THE BARTON SERIES

ACCORDING TO BARTON

BY

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(Ages 8 and over)

ACCORDING TO BARTON

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The Translation

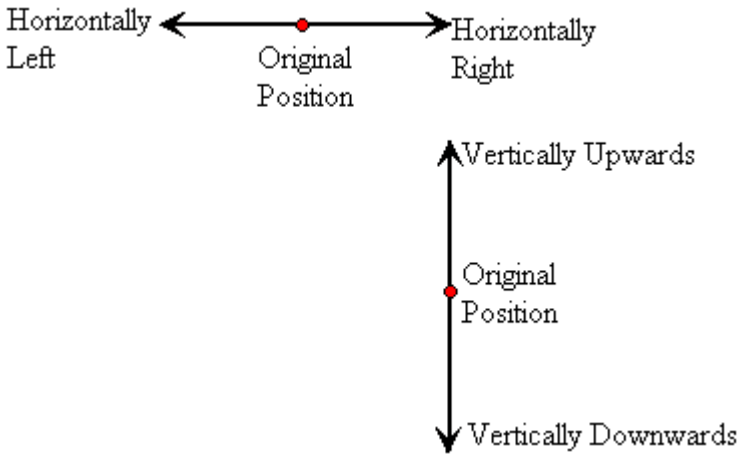
It was a most interesting mathematics class done by Miss on one particular morning, that had young Barton in deep thought for much of the rest of the day. He ambled around the school for a while before he sat down to his lunch.

As Barton munched on his sandwiches, he toyed with new ideas as he traded them for improved models. Then Barton created even sounder and better ones. The novel thoughts danced and teased him, much like a gentle breeze against a candle flame. Barton A. Sandiford was in a creative mood and he thoroughly enjoyed the feeling.

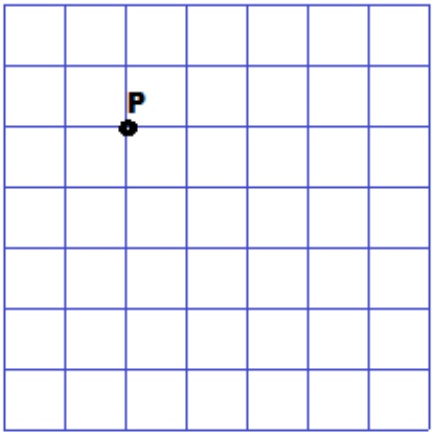
All of this had stemmed from the morning mathematics lesson that Miss taught on the topic of translation.

Miss, had first drawn a grid on the board and then she carefully explained the mathematical procedure of translation. It was the movement of any plane object or even a single point from one position on the grid to some other position on the grid.

This had sounded simple enough, but there was a special rule to describe this movement. The movement from one position to another must be performed by shifting horizontally to the left or to the right and then afterwards shifting vertically up or vertically down. Still, the order, though, was not important in the sense that first shifting vertically and then horizontally, or first shifting horizontally and then vertically, either produced the same eventual position.

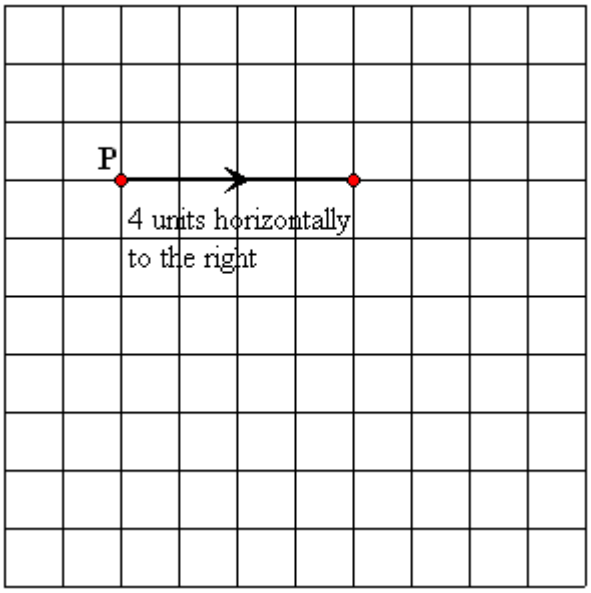


Miss demonstrated the full procedure by using a single point. She had named the point P and she drew its position on the grid. Each block on the grid was considered as a single unit. It was, therefore, easy to check the number of units for the horizontal shift and also the number of units for the vertical shift.

