$S\hat{U}T = 180° - (120° + 40°)$ (sum of angles in a triangle total 180°)
$= 20°$
$\frac{15}{\sin 20°} = \frac{UT}{\sin 120°}$ (sine rule)

$$\therefore UT = \frac{15 \times \sin 120°}{\sin 20°}$$
$$= 37.9\underline{8} \text{ m}$$
$$= 38.0 \text{ m to 1 decimal place}$$

(iii) **Required To Calculate:** the length of RU
Solution:

$$\sin 40° = \frac{RU}{38}$$
$$\therefore RU = 37.98 \sin 40°$$
$$= 24.4\underline{1} \text{ m}$$
$$= 24.4 \text{ m to 1 decimal place}$$

b. **Data:** Diagram showing a circle, centre O. $LMNR$ is a tangent. $LSOP$, NOQ and MSQ are straight lines. $S\hat{P}N = 35°$



(i) **Required to Calculate:** Angle SON
Solution:

$$S\hat{O}N = 2(35°)$$
$$= 70°$$

(The angle subtended by a chord at the centre of a circle is twice the angle subtended at the circumference, standing on the same arc.)

(ii) **Required To Calculate:** Angle NMQ
Solution:

$$O\hat{N}M = 90°$$

(The angle made by the tangent to a circle and radius, at the point of contact is 90°)

$$\therefore O\hat{N}M = 90°$$
$$S\hat{Q}N = 35°$$

(The angles subtended by chord *SN*, at the circumference of a circle, standing on the same arc are equal.)

$\therefore N\hat{M}Q = 180° - (90° + 35°)$

$= 55°$

(Sum of the angles in a triangle is 180°)

(iii) **Required To Calculate:** $\angle PLN$

Solution:

In ΔOLN

$O\hat{L}N = 180° - (90° + 70°)$

$= 20°$

(Sum of the angles in a triangle is 180°.)

(iv) **Required To Calculate:** $\angle SNM$

Solution:

$S\hat{N}M = 35°$

(The angle made by the tangent to a circle and a chord, at the point of contact, is equal to the angle in the alternate segment.)

12.a. **This part is not done since it involves latitude and longitude (Earth Geometry) which has been removed from the syllabus.**

12. b. $y = 2 - \cos x$

(i) **Required To Complete:** the table of values for $y = 2 - \cos x$

Solution:

x	0°	30°	60°	90°	120°	150°	180°
y	(1)	1.1	1.5	(2)	2.5	(2.9)	3

When $x = 0$ $\quad y = 2 - \cos(0)$

$= 2 - 1$

$= 1$

When $x = 90°$ $\quad y = 2 - \cos(90°)$

$= 2 - 0$

$= 2$

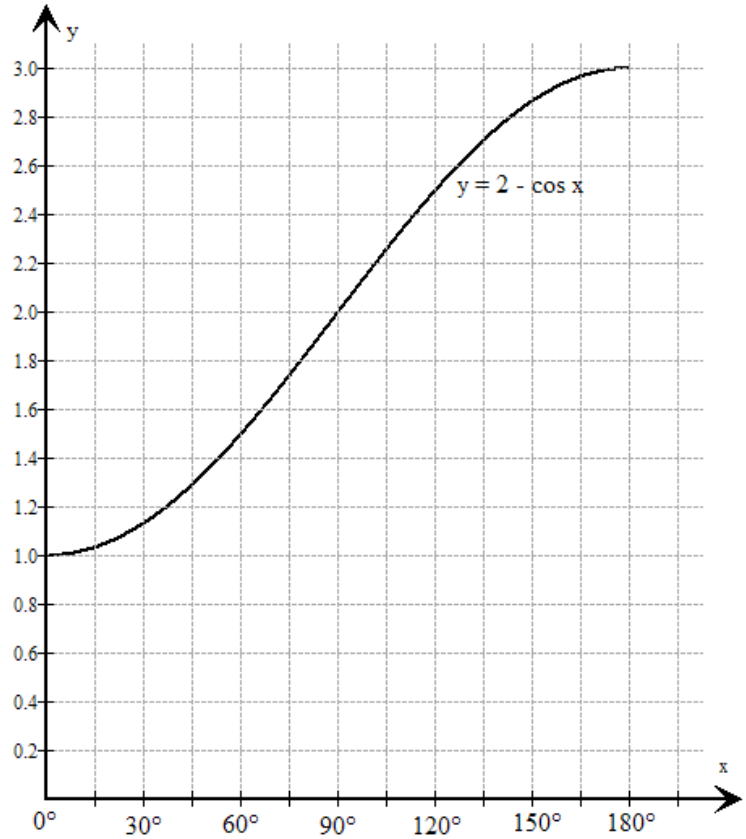
When $x = 150°$ $\quad y = 2 - \cos(150°)$

