THE BARTON SERIES

BARTON'S UNUSUAL ENCOUNTER

BY

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(Ages 8 and over)

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THE SECRET

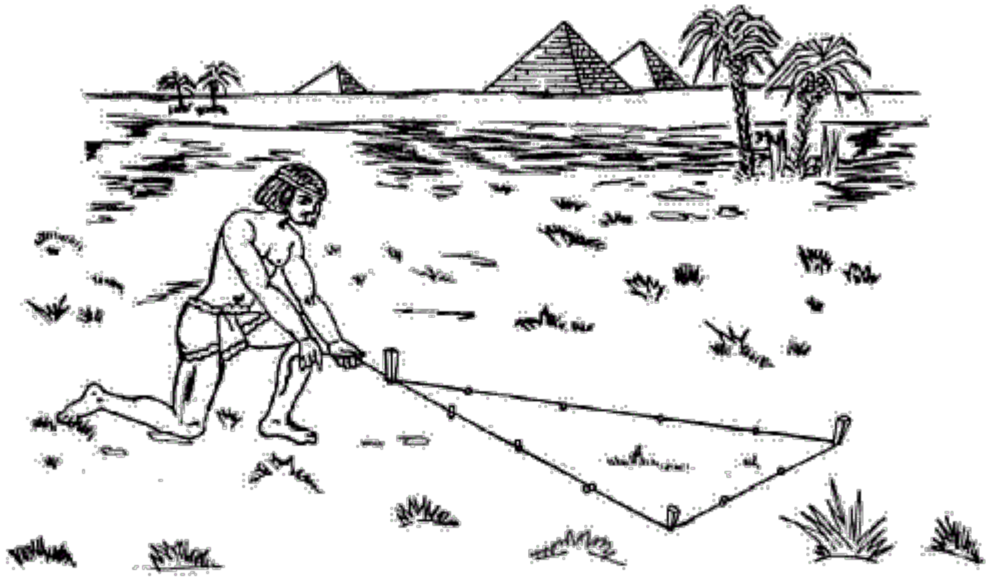
Barton and Kwame had earlier on met a brilliant little girl, Julie. She had both boys engrossed in an interesting, historical lecture on the Greek mathematician, Pythagoras. They were joined by five more of their friends and shortly afterwards, about half a dozen other children stopped by. All were enjoying Julie's discourse and Julie was justifiably proud.

When Julie told her engrossed listeners that Pythagoras had learned a great secret from a special group of Egyptians called the 'rope stretchers', they could not wait to hear about this secret. This secret would have led to the famous Pythagoras' theorem.

Julie continued with her lecture. All eyes were upon the little knowledgeable girl.

"The secret was a rope tied in a circle and which had twelve evenly spaced knots," said Julie. "It turned out that if the rope was pegged to the ground to form a triangle with the dimensions of the three sides being three knots long, four knots long and five knots long, a right angle would always be opposite the longest side of the triangle," Julie told the amazed group of listeners.

"The right angle was very important in their measurement and the division of the land and especially useful in their constructions of buildings. In both cases, precision was very important," Julie added.



“Amazing!” exclaimed the group of students.

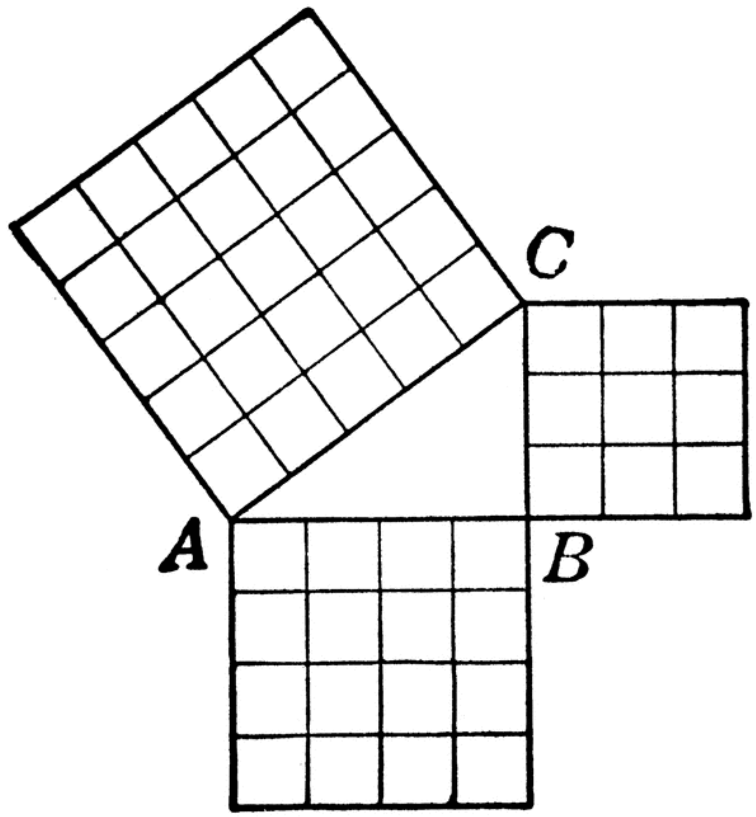
“Why is this always so,” asked a few members, anxious to get on with the story.