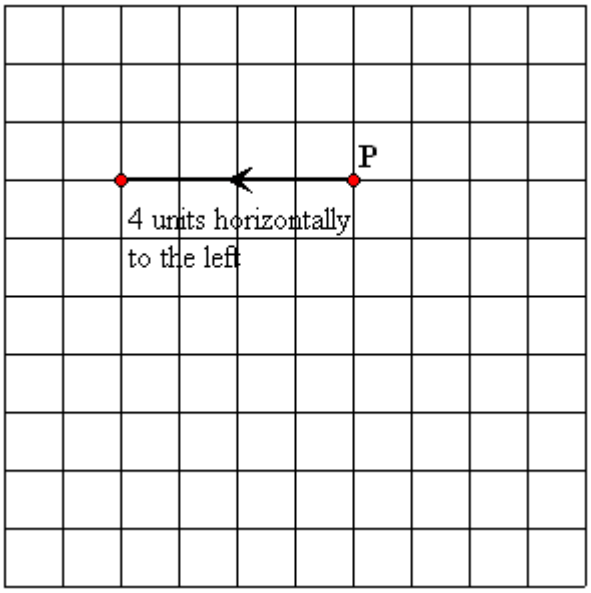


Miss further explained that under a translation, the object may also shift in only one direction, like horizontally alone or vertically alone.

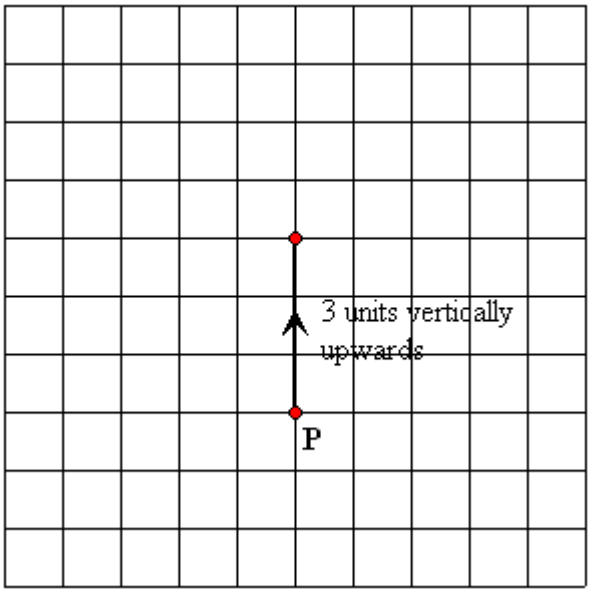
If for example, the point, P is to be shifted 4 units horizontally to the right, then we simply check 4 units to the right in order to find the new position for P.



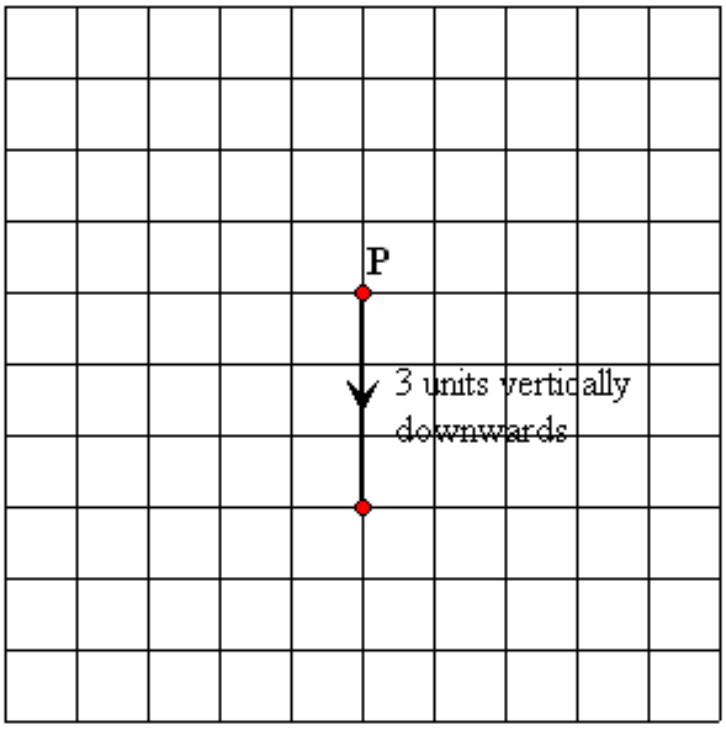
If the point, P, is to be shifted 4 units horizontally to the left, then we check 4 units to the left, to obtain the new position for P.



