## THE BARTON SERIES

## ON THE ROAD WITH BARTON



BY
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(Ages 8 and over)

# ON THE ROAD WITH BARTON 

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## THE CREATION

Barton sat in the kitchen with Mom as he related the events of his day at school. Mom was baking bread and Barton was hoping that the first set of loaves might be out of the oven before he went upstairs to change. Next to Mrs Winslow, Mom's homemade bread was the best that he had ever tasted. He never admitted this to Mom though, in case she felt disappointed.

"I find Mrs Winslow bakes really tasty bread," was all he would say.
Mother knew her son's secret thoughts about waiting for the first set of baked loaves. She continued to engage him in dialogue so that his wish would come true.

At school, Barton was an excellent student in the academics and both his parents and two siblings were quite proud of him. Young Barton too was also
an excellent sportsman and together with his close friend, Dane, they were often called the sports twins of the school.

Today, however, it was Barton's day. It was during the afternoon period when he represented the school in a sporting event, Barton had excelled. The school team was victorious and his performance was so magnificent, that his teammates carried him off the field on their shoulders. The elated Barton A. Sandiford received thunderous applause from the staff and students of his school and even from the rival team members and their supporters.

As Barton reiterated the events of the day, Mom ruffled his hair as she congratulated him and then she hugged her firstborn.

But proud as Barton was that evening, his mind was not entirely on the fabulous sporting event of the day. He remembered just a few days ago, together with some of his friends, they successfully solved three questions from a book on 'investigating mathematics'. Barton had thought a lot about the type of questions in the book and wondered if he could create one, similar to them. He glanced at some boxes of matches on the kitchen shelf and at their side, Barton saw two boxes of toothpicks.

"Mom," said Barton, "I would like to use a box of toothpicks for a small project that I have in mind."

Mom, without question, fetched the box of toothpicks for him, as it was on a high shelf.
"After I change and when my chores and homework are done, I would attempt to create a sequence with geometrical figures and even a question on this pattern. I want it to be just like the ones I saw in the book," he said to Mom.
"I would love to see it when you are through," said Mom, as Barton slipped off the stool and headed for his room.
"Wouldn't you like to try some slices of hot, homemade bread and butter, Barton?" Mom asked. "The first set of loaves is just about ready to be taken out of the oven."

The offer was graciously accepted and several slices of hot homemade bread, dripping with the spread of tasty butter, disappeared before Barton left Mom's kitchen that evening.


The young boy eventually settled down at the desk in his room. Earlier on, he had raked the fallen leaves from around the house, emptied the trash and then went inside to complete his homework. He completed it satisfactorily and was rather pleased with his efforts. Together with the rest of the family, they all kept up-to-date with the current local and international news. This was an activity that the Sandifords engaged in as often as possible.


As Barton was about to enter his room, his two younger siblings sought advice on homework of their own. As usual, he was only too pleased to assist them and they left contented and wiser. The two were, like their older brother, both outstanding students at school and Barton's younger brother was also showing great promise of being an excellent sportsman, just as he was.

Finally, Barton sat in front of his desk and he opened the box of toothpicks. He took a few of them in his hand and twiddled them around.

Barton A. Sandiford began to arrange the toothpicks on his desktop and to form plane geometrical patterns with them. He created triangles, squares and various other types of polygons. Suddenly, a thought entered his head. He took four toothpicks and formed a single square with them.


Then, Barton took a few more toothpicks and formed two squares, side by side. As he looked at the second creation, Barton noticed that one of the sides was common for both squares.


So, instead of using eight toothpicks as he would have used with two separate squares, he had used only seven to form the joined squares. This is because they shared a common side.
"Interesting," said Barton to himself as he took more toothpicks and formed three squares joined side by side. For this third figure, Barton needed only ten toothpicks. For three separate squares, however, he would have needed $4 \times 3=12$ toothpicks.


Barton began to write on a blank sheet of paper.

It was at that very moment Mrs Sandiford invited herself into Barton's room.
"And how is my little mathematics investigator?" she asked as she sat next to him.

The proud and excited boy showed his mother the squares that he had created with the toothpicks and the table which he had drawn up showing the number of squares and the number of toothpicks used to construct them.
"And, did you discover any relationship with the number of squares and the number of toothpicks used?" asked Mrs Sandiford.
"I have indeed found a pattern," said Barton to Mom, as he showed the sheet of paper to her.
"The first square has four sides, as expected. But as each new square is added, the number of toothpicks used is increased by three."