“Well,” said Julie, “Pythagoras thought long and hard about the reason why this was so and one day whilst he was drawing in the sand, this is what he found.”

The children drew closer up to her as she spoke.

“Pythagoras found that if he drew squares from each side of the 3-4-5- sided triangle, the area of the largest square was exactly equal to the total area of the two smaller squares.”

Julie had a picture that illustrated this fact and she took it out of her notebook. All crowded around to see the geometrical magic. Julie explained further as she pointed to the diagram.



“Notice,” she said, “that the square of side 5 units has an area of 5 x 5 = 25 square units and we can see the 25 square blocks. So too, the square of side 4 units has an area of 4 x 4 = 16 square units and we can again see the 16 square blocks. Finally, the square of side 3 units has an area of 3 x 3 = 9 square units and we can also see the 9 square blocks.”

The observers gasped at the figure in almost disbelief.

“I got goosebumps,” revealed Shanna.

The blocks were all numbered on Julie’s diagram and all the children looked closely at it. Some even counted the individual square blocks.

“Now let us look at the total area of the two smaller squares,” said Julie.

“We shall obtain 9 + 16 = 25 square units,” said the group members.

“And what is the area of the larger square,” asked Julie?

“It is 25 square units,” said the group, together, as they had already counted them.

“There you are,” said Julie, “the square of 3 added to the square of 4 is equal to the square of 5. This may be written as $3^2 + 4^2 = 5^2$,” added Julie, as she wrote it for all to see.

“Does it work for other right-angled triangles?” asked Kwame.

“That is exactly what Pythagoras was able to prove,” said Julie, “and so he was honoured with having the theorem named after him.”

“Just astounding,” said Shanna and Sian. “Imagine having a theorem named after you.”

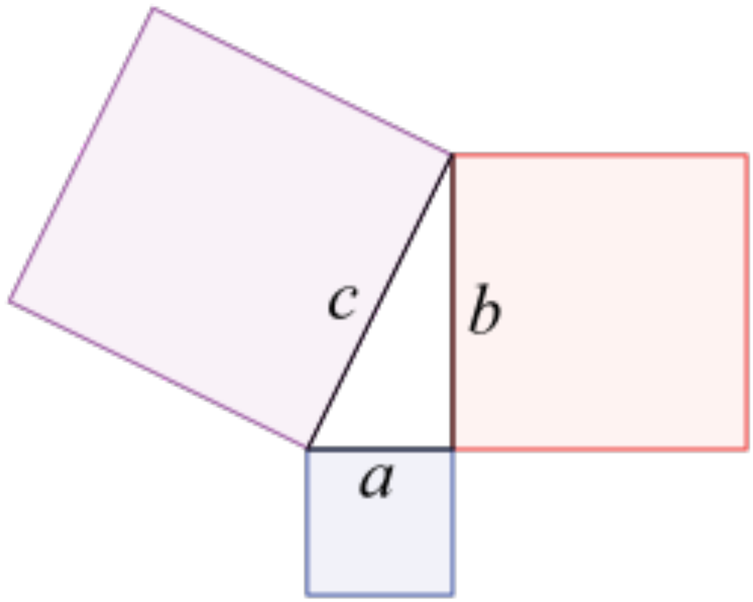
“And I felt that having a place named after you was a great thrill,” whispered Kwame, “for example Kwame’s Restaurant.”

Kwame usually thought much about food and this new thought of his, was not too surprising.

“Pythagoras’ theorem states that the square of the longest side of any right-angled triangle is always equal to the sum of the squares of the remaining two sides,” announced Julie.

“And Pythagoras was able to prove this?” asked the students.

“He most certainly did,” said Julie.



Julie called for silence as she showed an example of Pythagoras' theorem in use.

"Let us have a look at this question," she said.

Question

"The two shorter sides of a right-angled triangle are 6 cm and 8 cm. Calculate the length of the longest side."

Julie began to write for all to see.