If the point P is to be shifted 3 units vertically upwards, we locate P and then check 3 units upwards, to locate the new position for P.



If the point, P, is to be shifted 3 units vertically downwards, we locate P and then check 3 units downwards, to locate the new position for P.

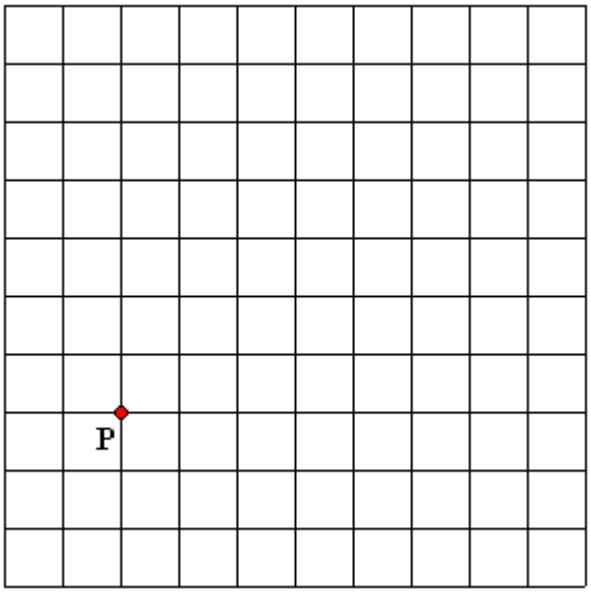


“We may,” said Miss, “even have the combination of both a horizontal and a vertical shift.”

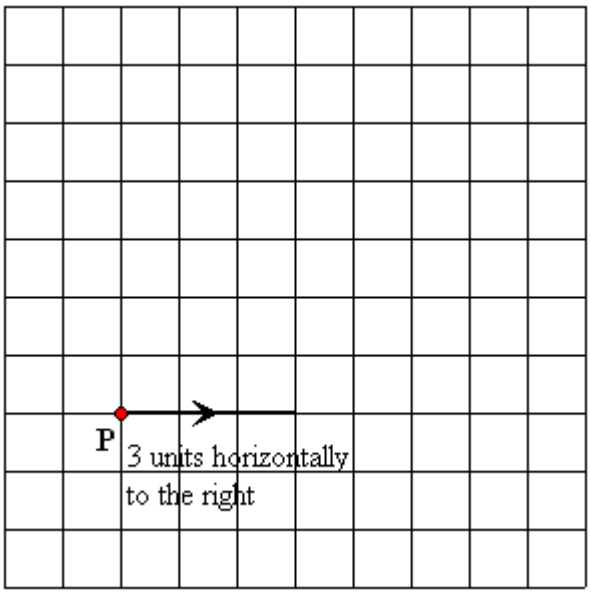
Miss proceeded to illustrate the procedure with an example to the class.

