## THE BARTON SERIES

## LOOKING FOR BARTON



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## THE RED CUBE

It was a quiet evening at the Sandifords' and the family had just finished dinner. They sat comfortably in the drawing-room as they engaged in conversation. Barton looked at his little brother building shapes with wooden cubes.


As he watched the little boy at play, he remembered all the properties of the cube that he had learned at school. He reached across and picked up one of them and he gazed at the red solid object.

Barton touched all of the six faces and he counted the twelve edges of the cube. He ran his fingers along any three edges as they met at a point called the vertex. Then he counted the total of eight vertices in the cube. All cubes have these same properties, he remembered, regardless of their sizes.

Not too far from where Barton sat was a board game in a box. Inside the box was a pretty red die. Barton reached across and took out the die and he looked at the six different numbers notched in the six faces of it.
"Do you know that the numbers on each of the three sets of opposite faces on a die total seven?" he asked.
"That's interesting," replied Dad, pretending not to know.
"Could you list the pairs of numbers on the opposite faces without looking at the die?" challenged Dad.
"I can," replied Barton without hesitation.
"They are one and six, two and five and three and four."
"That is correct," answered Mom and Dad together.
Barton looked on at the faces of the cube in his hands and he thought about something interesting. He first remembered a lesson on squares and square roots that he had learned at school.

The area of any one face of the die was found by multiplying the length of one side by itself.
"Miss would say, the length of a side multiplied by the length of another side OR simply side by side."

Miss had introduced the idea of 'squaring' soon afterwards and the class had learned that 'side multiplied by side' could have been written as side ${ }^{2}$, which was read as 'side squared' or the 'square of the side.'
"It is the squaring of the number that measured the length of the side, which would give the result for the area of the square's face," she told the class.

The class always remembered that Miss would tell them to write the number that is to be squared within brackets.
"If you have to find the area of a square of side 5 cm then you would multiply the length 5 cm by itself, which is $5 \mathrm{~cm} \times 5 \mathrm{~cm}=25 \mathrm{~cm}^{2}$. We can also express this computation as the square of the number 5 or $5^{2}$."

The enthusiastic and attentive class had immediately written the result as 25 square cm or $\mathrm{cm}^{2}$. However, although the process of squaring was easy to remember, there was something mysterious which Miss insisted upon. Miss always wanted the number that was going to be squared to be written within brackets.
"The number 6 multiplied by 6 can be expressed as six to be squared and written as $6^{2}$. However, it is best written as (6) ${ }^{2}$. The result can easily be obtained by all of us to get 36 ," Miss said.

Miss encouraged discussion and questioning in class and so it was not unusual when Malaika wanted clarification on something that puzzled her. She had first asked Barton and he also seemed a bit confused.
"Miss," asked 'Malikes,' as she was often called, "is it not true that the results of $6^{2}$ and $(6)^{2}$ are both the same? Why do we need to place the brackets?"
"You are right," said Miss, "the brackets do not affect the results in this case."
Miss smiled her mysterious smile as she added.
"One day you shall discover a different type of number, and not placing the brackets can lead to horrible and costly mistakes. I am inculcating good mathematical habits in you even at these earlier levels."

Everyone trusted Miss and though they could not quite understand what she spoke about, they all readily adopted the practice.

Sometime later, Miss had shown the class about the reverse process of squaring, called finding the square root.
"The square root of a number is the number which, when multiplied by itself, gives the number," Miss had told them.

Miss had shown the class the square root sign and they had done many examples.
"The square root of 25 is written as $\sqrt{ } 25$ and is 5 . This is because $5 \times 5=25$."

The class squared many numbers, including fractions, and found the square roots of several numbers as well. All the students had become very adept at squaring numbers and finding the square roots of numbers. They practised their mental arithmetic skills by asking each other to find the squares and square roots of numbers and working the answers in their heads only.

All these events ran through Barton's active mind as he looked at the cube. In his mind, however, was the thought of finding the volume of a cube. He knew that it was found by multiplying the area of a face by the length of the face. He knew this to be the (length of a side) x (length of a side) x (length of a side) or simply as Miss would say, side by side by side.

Barton thought long and hard. The area of a face is found by squaring the length of the side and is expressed in square units. The volume is expressed in cubic units and is the length of a side multiplied by itself and then multiplied by itself again. Barton's forehead frowned in deep thought. If the square of the number 4 is $4 \times 4$ and written as $4^{2}$, then $4 \times 4 \times 4$ might be written as $4^{3}$ and is called 4 cubed. Perhaps the cube of 5 which is $5 \times 5 \times 5=125$ and is written as $5^{3}$ and called 5 cubed.

Barton's eyes brightened. If $5 \times 5 \times 5=125$ is the result of 5 cubed, then 5 might even be called the cube root of 125 . Perhaps, he thought again, $2 \times 2 \times 2=2^{3}$ and is 2 cubed which is 8 and so the cube root of 8 is 2 . I wonder, thought Barton, if I am right.
"Dad," said Barton, "I have a question to ask."
Mr Sandiford watched the curious face of Barton and the frown on his forehead and he smiled.

