THE BARTON SERIES

BARTON FOLLOWS A CLUE

BY

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(Ages 8 and over)

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WHAT COMES NEXT?

As Kwame crunched on a cookie and sipped more hot cocoa, he listened and looked at the rain that still was falling heavily. He could see growing puddles of water all around. Kwame pointed his friends to a few visiting wild ducks nearby, who seemed to be enjoying a bath in one of the larger puddles. They flapped their wings as they welcomed the downpour and they 'quacked' their gratitude rather loudly, as Kwame put it, looking at them as they waded about in the shallow pool.

The few vehicles that passed by were being driven slowly, as the drivers were cautious of the slippery road. Kwame, like the rest of his friends, felt warm and safe in the treehouse.

The small group had successfully solved a question from a book on 'investigating mathematics' that Sian had brought. Now they were feasting on some savoury snacks which the members had brought along and they continued to enjoy each other's company.

Sian, Shanna, Malaika, Alfredo, Kwame, Dane, and Barton were very close friends and they all enjoyed being together when this was possible. They all belonged to the same class at school and were excellent students in academics. Barton and Dane were also great sportsmen as well. They were often referred to as the sports twins of the school. The seven members of the group met often after school and together enjoyed many wonderful activities together. The friends had formed the small club and their newest addition was Dane. He had returned to the country after living abroad for several years.

As they ate, Barton flipped through the pages of Sian's book on 'investigating mathematics'.

"Here's an interesting question," he called out to the group. "Are we willing to test our skills again?" he questioned.

"Why not?" was the unanimous reply.

"It was indeed a lot of fun doing the first one," admitted Barton.

The group saw what appeared to be an interesting method for squaring numbers which ended with the digit 5. They all looked carefully at the table of values given and then they tried to decode the rule that was being applied. At first glance, they could not see the pattern. They continued to look more intently.

SQUARING NUMBERS THAT END WITH 5.

Number ending in 5	Square of the number	Result
1 5	{1 x (1 + 1)}25	225
2 5	{2 x (2 + 1)}25	625
3 5	{3 x (3 + 1)}25	1225
4 5	(a)	(b)
(c)	{6 x (6 + 1)}25	(d)
(e)	(f)	9025
495	(g)	(h)
N	(i)	(j)

The children were spellbound by the method used, although the formula was not yet fully understood by them.

"I am going to check if their answers are correct, though I have no doubt they are," said Barton. "Work along with me," he invited the others.

The other children began to overseer Barton's quick multiplication of the numbers as he squared them. Barton was an excellent student of mathematics. He could perform all these multiplications in his head, but, decided to write them down to show his work to the others. Also, he was determined not to make any errors.

First, he squared the number 15 that started the first row of the table.

The small group looked on as Barton multiplied 15 by 15. His answer was 225, just as given. Next, Barton squared the number 25, the number that started the second row of the table.

Again, the small group looked on as the boy multiplied 25 by 25. His answer was 625, just as was given in the table.

Finally, Barton squared the number 35, the number that started the third row of the table. The group looked on as Barton multiplied 35 by 35 to obtain the correct answer of 1 225. In awe, they saw again, that this was the same answer given in the table.

"Very interesting," they exclaimed! "This is just too amazing. This brilliant rule in mathematics will have to be shared."

"We need to carefully examine how the formula was created," suggested Shanna. "Let us approach this question in a manner similar to what we did in the last one."

"It appears that we have little to do with the first number of the rows and the last number of the rows," claimed Sian. "The first number is just the number that is being squared,

Number ending in 5

1 5

2 5

3 5

and the last number is the result of the squaring."

Result

225

625

1225

The group members cautiously followed Sian's argument and were in full agreement at the end of it.

"You are right, Sian," they said, "We need to focus on the middle column and decipher how the pattern was created."

"Let us start with the number 15," volunteered Barton.

"To square the number 15, that is to obtain 15 x 15 or $(15)^2$, first, we notice that the number 1, which is the number just before the 5, was written down and then it was multiplied by the sum of 1 and 1. This gave 1 x (1 + 1) and which is 2," observed Barton.

The group listened but did not comment. Barton continued with his observation and explanation.

"To square the number 25, that is to obtain 25 x 25 or $(25)^2$, we again notice that the number 2, which is the number just before the 5, was written down and then it was multiplied by the sum of 2 and 1. This gave 2 x (2 + 1) and which is 6," claimed Barton.

Barton ran his finger along the row of numbers to back up his deductive reasoning. There were now the nods of approval from all the keen observers.