If the point P is to be translated by a horizontal shift of 3 units to the right and 4 units upwards, the steps would be,

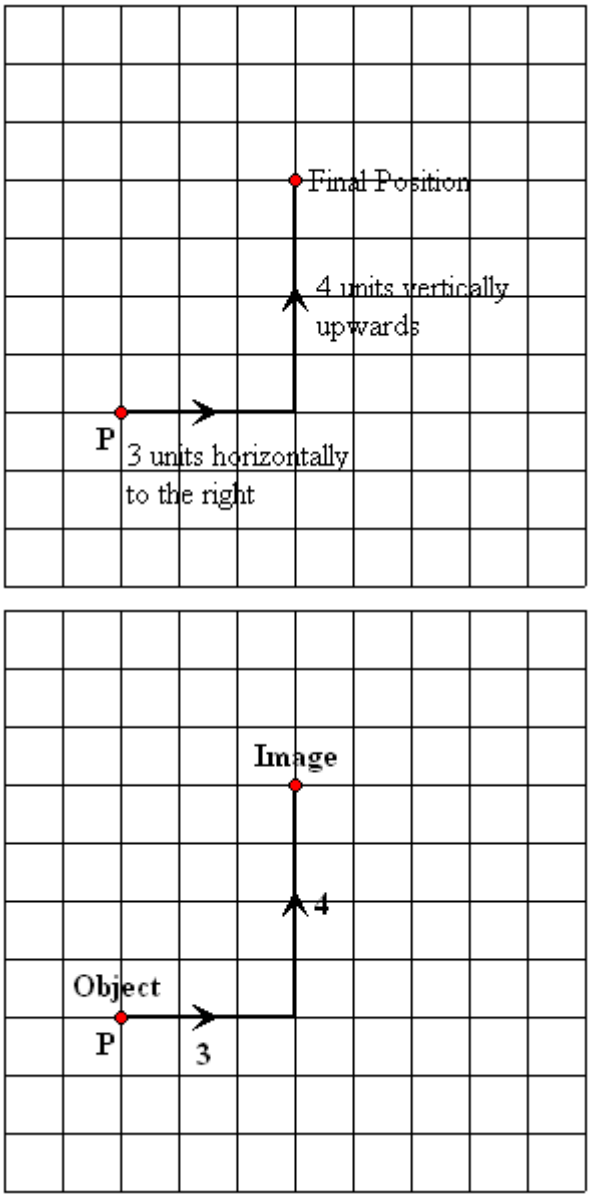
(i) Locate P



(ii) Shift P 3 units horizontally to the right, by checking 3 blocks to the right



(iii) After step (ii) is done, the shift is now to be 4 units upwards. This would be done by checking 4 blocks upwards.



The class was quite engrossed and the students clearly understood the illustration. They loved questions involving drawings and were anxious to translate a point now.

Miss further explained that the point, P, in its original position, was called the object and the point, P, in its new position, was called the image.

In a second question, Miss then gave the students a new point which she called Q. She then asked the class to use the same grid and shift the point Q to four new positions by:

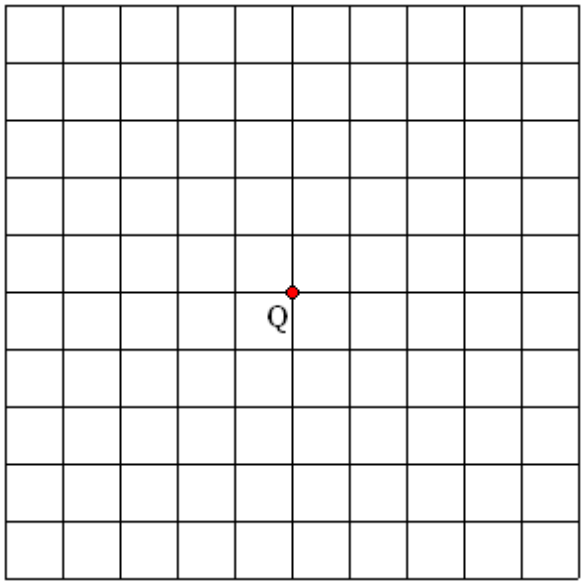
(i) 2 units to the right and 3 units up, calling the image R.

(ii) 2 units to the left and 3 units up, calling the image S.

(iii) 2 units to the right and 3 units down, calling the image T.

(iv) 2 units to the left and 3 units down, calling the image U.

The class looked at the given diagram with the point Q marked on the grid.



Then, working on one part at a time, each class member carefully checked the units in the appropriate direction for all four parts of the question, from (i) to (iv).