The square of the two shorter sides is 6 x 6 = 36 cm^2 and 8 x 8 = 64 cm^2

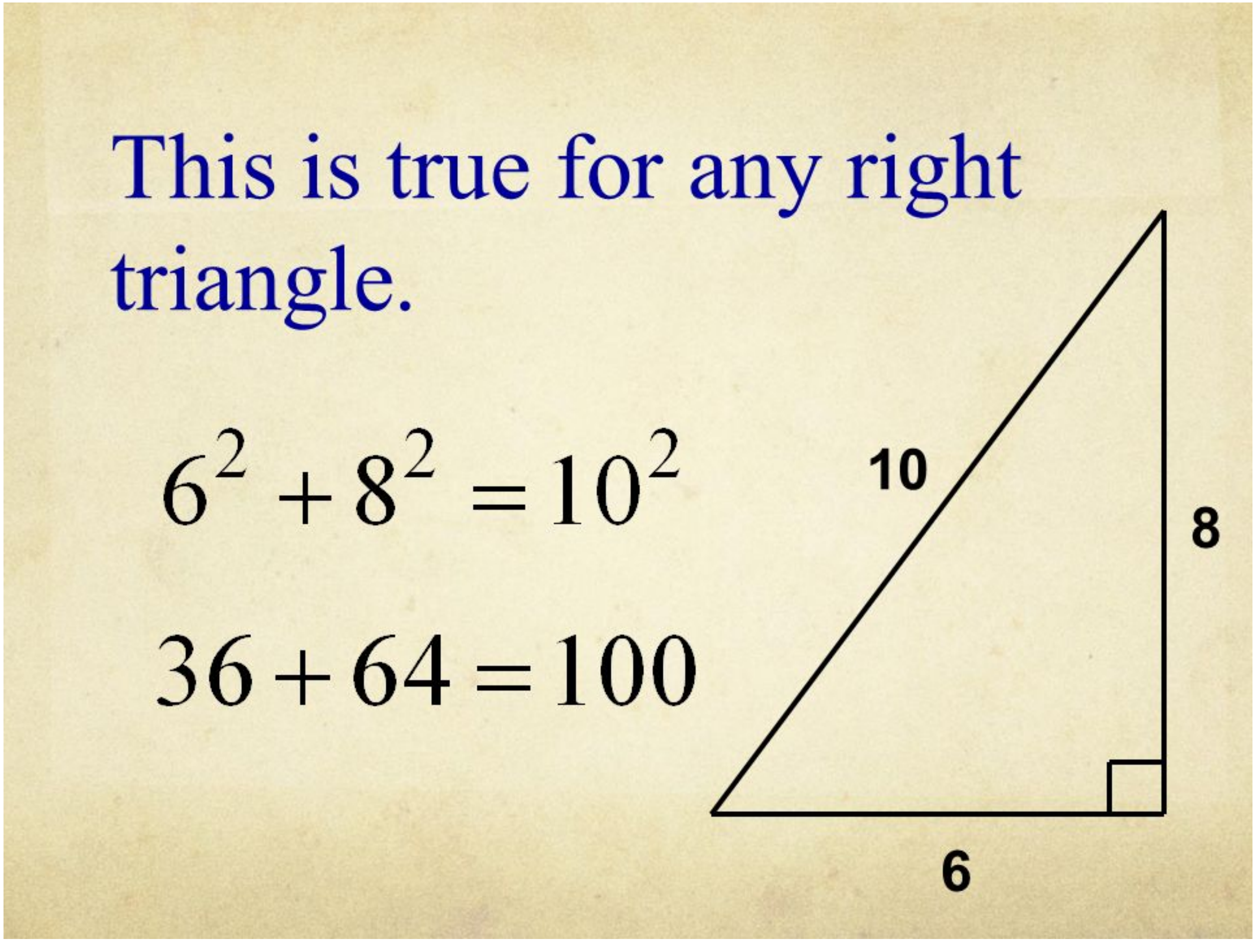
We now total the squares of the two shorter sides to get 36 + 64 = 100

Hence, the length of the longest side when squared is 100.

I now ask which number, when squared is 100?

This number is 10 since 10 x 10 = 100

Hence, the longest side is 10 cm long.



Kwame pretended to faint and two little girls rushed to his side. One of them began to fan him with a notebook and was overjoyed to see Kwame immediately rise and feeling rather proud of their training in first aid.

"We did it," they whispered to each other.

"Are you all right?" asked the other.

Kwame thanked them, not wishing to hurt their feelings and minimise their gallant efforts.

The small girls had failed miserably, though, in trying to lift the heavy one to a sitting position and were rather relieved when he rose on his own.

The group of students, including the recently revived Kwame, were more than impressed with Julie.