$= 2 - (-0.87)$

$= 2.9$

(iii) **Required To Draw:** the graph of $y = 2 - \cos x$

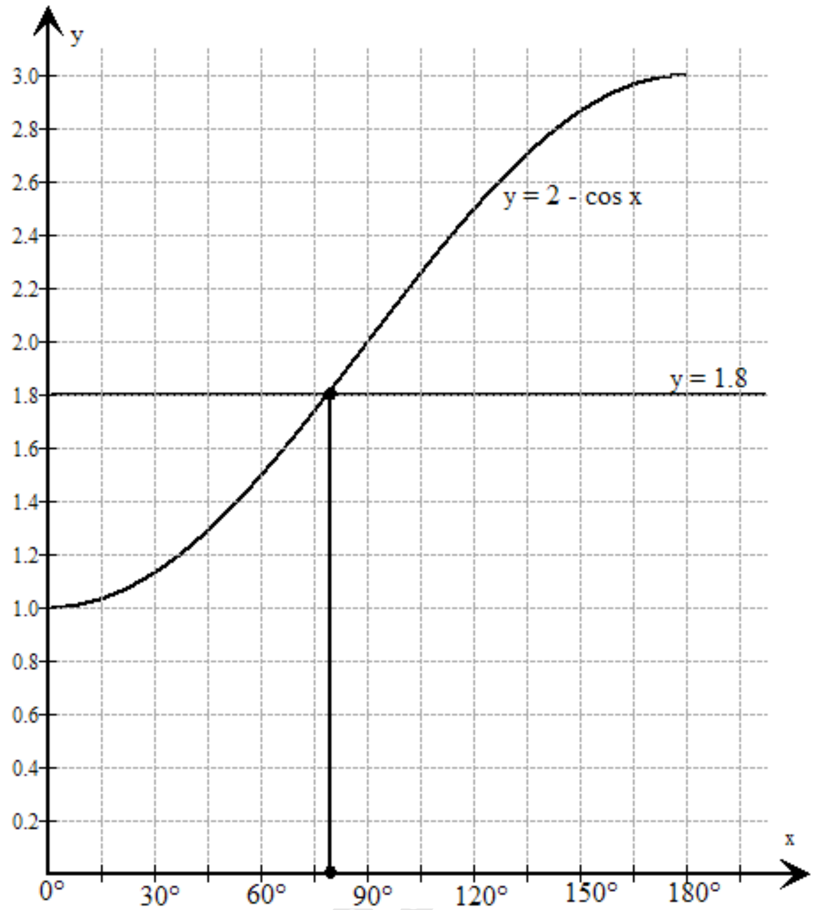
Solution:



(ii) **Required To Find:** the value of x for which $2 - \cos x = 1.8$.

Solution:

Draw the horizontal, $y = 1.8$



$2-\cos x = 1.8$ at $x = 78.5°$

13. a. **Data:** $\overrightarrow{OP} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $m = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $n = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ and $\overrightarrow{PQ} = m + 2n$

(i) **Required To Calculate:** $\overrightarrow{PQ}$ giving the answer in the form $\begin{pmatrix} x \\ y \end{pmatrix}$

Solution:

$$\overrightarrow{PQ} = m + 2n$$
$$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 1 \\ -3 \end{pmatrix}$$
$$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -6 \end{pmatrix}$$
$$= \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$

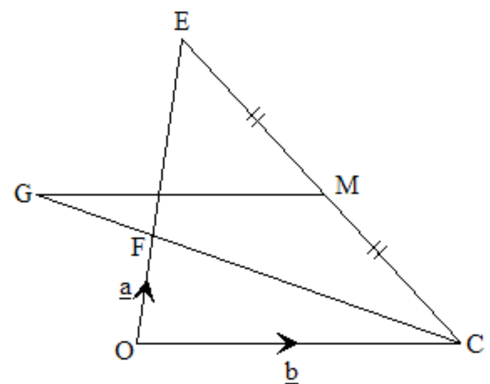
where $x = 4$ and $y = -3$

(ii) **Required To Calculate:** $\left|\overrightarrow{PQ}\right|$

Solution:

$$\left|\overrightarrow{PQ}\right| = \sqrt{(4)^2 + (-3)^2}$$
$$= \sqrt{25}$$
$$= 5 \text{ units}$$

b. **Data:** Vector diagram with M the midpoint of CE, $\overrightarrow{OF}$ and $\overrightarrow{FE} = 2\overrightarrow{OF}$



If $\overrightarrow{FE} = 2\overrightarrow{OF}$, then $\overrightarrow{FE} = 2\underline{a}$ and $\overrightarrow{OE} = \underline{a} + 2\underline{a} = 3\underline{a}$

(i) **Required To Express:** $\overrightarrow{CF}$ in terms of $\underline{a}$ and $\underline{b}$

Solution:

$$\overrightarrow{CF} = \overrightarrow{CO} + \overrightarrow{OF}$$
$$= -(\underline{b}) + \underline{a}$$
$$= \underline{a} - \underline{b}$$

(ii) **Required To Express:** $\overrightarrow{CE}$ in terms of $\underline{a}$ and $\underline{b}$

Solution:

$$\overrightarrow{CE} = \overrightarrow{CO} + \overrightarrow{OE}$$
$$= -\underline{b} + 3\underline{a}$$
$$= 3\underline{a} - \underline{b}$$