Finally, each arrived at the four correct locations for the images of R, S, T, and U. The students all completed their diagram to show the object point Q and the four image points R, S, T, and U under each of the four different translations.

As Miss walked around and looked at her students' completed diagrams, she was quite pleased.

Miss continued with her lesson.

"If we can translate a point, then we can translate an entire shape," she said. "All we need to do is to translate each vertex point of the shape, in turn."

The class noted everything that was being said by Miss.

“For example,” continued Miss, “if we have to translate a triangle named ABC, then we translate firstly the point or vertex, say A, then secondly the point or vertex, B, and finally the point or vertex, C. When we join the three images of A, B and C, we shall have the image of the original triangle, ABC. In general, we name the image differently from the object. We may choose entirely different letters such as say, PQR. However, some may choose to use a small stroke above the letters to indicate a difference but yet show some resemblance and connectivity between the object and its newly formed image. For example, in the case of the object ABC, we may consider the image to be named A'B'C'.

Miss said that it was usual to give the image formed another name besides that of the object.

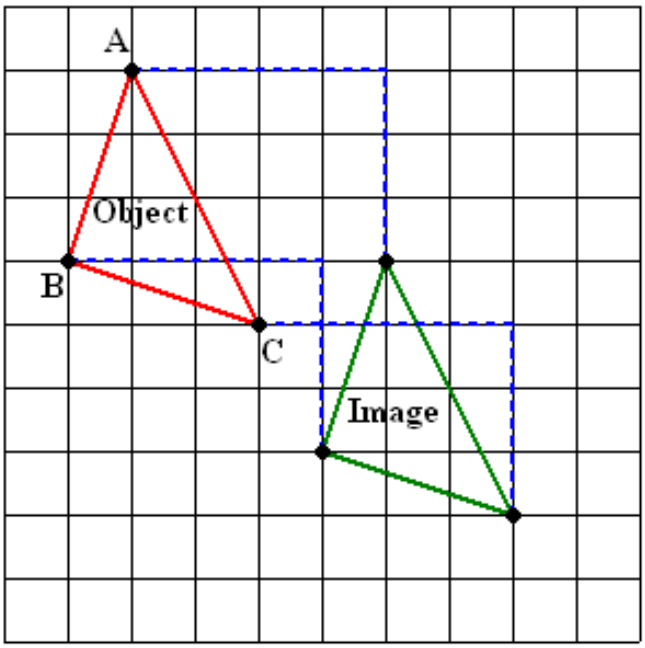
“Could anyone give me a reason why?” asked Miss.

Alfredo was quick to respond. The rest of the class looked at him as he spoke.

“Miss,” he said, seemingly a bit hesitant when he realised that all eyes were focused on him, “we ought not to have two figures in different positions and on the same grid and bearing the same name.”

“Yes,” replied Miss. “You are quite right. Figures in different positions should be named differently. We must show there is a difference between them and a different name is a very simple way to indicate that difference.”

The class was anxious to try the translation of a triangle ABC, given by Miss. The shift was 4 units to the right and then 3 units down.



The students of the class were even able to correctly suggest that if a line had to be translated, they would translate each of the two points at the ends of the line. The students went on to deduce that for a quadrilateral to be translated, they would translate each of the four vertices in turn. Then they would join all four image points and obtain the image of the quadrilateral.

“We must not forget to name the image differently from the object,” reminded Alfredo.

All laughed and even Miss found it hard to suppress a smile.

Miss was very pleased that the class understood the topic of translation so quickly and so well. Maybe I will test their thinking skills on the topic, Miss thought. She smiled to herself as she looked at their eager faces, asking for more.

Miss then asked a question concerning the object of a figure, which had just undergone a translation and produced an image.

“Let us look at our triangle ABC, called the object, and then look at the image of the triangle A'B'C'. Apart from their name, how do they differ?” she asked.

The class thought long and hard, but could not think about an answer. The only feasible answer to the question from the class was that the object and the image were in different positions and their names were different.

Miss agreed that this was indeed a very sound observation, but she teased them a little more. She wanted them to search for any difference between the object and the image and to do so for themselves.

Still, the class could not arrive at the answer that Miss sought.