"To square the number 35, that is to obtain 35 x 35 or $(35)^2$, we again notice that the number 3, which is the number just before the 5, was written down and then it was multiplied by the sum of 3 and 1. This gave 3 x (3 + 1) and which is 12," suggested Barton.

All agreed.

"I can see exactly what is happening at the end," said Alfredo, jumping in delight. "All the answers end with, 25 and so 25 is just attached to the end of the multiplication that was just created."

"A deeper explanation would be preferred," insisted the group.

Alfredo bowed before them and he obliged.

SQUARING NUMBERS THAT END WITH 5.

Number ending in 5	Squaring the number	Result
1 5	1 x (1 + 1) = 1 x 2 = 2	
	Now attach 2 to 25 and get 225	225
2 5	2 x (2 + 1) = 2 x 3 = 6	
	Now attach 6 to 25 and get 625	625
3 5	3 x (3 + 1) = 3 x 4 = 12	
	Now attach 12 to 25 and get 1225	1225

"That was absolutely brilliant, Alfredo," said the group. "You have now made it easy for us to fill in the missing numbers on the table and to complete the table."

"The first incomplete column is quite easy to complete," said Dane.

4 5 (a) (b)

"It shall be,

4 5 4 x (4 + 1) = 4 x 5 = 20 2025

Now attach 20 to 25 and get 2025

"The answer to (a) and to (b) can now be inserted," Dane concluded.

Barton opted for the completion of the next row.

(c) 6 x (6 + 1)25 (d)

"Since the middle column started with the number, 6 then the number that is to replace, (c), has to be 65. Next, the answer to (d) will be found by first multiplying 6 by 7 to get 42 and then attaching 42 to 25 to get the answer of 4 225."

"That was a bit too simple, Barton," said a few members as they laughed with their friend.

"The completion of that row did not require too much skill," they teased.

"Your secret message to me is loud and clear," laughed Barton, "I shall undertake the task of completing the next row."

"I'll help you, Barton," offered Malaika, "and together we shall complete the entire table."

"Your offer is graciously accepted, my dear friend," said Barton, as he gave the girl a friendly hug.

Together the two sat down next to each other and sought to put their thoughts together. And, it was not too long for the two brilliant mathematics students to see the light.

"In the next row, we have,

(e) (f) 9 025,"

read Barton, as the others looked on.

"The row ends with 9025. So let us ignore the last two digits of 25 and concentrate on the number 90. The product of a number and the number when added to 1, is to be 90," he said confidently.

"In other words, the product of two consecutive numbers is to be 90," added Malaika.

"These numbers can only be 9 and 10 or as the table requires 9 x (9 + 1), calculated Barton quickly and correctly. Hence, the number that is to replace, (e), is sure to be 95. The completed row can now be seen as,

95 9 x (9 + 1)25 9 025"

"Brilliant," said the group, patting the two on their shoulders.

"In the next row we have," said Malaika.

49 5 (g) (h)

"Again, this is a rather simple exercise to complete this row," she confessed. "It is now to be:

49 5 49 (49 + 1)25 245 025"

"And now, my dear students," said Dane, "Professor Barton, the Smart and Professor Malika, the Magnificent, shall conclude the proceedings with the completion of the final row."

N (i) (j)

The young boy looked at his friends as they waited in eager anticipation and he began to explain.

“The row does not start with a number, but starts with a symbol, N, which I assume is a number ending with 5,” started Barton.

“This is, of course, is an unknown quantity,” he continued. “Hence, N must be multiplied by 1 more than N and then this product is to have the number 25 attached to it for completion. This would look firstly like N x (N + 1) and then finally like {N x (N + 1)}25.”

“Marvellous,” exclaimed the impressed friends, looking on.

I cannot compute the product of N and N + 1, as I do not know what numbers they represent. So, I shall leave the table as,

N	{N x (N + 1)}25	{N x (N + 1)}25

“Not bad, not bad at all,” admitted the rest of the group.

“I now consider the table, as well as the question, to be completed, thank you all,” said Kwame.

Barton and Malaika laughed as Kwame pretended to seek their autographs. There was more laughter as Malaika gave it.

“Bravo to this fine duo,” shouted Dane and Alfredo. “Bravo! Bravo,” they applauded.

But soon after the applause had died down, Kwame stood up and called for silence. He paced the floor for a short time, creating anxiety among the rest. They giggled at his antics.

“My dear and cherished friends,” he began, “did you know that the procedure by which the 25 was attached to the product of the two numbers,

3 x (3 + 1)25 = (3 x 4)25 = 1225 to obtain the square of the number, is called ‘concatenation’?