Barton held Mom's hand as he showed her the table he had drawn up.
"Look at my table, Mom, and see how I found this out," he said proudly.

Number of toothpicks required

$$
\begin{aligned}
& \mathbf{4 + 3}=7 \\
& 7+\mathbf{3}=\mathbf{1 0}
\end{aligned}
$$

"Mom," said Barton, "I think that I have made a great discovery."
Mrs Sandiford appeared to be very surprised and pleased with her son's discovery. She drew closer to Barton.
"Can you tell your Mom about it, unless it is your secret and should not be revealed?" she asked.
"Mom," said Barton, looking rather amused, "I would not keep a secret from you, and also, it is not a secret. It is right here on this table which I have drawn."

Barton laughed at Mom for thinking the way she did and she ruffled his hair. "If the number of toothpicks used to construct each additional square, increases by three, as my working from the table shows, then the number of toothpicks that should be used to construct four adjacent squares, should be three more than what was used for three adjacent squares," suggested the young boy. "This number should be $10+3=13$ toothpicks," he concluded.
"How might you confirm that you are correct?" asked Mrs Sandiford, sounding rather curious and almost as excited as her son.
"I propose to construct a set of four adjacent squares and then I shall count the number of toothpicks used to make them," replied the young 'Euclid'.
"That sounds quite feasible," said Mrs Sandiford, nodding her head.

Barton took some more toothpicks from the box and began his small construction task. He meticulously began to place the toothpicks, in the required formation.
"Mom," he said, "I shall quickly construct these four adjacent squares and then I can easily count the number of toothpicks used."

As he worked, Barton paused for a brief moment. He held Mom's hand.
"Mom," he said softly, "when I am counting the number of toothpicks, I want you to count along with me. In this way, if I am correct, then we will be correct together."

Mrs Sandiford could not refuse the generous offer which overflowed with love. Soon after their elation was boundless, as they both counted the total of thirteen toothpicks. Mom hugged Barton and there was a tender moment between the mother and her son.
"I can easily calculate that the number of toothpicks required to construct five adjacent squares would be, $13+3=16$," suggested Barton, as he updated his table.

Number of squares
Number of toothpicks required

1
2
3
4
5

4

$$
4+3=7
$$

$$
7+\mathbf{3}=\mathbf{1 0}
$$

$$
10+3=13
$$

$$
13+3=16
$$

"That is excellent reasoning," said Mrs Sandiford, as she complimented the young mathematician.
"Can you see a rule that relates the number of squares and the number of toothpicks used to construct them?" she inquired.
"Why would we want a rule that connects the number of squares with the number of toothpicks?" queried Barton. "Do you think such a rule is important?" he asked further.
"Rules usually are," replied Mrs Sandiford. "Don't we need and use rules to find, for example, the area of a rectangle or a square?"
"Yes we do," Barton nodded in agreement as Mom continued to speak.
"We need rules to find the simple interest on a sum of money, the circumference of a circle and the volume of a box. Even the simplest of operations like addition, subtraction, multiplication, and division, require rules," Mrs Sandiford added.
"I know all of those rules," Mom, claimed Barton, anxious to reconfirm his mother's strong belief in his mathematical skills.

He insisted on recalling them for her. The young boy took a clean page from his notepad and wrote, as Mom looked on.

The area of a rectangle is length x width
The area of a square is side x side
Simple interest $=($ Principal $\times$ Rate $\times$ Time $) \div \mathbf{1 0 0}$
The circumference of a circle $=2 \times \pi \times r$, where $r$ is the radius and $\pi$ is to taken as 22/7.

The volume of a box $=$ length x width x height.
"They are all correct," confirmed Mrs Sandiford. "I am proud of you," she said to her beaming son.
"The reason why I ask if you have a rule that relates the number of adjacent squares and the number of toothpicks used to construct them, is similar to all of this," said Mom to Barton.

The young boy listened attentively. Over the years Mom taught him well and many of the topics which he now studied at school, he had learned from a long time before.
"Suppose," Mom continued, "I asked how many toothpicks would be required to make six adjacent squares; what would your answer be?"

Barton listened and then he laughed and laughed as Mom looked a bit surprised.
"Mom," said Barton, placing his written table in front of her, "the number of toothpicks required for five adjacent squares is 16 . By looking at the sequence, we would need three more to complete the sixth adjacent square. The requirement will be $16+3=19$ toothpicks."
"That's quite good," replied Mrs Sandiford, as she confirmed Barton's sound reasoning.