Miss casually stood in front of the class and asked them to look at her and the grid of tiles on the classroom floor. She shifted herself three 'tiles' horizontally to the right and then four 'tiles' vertically upwards, just as was performed with the translation of the triangle. Miss stood in this new position for a while and looked at the class inquiringly. She hoped that her demonstration could help the class members to deduce the property that she wanted them to see.

Sadly, no one came forth with the answer or even responded. Miss was a little disappointed. Yet, it was a new topic and it was not fully 'digested', in a manner of speaking, thought Miss. Still, Miss was patient and loved to teach by allowing her students to make discoveries.

Finally, Miss asked her class a question.

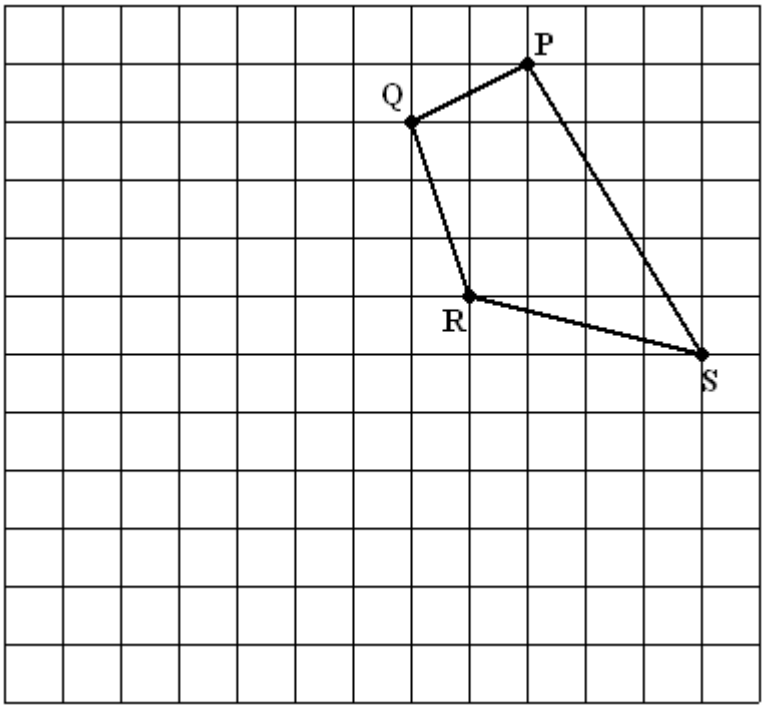
"How different am I, now that I have shifted my position or been translated from my original position?"

"There is no difference," said some members of the class shyly. They were still not too sure of the point that Miss was making.

"This is exactly what I am trying to show you," said Miss. "After a translation is performed, both the object and the image are identical. In mathematics, we say that both the object and the image are congruent and which means very much the same thing."

The class now clearly understood the teacher's point and a fundamental concept in the operation of translation. They looked forward to translating a

quadrilateral named PQRS, which Miss had given them for homework. First, she drew a grid on the board for the class and then she wrote the question.



The question was to translate the quadrilateral PQRS by shifting 3 units horizontally to the left and 4 units vertically downwards.

Barton was a deep thinker and he had read many books from the school library. The young boy also read several others which his parents had bought for him and which he kept in the bookshelves at home. He liked to do further research and reading, especially in mathematics and the science topics taught at school. Together, all of these helped his numeracy and literacy skills to be quite acute and very sound.

Barton would always try to understand the basic laws and the fundamentals of a lesson, and in so doing, would easily master the topic.

Thus, it all started with the lesson in translation, the shifting of a point to the left or to the right and either up or down that teased the young boy. A game began to meander in his head. It was a game about shifting and the rules of the game began to float around him.

Barton was in a deep thinking mood and was anxious to get home. His homework had to be completed, a few household chores had to be done, and his little chickens had to be tended. But, there was a fresh and very interesting thought bubbling in his active mind and Barton would not rest until these thoughts were realised. Barton was going to invent a new game.

Maybe this game will make me famous and I will be known all around the world. Maybe I'll call it 'Bartonpoly', he smiled.