As the grateful students left, much more learned than before, Kwame and Barton stayed back with Julie.

“At the beginning, Julie, you mentioned to us that if you knew the dimensions of any two sides of any right-angled triangle. then you can calculate the exact length of the third side, isn’t that true?” asked Kwame.

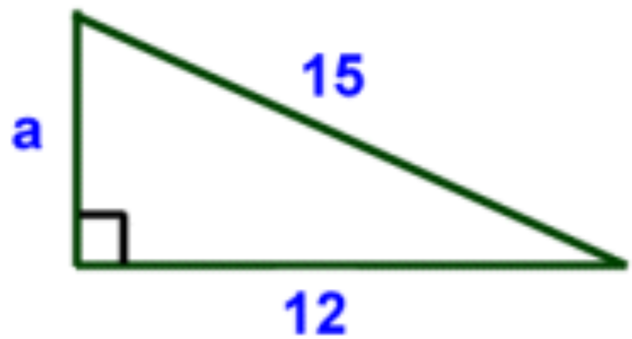
“I did so,” replied Julie, looking closely at Kwame.

“You gave us two examples in which you calculated the longest side and which is the hypotenuse of the right-angled triangle when given the lengths of the two shorter sides,” recalled the plump boy.

“Relax, my worried one,” responded Julie with a broad smile on her face. “I know exactly what you wish to ask,” she said. “You would like to know that if the length of the longest side, and which is the hypotenuse, is known and the length of only one of the shorter sides is known, how is it that we calculate the length of the third side.”

“I do most certainly,” replied Kwame, thinking how smart Julie must be to realise his question before he could even announce it.

Julie neatly drew a triangle on her notepad and showed it to both Barton and Kwame.



“The hypotenuse of this right-angled triangle is 15 units and one of the two shorter sides is 12 units,” said Julie, pointing them out to both boys.

“I shall show you two, just how easy it is to calculate the length of the other shorter side,” she said with oozing confidence.

“In fact,” Julie suggested, “I shall show both of you that you could do it yourself and I’ll just guide you along.”

“I’m game,” replied Barton. “I am too,” said the supportive Kwame.

“In your own words tell me what is Pythagoras’ theorem,” asked Julie.

Barton volunteered to answer.

“First of all, Pythagoras’ theorem applies only to right-angled triangles,” started Barton. “Such triangles have a right angle and its longest side is called the hypotenuse. The hypotenuse always lies opposite to the right angle.”

Julie nodded approvingly and patted Barton on the back. The act was a reversal of roles as Barton was accustomed to patting a smart worker on the back. Barton smiled to himself.

“Not bad, Barton, there is hope for you still,” laughed Julie.

Barton smiled and continued.

"If we square the hypotenuse, the result is the same as the sum of the squares of the two shorter sides," said the confident Barton.

"Can you apply this theorem to our triangle?" asked Julie.

Barton wrote,

(Length of hypotenuse x Length of the hypotenuse) = (length of the first shorter side x length of the first shorter side) + (length of the second shorter side x length of the second shorter side)

"Let us now apply the arithmetic, Barton," coaxed Julie.

This should be (15 x 15) = (12 x 12) + (the unknown shorter side) ^{2} wrote Barton, after carefully studying the diagram.

Kwame was quick to calculate and Barton wrote,

225 = 144 + (the unknown shorter side) ^{2}

"You are on your own, Barton," said Julie, as she sat down and continued with her work that she was doing, just before Barton and Kwame had come along.

Barton thought deeply about the problem in front of him and decided to rewrite it in the reverse order.

(The unknown shorter side) ^{2}+ 144 = 225

Therefore,

(The unknown shorter side) ^{2} = 225 -144

(The unknown shorter side) ^{2} = 81

(The unknown shorter side) = the square root of 81 and which is 9

Barton and Kwame were both overjoyed.

“The answer is indeed 9 units,” said Julie from her slightly distant sitting position.

Both boys hadn’t realised that Julie was not even paying them any mind. They walked across to her and sat down as Julie continued with her work.

Julie neither lifted her eyes nor her head from her work as she asked, “Barton A. Sandiford and D. O. P. Kwame, what can I do for you?”

“My name is not D. O. P. Kwame,” replied Kwame, looking a bit surprised at Julie. “Where did you get those initials of D, O and P?” he asked.

“They stand for ‘Descendant of Pythagoras’,” replied the serious-looking Julie. “You are a descendant of Pythagoras, aren’t you Kwame?” she asked.

Suddenly both boys burst out laughing as Julie looked at them. Then she looked down at the two boys who were rolling in laughter and she continued to write.