(iii) **Required To Express:** $\overrightarrow{CM}$ in terms of $\underline{a}$ and $\underline{b}$

Solution:

$$\overrightarrow{CM} = \frac{1}{2}\overrightarrow{CE}$$
$$= \frac{1}{2}(3\underline{a} - \underline{b})$$
$$= 1\frac{1}{2}\underline{a} - \frac{1}{2}\underline{b}$$

(iv) **Required To Express:** $\overrightarrow{MG}$ in terms of $\underline{a}$ and $\underline{b}$ and k

Solution:

$$\overrightarrow{CG} = k\overrightarrow{CF}$$
$$\therefore \overrightarrow{CG} = k(\underline{a} - \underline{b})$$
$$\overrightarrow{MG} = \overrightarrow{MC} + \overrightarrow{CG}$$
$$= -\left(1\frac{1}{2}\underline{a} - \frac{1}{2}\underline{b}\right) + k(\underline{a} - \underline{b})$$
$$= -1\frac{1}{2}\underline{a} + \frac{1}{2}\underline{b} + k\underline{a} - k\underline{b}$$
$$= \left(k - 1\frac{1}{2}\right)\underline{a} + \left(\frac{1}{2} - k\right)\underline{b}$$

(v) **Required To Calculate:** the value of k for which $\overrightarrow{MG} = \overrightarrow{CO}$

Solution:

$$\overrightarrow{MG} = \overrightarrow{CO}$$
$$\therefore \left(k - 1\frac{1}{2}\right)\underline{a} + \left(\frac{1}{2} - k\right)\underline{b} = -b$$
$$\equiv 0\underline{a} + (-\underline{b})$$

Equating components

$$k - 1\frac{1}{2} = 0$$
$$k = 1\frac{1}{2}$$

OR

$$\frac{1}{2} - k = -1$$
$$k = 1\frac{1}{2}$$

14. a. (i) **Data:** $M = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$

Required To Find: M^{-1}
Calculation:
Det $M = (2\times 3) - (1\times -1)$
$= 6 + 1 = 7$

$$M^{-1} = \frac{1}{7}\begin{pmatrix} 3 & -(1) \\ -(-1) & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{7} & -\frac{1}{7} \\ \frac{1}{7} & \frac{2}{7} \end{pmatrix}$$

(ii) **Required To Calculate:** the values of x and y for which $M\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 1 \end{pmatrix}$

Solution:

$$M\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 1 \end{pmatrix}$$

$\times M^{-1}$

$$M \times M^{-1}\begin{pmatrix} x \\ y \end{pmatrix} = M^{-1}\begin{pmatrix} 12 \\ 1 \end{pmatrix}$$

$$I\begin{pmatrix} x \\ y \end{pmatrix} = M^{-1}\begin{pmatrix} 12 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{7} & -\frac{1}{7} \\ \frac{1}{7} & \frac{2}{7} \end{pmatrix}\begin{pmatrix} 12 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \left(\frac{3}{7}\times 12\right) & \left(-\frac{1}{7}\times 1\right) \\ \left(\frac{1}{7}\times 12\right) & \left(\frac{2}{7}\times 1\right) \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

Equating corresponding entries,
$x = 5$ and $y = 2$

b. **Data:** Transformation matrix, $T = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$

A and B are mapped onto A' and B' $\quad A \xrightarrow{T} A'$

(i) **Required To Calculate:** the values of p, q, r and s.

Solution:

$$\begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} -4 \\ 2 \end{pmatrix}=\begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} -4p+2q \\ -4r+2s \end{pmatrix}=\begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

Equating corresponding entries

$-4p+2q=-2$

$\div -2$

$2p-q=1 \ldots(1)$

$-4r+2s=4$

$\div -2$

$2r-s=-2 \ldots(2)$

Similarly

$B \xrightarrow{T} B'$

$$\begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} -2 \\ 5 \end{pmatrix}=\begin{pmatrix} -5 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} -2p+5q \\ -2r+5s \end{pmatrix}=\begin{pmatrix} -5 \\ 2 \end{pmatrix}$$

Equating corresponding entries

$-2p+5q=-5 \ldots(3)$

$-2r+5s=2 \ldots(4)$

(1) + (3)

$$\begin{aligned} 2p-q&=1 \\ \underline{-2p+5q}&\underline{=-5} \\ 4q&=-4 \end{aligned}$$

$q=-1$

Subsituting in (1)

$-4p+2=-2$

$-4p=0$

$p=0$

(2) + (4)

$$\begin{aligned} 2r-s&=-2 \\ \underline{-2r+5s}&\underline{=2} \\ 4s&=0 \end{aligned}$$

$\therefore s=0$

Substitute $s = 0$ into (2)

$2r - 0 = -2$

$r = -1$

$\therefore\ p = 0,\ q = -1,\ r = -1$ and $s = 0$

(ii) **Required To Calculate:** The coordinates of the point *C*.

Solution:

$$C \xrightarrow{T} C'$$

$$\therefore \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} (0 \times -2) + (-1 \times 2) \\ (-1 \times -2) + (0 \times 2) \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$\therefore C' = (-2,\ 2)$